

Price Gouging and the Monopoly Option*

Samuel Häfner

University of St. Gallen
samuel.haefner@unisg.ch

Curtis R. Taylor

Duke University
curtis.taylor@duke.edu

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Abstract

We investigate a dynamic oligopoly model with two phases: a normal phase where an essential input is available, and a disruption phase where firms must rely on inventories. Disruptions create an overcutting pricing incentive arising from the option to forgo sales and monopolize the market once rivals stock out. This monopoly option causes the price to jump to a supracompetitive level at the advent of the disruption and to remain above the competitive price path throughout. Price caps raise welfare, but reduce inventory investment. Government reserves reduce prices and raise buffer stocks, as do reserve mandates when financially feasible.

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1 Introduction

If you run a Home Depot in Florida, how many 4x8 sheets of plywood do you keep around? Well, if you're allowed to sell them for \$100 each when the next hurricane is coming, a lot. If you must charge only the regular price until the shelves empty out, then not so much. Inventory is expensive. "Windfall" profits belong in the pantheon of saints along with price-gouging. In competitive industries, that's what encourages people to enter and offer new supply.¹

The received wisdom among much – perhaps most – of the economics profession regarding the occurrence of *price gouging* is colorfully reflected in the above blog post by the Grumpy Economist. In a nutshell, dramatic price increases in the event of a precipitous supply disruption or surge in demand are simply an equilibrium response to a sudden and extreme shift in market conditions. Moreover the prospect of obtaining extraordinary prices during a crisis serves as an efficient incentive for sellers to carry costly inventory in a well-functioning market. In this paper we present a theory of price gouging that both confirms and challenges this prevalent viewpoint.

The key observation is that an input supply outage changes the competitive environment when firms transition from spot-market sourcing to reliance on inventories. Alongside the familiar incentive to undercut rivals and capture demand, firms acquire an incentive to overcut—to hold back sales so as to monopolize the market once rivals' inventories are exhausted. We call this *the monopoly option*. Thus, while competitive pressure drives undifferentiated oligopolists to price at marginal cost under normal conditions, the monopoly option induces excessive prices when a disruption hits.

Our baseline model features $n \geq 2$ symmetric price-setting firms that sell a homogeneous good in continuous time. The market begins in a business-as-usual phase and transitions stochastically to a disruption phase, during which firms can no longer procure an essential input and must rely on inventories.² We assume the disruption is permanent, both to simplify the analysis and to show that non-competitive pricing arises without tacit collusion à la the Folk Theorem.³

We begin with a perfectly competitive benchmark. During the normal phase, price equals marginal cost; when the disruption hits, price jumps discontinuously by the ex-ante expected cost of carrying inventory through the business-as-usual phase. It then rises at the carrying-cost rate until the final unit is sold at the highest price the market

¹"Praise for Price Gouging," John H. Cochrane, *The Grumpy Economist*, August 8, 2024, <https://www.grumpy-economist.com/p/praise-for-price-gouging>.

²To fix ideas, consider gas-station pricing during the May 2021 week-long shutdown of the Colonial Pipeline due to a ransomware attack (Ahmed, January 21, 2021).

³In Online Appendix D, we show that our results continue to hold when the state switches between normal and disruption phases ad infinitum.

will bear. Thus, – as noted by the Grumpy Economist – neither an abrupt price increase at the advent of a supply shock nor eventually very high prices are inconsistent with perfect competition or zero expected profit.

We then turn to duopoly. Firms invest in buffer stocks before the disruption, and Bertrand competition keeps prices at marginal cost in the business-as-usual phase. Once the disruption hits, however, the monopoly option becomes relevant. Exercising it is costly: a firm that waits for its rival to stock out earns no revenue and bears carrying costs in the meantime. Nevertheless, a firm’s disruption-phase profit can be no less than the payoff from pursuing the monopoly option, and at equilibrium inventory levels this lower bound is strictly positive.

We characterize a *basic* minimum-profit symmetric equilibrium in which the disruption price jumps up by more than in the competitive benchmark, and the equilibrium price path remains above the competitive path until both firms simultaneously stock out. This is what we mean by *price gouging*: a relaxation of ordinary competitive pressures caused by scarce buffer stocks and the associated monopoly option.

For $n > 2$, the duopoly equilibrium generalizes in a manner analogous to static Cournot competition. Symmetric equilibrium profits decrease in n and converge to zero. Each firm’s inventory becomes infinitesimal, and the equilibrium price path converges pointwise to the competitive benchmark.

Turning to policy, we consider a regulator who places greater weight on consumer surplus than on producer surplus or government outlays. For any finite n , an optimally designed price cap raises welfare, but price caps also reduce firms’ inventory investment, so unduly stringent caps are harmful. We also study two reserve policies. A minimum-inventory mandate is effective so long as required reserves are not so large that firms incur losses. Alternatively, the government can hold strategic reserves and sell them to firms at competitive prices during the disruption. Both policies increase buffer stocks and reduce prices; the difference is whether firms or the government bear the cost of the extra inventory.

Finally, our baseline model assumes *box demand*: a unit mass of consumers, each of whom values one unit at v . This yields clean intuitive results but suppresses insights regarding demand elasticity. Therefore, Section 7 provides sufficient conditions for the existence of equilibrium duopoly disruption-pricing for a broad class of demand functions, and then characterizes pricing under linear demand. In that case, the unique equilibrium inventory profile, up to relabeling, is asymmetric: the larger firm explicitly exercises its monopoly option, waiting for its smaller rival to stock out before commandeering the market. Welfare is higher than under monopoly but lower than first best, due to distorted inventory investment and supracompetitive disruption pricing. An optimal reserve mandate restores symmetry and alleviates a substantial fraction of the deadweight loss.

2 Related Literature

Although our application to price gouging is novel, our theoretical contribution belongs to a longstanding literature on costly capacity or inventory choices in price competition. The seminal contribution by [Kreps and Scheinkman \(1983\)](#) shows that when firms first choose costly capacities and then compete in prices, the Cournot outcome ensues. Subsequent work explores the robustness of this result (e.g. [Deneckere and Kovenock, 1996](#); [Dana Jr, 1999](#); [Acemoglu et al., 2009](#)). More recently, [Montez and Schutz \(2021\)](#) show that if capacity choices are unobservable and inventories are fully recoverable, the Bertrand rather than Cournot outcome obtains. Most of this literature studies a static one-shot pricing subgame. We extend it by allowing firms to set prices continuously over time, so that inventory choices generate substantive equilibrium price dynamics.

The analysis of scarcity rents in exhaustible-resource markets goes back to [Hotelling \(1931\)](#), but the strategic residual-monopoly incentive in our model is more closely associated with subsequent work on exhaustible-resource competition. In particular, [Salant \(1976\)](#) studies a dominant cartel competing against a competitive fringe and shows that the cartel restricts sales so as eventually to become the sole supplier after the fringe exhausts its reserves.

Several papers study dynamic competition with fixed inventories or exhaustible capacities. [Dudey \(1992\)](#) studies duopolists serving a finite sequence of customers with unit demand, while [Dana Jr and Williams \(2022\)](#) analyze differentiated-goods competition over two periods. Both papers obtain equilibria with constant supracompetitive prices generated by an effect similar to our monopoly option. Continuous-time models are studied by [Harris et al. \(2010\)](#), who analyze resource depletion in an N -firm Cournot game with exhaustible supplies, and by [Ledvina and Sircar \(2011\)](#), who study Bertrand competition with finite capacities, substitutable products, and uncertain demand. These papers show how fixed stocks shape dynamic pricing and depletion, but they take inventories or capacities as primitives. Our contribution is to study how the residual-monopoly incentive feeds back into firms' ex ante buffer-stock choices, how those choices determine disruption pricing and stockout dynamics, and how policy can improve the resulting allocation.

In our model, a disruption causes price to jump from marginal cost to a supracompetitive level and then rise further as inventories are depleted. These dynamics are driven by carrying costs: higher carrying costs make waiting more expensive, producing a larger initial jump and a steeper subsequent price path. Carrying costs also make market structure consequential, since the number of firms affects both the size of the jump and the slope of the price path, thereby shifting surplus between consumers and producers.

Also related is revenue management (cf. [Board and Skrzypacz, 2016](#); [Dilme and Li, 2019](#); [Martínez-de Albéniz and Talluri, 2011](#); [Betancourt et al., 2024](#), among others).

These papers study dynamic pricing when firms sell a fixed stock before an exogenous deadline, such as seats on an airplane. Board and Skrzypacz (2016) and Dilme and Li (2019) analyze the monopoly case: in the former, commitment leads to an auction at the deadline; in the latter, the seller lacks commitment and runs a sequence of flash sales. Martínez-de Albéniz and Talluri (2011) study oligopoly price competition with fixed capacities over a finite horizon. As in our disruption subgame, their equilibrium features an endogenous stockout order: the lower-capacity firm sells first, after which the remaining firm becomes a residual monopolist. Betancourt et al. (2024) study a Bertrand oligopoly with differentiated goods, demand uncertainty, limited capacities, and a common deadline. They also find that the lower-capacity firm sells first, but through a different mechanism: demand uncertainty and product differentiation generate a Bertrand scarcity trap, in which firms price inefficiently low because they fail to internalize the option value of preserving capacity for future buyers.

The main distinction is that these models take capacities and the sales horizon as primitives. By contrast, there is no exogenous deadline at which the good perishes in our model; stockouts occur endogenously, with depletion times determined by firms' buffer-stock choices. Moreover, whether under box demand with extra weight on consumer surplus or under linear demand and surplus maximization, the firms in our model choose inefficiently low initial inventories and price inefficiently high throughout the disruption.

There is also a distinct literature on inventory holding under stochastic demand (e.g. Deneckere et al., 1996; Dana, 2001; Lippman and McCardle, 1997). In these models, firms carry inventory to increase availability or service probability when demand is uncertain. Our inventory motive is different. Demand before and during the disruption is deterministic; uncertainty concerns the arrival of the supply outage. Inventories are therefore buffer stocks held as insurance, and their strategic value comes from the dynamic pricing game they induce once the disruption occurs.

Green et al. (2024) study the pricing and inventory decisions of a monopolist in a continuous-time setting where consumers decide when and whether to stockpile in the face of a supply shock. They show that consumer storage generates a preemption game in which consumers stockpile slowly and the monopolist raises price from low to high randomly, typically increasing consumer surplus, decreasing monopoly profit, and reducing total welfare. They also study policy and conclude that government strategic reserves with commitment power are fully corrective in their setting.

In a recent working paper, Kominers and Dworczak (2025) propose a definition of price gouging based on redistributive motives arising from a utilitarian welfare function that assigns different weights to market participants. Like us, they argue that price caps can raise welfare if consumers receive greater weight than producers. Their model, however, is static and therefore cannot address the issues at the core of our dynamic setting.

Finally, there is a small but growing empirical literature on price gouging, with mixed

evidence. Cabral and Xu (2021) study third-party sellers of 3M masks on Amazon during the COVID-19 pandemic and find prices roughly 2.4 times higher than Amazon’s 2019 price. They also show that price increases depend negatively on a seller’s review count, which they use as a proxy for reputation. Changes in markups are also studied by Neilson (2009) and Beatty et al. (2021), who analyze gas-station pricing during hurricanes. Neither finds significant increases in retail margins, though Neilson (2009) does find a significant increase in wholesale gasoline prices.⁴

Recent episodes widely perceived as “price gouging,” documented in the popular press, have spurred public concern and policy responses. In 2020, the UK Competition and Markets Authority set up a “price gouging taskforce” to monitor possible cases, especially for goods like hand sanitizer (Jack and Gavin, February 13, 2025). As energy prices soared following Russia’s invasion of Ukraine in 2022, the EU capitals agreed on a gasoline price cap, which was never activated, however, and expired in 2025 (Butler, March 20, 2020). Further, refinery outages in 2023 drove a prolonged spike in California gas prices, prompting new reserve requirements for refiners (Cummings and Mahoney, September 30, 2024; Risser, October 10, 2024). Similar concerns of gouging arose in the US during the Colonial Pipeline ransomware shutdown, the 2022 infant formula shortage, and the 2023–24 surge in egg prices (Ahmed, January 21, 2021; Stempel, March 12, 2024; Kim, Apr 30, 2024).

3 Model Setup

There is a market for a homogeneous good consisting of n identical firms, indexed by $i \in \{1, \dots, n\}$, and a continuum of consumers with total measure normalized to 1. Time is continuous; information is complete; and all parties are risk-neutral, infinitely-lived, and (purely for ease of exposition) do not discount the future.

Each consumer wishes to purchase a flow of one unit of a good per period and has flow value $v > 0$. Hence, market demand at any instant is a *box* with width of 1 and height of v .⁵ The firms purchase an essential input (i.e., one for which substitution is not possible) at unit cost $c < v$ and costlessly convert each unit of input into a unit of output. The essential input is non-perishable and consumers have no storage capacity.

At any moment, the market is in one of two phases, designated as *the business-as-usual (BAU) phase* and the *disruption phase*. The game begins in the BAU phase, where the input is readily available. A supply disruption arrives stochastically, after which the input is no longer available. The disruption arrives at a constant hazard rate $\lambda > 0$ and

⁴Anecdotaly, there are documented cases of profiteering at the retail level. For instance, the North Carolina attorney general reached a \$10,000 settlement with a Durham gas station for raising its premium fuel price by 67% during the Colonial pipeline shutdown in May 2021.

⁵In Section 7, we characterize duopoly equilibrium for more general demand functions including linear demand.

is permanent.⁶

At any time $t \geq 0$ during the BAU phase, the firms can adjust their inventories k_{it} in anticipation of the supply disruption by buying or selling the input at price c . During the disruption phase, firms can reduce their inventories only through selling to the market. The marginal flow cost of carrying inventory is $\mu > 0$.⁷

Each firm i also sets a price p_{it} at every time $t \geq 0$ prior to the disruption and at every time during the disruption that it has inventory to sell. If a single firm chooses the lowest price (no greater than v), then it serves the entire market. If there is a tie, market shares are determined by a commonly known exogenous tie-breaking rule. Below we specify a tie-breaking protocol under which equilibria always exist. For technical convenience, we assume demand is defined for negative prices; although the firms will never set negative prices in equilibrium.

In the disruption phase, we restrict firms to Markovian strategies determining p_{it} that condition on the current inventories, $\vec{k} = (k_1, \dots, k_n)$. In the BAU phase, we focus on stationary pricing and inventory strategies, determining p_{it} and k_{it} independent of time and the past history of play.⁸ We formally define the strategies and equilibrium with Definitions 1–3 in Section 5. First, we solve for the perfectly competitive benchmark outcome of the model.

4 The Competitive Benchmark

We begin by deriving the competitive price path before and during the disruption and the competitive level of inventory investment.

Once the disruption hits, our setting is analogous to the extraction of an exhaustible resource. Following the classic work of Hotelling (1931), each firm holding inventory in a competitive equilibrium must be indifferent at each moment t between selling a unit at the current price $p^\dagger(t)$ and waiting to sell it at $p^\dagger(t + dt)$. The cost of waiting to sell is the carrying cost μdt . From this, we deduce that the competitive price must rise at rate μ during the disruption

$$\frac{dp^\dagger}{dt} = \mu \implies p^\dagger(t) = \mu t + C.$$

Next, let τ be the random time at which the disruption occurs and K_0 be the economy's initial inventory holdings. Because inventory is drawn down at rate 1 throughout the disruption, the remaining stock K at time t is simply $K = K_0 - (t - \tau)$. Using this to

⁶In Online Appendix D, we also consider the case in which the input good remains available but its price increases to some $c_s \in (c, v)$ in case of a shock.

⁷In Online Appendix D, we also consider asymmetric and convex carrying costs.

⁸Since the input can always be procured at c during the BAU phase, storing costly inventory to produce for the market during this phase is dominated by sourcing the input continuously on the spot market. Consequently, we take firms to do so throughout the BAU phase and do not consider the possibility of selling to consumers from inventory prior to the disruption.

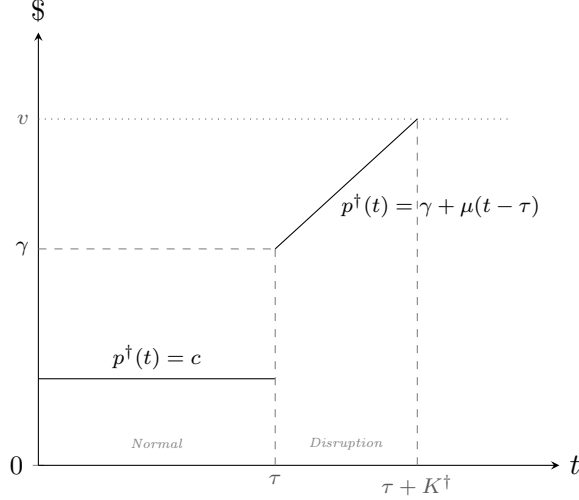


Figure 1: The supply disruption hits at random time τ . The competitive price path is $p^\dagger(t)$.

substitute for t in the previous expression gives $p^\dagger(\tau + K_0 - K) = \mu(\tau + K_0 - K) + C$. Now, the last unit of inventory must sell for v , because that is its marginal social value – so we have $p^\dagger(\tau + K_0) = v \implies C = v - \mu(\tau + K_0)$. Thus, during the disruption, the competitive price path as a function of the economy's remaining stock is

$$p^\dagger(\tau + K_0 - K) = v - \mu K.$$

For the extraction of an exhaustible resource, this would be the end of the story. Specifically, each unit of scarce resource would command rent equal to the initial price $p^\dagger(\tau) = v - \mu K_0$, which depends on the economy's exogenous endowment. In our setting, however, K_0 is determined by the firms' inventory investment decisions.

Because the disruption arrives at rate λ , the expected duration of the BAU period is $E[\tau] = \frac{1}{\lambda}$. Hence, the expected cost of procuring a unit of inventory and carrying it until the disruption hits is $c + \frac{\mu}{\lambda} \equiv \gamma$. In a competitive equilibrium, firms invest in inventory up to the point when expected profit on each unit is zero. Thus, assuming (as we do throughout) that $v > \gamma$, we have $p^\dagger(\tau) = \gamma$, implying that aggregate competitive inventory investment is

$$K^\dagger \equiv \frac{v - \gamma}{\mu}.$$

During the BAU phase, the competitive price coincides with the marginal cost of production, $p^\dagger(t) = c$. Therefore, a firm producing for the spot market and holding inventory in anticipation of the supply disruption makes zero expected profit.

In sum, the competitive price path is constant at c until the disruption hits. Then it jumps up to $c + \frac{\mu}{\lambda} \equiv \gamma$. Thereafter it increases at rate μ until the last unit sells for

its marginal social value of v . Figure 1 illustrates. The jump in the competitive price at the onset of the disruption can be viewed as the *good* kind of price gouging that induces inventory investment praised by the Grumpy Economist. What we show below is that oligopolists generally engage in *bad* price gouging in the sense that the jump in the equilibrium price is excessive.

5 Duopoly

To fix ideas and facilitate exposition, we begin with symmetric duopoly. As usual, we solve the game backward beginning with the disruption phase and then consider pricing and inventory decisions in the BAU phase.

5.1 Disruption Phase

We first define the firms' payoffs and strategies. Let $g_i(\vec{p}, \vec{k})$ be the flow profit of firm i when the current prices are $\vec{p} = (p_1, p_2)$ and the current state is $\vec{k} = (k_1, k_2)$. To define $g_i(\cdot, \cdot)$, we write $D(p)$ for the market flow demand at price p . Market demand satisfies $D(p) = 1$ when $p \leq v$ and zero otherwise. Further, we let $q_i(k_1, k_2)$ be the share of demand allocated to firm i when both firms set the same price; $q_i(k_1, k_2) + q_j(k_1, k_2) = 1$, where $i \in \{1, 2\}$ indexes an arbitrary firm and $j \neq i$. (We identify feasible $q_i(k_1, k_2)$ in Definition 2 below.) For $k_1, k_2 > 0$, we have

$$g_i(\vec{p}, \vec{k}) = \begin{cases} -\mu k_i & \text{if } p_i > p_j \\ q_i(\vec{k})p_i D(p_i) - \mu k_i & \text{if } p_i = p_j \\ p_i D(p_i) - \mu k_i & \text{if } p_i < p_j. \end{cases}$$

In all cases, the firms bear the contemporaneous costs of carrying their inventories. When overcutting its opponent's price, a firm surrenders the entire market demand; when undercutting, it captures all demand; and when setting the same price as its opponent, demand is allocated according to the tie-breaking rule. For $k_i = 0$, we have $g_i(\vec{p}, \vec{k}) = 0$ and when $k_i > k_j = 0$, then $g_i(\vec{p}, \vec{k}) = p_i D(p_i) - \mu k_i$.

The strategy of firm i in the disruption phase consists of a pricing function $p_i(\vec{k})$. The pricing functions and tie-breaking rule also determine the inventory dynamics. Let $f_i(\vec{p}, \vec{k})$ be the marginal change of firm i 's inventory. Then, for any initial state $\vec{k}_0$, the ensuing dynamics $\vec{k}_s$ of the inventories satisfy, for all $s \geq 0$,

$$k_{is} = k_{i0} + \int_0^s f_i((p_1(\vec{k}_x), p_2(\vec{k}_x)), \vec{k}_x) dx, \quad i = 1, 2, \quad (1)$$

where the marginal inventory changes are given by

$$f_i(\vec{p}, \vec{k}) = \begin{cases} 0 & \text{if } p_i > p_j \\ -q_i(\vec{k})D(p_i) & \text{if } p_i = p_j \\ -D(p_i) & \text{if } p_i < p_j. \end{cases}$$

when $k_1, k_2 > 0$. When $k_i = 0$, then $f_i(\vec{p}, \vec{k}) = 0$ and when $k_i > k_j = 0$, then $f_i(\vec{p}, \vec{k}) = -D(p_i)$.

Throughout, we consider pricing function profiles $(p_1(\vec{k}), p_2(\vec{k}))$ and a tie-breaking rule for which (1) has a unique forward solution for any $\vec{k}_o$, which is required for the firms' profits to be well-defined.⁹ In particular, the profit of firm i in state $\vec{k}_o$ under the Markov pricing functions $(p_1(\cdot), p_2(\cdot))$ is given by

$$\Pi_i(p_1(\cdot), p_2(\cdot), \vec{k}_o) = \int_0^\infty g_i(p_1(\vec{k}_s), p_2(\vec{k}_s), \vec{k}_s) ds,$$

where $\vec{k}_s$ is the unique forward solution to (1). (2)

Then, a (continuation) equilibrium in the disruption phase is a tuple of pricing functions such that, given the behavior of the other firm, each firm chooses optimally among all those pricing functions that yield a unique forward solution to (1).

Definition 1 (Continuation Equilibrium in the Disruption Phase). *A continuation equilibrium in the disruption phase is a tuple of pricing functions $(p_1^*(\vec{k}), p_2^*(\vec{k}))$ such that for any $\vec{k}_o \in \mathbb{R}_+^2$, we have*

$$p_1^*(\cdot) \in \arg \max_{p(\cdot)} \Pi_1(p(\cdot), p_2^*(\cdot), \vec{k}_o) \quad \text{and} \quad p_2^*(\cdot) \in \arg \max_{p(\cdot)} \Pi_2(p_1^*(\cdot), p(\cdot), \vec{k}_o)$$

Definition 1 imposes Markov perfection because it requires a price function for a firm to be optimal for any state $\vec{k}_o$. For a continuation equilibrium $(p_1^*(\cdot), p_2^*(\cdot))$, we say that $V_i(\vec{k})$ is the corresponding value function of firm i if $V_i(\vec{k}) = \Pi_i(p_1^*(\cdot), p_2^*(\cdot), \vec{k})$ for all $\vec{k} \in \mathbb{R}_+^2$.

⁹Existence of a solution to (1) is delicate due to the discontinuities in f_i . Moreover, the integral equation alone need not uniquely determine the outgoing inventory path at an isolated switching state (i.e., where f_i is discontinuous). In Online Appendix C, we therefore define a forward solution, which requires the outgoing inventory dynamics at such states to be consistent with the contemporaneous marginal inventory changes, and provide mild regularity conditions under which the forward solution exists and is unique for general demand functions $D(p)$ and the *bottom-down* tie-breaking rule $q_i(\vec{k})$ given in Definition 2 below.

5.1.1 The Monopoly Option

We begin by observing that, in any continuation equilibrium in the disruption phase, the firms' profits in a state (k_1, k_2) are bounded below by the *monopoly option* payoff.

To derive this payoff, we first observe that if firm i with $k_i > 0$ can act as a monopolist, it will sell the good at price v . Hence, its profit will be

$$\int_0^{k_i} v - \mu(k_i - t) dt = vk_i - \frac{1}{2}\mu k_i^2. \quad (3)$$

Second, we note that on the equilibrium path, when $k_1, k_2 > 0$, both firms must post the same price (weakly below v) at almost every instant. Else a firm posting a lower price for a non-negligible set of time could profitably deviate by raising its price on that set by a small amount.

Third, if the competitor $j \neq i$ also holds inventory, $k_j > 0$, then firm i can always undercut j 's price and concede the market until j stocks out. Given that sales occur at rate 1, this will take time k_j , implying that exercising the monopoly option for firm i involves a waiting cost $\mu k_i k_j$. We thus conclude the following without further proof.

Lemma 1 (Monopoly Option). *In any continuation equilibrium in the disruption phase, the value of firm i satisfies*

$$V_i(k_1, k_2) \geq vk_i - \frac{1}{2}\mu k_i^2 - \mu k_1 k_2, \quad i = 1, 2. \quad (\text{MOP})$$

5.1.2 Tie-Breaking and the Basic Equilibrium

As noted earlier, whether an equilibrium of the BAU phase exists hinges critically on the form of the tie-breaking rule stipulated when the firms set the same price. The following protocol turns out to be crucial.

Definition 2 (Symmetric Bottom-Down Tie-Breaking). *A tie-breaking rule (when the firms set the same price) is called symmetric bottom-down (SBD) if $k_i > k_j > 0$ implies $q_j(k_1, k_2) = 1$ and $k_i = k_j > 0$ implies $q_j(k_1, k_2) = \frac{1}{2}$.*

Thus, under SBD tie-breaking, the firm with lower inventory serves all the demand until it stocks out, after which the other firm sells its inventory as a monopolist. If the firms have the same inventories, then they share the market by splitting demand equally until they simultaneously stock out.

Remark 1 (SBD Tie-Breaking). *To see why breaking ties in favor of the lower-inventory firm is crucial for equilibrium existence, suppose that $k_i > k_j > 0$ and that inventories are in the region where the monopoly option is valuable (i.e., $k_i + k_j < \frac{2v}{\mu}$). If firm i allows*

firm j to sell first, then i carries k_i units for time k_j , whereas if firm j allows firm i to sell first, then j carries k_j units for time k_i . In either case, the total holding-cost sacrifice from waiting is the same, namely $\mu k_i k_j$. The gain from waiting, however, is larger for the high-inventory firm, because once its rival stocks out it has more remaining units to sell along the monopoly path. Thus, when $k_i > k_j$, firm i has a strictly stronger incentive than firm j to be the firm that waits.

At the same time, actual overcutting cannot occur in equilibrium (for non-negligible time). If $p_i > p_j$, then firm j can profitably raise its price slightly without losing demand. Hence, in any candidate equilibrium with asymmetric inventories, prices cannot be strictly ordered. The only way to reconcile this with the high-inventory firm's stronger desire to wait is for ties at a common price to be broken in favor of the low-inventory firm.

This tie-breaking convention is analogous to the one used in the standard static Bertrand model with asymmetric marginal costs. There, if both firms charge the high-cost firm's price, equilibrium requires that all demand be allocated to the low-cost firm. In both models, the tie-breaking rule is a continuous-price counterpart of the small price adjustment that would arise with a discrete price grid. If prices were quoted in pennies, the advantaged firm would instead make the smallest feasible price move in its preferred direction: in the present model, the high-inventory firm would overcut its rival by one cent, while in the static Bertrand model the low-cost firm would undercut by one cent. Moreover, the tie-breaking convention appears to be essential for equilibrium existence: as shown in Online Appendix Section G, no equilibrium exists if ties are instead broken in favor of the high-inventory firm, or if asymmetric firms both receive positive demand shares at a common price.

The following lemma presents an equilibrium under SBD tie-breaking which we refer to below as the *basic* (continuation) equilibrium of the disruption phase. In this equilibrium, both firms always obtain their monopoly-option payoffs, whether selling first, second, or simultaneously.¹⁰

Lemma 2 (Basic Equilibrium of the Disruption Phase). *Under SBD tie-breaking it is an equilibrium for both firms to set $p_1^*(k_1, k_2) = p_2^*(k_1, k_2) = p^*(k_1, k_2)$, where*

$$p^*(k_1, k_2) = v - \mu \cdot \max\{k_1, k_2\},$$

when $k_1, k_2 > 0$, and

$$p_1^*(k_1, 0) = p_2^*(0, k_2) = v,$$

when the rival firm has no stock. The value functions for either firm i and any $(k_1, k_2) \in$

¹⁰The basic equilibrium pricing function of the disruption subgame is not unique. Online Appendix Section H gives two examples of other equilibrium pricing functions under SBD tie-breaking. In these examples, the high-inventory firm continues to earn exactly its monopoly-option payoff, while the active low-inventory firm's payoff is higher than its monopoly option payoff.

$\mathbb{R}_+^2$ are

$$V_i(k_1, k_2) = vk_i - \frac{1}{2}\mu k_i^2 - \mu k_1 k_2. \quad (4)$$

To help with intuition, suppose $k_i > k_j > 0$. Then the fact that each firm receives its monopoly option payoff follows directly from the pricing function and SBD tie-breaking. Specifically, because $k_i > k_j$, i will have to wait until j stocks out before monopolizing the market. Further, by selling all k_j units at price $v - \mu k_i$, j also obtains its monopoly option payoff, even though it sells first. As a consequence, firms are indifferent between selling at $p(k_1, k_2)$ and overcutting in order to cede the market to the opponent, and strictly prefer waiting to undercutting when not currently serving the market. Indeed, the equilibrium duopoly price increases at rate μ when inventories are asymmetric and at rate $\mu/2$ when they are symmetric, so that in either case the flow opportunity cost of serving the market exactly matches the flow saving in carrying costs from reducing inventory.

Without further reference, in what follows we fix the SBD tie-breaking rule both in the disruption phase and the BAU phase and assume that firms play the basic equilibrium of the disruption subgame given in Lemma 2.

5.2 The business-as-usual (BAU) Phase

Now consider the BAU phase prior to the disruption. Recall that we assume that at any instant either firm can buy or sell as much of the essential input as it wishes at a factor price of c . That is, each firm chooses its level of inventory k_{it} at any instant. The firms also set output prices p_{it} at every point in time.

Because the supply disruption arrives randomly at a constant rate, the BAU phase has indefinite duration. This allows for the possibility of equilibria involving implicit collusion à la the Folk Theorem. But our focus is on the existence of non-competitive pricing that does not rely on implicit collusion. Accordingly, we consider equilibria involving stationary strategies in the BAU phase. We thus collect the relevant strategies in a vector $(p_1, p_2, k_1, k_2) \in \mathbb{R}_+^4$ meaning that each firm sets $p_{it} = p_i$ and $k_{it} = k_i$ for all $t \in [0, \tau)$. Together with the continuation equilibrium in the disruption phase, this finally allows us to define our notion of equilibrium for the entire game.

Definition 3 (Equilibrium of the Game). *We say that $(p_1^*, p_2^*, k_1^*, k_2^*, p_1^*(\vec{k}), p_2^*(\vec{k}))$ is an equilibrium of the game, if*

- (i) $(p_1^*(\vec{k}), p_2^*(\vec{k}))$ is a (continuation) equilibrium in the disruption phase; and
- (ii) setting $p_{it} = p_i^*$ and $k_{it} = k_i^*$ is mutually optimal for the firms throughout the BAU phase, i.e., for all $t \in [0, \tau)$, irrespective of the history of past play.

Not surprisingly, the best response to a stationary strategy, is also stationary. In particular, the pricing decision in the BAU phase reduces to static Bertrand competition in any stationary equilibrium, implying $p_1^* = p_2^* = c$. Notably, marginal-cost pricing obtains irrespective of the equilibrium inventories k_1^* and k_2^* . Moreover, the firms optimally choose to hold (k_1^*, k_2^*) from the start of the game, where (k_1^*, k_2^*) satisfies

$$k_1^* \in \arg \max_{k_1} -\gamma k_1 + V_1(k_1, k_2^*) \quad \text{and} \quad k_2^* \in \arg \max_{k_2} -\gamma k_2 + V_2(k_1^*, k_2), \quad (5)$$

where $\gamma = c + \frac{\mu}{\lambda}$. We collect the above observations in the following lemma. Point (i) and the final claim follow directly; we obtain point (ii) from the first-order conditions from (5). Recall that $K^\dagger$ is the aggregate competitive inventory.

Lemma 3. *Fix the basic equilibrium of the disruption phase from Lemma 2. Then, there are mutually optimal strategies described by $(p_1^*, p_2^*, k_1^*, k_2^*)$ in the BAU phase under which:*

- (i) *Each firm $i = 1, 2$ sets its price equal to marginal cost, $p_i^* = c$;*
- (ii) *each firm $i = 1, 2$ sets its inventory equal to $\frac{v-\gamma}{2\mu} = k_i^* = \frac{1}{2}K^\dagger$.*

Moreover, marginal cost pricing must obtain in any equilibrium in stationary strategies of the BAU phase.

Combining Lemmas 2 and 3 yields the following result that we state without further proof.

Proposition 1 (Basic Equilibrium). *There is a symmetric equilibrium of the game, the basic equilibrium, fully characterized by Lemmas 2 and 3. Behavior on the equilibrium path is as follows:*

- (i) *At the beginning of the game, firms choose inventories $k_i^* = \frac{v-\gamma}{2\mu}$ and do not change them throughout the BAU phase.*
- (ii) *During the BAU phase, both firms charge the same price equal to c .*
- (iii) *During the disruption phase, firms behave as described in Lemma 2. In particular, if the disruption hits at time $\tau > 0$, then on the equilibrium path the market price p_t^* at $t \geq \tau$ is given by*

$$p_t^* = v - \mu \left(\frac{v-\gamma}{2\mu} - \frac{t-\tau}{2} \right) = \frac{v+\gamma}{2} + \frac{\mu(t-\tau)}{2},$$

until both firms have stocked out at $t = \tau + K^\dagger$.

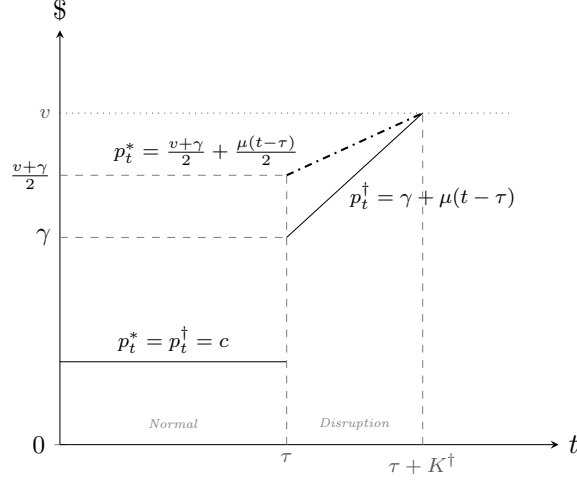


Figure 2: The competitive price path is $p_t^\dagger$. The symmetric duopoly equilibrium price path is p_t^* . Before the disruption hits at τ , the two coincide. After τ , p_t^* lies above $p_t^\dagger$.

Because aggregate inventory corresponds to the socially optimal level, $K^\dagger$, the depletion of the firms' inventories takes the same time as under the competitive benchmark.¹¹ However, unlike the competitive benchmark, the duopolists make a strictly positive expected profit on each unit of inventory they sell (their realized profit depends on the random duration of the BAU phase, the amount of time they carry stock before selling it).

We obtain the duopoly price path p_t^* during the disruption phase in Part (iii) by substituting the individual firms' inventories into the equilibrium pricing function $p(k, k)$ from Lemma 2. Observe that the duopoly price p_t^* jumps up by more than the competitive benchmark price when the disruption hits, namely from c to $(v + \gamma)/2$ rather than from c to γ . Moreover, the duopoly price remains strictly above the competitive price until the firms simultaneously stock out. See Figure 2, where the duopoly price path bisects the surplus available in the disruption, yielding the top half to consumers and the bottom half to the firms.

Remark 2 (Asymmetric Equilibria). Lemmas 2 and 3 actually identify a continuum of equilibria. Specifically, the best-response functions from (5) imply any k_1 and k_2 such that $k_1 + k_2 = K^\dagger$ can be supported as stationary equilibrium inventories. If $k_i > k_j$, then i waits for j to stock out once the disruption hits. The symmetric equilibrium yields the lowest aggregate profit because $k_1^* = k_2^* = \frac{1}{2}K^\dagger$ is a saddle point of $V_1(k_1, k_2) + V_2(k_1, k_2) - \gamma(k_1 + k_2)$ as can easily be checked. Therefore, the more asymmetric are the inventories, the higher is aggregate profit. This suggests that both firms could benefit if one sold its stock to the other. Though intriguing, we do not consider this sort of merger below.

¹¹For a linear demand function, equilibrium inventory investment is inefficiently low and depletion inefficiently slow as we illustrate in Section 7.

6 Oligopoly: Equilibrium, Welfare, and Policy

We now turn to the oligopoly model in which a general number $n \geq 2$ symmetric firms compete. We first derive the symmetric equilibrium. Then we turn to policy analysis.

6.1 The Basic Oligopoly Equilibrium

The definition of the (continuation) equilibrium in the disruption phase is analogous to that of the duopoly case. We thus refrain from giving a formal definition and turn right to characterizing the equilibrium analogous to the basic duopoly one given in Lemma 2.

We begin by generalizing the definition of SBD tie-breaking to oligopolistic markets. The idea is as before: If two or more firms tie at the lowest price, then those firms with the lowest inventories share the market equally. Formally,

Definition 4 (SBD Tie-Breaking in Oligopoly). *Let $M \subseteq \{1, \dots, n\}$ be the firms tying at the lowest price. Among the firms in M , let $\underline{M} \subseteq M$ be the firms with the lowest inventories, $i \in \underline{M} \iff k_i \leq k_j, \forall j \neq i \in M$. Then, the market share of any firm $i \in \underline{M}$ is $1/|\underline{M}|$ and for all $i \in \{1, \dots, n\} \setminus \underline{M}$, it is zero.*

Now, for any vector of inventories $\vec{k} = (k_1, \dots, k_n)$, let $N_+(\vec{k}) = \{j \in \{1, \dots, n\} : k_j > 0\}$ be the set of firms with strictly positive inventories and let $\underline{N}(\vec{k}) \subseteq N_+(\vec{k})$ be the set of firms that have the lowest inventories among the firms with strictly positive stock; $i \in \underline{N}(\vec{k}) \iff k_i \leq k_j, \forall j \neq i \in N_+(\vec{k})$. Let $\kappa > 0$ be the inventory of the firms in $\underline{N}(\vec{k})$, $\kappa = k_i$ for all $i \in \underline{N}(\vec{k})$.

Lemma 4 (Basic Equilibrium of the Disruption Phase). *Fix the SBD tie-breaking rule for $n \geq 2$ firms from Definition 4. In the basic equilibrium of the disruption phase, all firms charge price*

$$p^*(\vec{k}) = v - \mu \left[\sum_{j \in N_+(\vec{k}) \setminus \underline{N}(\vec{k})} k_j + (|\underline{N}(\vec{k})| - 1)\kappa \right],$$

and the value function of any firm i is given by

$$V_i(\vec{k}) = vk_i - \frac{1}{2}\mu k_i^2 - \mu k_i \sum_{i \neq j} k_j.$$

The disruption phase equilibrium under oligopoly is a straightforward generalization of the basic duopoly equilibrium. When all firms have the same inventory, $k_1 = \dots = k_n$, they share the market and deplete their inventories at rate $\frac{1}{n}$. In the case of asymmetric inventories, the group of firms with the lowest, identical inventories stocks out together first, then the group with the next-lowest inventories follows suit, and so on. The market

price being charged at any state depends on the sum of the inventories by all firms except one of the lowest-inventory firms. This is consistent with the market pricing function in the duopoly case, which is a function of the highest inventory held by either firm. Finally, the monopoly payoff in the firms' value functions V_i is the same as in the duopoly case, namely $vk_i - \frac{1}{2}\mu k_i^2$. But the cost of waiting to monopolize the market depends now on the time until all other firms stock out, $\mu k_i \sum_{j \neq i} k_j$. With this characterization in hand, we can apply the same arguments underpinning Lemma 3 above to obtain Parts (i) and (ii) of the following result, which we state without further proof.

Proposition 2. *With $n \geq 2$ firms, there is a symmetric equilibrium of the game characterized as follows:*

- (i) *At the beginning of the game, firms choose inventories $k_i^* = \frac{v - \gamma}{n\mu}$ and do not change them throughout the BAU phase.*
- (ii) *During the BAU phase, all firms charge the same price equal to c .*
- (iii) *During the disruption phase, firms behave as described in Lemma 4. In particular, if the disruption hits at time $\tau > 0$, then the on-path market price p_t^* at $t \geq \tau$ is given by*

$$p_t^* = v - \mu(n-1) \left(\frac{v - \gamma}{n\mu} - \frac{t - \tau}{n} \right) = \frac{1}{n}v + \frac{n-1}{n}\gamma + \frac{n-1}{n}\mu(t - \tau),$$

until all firms have stocked out at $t = \tau + K^\dagger$.

Once again, we obtain the oligopoly price path p_t^* during the disruption phase by substituting the individual firms' inventories into the equilibrium pricing function $p(k, \dots, k)$ from Lemma 4. Observe that, as under duopoly, the price p_t^* jumps up more than the competitive price when the disruption hits. Yet, as the number of firms grows large, the jump converges to that of the competitive benchmark. Moreover, the oligopoly price path approaches pointwise that of the competitive benchmark during the disruption phase.

6.2 Policy Analysis

Because box demand is completely inelastic for prices less than v , aggregate equilibrium investment in inventory $nk^* = K^\dagger$ is efficient if consumer surplus and producer surplus are weighted equally when computing welfare. Regulators, however, often place more weight on consumers than on producers. Hence, in this section we let $\phi \in [0, 1]$ be extra weight placed on consumer surplus¹² and investigate some common regulatory policies

¹²Formally, $\phi < \frac{n}{n-1}$ is required for socially optimal inventory to be finite as is shown below.

for improving the welfare function

$$W = (1 + \phi)CS + PS + GS, \quad (6)$$

where CS is consumer surplus during the disruption, PS is producer surplus including the cost of procuring and holding inventory during the BAU phase, and GS is the net change in government revenue associated with the policy in question.¹³ Welfare in the basic symmetric equilibrium without any interventions is given by

$$W^* = \frac{(v - \gamma)^2}{2n\mu} (n + \phi(n - 1)).$$

Throughout the analysis we assume that the n firms play the basic symmetric equilibrium as altered by the policy in question. In particular, we suppose that the regulatory authority has only a limited set of policy instruments at its disposal: price caps, minimum inventory requirements, and government held reserves.¹⁴

6.2.1 Price Caps

The first policy we investigate is a cap on the price, $\bar{p}$, that firms are allowed to charge during the disruption. To the firms, this manifests as a reduction in the monopoly price from v to $\bar{p}$ as is evident from the following lemma.

Lemma 5. *Suppose firms are prohibited to charge a price higher than $\bar{p} \in [\gamma, v]$. Then, there exists an equilibrium in which the firms share the market equally on-path. The price in the BAU phase is c and in the disruption phase it is $\bar{p} - \mu(n - 1)k$. The symmetric inventories chosen at the beginning of the game are*

$$k_{PC} = \frac{\bar{p} - \gamma}{\mu n}.$$

Consumer surplus CS and producer surplus PS are given by

$$CS = \frac{(v - \bar{p})(\bar{p} - \gamma)}{\mu} + \frac{(n - 1)(\bar{p} - \gamma)^2}{2n\mu} \quad \text{and} \quad PS = \frac{(\bar{p} - \gamma)^2}{2n\mu}. \quad (7)$$

The pricing, inventory, and profits of the firms are the same as in the baseline model with $\bar{p}$ replacing v . Hence, producer surplus is not surprisingly increasing in $\bar{p}$. However, consumer surplus is concave in $\bar{p}$ and strictly decreasing for high $\bar{p}$. So, whenever the regulator puts extra weight on consumers, the optimal price cap is below v .

¹³We simplify notation by omitting the constant part of expected consumer surplus earned during the BAU phase, $\frac{v-c}{\lambda}$.

¹⁴Rationing (restricting the sales rate) in this model raises holding costs, causing the firms to invest in less inventory, which reduces welfare relative to laissez faire.

Proposition 3 (Optimal Price Cap). *The optimal price cap is*

$$\bar{p}^* = \frac{(1 + \phi)nv + \phi\gamma}{(1 + \phi)n + \phi}.$$

This implements aggregate initial inventory K_{PC} and generates welfare W_{PC} , where

$$K_{PC} = \frac{(1 + \phi)n}{(1 + \phi)n + \phi} K^\dagger \text{ and } W_{PC} = \frac{n((1 + \phi)(v - \gamma))^2}{2\mu((1 + \phi)n + \phi)}.$$

The direct effect of a price cap is to raise consumer surplus and lower producer surplus by reducing the value of the monopoly option. However, because producer surplus is reduced, firms also invest less in initial inventory. Hence, the optimal price cap $\bar{p}^*$, which decreases in ϕ , trades off the reduction in initial firm inventories with higher consumer surplus during the disruption. Note that $\bar{p}^*$ is increasing in n because the more competitive the market the higher is consumer surplus and the less a price cap is needed.¹⁵

6.2.2 Minimum Reserve Mandates

Next we investigate a policy that requires firms to hold minimum inventories higher than the equilibrium level during the BAU phase. For instance, to mitigate gasoline price spikes due to refinery outages, California requires refiners to hold buffer stocks of gasoline corresponding to at least 25 days of average use (Risser, October 10, 2024). Australia, Japan, South Korea, and several EU countries impose similar minimum fuel inventory requirements on refiners.

We focus on symmetric minimum requirements. From the value functions (4), the inventory choices are strategic substitutes. Consequently, any symmetric minimum requirement that asks firms to jointly hold more than $K > K^\dagger$ will induce each firm to choose exactly K/n . Further, such a policy both lowers the equilibrium price path and raises the stock available for consumption during the disruption. This benefits consumers and harms firms, but raises welfare when $\phi > 0$. To see this, observe that equilibrium consumer and producer surplus during the disruption are

$$CS(K) = \frac{\mu(n - 1)K^2}{2n} \quad \text{and} \quad PS(K) = (v - \gamma)K - \frac{\mu(2n - 1)K^2}{2n}.$$

Therefore, welfare is

$$W(K) \equiv (1 + \phi)CS(K) + PS(K) = (v - \gamma)K - \frac{\mu(n - \phi(n - 1))K^2}{2n}.$$

¹⁵In deadline models such as Betancourt et al. (2024), a regulator would impose a *price floor* to help preserve capacity for late arriving consumers, rather than a *price cap* to redistribute surplus from producers to consumers.

This is evidently maximized at

$$K^\dagger \equiv \frac{nK^\dagger}{n - \phi(n-1)}.$$

Observe that $\phi = 0$ implies $K^\dagger = K^\dagger$, and that $K^\dagger$ is increasing in ϕ . There is, however, a potential problem with implementing this reserve mandate. For large enough ϕ or n , requiring each firm to carry $\frac{K^\dagger}{n}$ would result in losses. To see this, note that when each firm holds buffer stock k , firm profit is given by the monopoly option: $(v - \gamma)k - \frac{1}{2}\mu k^2 - \mu(n-1)k^2$. This is zero at $k^0 = \frac{2K^\dagger}{2n-1}$ and negative for higher inventories. Hence, mandating minimum reserves of $\frac{K^\dagger}{n}$ will result in losses by the firms iff $\frac{K^\dagger}{n} > k^0$, or, equivalently, if

$$2\phi(n-1) > 1. \quad (8)$$

For any $\phi > 0$, (8) must hold for n sufficiently large, and even duopolists would incur losses for $\phi > \frac{1}{2}$. Hence, when (8) holds, the government must compensate the firms if it wishes to implement the optimal reserve mandate.

6.2.3 Government Reserves

Last we consider a reserve policy whereby the government carries inventory Q during the normal phase and sells it at a competitive price in equal parts to interested firms once the disruption hits. (When the US government releases oil from the strategic petroleum reserve, it is sold to refiners in an auction.) Suppose that the government has the same cost μ per period of carrying inventory. Assume that if more than one firm wishes to purchase reserves at the posted price r , then the government allocates its holdings equally among them. Finally suppose that the government publicly procures reserves on the open market (i.e., at price c) prior to the firms' simultaneous inventory acquisition decisions.

Proposition 4 (Optimal Reserves).

- (i) For all $Q \leq K^\dagger$, aggregate firm inventory investment is $K^\dagger - Q$; so welfare is constant at $W(K^\dagger) = W^*$.
- (ii) The optimal reserve policy is $Q^* = K^\dagger$, which induces firms to invest in zero inventory during BAU and generates welfare $W(K^\dagger)$.
- (iii) The optimal reserve policy involves losses by the government iff (8) holds.

For $Q < K^\dagger$, there is a one-for-one substitution between private and government investment in inventory. Once $Q = K^\dagger$, firm investment hits zero, and additional government reserves raise aggregate investment. This causes consumer surplus to rise and welfare to increase up to the point where $Q = K^\dagger$, which is optimal.

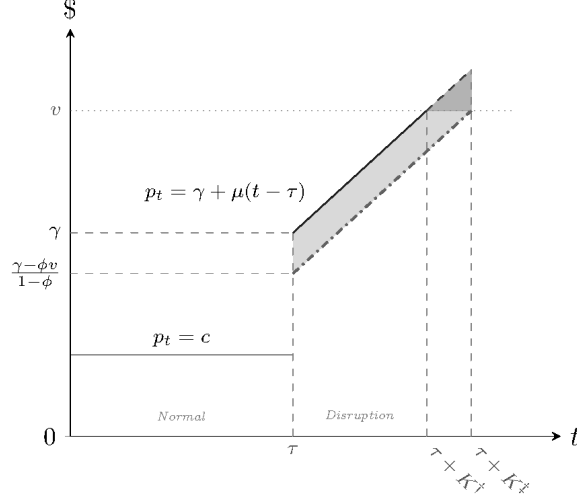


Figure 3: The competitive price path with $Q \leq K^\dagger$ is p_t . The competitive price path with $Q = K^\dagger$ corresponds to p_t before τ . It corresponds to the dashed-dotted line below p_t after τ . The light shaded area is the change in CS. The light shaded area and the dark shaded area are $-GS$.

In short, the optimal level of reserves is the same whether held by firms or the government, and the policies differ only with respect to the distribution of surplus between these parties. Moreover, it is straightforward to verify that an optimal reserve policy generates higher welfare than an optimal price cap. Both regulations reduce prices during the disruption, but a reserve policy results in higher aggregate inventory whereas a price cap suppresses inventory investment.

Figure 3 illustrates the optimal reserve policy under perfect competition ($n = \infty$), when (8) must hold. The increase in aggregate inventory lowers the equilibrium price path: the first unit is sold for $\frac{\gamma - \phi v}{1 - \phi}$ rather than γ . It also increases the stock available for consumption during the disruption from $K^\dagger$ to $K^\ddagger$. The increase in consumer surplus is less than the cost to the government, but consumer surplus receives extra weight of ϕ in welfare.

7 General and Linear Demand

This section extends the disruption-pricing analysis from box demand to more general demand functions and then explores the case of linear demand in particular. Let market demand be given by $D(p)$, where D is weakly decreasing, twice differentiable, and strictly log-concave on the region where it is positive. The choke price is $v > 0$: $D(p) > 0$ for $p < v$ and $D(p) = 0$ for $p \geq v$.

We proceed by giving sufficient conditions for a bottom-down duopoly pricing equilibrium under general demand in the next subsection. Then, we specialize to linear demand,

solve explicitly for the equilibrium price path and the associated inventory choices, and compare them with the monopoly and socially optimal benchmarks given in the following result.

Lemma 6 (Monopoly and Surplus Maximization). *Let $\gamma \equiv c + \frac{\mu}{\lambda} < v$.*

(i) *Monopoly:*

- (a) *The monopoly pricing function during the disruption is $p^M(K) = v$ if $D(v)^2 + D'(v)\mu K \geq 0$. Otherwise, it is defined implicitly by the $p < v$ solving $-\frac{D(p)^2}{D'(p)} = \mu K$.*
- (b) *The monopoly value function during the disruption is $V_M(K) = \int_0^K p^M(x) - \frac{\mu x}{D(p^M(x))} dx$.*
- (c) *The optimal initial inventory for a monopoly, K^M , is defined implicitly by $p^M(K) - \frac{\mu K}{D(p^M(K))} = \gamma$.*

(ii) *Social Planner:*

- (a) *The surplus-maximizing pricing function during the disruption, $p^{SP}(K)$, is defined implicitly by $\int_p^v D(y) dy = \mu K$.*
- (b) *The value function for social surplus during disruption is $V_{SP}(K) = \int_0^K p^{SP}(x) dx$.*
- (c) *The socially optimal initial inventory, K^{SP} , is defined implicitly by $p^{SP}(K) = \gamma$.*

This lemma highlights the difference between private and social drawdown rules. A monopolist raises price to increase revenue per unit, but doing so slows demand and increases the expected carrying cost of the remaining inventory. The monopoly pricing rule therefore balances these two forces. The social planner faces the same physical carrying cost, but also internalizes consumer surplus. As a result, price increases are socially valuable only insofar as they slow depletion and reduce carrying costs, not because they transfer surplus from consumers to producers. The value functions reflect these marginal drawdown payoffs: monopoly value integrates the firm's net private return from selling successive units, whereas social value integrates the marginal social value of inventory. Finally, the initial inventory conditions have the usual marginal-value interpretation: inventories are accumulated until marginal continuation value equals the expected marginal procurement and carrying cost γ .

Throughout this section we restrict attention to duopoly and assume bottom-down tie-breaking. In contrast to the base model, however, we do not impose equal sharing when inventories are symmetric. Instead, one of the firms captures the entire demand when inventories are equal.¹⁶ Concretely,

¹⁶Our equilibrium existence result requires that the value functions and the pricing function are continuous at equal inventories. But imposing equal sharing at equal inventories can create pricing discontinuities, as in our characterization of equilibrium under linear demand below.

Definition 5 (Asymmetric Bottom-Down (ABD) Tie-Breaking). *If $k_i > k_j > 0$, then $q_i(\vec{k}) = 0$. If $k_1 = k_2 > 0$, then $q_1(\vec{k}) = 1$ and $q_2(\vec{k}) = 0$.*

As noted in Remark 1, the cost of waiting is identical for both firms when inventories are equal, so any division of the market is consistent with their incentives to undercut. Indeed, for the value function in the continuation equilibrium following a supply disruption, it is immaterial which firm serves the market when inventories are equal. Moreover, when we consider the case of linear demand below, equilibrium inventory investments are asymmetric, so the specific tie-breaking rule for equal inventories is also irrelevant for equilibrium behavior.

7.1 The Supply Disruption

As with box demand, both firms must choose the same price at any interior state, which we denote by $p(k_1, k_2)$. Under ABD tie-breaking, the equilibrium value function for firm 1 is

$$V_1(k_1, k_2) = \begin{cases} \int_0^{k_1} p(x, 0) - \frac{\mu x}{D(p(x, 0))} dx - \int_0^{k_2} \frac{\mu k_1}{D(p(k_1, x))} dx & \text{if } k_1 > k_2 \\ \int_0^{k_1} p(x, k_2) - \frac{\mu x}{D(p(x, k_2))} dx, & \text{if } k_1 \leq k_2, \end{cases} \quad (9)$$

Firm 2's value function $V_2(k_1, k_2)$ is analogous except that ties are broken in favor of firm 1 when $k_1 = k_2$. The following result provides sufficient conditions such that $p(k_1, k_2)$ is an equilibrium pricing function in the disruption phase under ABD tie breaking.

Lemma 7. *Suppose the pricing function $p(k_1, k_2)$ is continuous on $\mathbb{R}_{++}^2$ and the value functions $V_i(k_1, k_2)$ are continuous on $\mathbb{R}_+^2$ and continuously differentiable on $\mathbb{R}_{++}^2 \setminus \{(k_1, k_2) : k_1 = k_2\}$. Under ABD tie-breaking, the pricing function $p(k_1, k_2)$ constitutes an equilibrium if:*

(i) *For all $k_i > k_j > 0$,*

$$(a) \text{ For all } p < p(k_1, k_2): p - \frac{\partial V_i(k_1, k_2)}{\partial k_i} + \frac{\partial V_i(k_1, k_2)}{\partial k_j} \frac{D(p(k_1, k_2))}{D(p)} \leq 0,$$

$$(b) p(k_1, k_2) - \frac{\partial V_j(k_1, k_2)}{\partial k_j} + \frac{\partial V_j(k_1, k_2)}{\partial k_i} \geq 0,$$

$$(c) p(k_1, k_2) \in \arg \max_{p \in (-\infty, p(k_1, k_2)]} D(p)(p - \partial V_j(k_1, k_2)/\partial k_j).$$

(ii) *For all $k_i > k_j = 0$, $p(k_1, k_2) = v$ if $D(v)^2 + D'(v)\mu k_i \geq 0$. Otherwise, $p(k_1, k_2)$ is defined implicitly by the $p < v$ solving $-\frac{D(p)^2}{D'(p)} = \mu k_i$.*

In point (i), the first constraint is the undercutting constraint of the firm that does not receive any demand, ensuring that it has no incentive to set a lower price than its competitor. In the baseline box-demand model, this constraint is automatically satisfied when the inequality holds at $p = p(k_1, k_2)$. With a general demand function, however,

lowering the price raises demand and may therefore raise revenue at the margin. The second constraint ensures that the firm serving the entire market has no incentive to raise its price and cede demand to its competitor. If demand $D(p)$ is strictly decreasing, the third constraint in (i) guarantees that the firm serving the market also has no incentive to lower its price. In the box-demand model, this condition is automatically satisfied, since lowering the price does not increase demand and therefore strictly reduces revenue. The condition in (ii) implicitly defines the monopoly price path, as given in Lemma 6.

7.2 Linear Demand

In this section we apply Lemma 7 to establish an equilibrium for linear demand, $D(p) = 1 - p$.

7.2.1 Supply Disruption

We start by characterizing the equilibrium in the supply disruption, contrasting it with the monopoly price path and the price path inducing the socially optimal drawdown of inventories.

Lemma 8 (Disruption Pricing with Linear Demand). *Suppose demand at every instant is $D(p) = 1 - p$.*

- (i) *The monopoly pricing function is $p^M(K) = 1 - \sqrt{\mu K}$.*
- (ii) *The surplus-maximizing pricing function is $p^{SP}(K) = 1 - \sqrt{2\mu K}$.*
- (iii) *In the disruption phase under ABD tie-breaking, it is an equilibrium for both firms to choose $p^*(k_1, k_2) = 1 - \sqrt{\beta\mu(k_1 + k_2)}$ for $(k_1, k_2) \in \mathbb{R}_{++}^2$, where $\beta \in (1, 2)$ is the unique positive root of $(2\sqrt{2} - 1)\beta - 2\sqrt{\beta} + (5 - 4\sqrt{2}) = 0$.*

Parts (i) and (ii) are the linear-demand prices given in Lemma 6. For part (iii), the value of β (which is approximately 1.845) is chosen to ensure continuity of the duopoly value function along the 45-degree line. The proof then verifies that, for this value of β , conditions (i.a–c) of Lemma 7 are satisfied.

Unlike the basic equilibrium with box demand, if the firms have asymmetric inventories, the one that sells first enjoys an increasing price path. The maximum price it receives is, however, bounded by the inventory held by its rival. By holding the firm that sells first to a relatively low price, the firm that sells second shortens the amount of time it has to wait before monopolizing the market. This effect is not present in the box-demand specification where the sales rate is constant.

Finally, when the first firm stocks out, there is a discontinuous jump in the price from the duopoly level $1 - \sqrt{\beta\mu k_i}$ to the monopoly level $1 - \sqrt{\mu k_i}$. From that point on,

the monopoly price continues to rise until the last unit is sold at the choke price of 1. At equal aggregate inventories, both the monopoly and the duopoly prices lie above the surplus-maximizing price, implying that inventory depletion is generally too slow. We return to the question of efficiency once we have determined the initial inventory choices for each regime.

Under the pricing function delineated in Lemma 8, the value function of firm 1 is

$$V_1(k_1, k_2) = \begin{cases} k_1 - \frac{4}{3}\sqrt{\mu}k_1^{3/2} - \frac{2\sqrt{\mu}k_1}{\sqrt{\beta}}(\sqrt{k_1 + k_2} - \sqrt{k_1}) & \text{if } k_1 > k_2 \\ \int_0^{k_1} 1 - \sqrt{\beta\mu(x + k_2)} - \frac{\mu x}{\sqrt{\beta\mu(x + k_2)}} dx, & \text{if } k_1 \leq k_2. \end{cases} \quad (10)$$

Firm 2's value function $V_2(k_1, k_2)$ is analogous except that ties are broken in favor of firm 1 when $k_1 = k_2$.

7.2.2 Inventory Choices

Recall that the equilibrium inventory investments (k_1^*, k_2^*) satisfy

$$k_1^* \in \arg \max_{k_1} -\gamma k_1 + V_1(k_1, k_2^*) \quad \text{and} \quad k_2^* \in \arg \max_{k_2} -\gamma k_2 + V_2(k_1^*, k_2), \quad (11)$$

Our first result for linear demand establishes that there is no symmetric equilibrium investment choices.

Lemma 9 (Failure of Symmetry). *There is no pure-strategy equilibrium with symmetric inventory investment, $k_1^* = k_2^* = k^* > 0$.*

The proof is by contradiction. It shows that, at any symmetric inventory pair, each firm has a profitable local inventory adjustment. This conclusion is not an artifact of the asymmetric tie-breaking rule. Instead, it is established by evaluating the one-sided partial derivatives of a firm's value function with respect to its own inventory as the 45-degree line is approached from below and from above.

Next we characterize the unique asymmetric equilibrium inventory choices up to re-labeling of the firms.

Proposition 5 (Asymmetric Equilibrium). *Let $y^* \in (1, \sqrt{2})$ be the unique root of $(\beta - 1)y^2 + (3 - 2\sqrt{\beta})y - 2 = 0$ and set*

$$h^* = \frac{(1 - \gamma)^2}{\mu(A^*)^2}, \quad \ell^* = ((y^*)^2 - 1)h^*, \quad \text{where } A^* = \sqrt{\beta}y^* + \frac{(y^*)^2 - 1}{\sqrt{\beta}y^*}.$$

Then $h^* > \ell^* > 0$, and the only pure-strategy equilibrium inventory choices are

$$(k_1^*, k_2^*) = (h^*, \ell^*) \quad \text{and} \quad (k_1^*, k_2^*) = (\ell^*, h^*).$$

Comparison to Monopoly and the Efficient Inventories Specializing to linear demand in Lemma 6, the monopoly and surplus-maximizing value functions during the disruption are

$$V_M(K) = K - \frac{4}{3}\sqrt{\mu} K^{3/2}. \quad (12)$$

$$V_{SP}(K) = K - \frac{2\sqrt{2}}{3}\sqrt{\mu} K^{3/2}. \quad (13)$$

Subtracting γk from these expressions and maximizing gives the values for K^M and K^{SP} in the following result.

Proposition 6 (Aggregate investment ordering). *Aggregate investment is highest under surplus maximization (K^{SP}), intermediate under the asymmetric duopoly equilibrium (K^D), and lowest under monopoly (K^M):*

$$\frac{(1-\gamma)^2}{2\mu} = K^{SP} > K^D = h^* + \ell^* > K^M = \frac{(1-\gamma)^2}{4\mu}.$$

Proposition 6 not only ranks aggregate inventory investments, but when combined with Lemma 8 pins down the equilibrium price path. Figure 4 plots the three disruption price paths in normalized units with $(p-\gamma)/(1-\gamma)$ on the vertical axis. The planner's path is steepest and extends furthest, ending at K^{SP} . The monopoly path ends at K^M . The duopoly path coincides with monopoly while only the large firm has inventory remaining, then drops discontinuously at $K = h^*$ when the smaller firm is also active, and finally ends at $K^D = h^* + \ell^*$. Observe that the duopoly engages in price gouging due both to the presence of the monopoly option and the inefficiently low inventory investment.

Welfare and Policy With box demand and linear holding costs, there is no distortion in equilibrium inventory investment or disruption pricing when consumer and producer surplus receive equal welfare weight ($\phi = 0$). With linear demand, however, laissez-faire duopoly is inefficient: firms carry too little aggregate inventory and price too high during the disruption relative to the surplus-maximizing benchmark (see Figure 4). Indeed, at equal levels of aggregate inventory, $p^D(K) > p^{SP}(K)$ because the firms maximize profit without regard to consumer surplus. More, since $p^D(K) > p^{SP}(K)$, the duopoly has a lower sales rate and correspondingly higher cost of carrying inventory because they take longer to stock out. This leads the firms to invest in less than the efficient level of aggregate inventory in the first place, $K^D < K^{SP}$.

Although space constraints prohibit a comprehensive policy analysis of the linear demand duopoly model, it is straightforward to characterize the optimal reserve mandates that a surplus-maximizing regulator would impose on the two firms.

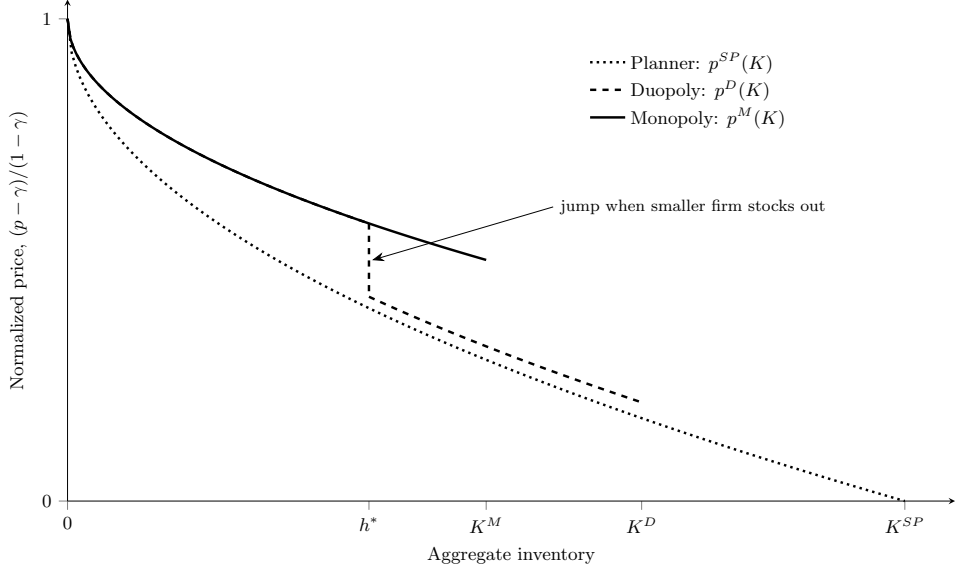


Figure 4: Disruption price paths as a function of aggregate inventory under linear demand. The figure uses normalized price units with $(p - \gamma)/(1 - \gamma)$ on the vertical axis, where $\gamma = c + \mu/\lambda$. It highlights that $p^{SP}(K) < p^D(K) < p^M(K)$ when both duopolists are active, while $p^D(K) = p^M(K)$ after the smaller firm stocks out.

Proposition 7 (Socially Optimal Duopoly Inventory Requirements). *Suppose the regulator can impose initial inventory requirements (k_1, k_2) on the two firms, but that after the supply disruption hits the firms price according to the duopoly equilibrium described in Lemma 8. Then the socially optimal firm inventory requirement is symmetric, $k_1^R = k_2^R = \frac{K^R}{2}$, where the optimal aggregate requirement is*

$$K^R = \frac{4(1 - \gamma)^2}{9(a_D + (1 - a_D)2^{-3/2})^2 \mu} \quad \text{with} \quad a_D = \frac{2}{3} \left(\frac{\sqrt{\beta}}{2} + \frac{1}{\sqrt{\beta}} \right).$$

Moreover, $K^D < K^R < K^{SP}$. Thus, the socially optimal duopoly requirement is larger than laissez-faire duopoly investment, but it remains strictly below the first-best planner's aggregate inventory.

The inventory mandate raises welfare because laissez-faire duopoly inventories are distorted in two ways. First, firms carry too little aggregate inventory because they do not internalize the consumer-surplus value of keeping disruption prices low. Second, asymmetric inventories are socially costly: once the smaller firm stocks out, the larger firm becomes a monopolist and the price jumps up. For any fixed aggregate inventory, a symmetric mandate minimizes the inventory left for this monopoly phase, thereby shortening the high-price part of the disruption. But the regulator still cannot force firms to price efficiently during the duopoly phase, so the optimal mandate raises aggregate inventory above laissez-faire duopoly investment but stops short of the first-best planner's inventory (see Figure 5).

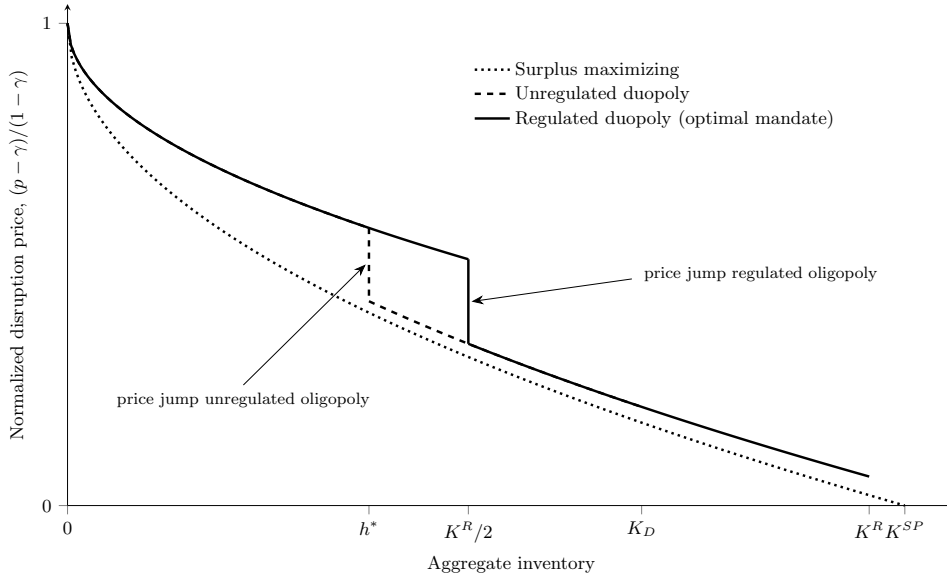


Figure 5: Disruption price paths under linear demand with and without the optimal reserve mandate. The figure uses normalized price units, with $(p - \gamma)/(1 - \gamma)$ on the vertical axis. The unregulated duopoly path ends at the laissez-faire aggregate inventory K^D and features a jump to the monopoly price after the smaller firm stocks out. The regulated path is the duopoly equilibrium price path induced by the optimal reserve mandate, so it jumps at $K^R/2$ and ends at K^R . The mandate raises aggregate inventory and shortens the monopoly-pricing phase relative to laissez faire, while the surplus-maximizing path remains below both duopoly paths.

Finally, the optimal symmetric inventory mandate strictly increases total surplus relative to laissez-faire duopoly investment because it raises aggregate inventories and shortens the monopoly-pricing phase following the smaller firm's stockout. In the linear-demand case, the mandate closes a fixed – parameter-independent – fraction of the gap between laissez-faire duopoly surplus and first-best surplus. Specifically, it can be shown that a reserve mandate eliminates approximately 60 percent of the dead-weight loss generated by the unregulated duopoly (see Section F of the Online Appendix).

8 Conclusion

We investigate a continuous-time homogeneous good oligopoly model with two phases. In the business-as-usual phase, Bertrand competition prevails and the firms price at marginal cost. A disruption arrives randomly in which the firms are unable to source an essential input and must rely on inventories to continue to produce.

For a finite number of firms, the arrival of the supply disruption engenders a shift in the competitive environment associated with an incentive to forgo current sales in order to sell inventory as a monopoly once the other firms stock out. This *monopoly option* causes the equilibrium price to jump to a supracompetitive level at the advent of

the disruption and to remain above the competitive price path until all inventories are exhausted. In this sense, the equilibrium dynamics exhibit *price gouging*.

Anticipating the supply disruption, firms may – depending on market demand – choose inventory levels that are inefficiently low, asymmetric, or neither of the two.¹⁷ For our baseline model, we showed that an optimally designed price cap suppresses the value of the monopoly option, thereby reducing equilibrium prices. However, price caps always lead to lower inventory investment: so must be implemented judiciously. An optimal reserve mandate or strategic reserve policy raises consumer surplus by more than a price cap, but may require significant government oversight or infrastructure.

The main message of this paper is somewhat nuanced. We do find that supply disruptions can lead to supracompetitive pricing that is especially severe in industries with a small number of asymmetric firms. Nevertheless, we show that competitive pricing dynamics can be similar in appearance to non-competitive pricing, differing primarily in the magnitude of the price jump when the disruption hits. Hence, it may be difficult to detect and effectively regulate price gouging as such. Perhaps the most robust prescription we can offer in this light is one already familiar to industrial economists, namely to promote competition among as many symmetric firms as possible.

A Proofs for Sections 5 and 6

Proof of Lemma 2. For $k_1 \neq k_2$, the result follows from the proof of Lemma 7, specialized to box demand, since SBD and ABD tie-breaking coincide off the diagonal.

It remains to verify the case $k_1 = k_2 = k > 0$. From (4),

$$\frac{\partial V_i(k, k)}{\partial k_i} = v - 2\mu k, \quad \frac{\partial V_i(k, k)}{\partial k_j} = -\mu k.$$

At the equilibrium price $p^*(k, k) = v - \mu k$, SBD assigns each firm one half of demand. Under the forward selection (Definition C.1 in Online Appendix C), both inventories therefore decline at rate $1/2$, and

$$\frac{1}{2}p^*(k, k) - \mu k - \frac{1}{2}\frac{\partial V_i(k, k)}{\partial k_i} - \frac{1}{2}\frac{\partial V_i(k, k)}{\partial k_j} = 0.$$

If firm i instead chooses $p > p^*(k, k)$, it receives no demand while its rival serves the entire market, and hence

$$-\mu k - \frac{\partial V_i(k, k)}{\partial k_j} = 0.$$

If, on the other hand, it chooses $p < p^*(k, k)$, it serves the entire market, yielding

¹⁷While we have not establish this formally, in the demand specifications we have examined, equilibrium inventories are never inefficiently high.

$$p - \mu k - \frac{\partial V_i(k, k)}{\partial k_i} = p - p^*(k, k) < 0.$$

Thus the HJB condition also holds on the diagonal. The sufficiency argument in the proof of Lemma 7 then applies unchanged. $\square$

Proof of Lemma 4. For the proof it is convenient to write $\{N_1, \dots, N_m\}$ for the partition of $N_+(\vec{k})$ such that all firms in an element N_ℓ have the same inventories, $k_i = \kappa_\ell$ for all $i \in N_\ell$, and the inventories are strictly ordered, $\kappa_1 < \kappa_2 < \dots < \kappa_m$. Note that $N_1 = \underline{N}(\vec{k})$ by definition.

We begin by verifying the value functions. In the conjectured equilibrium, the value function for a firm $i \in N_\ell$ for some $\ell \in \{1, \dots, m\}$ can be expressed as

$$\begin{aligned} V_i(\vec{k}) &= \int_0^{\kappa_\ell} \left[v - \mu \left[\sum_{j \in \bigcup_{q > \ell} N_q} k_j - (|N_\ell| - 1)\kappa \right] - \mu |N_\ell| \kappa \right] d\kappa - \kappa_\ell \sum_{j \in \bigcup_{q < \ell} N_q} k_j \\ &= v\kappa_\ell - \mu\kappa_\ell \sum_{j \in \bigcup_{q \neq \ell} N_q} k_j - \frac{\mu}{2}(2|N_\ell| - 1)\kappa_\ell^2 = vk_i - \mu k_i \sum_{j \neq i} k_j - \frac{\mu}{2}k_i^2, \end{aligned}$$

which is $V_i(\vec{k})$ from the statement.

Next, we verify that, given the other firms follow the equilibrium pricing rule, the following HJB equations hold for every firm $i \in N_\ell$ and $\ell \in \{1, \dots, m\}$,

$$0 = \sup_p \left\{ g_i(p, \vec{k}) + \partial V_i(\vec{k}) \cdot f_i(p, \vec{k}) \right\}, \quad (14)$$

where $\partial V_i(\vec{k})$ is the vector of partial derivatives of V_i , the function $g_i(p, \vec{k})$ returns the flow payoff when setting a price p in state k while the other firms set $p(\hat{k})$,

$$g_i(p, \vec{k}) = \begin{cases} -\mu k_i & \text{if } p > p(\vec{k}) \\ -\mu k_i & \text{if } p = p(\vec{k}) \text{ and } \ell \neq 1 \\ \frac{1}{|N_1|}p(\vec{k}) - \mu k_i & \text{if } p = p(\vec{k}) \text{ and } \ell = 1 \\ p - \mu k_i & \text{if } p < p(\vec{k}), \end{cases}$$

and $f_i(p, \vec{k}) = (f_{i1}(p, \vec{k}), \dots, f_{in}(p, \vec{k}))$ describes the marginal changes in inventory of the respective firms when firm i chooses a price p while all other firms choose $p(\vec{k})$. In

particular, for firm $i \in N_\ell$ and firm $j \in N_q$, $i \neq j$, we have

$$f_{ij}(p, \vec{k}) = \begin{cases} -\frac{1}{|N_q|-1} & \text{if } p > p(\vec{k}) \text{ and } \ell = q = 1 \\ -\frac{1}{|N_q|} & \text{if } \begin{aligned} & \text{(i) } p > p(\vec{k}) \text{ and } \ell > q = 1, \text{ or} \\ & \text{(ii) } p > p(\vec{k}) \text{ and } |N_\ell| = \ell = 1 \text{ and } q = 2, \text{ or} \\ & \text{(iii) } p = p(\vec{k}) \text{ and } q = 1 \end{aligned} \\ 0 & \text{if } \begin{aligned} & \text{(i) } p > p(\vec{k}) \text{ and } q > 1 \text{ and } \ell \neq 1, \text{ or} \\ & \text{(ii) } p > p(\vec{k}) \text{ and } q > 1 \text{ and } |N_\ell| > 1, \text{ or} \\ & \text{(iii) } p = p(\vec{k}) \text{ and } q > 1, \text{ or} \\ & \text{(iv) } p < p(\vec{k}). \end{aligned} \end{cases}$$

First, consider $i \in N_1$. If $|N_1| > 1$, then

$$\begin{aligned} & g_i(p, \vec{k}) + \partial V_i(\vec{k}) \cdot f_i(p, \vec{k}) \\ &= \begin{cases} -\mu k_i + \sum_{j \in N_1 \setminus i} \mu k_j \frac{1}{|N_1|-1} & \text{if } p > p(\vec{k}) \\ \frac{1}{|N_1|} p(\vec{k}) - \mu k_i - \frac{1}{|N_1|} \left[v - \mu k_i - \mu \sum_{j \neq i} k_j \right] + \sum_{j \in N_1 \setminus i} \mu k_j \frac{1}{|N_1|} & \text{if } p = p(\vec{k}) \\ p - \mu k_i - \left[v - \mu k_i - \mu \sum_{j \neq i} k_j \right] & \text{if } p < p(\vec{k}). \end{cases} \end{aligned}$$

The first case is zero. Plugging in for $p(\vec{k})$, we further see that the second case is zero and the third case is strictly negative. The same holds true when $|N_+(\vec{k})| > |N_1| = 1$, for which we obtain

$$g_i(p, \vec{k}) + \partial V_i(\vec{k}) \cdot f_i(p, \vec{k}) = \begin{cases} -\mu k_i + \sum_{j \in N_2} \mu k_j \frac{1}{|N_2|} & \text{if } p > p(\vec{k}) \\ p(\vec{k}) - \mu k_i - \left[v - \mu k_i - \mu \sum_{j \neq i} k_j \right] & \text{if } p = p(\vec{k}) \\ p - \mu k_i - \left[v - \mu k_i - \mu \sum_{j \neq i} k_j \right] & \text{if } p < p(\vec{k}). \end{cases}$$

And finally, if $|N_+(\vec{k})| = |N_1| = 1$, it holds

$$g_i(p, \vec{k}) + \partial V_i(\vec{k}) \cdot f_i(p, \vec{k}) = \begin{cases} -\mu k_i & \text{if } p > p(\vec{k}) \\ p(\vec{k}) - \mu k_i - [v - \mu k_i] & \text{if } p = p(\vec{k}) \\ p - \mu k_i - [v - \mu k_i] & \text{if } p < p(\vec{k}), \end{cases}$$

which also gives us the desired inequalities.

Next, consider a firm $i \in N_\ell$ with $\ell \geq 2$. We have

$$g_i(p, \vec{k}) + \partial V_i(\vec{k}) \cdot f_i(p, \vec{k}) = \begin{cases} -\mu k_i + \sum_{j \in N_1} \mu k_i \frac{1}{|N_1|} & \text{if } p > p(\vec{k}) \\ -\mu k_i + \sum_{j \in N_1} \mu k_i \frac{1}{|N_1|} & \text{if } p = p(\vec{k}) \\ p - \mu k_i - \left[v - \mu k_i - \mu \sum_{j \neq i} k_j \right] & \text{if } p < p(\vec{k}). \end{cases}$$

The first two cases are evidently zero. To see that the last case is strictly below zero, observe

$$\begin{aligned} p - \mu k_i - \left[v - \mu k_i - \mu \sum_{j \neq i} k_j \right] &< p(\vec{k}) - \mu k_i - \left[v - \mu k_i - \mu \sum_{j \neq i} k_j \right] \\ &= -\mu \left[\sum_{j \in N_+(\vec{k}) \setminus \underline{N}(\vec{k})} k_j + (|\underline{N}(\vec{k})| - 1)\kappa - \sum_{j \neq i} k_j \right] < 0, \end{aligned}$$

where the inequality follows because κ is the inventory of a firm in the group with the lowest inventories, $\underline{N}(\vec{k})$, and hence $\kappa < k_i$.

To finish the proof, we must show that the HJB equations (14) are sufficient for optimality. This can be done with an analogous arguments as we use in the proofs of Lemma 2 and Lemma 7 for the duopoly case, and we refrain from doing so here explicitly. $\square$

Proof of Lemma 5. The argument for the prices during the BAU phase is as in the arguments leading to Part (ii) of Proposition 2. The equilibrium price evolution in the disruption phase and the inventory decisions of the firms can be derived with the same arguments as in the proof of Lemma 4, replacing v with $\bar{p}$. Specifically, under a price cap $\bar{p} \in [\gamma, v]$, the firms will choose a price $\hat{p}(x) = \bar{p} - (n-1)\mu x$ whenever $k_i = x$, for all firms i .

For the surplus expressions, we use the continuation profit of any firm i in the disruption phase, $V_i(\vec{k}) = \bar{p}k_i - \frac{1}{2}\mu k_i^2 - \mu k_i \sum_{j \neq i} k_j$, to obtain

$$PS = n(\bar{p} - \gamma) \frac{\bar{p} - \gamma}{n\mu} - n \frac{1}{2} \mu \left(\frac{\bar{p} - \gamma}{n\mu} \right)^2 - n(n-1)\mu \left(\frac{\bar{p} - \gamma}{n\mu} \right)^2 = \frac{(\bar{p} - \gamma)^2}{2\mu n},$$

as desired. For consumer surplus, we obtain

$$CS = n \int_0^{k_{PC}} v - \hat{p}(x) dx = n \int_0^{k_{PC}} v - \bar{p} + (n-1)\mu x dx.$$

Performing the integration and evaluating the result at k_{PC} yields the claim. $\square$

Proof of Proposition 3. Using the expressions for CS and PS from Lemma 5, the objec-

tive is

$$W_{\text{PC}} = \frac{(1 + \phi)(v - \bar{p})(\bar{p} - \gamma)}{\mu} + \left(1 + \frac{\phi(n-1)}{n}\right) \left(\frac{(\bar{p} - \gamma)^2}{2\mu}\right). \quad (15)$$

Maximization yields the expression for the optimal price cap $\bar{p}^*$. Substituting the expression for $\bar{p}^*$ back into the objective yields the expression for equilibrium welfare. $\square$

Proof of Proposition 4. Consider a symmetric equilibrium in which (on-path) each firm invests $k^*(Q)$ and is allocated Q/n of the total reserves. To compute the price each firm is willing to pay for a share of the reserves we subtract the value function associated with not purchasing reserves from the one associated with purchasing, assuming the other $n-1$ firms purchase reserves. In particular, we have

$$\begin{aligned} & v(k^*(Q) + Q/n) - \frac{1}{2}\mu(k^*(Q) + Q/n)^2 - \mu(n-1)(k^*(Q) + Q/n)^2 \\ - & \quad vk^*(Q) - \frac{1}{2}\mu(k^*(Q))^2 - \mu(n-1)k^*(Q)(k^*(Q) + Q/(n-1)) \\ = & \quad v\frac{Q}{n} - \frac{\mu(2n-1)}{2} \left(\frac{Q}{n}\right)^2 - \mu(n-1)k^*(Q)\frac{Q}{n}. \end{aligned}$$

Dividing through by Q/n gives the highest price at which the government can sell all reserves

$$r^* = v - \frac{\mu(2n-1)Q}{2n} - \mu(n-1)k^*(Q). \quad (16)$$

Hence, when acquiring inventory each firm solves

$$\max_{k \geq 0} (v - \gamma)k - \frac{1}{2}\mu k^2 - \mu k((n-1)k^*(Q) + Q).$$

This results in symmetric inventory investment of

$$k^*(Q) = \max \left\{ \frac{v - \gamma}{\mu n} - \frac{Q}{n}, 0 \right\}.$$

In other words, each firm reduces its investment by the amount of reserves it anticipates purchasing when the disruption hits. Consumer surplus will be

$$CS = n \int_0^{k^*(Q) + Q/n} \mu(n-1)x \, dx = \begin{cases} \frac{(n-1)(v-\gamma)^2}{2\mu n} & \text{if } Q < K^\dagger \\ \frac{\mu(n-1)Q^2}{2n} & \text{if } Q \geq K^\dagger. \end{cases}$$

That is, if $Q < K^\dagger$, then consumer surplus is as in the case without strategic reserves. Consumer surplus is affected only when strategic reserves completely crowd out firm inventories.

To compute producer surplus, observe that individual firm profit is

$$(v - \gamma)k^*(Q) - \frac{\mu(2n - 1)}{2}(k^*(Q))^2 - \mu k^*(Q)Q.$$

Note that this corresponds to waiting to sell $k^*(Q)$ units at a price of v once all other sources of inventory have been exhausted, private holdings of $(n - 1)k^*(Q)$ and government reserves of Q . Substituting for $k^*(Q)$, collecting terms, and multiplying by n gives producer surplus as

$$PS = \begin{cases} \frac{(v - \gamma - \mu Q)^2}{2\mu n} & \text{if } Q < K^\dagger \\ 0 & \text{if } Q \geq K^\dagger. \end{cases}$$

Intuitively, because government reserves crowd out private inventory and the government has full bargaining power when selling its reserves, producer profits decrease in the reserves and hit zero once firms stop acquiring inventory themselves.

Finally, government surplus is

$$\begin{aligned} GS &= (r - \gamma)Q = (v - \gamma)Q - \frac{\mu(2n - 1)}{2n}Q^2 - \mu(n - 1)k^*(Q)Q \\ &= \begin{cases} \frac{(v - \gamma)Q}{n} - \frac{\mu Q^2}{2n} & \text{if } Q < K^\dagger \\ (v - \gamma)Q - \mu \frac{2n - 1}{2n}Q^2 & \text{if } Q \geq K^\dagger. \end{cases} \end{aligned}$$

Combining the surpluses given above, the government's program is

$$\max_{Q \geq 0} \begin{cases} \left(1 + \frac{\phi(n-1)}{n}\right) \left(\frac{(v-\gamma)^2}{2\mu}\right) & \text{if } Q < K^\dagger \\ (v - \gamma)Q - \frac{\mu(n - \phi(n-1))Q^2}{2n} & \text{if } Q \geq K^\dagger. \end{cases}$$

The first line does not depend on Q , establishing the first claim. The second line evidently is maximized at $Q^* = K^\dagger$. Substituting the solution back into the objective yields welfare $W(K^\dagger)$. $\square$

B Proofs for Section 7

Proof of Lemma 6. Part (i): We begin with the monopolist's value function. Let $p^M(K)$ be the price the monopolist sets when the inventory is K . Then, the monopolist's value function can be written as

$$V_M(K) = \int_0^K p^M(x) - \frac{\mu x}{D(p^M(x))} dx,$$

which is i.b) in the statement. Statement i.c) then follows immediately by appreciating that the initial inventory for the monopoly satisfies the first-order condition

$$p^M(K_M) - \frac{\mu K_M}{D(p^M(K_M))} = \gamma.$$

Next, we turn to the optimal price, $p^M(K)$. The HJB for monopoly is

$$0 = \sup_p pD(p) - \mu K - V'_M(K)D(p), \quad (17)$$

and the derivative of the objective function on the right in (17) is $D(p) + D'(p)[p - V'_M(K)]$.

From $D(p) > 0$ for all $p < v$, the price p_M solving the right side in (17) is, thus, either in $(-\infty, v)$, in which case $D'(p_M) < 0$ holds and p_M corresponds to the p satisfying the first-order condition

$$V'_M(K) = p + \frac{D(p)}{D'(p)}, \quad (18)$$

or it is $p_M = v$ and, using $V'_M(K) = v - \frac{\mu K}{D(v)}$ in that case, we have $D(v) + D'(v)\frac{\mu K}{D(v)} \geq 0$.

So, consider the interior solution. Substituting (18) into (17) gives

$$\frac{1}{\mu K} = -\frac{D'(p)}{D(p)^2} \equiv H(p). \quad (19)$$

Observe that because $D(p)$ is strictly log-concave we have

$$H'(p) = -\frac{D''(p)D(p) - 2D'(p)^2}{D(p)^3} > 0,$$

for all $p \in (\infty, v)$. Therefore $H(p)$ is strictly increasing and invertible on (∞, v) . Recall $K > 0$. Consequently, (19) describes the optimal (interior) price provided the first-order condition (18) identifies a maximum. But this holds because the right-hand side of (18) increases, implying that the objective function (17) is unimodal.

Finally, for (19) to indeed describe the monopoly price path $p^M(K)$, we need to verify that the HJB equation (17) is sufficient for optimality, which we postpone to the proof of Lemma 7 below.

Part (ii): The HJB for the social planner is

$$0 = \sup_p pD(p) + \int_p^1 D(y) dy - \mu K - V'_{SP}(K)D(p). \quad (20)$$

Here, the first-order necessary condition simply gives $V'_{SP}(K) = p$. (Sufficiency can be established with a similar argument as for the monopolist.) Substituting this into (20) gives the implicit definition of the optimal price function $\int_{p^{SP}(K)}^1 D(y) dy = \mu K$. Moreover, the value function is $V_{SP}(K) = \int_0^K p^{SP}(x) dx$. The optimal initial inventory then simply

satisfies the first-order condition $V'_{SP}(K^{SP}) = \gamma$. $\square$

Proof of Lemma 7. The proof proceeds in two steps. First, we verify that the value functions satisfy the following HJB equations:

(i) For $k \in \mathbb{R}_+^2 \setminus \{(k_1, k_2) : k_1 = k_2\}$, we have

$$0 = \sup_p \left\{ g_i((p, p(\vec{k})), \vec{k}) + \sum_j \frac{\partial V_i(\vec{k})}{\partial k_j} f_j((p, p(\vec{k})), \vec{k}) \right\}, \quad i = 1, 2. \quad (21)$$

(ii) For $k \in \{(k_1, k_2) \in \mathbb{R}_{++}^2 : k_1 = k_2\}$, we have

$$0 = \sup_p \left\{ g_i((p, p(\vec{k})), \vec{k}) + \lim_{\kappa \uparrow k_1} \frac{\partial V_i(\kappa, k_2)}{\partial k_1} f_1((p, p(\vec{k})), \vec{k}) + \lim_{\kappa \uparrow k_2} \frac{\partial V_i(k_1, \kappa)}{\partial k_2} f_2((p, p(\vec{k})), \vec{k}) \right\}, \quad i = 1, 2. \quad (22)$$

As in the text, $f_i(\vec{p}, \vec{k})$ describes the marginal inventory change of firm i when the firms choose prices $\vec{p}$ at inventories $\vec{k}$ and $g_i(\vec{p}, \vec{k})$ describes the flow payoff of firm i . In a second step, we then show that these conditions are sufficient for mutual optimality.

Step 1: HJB Equations. First, we consider (k_1, k_2) satisfying $k_1 > k_2 > 0$ (the argument for $k_2 > k_1 > 0$ is symmetric). For Firm 1, we obtain

$$g_1((p, p(\vec{k})), \vec{k}) + \sum_j \frac{\partial V_1(\vec{k})}{\partial k_j} f_j((p, p(\vec{k})), \vec{k}) = \begin{cases} -\mu k_1 - D(p(\vec{k})) \frac{\partial V_1}{\partial k_2} & \text{if } p > p(\vec{k}) \\ -\mu k_1 - D(p(\vec{k})) \frac{\partial V_1}{\partial k_2} & \text{if } p = p(\vec{k}) \\ pD(p) - \mu k_1 - D(p) \frac{\partial V_1}{\partial k_1} & \text{if } p < p(\vec{k}). \end{cases}$$

Now, from $\frac{\partial V_1}{\partial k_2} = -\frac{\mu k_1}{D(p(\vec{k}))}$, the first two cases are zero. Further, we obtain

$$\begin{aligned} pD(p) - \mu k_1 - D(p) \frac{\partial V_1}{\partial k_1} &= pD(p) + D(p(\vec{k})) \frac{\partial V_1}{\partial k_2} - D(p) \frac{\partial V_1}{\partial k_1} \\ &= D(p) \left[p + \frac{D(p(\vec{k}))}{D(p)} \frac{\partial V_1}{\partial k_2} - \frac{\partial V_1}{\partial k_1} \right], \end{aligned}$$

which is negative under condition (i.a) in the statement. Consequently, (21) holds for $k_1 > k_2 > 0$. For Firm 2, we obtain

$$g_2((p(\vec{k}), p), \vec{k}) + \sum_j \frac{\partial V_2(\vec{k})}{\partial k_j} f_j((p(\vec{k}), p), \vec{k})$$

$$= \begin{cases} -\mu k_2 - D(p(\vec{k})) \frac{\partial V_2}{\partial k_1} & \text{if } p > p(\vec{k}) \\ p(\vec{k}) D(p(\vec{k})) - \mu k_2 - D(p(\vec{k})) \frac{\partial V_2}{\partial k_2} & \text{if } p = p(\vec{k}) \\ p D(p) - \mu k_2 - D(p) \frac{\partial V_2}{\partial k_2} & \text{if } p < p(\vec{k}). \end{cases}$$

Using $\frac{\partial V_2}{\partial k_2} = p(\vec{k}) - \frac{\mu k_2}{D(p(\vec{k}))}$, we obtain

$$-\mu k_2 - D(p(\vec{k})) \frac{\partial V_2}{\partial k_1} = D(p(\vec{k})) \left[-p(\vec{k}) + \frac{\partial V_2}{\partial k_2} - \frac{\partial V_2}{\partial k_1} \right] \leq 0,$$

where the inequality follows from condition (i.b) in the statement. Hence, the term in the first case is non-positive. Using $\frac{\partial V_2}{\partial k_2} = p(\vec{k}) - \frac{\mu k_2}{D(p(\vec{k}))}$ again, we obtain that the term in the second case is zero. Finally, we observe that the expression in the third case is negative whenever condition (i.c) holds in the statement. So, equation (21) holds for Firm 2, too.

Second, we consider (k_1, k_2) satisfying $k_1 = k_2 > 0$. For Firm 1, we obtain

$$\begin{aligned} g_1((p, p(\vec{k})), \vec{k}) + \lim_{\kappa \uparrow k_1} \frac{\partial V_1(\kappa, k_2)}{\partial k_1} f_1((p, p(\vec{k})), \vec{k}) + \lim_{\kappa \uparrow k_2} \frac{\partial V_1(k_1, \kappa)}{\partial k_2} f_2((p, p(\vec{k})), \vec{k}) \\ = \begin{cases} -\mu k_1 - D(p(\vec{k})) \lim_{\kappa \uparrow k_2} \frac{\partial V_1(k_1, \kappa)}{\partial k_2} & \text{if } p > p(\vec{k}) \\ p(\vec{k}) D(p(\vec{k})) - \mu k_1 - D(p(\vec{k})) \lim_{\kappa \uparrow k_1} \frac{\partial V_1(\kappa, k_2)}{\partial k_1} & \text{if } p = p(\vec{k}) \\ p D(p) - \mu k_1 - D(p) \lim_{\kappa \uparrow k_1} \frac{\partial V_1(\kappa, k_2)}{\partial k_1} & \text{if } p < p(\vec{k}). \end{cases} \end{aligned}$$

Now, from $\lim_{\kappa \uparrow k_2} \frac{\partial V_1(k_1, \kappa)}{\partial k_2} = -\frac{\mu k_1}{D(p(\vec{k}))}$, the first case is zero. From $\lim_{\kappa \uparrow k_1} \frac{\partial V_1(\kappa, k_2)}{\partial k_1} = p(\vec{k}) - \frac{\mu k_1}{D(p(\vec{k}))}$, the second case is also zero. Consequently, the third case is non-positive if, for $k_1 = k_2 > 0$,

$$p(\vec{k}) = \arg \max_{p \in [0, p(\vec{k})]} D(p) \left(p - p(\vec{k}) + \frac{\mu k_1}{D(p(\vec{k}))} \right).$$

By condition (i.c) above holds for all $k_2 > k_1 > 0$. The objective function is continuous in k_1 , as is the feasible set $(-\infty, p(\vec{k})]$. Moreover, the arg max is unique because $D(p)$ is strictly log-concave on $(-\infty, v]$. By the Berge Maximum theorem, the arg max of the problem is thus continuous in k_1 . Because the problem has a corner solution for all $k_2 > k_1 > 0$ it thus must also have a corner solution when $k_2 = k_1 > 0$, keeping k_2 fixed, as desired.

For Firm 2, we obtain

$$g_2((p, p(\vec{k})), \vec{k}) + \lim_{\kappa \uparrow k_1} \frac{\partial V_2(\kappa, k_2)}{\partial k_1} f_1((p, p(\vec{k})), \vec{k})$$

$$+ \lim_{\kappa \uparrow k_2} \frac{\partial V_2(k_1, \kappa)}{\partial k_2} f_2((p, p(\vec{k})), \vec{k}) = \begin{cases} -\mu k_2 - D(p(\vec{k})) \lim_{\kappa \uparrow k_1} \frac{\partial V_2(\kappa, k_2)}{\partial k_1} & \text{if } p > p(\vec{k}) \\ -\mu k_2 - D(p(\vec{k})) \lim_{\kappa \uparrow k_1} \frac{\partial V_2(\kappa, k_2)}{\partial k_1} & \text{if } p = p(\vec{k}) \\ pD(p) - \mu k_2 - D(p) \lim_{\kappa \uparrow k_2} \frac{\partial V_2(k_1, \kappa)}{\partial k_2} & \text{if } p < p(\vec{k}). \end{cases}$$

From $\lim_{\kappa \uparrow k_1} \frac{\partial V_2(\kappa, k_2)}{\partial k_1} = -\frac{\mu k_2}{D(p(\vec{k}))}$, we obtain that the first two cases are zero. Because $k_1 = k_2$, and $\lim_{\kappa \uparrow k_2} \frac{\partial V_2(k_1, \kappa)}{\partial k_2} = \lim_{\kappa \uparrow k_1} \frac{\partial V_1(\kappa, k_2)}{\partial k_1}$ when $k_1 = k_2$, the third case is identical to the third case for Firm 1, which we have shown to be non-positive.

Third, we verify the monopoly phase, $k_1 > k_2 = 0$ (again, the case $k_2 > k_1 = 0$ is symmetric). We obtain

$$g_1((p, p(\vec{k})), \vec{k}) + \sum_j \frac{\partial V_1(\vec{k})}{\partial k_j} f_j((p, p(\vec{k})), \vec{k}) = \begin{cases} pD(p) - \mu k_1 - D(p) \frac{\partial V_1}{\partial k_1} & \text{if } p > p(\vec{k}) \\ p(\vec{k})D(p(\vec{k})) - \mu k_1 - D(p(\vec{k})) \frac{\partial V_1}{\partial k_1} & \text{if } p = p(\vec{k}) \\ pD(p) - \mu k_1 - D(p) \frac{\partial V_1}{\partial k_1} & \text{if } p < p(\vec{k}). \end{cases}$$

The second case is zero by definition of $V_1(k_1, 0)$. The first and third case requires $pD(p) - \mu k_1 - D(p)[p(\vec{k}) - \mu k_1/D(p(\vec{k}))] \leq 0$, which holds for $\mu k_1 = -\frac{D(p(\vec{k}))^2}{D'(p(\vec{k}))}$ as shown in the proof of Lemma 6.

Step 2: Sufficiency. With these preliminary observations, we are now in a position to show that $p(\vec{k})$ is a best response for any firm when the other firm prices according to $p(\vec{k})$ from any initial condition $\vec{k}_0 \in \mathbb{R}_+^2$.

First consider the conjectured equilibrium pricing function $p(\cdot)$. By assumption, for any $\vec{k}_0$, there is a unique forward inventory path $\vec{k}_s^*$ when both firm price according to $p(\vec{k})$.¹⁸ So, appreciating that $\lim_{\kappa \uparrow k_1} \frac{\partial V_i(\kappa, k_2)}{\partial k_1} = \frac{\partial V_i(k_1, k_2)}{\partial k_1}$ and $\lim_{\kappa \uparrow k_2} \frac{\partial V_i(k_1, \kappa)}{\partial k_2} = \frac{\partial V_i(k_1, k_2)}{\partial k_2}$ whenever $V_i(k_1, k_2)$ is continuously differentiable at (k_1, k_2) , that the inventory dynamics either move downwards or leftwards in the (k_1, k_2) -plane when they are not, and that V_i is continuous, we may integrate both sides of the following equality,

$$0 = g_i((p(\vec{k}_s^*), p(\vec{k}_s^*)), \vec{k}_s^*) + \lim_{\kappa \uparrow k_{1s}^*} \frac{\partial V_i(\kappa, k_{2s}^*)}{\partial k_1} \cdot f_1((p(\vec{k}_s^*), p(\vec{k}_s^*)), \vec{k}_s^*) + \lim_{\kappa \uparrow k_{2s}^*} \frac{\partial V_i(k_{1s}^*, \kappa^*)}{\partial k_2} \cdot f_2((p(\vec{k}_s^*), p(\vec{k}_s^*)), \vec{k}_s^*), \quad (23)$$

to obtain

$$0 = \int_0^\infty g_i((p(\vec{k}_s^*), p(\vec{k}_s^*)), \vec{k}_s^*) ds + \lim_{s \rightarrow \infty} V_i(\vec{k}_s^*) - V_i(\vec{k}_0). \quad (24)$$

¹⁸As discussed in Footnote 9, Online Appendix C provides mild regularity conditions on $p(\cdot)$ under which the forward solution exists and is unique.

Now, we argue that the inventory path $\vec{k}_s^*$ satisfies $\vec{k}_s^* \rightarrow 0$. For the argument, observe that, because inventories are weakly decreasing under any pricing profile, every forward inventory path starting from $\vec{k}_o$ remains in the compact set $[0, k_{1o}] \times [0, k_{2o}]$. Hence, by continuity, V_i is bounded along every such path. So, suppose the inventory path $\vec{k}_s^*$ does not satisfy $\vec{k}_s^* \rightarrow 0$. Then at least one of the firms keeps a strictly positive inventory forever, $k_{it}^* \rightarrow \kappa$ for some $\kappa > 0$. Firm i 's total sales revenue is bounded above by $v k_{io}$, while its carrying costs eventually accrue at rate at least $\mu \kappa$. Hence $\lim_{T \rightarrow \infty} \int_0^T g_i((p(\vec{k}_s^*), p(\vec{k}_s^*)), \vec{k}_s^*) ds = -\infty$ as $T \rightarrow \infty$. This contradicts (24) because the two terms involving the value function V_i are bounded. Consequently, the relevant inventory is depleted, and continuity together with $V_i(0, k_j) = 0$ implies $\lim_{s \rightarrow \infty} V_i(\vec{k}_s^*) = 0$, giving us

$$\int_0^\infty g_i((p(\vec{k}_s^*), p(\vec{k}_s^*)), \vec{k}_s^*) ds = V_i(\vec{k}_o).$$

On the other hand, fix the same initial condition $\vec{k}_o$ but now take any pricing function $\hat{p}(\vec{k}) \neq p(\vec{k})$ such that the resulting pricing profile admits a unique forward solution (cf. Online Appendix C), and denote this solution by $\hat{k}_s = (\hat{k}_{1s}, \hat{k}_{2s})$. Again, $\lim_{\kappa \uparrow k_1} \frac{\partial V_i(\kappa, k_2)}{\partial k_1} = \frac{\partial V_i(k_1, k_2)}{\partial k_1}$ and $\lim_{\kappa \uparrow k_2} \frac{\partial V_i(k_1, \kappa)}{\partial k_2} = \frac{\partial V_i(k_1, k_2)}{\partial k_2}$ whenever $V_i(k_1, k_2)$ is continuously differentiable at (k_1, k_2) and the inventory dynamics either move downwards or leftwards in the (k_1, k_2) -plane when they are not. So, considering Firm 1 (the argument for firm 2 is analogous) and integrating both sides of

$$0 \geq g_1(\hat{p}(\hat{k}_s), p(\hat{k}_s), \hat{k}_s) + \lim_{\kappa \uparrow \hat{k}_{1s}} \frac{\partial V_1(\kappa, \hat{k}_{2s})}{\partial k_1} \cdot f_1((\hat{p}(\hat{k}_s), p(\hat{k}_s)), \hat{k}_s) + \lim_{\kappa \uparrow \hat{k}_{2s}} \frac{\partial V_1(\hat{k}_{1s}, \kappa)}{\partial k_2} \cdot f_2((\hat{p}(\hat{k}_s), p(\hat{k}_s)), \hat{k}_s), \quad (25)$$

then gives us

$$0 \geq \int_0^\infty g_1(\hat{p}(\hat{k}_s), p(\hat{k}_s), \hat{k}_s) ds + \lim_{s \rightarrow \infty} V_1(\hat{k}_s) - V_1(\vec{k}_o).$$

Now, we argue that the inventory path $\hat{k}_{it}$ must satisfy $\hat{k}_{it} \rightarrow 0$ if it were optimal. Suppose not, but $\hat{k}_{it} \rightarrow \kappa$ for some $\kappa > 0$. Then, the flow revenues of firm i converge to zero because we have a bounded choke price. But then, because carrying costs are positive, the firm will incur an infinite loss, which cannot be optimal. This implies $\lim_{s \rightarrow \infty} V_i(\hat{k}_s) = 0$, giving us

$$\int_0^\infty g_1(\hat{p}(\hat{k}_s), p(\hat{k}_s), \hat{k}_s) ds \leq V_1(\vec{k}_o),$$

as desired. $\square$

Proof of Lemma 8. Parts (i) and (ii) follow from Lemma 6 (i.a) and (ii.a) respectively. We now prove part (iii). Consider the value functions defined in (9) and suppose $k_1 > k_2 > 0$. Using $\frac{\partial V_1}{\partial k_2} = -\frac{\mu k_1}{D(p(k_1, k_2))}$, condition (i.a) from Lemma 7 is

$$p - \frac{\mu k_1}{1 - p} - \frac{\partial V_1(k_1, k_2)}{\partial k_1} \leq 0, \forall p < p(k_1, k_2). \quad (26)$$

Next, using $\frac{\partial V_2(k_1, k_2)}{\partial k_1} = \int_0^{k_2} \frac{\partial p(k_1, x)}{\partial k_1} - \frac{\mu x \frac{\partial p(k_1, x)}{\partial k_1}}{[1 - p(k_1, x)]^2} dx$ and $\frac{\partial V_2(k_1, k_2)}{\partial k_2} = p(k_1, k_2) - \frac{\mu k_2}{1 - p(k_1, k_2)}$, condition (i.b) is

$$\begin{aligned} p(k_1, k_2) - \frac{\partial V_2(k_1, k_2)}{\partial k_2} + \frac{\partial V_2(k_1, k_2)}{\partial k_1} &\geq 0 \iff \\ \frac{\mu k_2}{1 - p(k_1, k_2)} + \int_0^{k_2} \frac{\partial p(k_1, x)}{\partial k_1} \left[1 - \frac{\mu x}{[1 - p(k_1, x)]^2} \right] dx &\geq 0, \end{aligned} \quad (27)$$

and condition (i.c) requires

$$p(k_1, k_2) \in \arg \max_{p \in (\infty, p(k_1, k_2)]} D(p) \left(p - p(k_1, k_2) + \frac{\mu k_2}{1 - p(k_1, k_2)} \right). \quad (28)$$

Furthermore, continuity of the value function requires, for all k , $\lim_{k_1 \downarrow k} V_1(k_1, k) = V_1(k, k)$. Using the monopoly pricing function from (i), this can be expressed as

$$k - \frac{4\sqrt{\mu}}{3} k^{3/2} - \int_0^k \frac{\mu k}{1 - p(k, x)} dx = \int_0^k p(k, x) - \frac{\mu x}{1 - p(k, x)} dx.$$

The left and right sides are zero at $k = 0$. Consequently, the derivatives on either side must coincide for all $k \geq 0$,

$$1 - 2\sqrt{\mu k} - p(k, k) - \int_0^k \frac{\partial p(k, x)}{\partial k_1} dx = \mu \int_0^k \frac{1 - p(k, x) + (k - x) \frac{\partial p(k, x)}{\partial k_1}}{[1 - p(k, x)]^2} dx. \quad (29)$$

It is straightforward yet tedious to establish that there is $\beta \in (1, 2)$ such that $p(k_1, k_2) = 1 - \sqrt{\beta \mu (k_1 + k_2)}$ for $k_1, k_2 \in \mathbb{R}_{++}^2$ satisfy the conditions (26), (27), (28), and (29). We provide the computations in Online Appendix E.

Straightforward computation gives that the value functions under the equilibrium price functions are those in (10). Then, continuity of the value function $V_1(k_1, k_2)$ at $k_1 = k_2 = k$ implies

$$\frac{4}{3} + \frac{2(\sqrt{2} - 1)}{\sqrt{\beta}} = \frac{1}{3\sqrt{\beta}} (4\sqrt{2}\beta - 2\beta + 4 - 2\sqrt{2}).$$

Multiplying through by $3\sqrt{\beta}$ yields

$$(2\sqrt{2} - 1)\beta - 2\sqrt{\beta} + (5 - 4\sqrt{2}) = 0, \quad (30)$$

as claimed. $\square$

Proof of Lemma 9. Consider Firm 1 and a symmetric candidate $k_1 = k_2 = k > 0$. Define

$$D_-(k) \equiv -\gamma + \left. \frac{\partial V_1(q, k)}{\partial q} \right|_{q \uparrow k} = 1 - \gamma - \left(\sqrt{2\beta} + \frac{1}{\sqrt{2\beta}} \right) \sqrt{\mu k}$$

and

$$D_+(k) \equiv -\gamma + \left. \frac{\partial V_1(q, k)}{\partial q} \right|_{q \downarrow k} = 1 - \gamma - \left(2 + \frac{5/\sqrt{2} - 3}{\sqrt{\beta}} \right) \sqrt{\mu k}.$$

A symmetric interior best response at k requires

$$D_-(k) \geq 0 \geq D_+(k), \quad (31)$$

and we want to show that this cannot hold. To this end, write $t = \sqrt{\beta}$. Then,

$$D_+(k) - D_-(k) = \left(\sqrt{2}t + \frac{1}{\sqrt{2}t} - 2 - \frac{5/\sqrt{2} - 3}{t} \right) \sqrt{\mu k}.$$

After multiplying by $t > 0$, the coefficient in the brackets becomes

$$\sqrt{2}t^2 - 2t + (3 - 2\sqrt{2}). \quad (32)$$

Now, using (30) from the proof of Lemma 8, one can rewrite (32) as $\sqrt{2}t^2 - 2t + (3 - 2\sqrt{2}) = \frac{(3-\sqrt{2})(3-2t)}{7} > 0$, where the inequality follows from the relevant root t of (30) lying in $(1, \sqrt{2})$ by Claim (iii) in Lemma 8. Hence $D_+(k) > D_-(k)$ for every $k > 0$, contradicting (31). Therefore, no positive symmetric pure-strategy equilibrium exists. $\square$

Proof of Proposition 5. We consider an equilibrium candidate $(k_1^*, k_2^*) = (\ell, h)$ in which Firm 1 chooses ℓ and Firm 2 chooses $h > \ell$. Against a larger rival with inventory h , Firm 1 chooses $q < h$ to maximize $\Pi_1(q; h) = V_1(q, h) - \gamma q$. From (10), the first and second derivatives are

$$\begin{aligned} \Pi_1'(q; h) &= 1 - \gamma - \sqrt{\beta\mu(q+h)} - \frac{\mu q}{\sqrt{\beta\mu(q+h)}}, \\ \Pi_1''(q; h) &= -\frac{\sqrt{\mu}(\beta(h+q) + 2h + q)}{2\sqrt{\beta}(h+q)^{3/2}} < 0. \end{aligned} \quad (33)$$

So the smaller-firm problem is strictly concave on $[0, h]$.

Against a smaller rival with inventory ℓ , Firm 2 chooses $q > \ell$ to maximize $\Pi_2(q; \ell) = V_2(q, \ell) - \gamma q$. The first and second derivatives are

$$\begin{aligned}\Pi_2'(q; \ell) &= 1 - \gamma - \sqrt{\mu} \left[2\sqrt{q} - \frac{3\sqrt{q}}{\sqrt{\beta}} + \frac{q}{\sqrt{\beta}\sqrt{q+\ell}} + \frac{2\sqrt{q+\ell}}{\sqrt{\beta}} \right], \\ \Pi_2''(q; \ell) &= \frac{\sqrt{\mu}}{\sqrt{q}} \left[-1 + \frac{3-4z+z^3}{2\sqrt{\beta}} \right], \quad z \equiv \sqrt{\frac{q}{q+\ell}} \in \left(\frac{1}{\sqrt{2}}, 1 \right).\end{aligned}\tag{34}$$

Since $z \mapsto 3 - 4z + z^3$ is decreasing on $[1/\sqrt{2}, 1]$, $3 - 4z + z^3 \leq 3 - \frac{4}{\sqrt{2}} + \frac{1}{2\sqrt{2}} < 1$, so the bracket is strictly negative. Hence $\Pi_2''(q; \ell) < 0$ for all $q > \ell$, and the larger-firm problem is strictly concave on (ℓ, ∞) .

Next, we establish that the firms' first-order conditions have a unique solution. Let

$$y \equiv \sqrt{1 + \frac{\ell}{h}} \in (1, \sqrt{2}).$$

Then $\ell/h = y^2 - 1$. Writing $t = \sqrt{\beta}$, the smaller firm's first-order condition at $q = \ell$ from (33) becomes

$$1 - \gamma = \sqrt{\mu h} \left(ty + \frac{y^2 - 1}{ty} \right),\tag{35}$$

and the larger firm's first-order condition at $q = h$ from (34) becomes

$$1 - \gamma = \sqrt{\mu h} \left(2 - \frac{3}{t} + \frac{2y^2 + 1}{ty} \right).\tag{36}$$

Equating (35) and (36) yields

$$0 = (t^2 - 1)y^2 + (3 - 2t)y - 2 \equiv Q(y).\tag{37}$$

Because $t > 1$, then $\lim_{y \rightarrow \infty} Q(y) = \infty$. Also, $Q(1) = t^2 - 2t = t(t - 2) < 0$. At $y = \sqrt{2}$, $Q(\sqrt{2}) = 2t^2 - 2\sqrt{2}t - 4 + 3\sqrt{2}$. Using the equation (30) from the proof of Lemma 8 and recalling that $t \in (1, 3/2)$, this can be rewritten as $Q(\sqrt{2}) = \frac{(3\sqrt{2}-2)(3-2t)}{7} > 0$. Hence $Q(y)$ has a root in $(1, \sqrt{2})$. Since the constant term is $-2 < 0$, the product of the two roots is negative, so exactly one root is positive. Therefore the root in $(1, \sqrt{2})$ is unique. So, from (35) together with $\ell/h = y^2 - 1$ we obtain that there are unique $\ell^* < h^*$ solving the two first-order conditions.

To finish, it remains to show that ℓ^* is the global best response to h^* and that h^* is the global best response to ℓ^* . Because $\Pi_1(q; h^*)$ is strictly concave on $[0, h^*)$ and $\Pi_2(q; \ell^*)$ is strictly concave on (ℓ^*, ∞) , ℓ^* is the unique maximizer on the side $q < h^*$, and h^* is the unique maximizer on the side $q > \ell^*$.

We first rule out upward deviations by the smaller firm (Firm 1). By continuity and symmetry of the value functions, it is enough to study $\Pi_2(q; h^*)$ for $q \geq h^*$. Since that

branch is strictly concave, it suffices to check the one-sided derivative at the boundary $q \downarrow h^*$. For the following, let $A^* \equiv ty^* + \frac{(y^*)^2-1}{ty^*}$, where $Q(y^*) = 0$ in (37). Using the larger-firm first-order condition (36),

$$\frac{\Pi'_2(h^*; h^*)}{\sqrt{\mu h^*}} = A^* - \left(2 + \frac{5/\sqrt{2} - 3}{t}\right) = \frac{2y^* + 1/y^* - 5/\sqrt{2}}{t}.$$

Since $y^* \in (1, \sqrt{2})$ and $y \mapsto 2y + 1/y$ is increasing for $y > 1$, $2y^* + \frac{1}{y^*} < 2\sqrt{2} + \frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}}$. Therefore $\Pi'_2(h^*; h^*) < 0$. So the payoff is decreasing on the entire side $q \geq h^*$, and the smaller firm's global best response to h^* is ℓ^* .

Next, we rule out downward deviations by the larger firm (Firm 2). By continuity and symmetry of the value functions, it is enough to study $\Pi_1(q; \ell^*)$ for $q \leq \ell^*$. Since that branch is strictly concave, it suffices to check the one-sided derivative at the boundary $q \uparrow \ell^*$. Using the smaller-firm first-order condition (35),

$$\frac{\Pi'_1(\ell^*; \ell^*)}{\sqrt{\mu h^*}} = A^* - \left(\sqrt{2}t + \frac{1}{\sqrt{2}t}\right) \sqrt{(y^*)^2 - 1}.$$

To sign this expression, square both sides. A direct simplification gives

$$(A^*)^2 - \left(\sqrt{2}t + \frac{1}{\sqrt{2}t}\right)^2 ((y^*)^2 - 1) = \frac{(2 - (y^*)^2)((2t^4 - 1)(y^*)^2 + 1)}{2t^2(y^*)^2} > 0,$$

because $(y^*)^2 < 2$ and $t > 1$. Hence $\Pi'_1(\ell^*; \ell^*) > 0$. So the payoff is increasing on the entire side $q \leq \ell^*$, and the larger firm's global best response to ℓ^* is h^* .

Thus $(k_1^*, k_2^*) = (\ell^*, h^*)$ corresponds to a pure-strategy equilibrium. By symmetry of the underlying game, $(k_1^*, k_2^*) = (h^*, \ell^*)$ is also an equilibrium. $\square$

Proof of Proposition 6. Let total duopoly inventory in the asymmetric equilibrium be $K^D \equiv h^* + \ell^*$, where h^* and ℓ^* are the equilibrium inventories determined in the proof of Proposition 5. Recall that $\ell^*/h^* = (y^*)^2 - 1$, where y^* is also defined in that proof. So, we have $K^D = (y^*)^2 h^*$. For the following, it is convenient to define $B^* \equiv \frac{A^*}{y^*}$. Using the definition of A^* and recalling $t = \sqrt{\beta}$, we have $B^* = t + \frac{(y^*)^2-1}{t(y^*)^2} = t + \frac{1-1/(y^*)^2}{t}$. Moreover, using the defining equation for y^* , $(t^2 - 1)(y^*)^2 + (3 - 2t)y^* - 2 = 0$, this can be rewritten as $B^* = \frac{2}{y^*} + \frac{2(y^*-1/y^*)}{t}$. Hence, using (35), $K^D = \frac{(1-\gamma)^2}{\mu(B^*)^2}$. Maximizing $V_{SP}(K) - \gamma K$ and $V_M(K) - \gamma K$ we get $K^{SP} = \frac{(1-\gamma)^2}{2\mu}$, and $K^M = \frac{(1-\gamma)^2}{4\mu}$.

So it suffices to show $\sqrt{2} < B^* < 2$. Using $B^* = \frac{2}{y^*} + \frac{2(y^*-1/y^*)}{t}$, we have $2/y^* > \sqrt{2}$ because $y^* < \sqrt{2}$, and the second term is positive because $y^* > 1$ (cf. the proof of Lemma 8). Hence $B^* > \sqrt{2}$. Using instead $B^* = t + \frac{1-1/(y^*)^2}{t}$, and the fact that $(y^*)^2 < 2$, we obtain $1 - \frac{1}{(y^*)^2} < \frac{1}{2}$, so $B^* < t + \frac{1}{2t}$. From the proof of Lemma 9, we know that $t \in (1, 3/2)$, and therefore $B^* < \frac{3}{2} + \frac{1}{2} = 2$. Combining the two bounds yields $\sqrt{2} < B^* < 2$. Thus $K^{SP} > K^D > K^M$, as desired. $\square$

Proof of Proposition 7. Let $h \geq \ell$ denote the larger and smaller inventory requirements, and write $K = h + \ell$. Under the ABD, the firm with inventory ℓ sells first while the other firm waits. Let $p^D(x) = 1 - \sqrt{\beta\mu x}$ be the duopoly price. Writing $t = \sqrt{\beta}$, net social surplus from the duopoly disruption path is

$$\begin{aligned}\mathcal{W}(h, \ell) &= -\gamma K + \int_0^h \left[\int_0^{1-p^M(x)} [1-y] dy \right] \frac{1}{1-p^M(x)} dx + \int_h^{h+\ell} \left[\int_0^{1-p^D(x)} [1-y] dy \right] \frac{1}{1-p^D(x)} dx \\ &= -\gamma K + \int_0^h \left[1 - \frac{3}{2}\sqrt{\mu x} \right] dx + \int_h^{h+\ell} \left[1 - \left(\frac{t}{2} + \frac{1}{t} \right) \sqrt{\mu x} \right] dx \\ &= -\gamma K + h + \ell - \sqrt{\mu} \left\{ h^{3/2} + \frac{2}{3} \left(\frac{t}{2} + \frac{1}{t} \right) [(h+\ell)^{3/2} - h^{3/2}] \right\} \\ &= (1-\gamma)K - \sqrt{\mu} [a_D K^{3/2} + (1-a_D)h^{3/2}].\end{aligned}$$

We first show that the optimal requirement is symmetric. Fix aggregate inventory K and write $h = \theta K$, where $\theta \in [\frac{1}{2}, 1]$. Then

$$\mathcal{W}(h, \ell) = (1-\gamma)K - \sqrt{\mu}K^{3/2} [a_D + (1-a_D)\theta^{3/2}].$$

Because $t \in (1, \sqrt{2})$, $a_D = \frac{1}{3}(t + \frac{2}{t}) < 1$. Hence $1 - a_D > 0$, and so the coefficient $a_D + (1-a_D)\theta^{3/2}$ is strictly increasing in θ . For a fixed aggregate requirement K , social surplus is therefore maximized by choosing the smallest feasible θ , namely $\theta = \frac{1}{2}$. Thus the optimal requirement is symmetric: $h = \ell = \frac{K}{2}$.

Now, substituting $h = K/2$ gives net social surplus

$$\mathcal{W}^R(K) = (1-\gamma)K - C_R \sqrt{\mu} K^{3/2},$$

where we define $C_R = a_D + (1-a_D)2^{-3/2}$ for further reference. This objective is strictly concave in $K > 0$, and the first-order condition is $1-\gamma = \frac{3}{2}C_R\sqrt{\mu K}$. Solving gives $K^R = \frac{4(1-\gamma)^2}{9C_R^2\mu}$.

It remains to compare K^R with the first-best planner inventory K^{SP} and the unregulated duopoly inventory K^D . Consider $K^{SP} = \frac{(1-\gamma)^2}{2\mu}$ first. Since $t \in (1, \sqrt{2})$, the function $t + \frac{2}{t}$ is strictly decreasing on this interval and satisfies $t + \frac{2}{t} > 2\sqrt{2}$. Therefore $a_D = \frac{1}{3}(t + \frac{2}{t}) > \frac{2\sqrt{2}}{3}$. Also, since $a_D < 1$, it holds that $C_R = a_D + (1-a_D)2^{-3/2} > a_D > \frac{2\sqrt{2}}{3}$. Hence $9C_R^2 > 8$, which implies $K^R = \frac{4(1-\gamma)^2}{9C_R^2\mu} < \frac{4(1-\gamma)^2}{8\mu} = \frac{(1-\gamma)^2}{2\mu} = K^{SP}$.

Next, we turn to the unregulated duopoly and show $K^D < K^R$. Recall from Proposition 6 that $K^D = \frac{(1-\gamma)^2}{\mu(B^*)^2}$, where $B^* = t + \frac{1-1/(y^*)^2}{t}$ and $y^* \in (1, \sqrt{2})$ solves

$$(t^2 - 1)(y^*)^2 + (3 - 2t)y^* - 2 = 0. \quad (38)$$

Since $a_D < 1$ and $2^{-3/2} < 1$, we have $C_R = a_D + (1-a_D)2^{-3/2} < 1$. It is therefore

enough to show that $B^* > 3/2$. First, $y^* > \frac{5}{4}$. Indeed, evaluating the defining quadratic (38) at $y = 5/4$ gives $(t^2 - 1)\frac{25}{16} + (3 - 2t)\frac{5}{4} - 2 = \frac{25t^2 - 40t + 3}{16} < 0$ for all $t \in (1, \sqrt{2})$. Since the quadratic is strictly convex and y^* is its positive root, this implies $y^* > 5/4$. Hence, $1 - \frac{1}{(y^*)^2} > 1 - \frac{16}{25} = \frac{9}{25}$. Moreover, $y^* < \sqrt{2}$ implies $t > 13/10$. To see this, note that the defining quadratic (38) evaluated at $\sqrt{2}$ is $2t^2 - 4 + \sqrt{2}(3 - 2t)$, which is strictly increasing in t on $(1, \sqrt{2})$ and is negative at $t = 13/10$. Since $y^* < \sqrt{2}$, this expression must be positive, and therefore $t > 13/10$. Finally, using also $t < \sqrt{2}$, we obtain $B^* = t + \frac{1 - 1/(y^*)^2}{t} > \frac{13}{10} + \frac{9}{25\sqrt{2}} > \frac{3}{2}$, as desired. $\square$

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