Demographics, Wealth, and Global Imbalances in the Twenty-First Century

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Abstract

We use a sufficient statistic approach to quantify the general equilibrium effects of population aging on the returns to wealth, wealth accumulation, and global imbalances. Combining population forecasts with household survey data from 25 countries, we measure the *compositional effect* of aging: how a changing age distribution affects wealth-to-GDP, holding the age profiles of wealth and labor income fixed. In a baseline overlapping generations model this statistic, in conjunction with cross-sectional information and two standard macro parameters, pins down general equilibrium outcomes. Since the compositional effect is positive, large, and heterogeneous across countries, our model predicts that population aging will lower returns to wealth, increase wealth-to-GDP ratios, and widen global imbalances through the twenty-first century. These conclusions extend to a richer model in which bequests, individual savings, and the tax-and-transfer system all respond to demographic change.

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1 Introduction

The world is experiencing rapid demographic change. The share of the world population above 50 years of age has increased from 15% to 25% since the 1950s, and it is expected to rise further to 40% by the end of the twenty-first century (figure 1, panel A). There is a widespread view that this aging process has been an important driver of three key macroeconomic trends to date. According to this view, an aging population saves more, helping to explain why average returns on private wealth have fallen and private wealth-to-GDP ratios have risen (panels B and C).[1]

Insofar as this mechanism is heterogeneous across countries, it can further explain the rise of global imbalances (panel D).

Beyond this qualitative consensus lies substantial disagreement about magnitudes. For instance, structural estimates of the effect of demographics on interest rates over the 1970–2015 period range from a moderate decline of less than 1 percentage point (pp, Gagnon, Johannsen and López-Salido 2021) to a large decline of over 3 pp (Eggertsson, Mehrotra and Robbins 2019).[2] Turning to predictions for the future, economists are starkly divided about the direction of the effect. Some structural models predict falling interest rates going forward (see, e.g., Gagnon et al. 2021, Papetti 2021a). At the same time, an influential hypothesis argues, based on the dissaving of the elderly, that aging will eventually push savings rates down and interest rates back up. This argument, popular in the 1990s as the "asset market meltdown" hypothesis (Poterba 2001, Abel 2001), was recently revived under the name "great demographic reversal" (Goodhart and Pradhan 2020). It is central to Larry Summers's recent view that interest rates will be persistently high going forward (as quoted in Rubin 2023):

> Once people have aged and they're retiring, then they draw down their savings and spend. And so I think we're making a transition from more saving because of aging, to less saving because aging has happened.

Our paper refutes this argument and shows that, instead, demographics will continue to push strongly in the same direction, leading to falling rates of return and rising wealth-to-GDP ratios. The key to our results is the *compositional effect* of an aging population:

[1] Our measure of the average return on wealth in panel B is designed to capture the total payments to private wealth owners, divided by total private wealth. We will often refer to this measure as the "interest rate". Appendix A.1 describes the details of the calculation, and explains why we prefer it over other measures in the literature that suggest a stable or (since 1970) rising return to capital, as depicted in figure A.1C. As is well known, safe rates of return have also fallen, though their fall is most pronounced since the 1980s. Appendix D.3 provides a model in which demographic change influences the spread between average and safe returns.

[2] Appendix G presents a selective summary of findings in the literature and shows how to interpret them through the lens of this paper's framework.

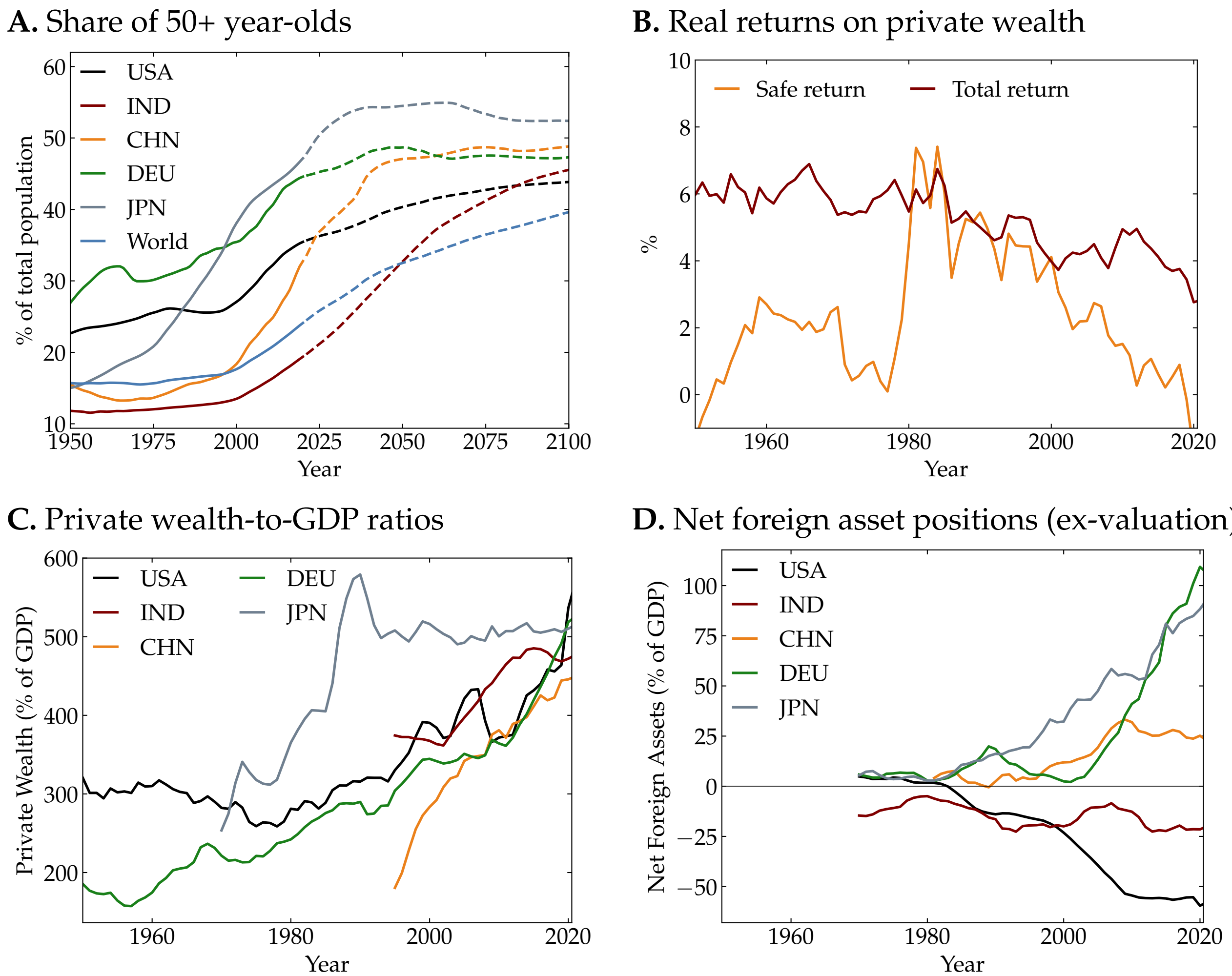

Figure 1: Demographics, interest rates, wealth, and global imbalances

Notes: Panel A presents the share of 50+ year-olds in the five largest economies by GDP and the world as a whole (source: 2019 UN World Population Prospects, the projection is the central scenario). Panel B presents a measure of the US total return on private wealth (red line) and of the US safe rate of return (orange line). Details on the construction of these series are in appendix A.1, with figure A.1C showing an alternative series that measures the return on capital (rather than wealth), which has increased since 1970. Panel C presents private wealth-to-GDP ratios (source: World Inequality Database). Panel D presents net foreign asset positions over GDP, constructed exclusive of valuation effects by cumulating the current account series from Lane and Milesi-Ferretti (2018), as explained in appendix A.2.

the direct impact of the changing age distribution on log wealth-to-GDP, holding the age profiles of wealth and labor income fixed. In a baseline overlapping generations (OLG) model, this is a sufficient statistic for the actual change in wealth-to-GDP for a small open economy. Further, for a world economy, the compositional effect—when aggregated across countries, and combined with elasticities of asset supply and demand that we obtain with other sufficient statistic formulas—fully pins down the general equilibrium effect on returns, wealth-to-GDP, and global imbalances.

We measure the compositional effect by combining population forecasts with household survey data from 25 countries over the period 2016–2100. We find that it is positive and large everywhere, but also heterogeneous, ranging from 17 log points in Sweden to 56 in India, with a global wealth-weighted average of 32 log points. Behind these numbers lies a key feature of age-wealth profiles: the old hold much more wealth than the young, and on average do not dissave much as they age. This feature, which is remarkably robust across countries and time, interacts with large and heterogeneous increases in the old-age share to produce our compositional effects.

Since the average effect is positive and large, our model predicts that there will be no great demographic reversal: through the twenty-first century, population aging will continue to push down global rates of return, with our central estimate being -1.07pp. Heterogeneity in aging across countries also implies large global imbalances: for instance, we find that India's net foreign asset position will grow until it reaches 179% of GDP in 2100, while the United States's and Germany's net foreign asset positions will decline to absorb this asset demand. Finally, global wealth-to-GDP will rise, but only by 8.9 log points (from 456% to 498% of world GDP), since declining returns attenuate the equilibrium response.

Our sufficient statistic framework offers a transparent way to compute the effect of a changing age distribution on key macroeconomic variables. General equilibrium outcomes can be obtained with limited information: other than the data required to compute the compositional effect, we only need data on macro aggregates and assumptions on two standard parameters, the elasticity of intertemporal substitution and the elasticity of substitution between capital and labor. Our framework also clarifies a key limitation of the great demographic reversal hypothesis, which focuses on the decline in one flow (savings) when another (investment) is also declining due to demographic change. In contrast, the compositional effect on stocks (rising wealth-to-GDP) unambiguously implies a falling rate of return.

Our baseline model allows for a broad range of savings motives, but rules out some potentially important mechanisms, such as the effects of rising life expectancy on individual savings. To evaluate these, we numerically simulate a richer structural model of demographic change. We find that the results are always the same qualitatively, and—aside from extreme fiscal adjustments—also close quantitatively to our baseline. Interestingly, we do not find that individuals save much more in anticipation of a longer life, even though they need to finance more old-age consumption. The reason is that to match limited dissaving in old age, most old-age assets must be held for reasons other than personal consumption—which the model captures with a bequest motive.

Existing literature has followed two broad approaches, which our paper combines, to quantify the impact of demographic change on macroeconomic outcomes. The first is reduced-form. One branch of this literature, following Mankiw and Weil (1989) and Poterba (2001), computes the effect of a changing age distribution over fixed asset profiles.[3] Another branch, following Cutler, Poterba, Sheiner and Summers (1990) and the "demographic dividend" literature (Bloom, Canning and Sevilla 2003), computes the effect of changing age distributions over fixed income profiles.[4] These "shift-share" calculations are very intuitive, but are not tied to specific general equilibrium counterfactuals. We show that a ratio of two such shift-shares is the driver of equilibrium outcomes in a fully specified OLG model.

The alternative approach is structural, relying on quantitative general equilibrium OLG models. This tradition, which originated in Auerbach and Kotlikoff (1987), has tackled effects of demographics on aggregate wealth accumulation,[5] asset returns,[6] and international capital flows.[7] We show how to decompose these quantitative results, tracing them back to key moments in the data. This helps to identify the source of conflicting estimates: for instance, the compositional effect in Gagnon et al. (2021) is about the same as in the data, while that in Eggertsson et al. (2019) is about triple that in the data. Relative to this literature, we also find a more central role for composition, and a smaller savings response to longevity. This distinction matters going forward, since a large share of population aging will come from falling fertility rather than longevity.

In this paper, we focus on the causal effect of demographic change in the twenty-first century. We do not explain the sources of this change; instead, we take demographic projections as given. We also rule out some indirect effects of aging, such as changes in technology or market structure.[8] In our main model, we assume that the rapidly growing gap between wealth and measured capital, associated with a rising return to measured capital

[3]There is also a tradition that computes the effect of changing age distributions over fixed age profiles of savings *rates* (Summers and Carroll 1987, Auerbach and Kotlikoff 1990, Bosworth, Burtless and Sabelhaus 1991, Mian, Straub and Sufi 2021). We relate to this literature and discuss its limitations in section 5. Jaimovich and Siu (2009) also explore the effect of changing age distributions on business cycle volatility.

[4]This accounting-based approach to aging has been systematized in the literature and practice of National Transfer Accounts (see, e.g., Lee and Mason 2011).

[5]E.g., İmrohoroğlu, İmrohoroğlu and Joines (1995), Kotlikoff, Smetters and Walliser (1999), De Nardi, Imrohoroğlu and Sargent (2001), and Kitao (2014).

[6]E.g., Abel (2003), Geanakoplos, Magill and Quinzii (2004), Carvalho, Ferrero and Nechio (2016), Eggertsson et al. (2019), Lisack, Sajedi and Thwaites (2021), Jones (2023), Papetti (2021a), Rachel and Summers (2019), Kopecky and Taylor (2022), Antunes and Ercolani (2020), Gagnon et al. (2021), and Peruffo and Platzer (2024).

[7]E.g. Henriksen (2002), Börsch-Supan, Ludwig and Winter (2006), Domeij and Flodén (2006), Attanasio, Kitao and Violante (2006), Attanasio, Kitao and Violante (2007), Krueger and Ludwig (2007), Backus, Cooley and Henriksen (2014), Papetti (2021b), Bonfatti, İmrohoroğlu and Kitao (2022), and Sposi (2022).

[8]For the effects of demographics on TFP, see the debate between Maestas, Mullen and Powell (2023) and Acemoglu and Restrepo (2017) for the effect of aging, and Jones (2022) for the effect of slower population growth. For models in which demographics can affect markups via either the structure of consumer demand or firm entry incentives, see Bornstein (2021) vs. Peters and Walsh (2022).

in recent years (see Gomme, Ravikumar and Rupert 2015), is the result of unmeasured capital, although in an extension we allow for rising markups.[9] Finally, although our baseline exercise holds government debt-to-GDP policy fixed, we show how rising government debt can mitigate or even undo the effect of demographic change on real interest rates, while increasing the effect on wealth-to-GDP.[10]

The compositional effects we identify are large in the past as well as in the future. This suggests that demographic change has been a key historical driver of macro trends. Indeed, according to our model, it can explain roughly half the decline in interest rates since 1950, as well as a smaller share of the rise in wealth-to-GDP. To fully rationalize both time series, substantial upward shifts in both asset demand and supply are needed. We calculate that demographic change was a primary driver of the shift in asset demand, accounting for 30% of the overall increase in our central case; the remainder is consistent with a large literature documenting other forces that pushed up asset demand and supply.[11]

The paper proceeds as follows. In section 2, we describe our baseline model, define the compositional effect, and prove our main sufficient statistic results. In section 3, we turn to measurement, documenting compositional effects across countries and calculating their general equilibrium implications. In section 4, we extend the baseline model to capture additional macroeconomic effects of population aging and show that the results from section 3 are a close fit in nearly all cases. Finally, in section 5 we show why the great demographic reversal hypothesis's focus on savings rates is misleading.

2 The compositional effect of demographics

In this section, we set up a benchmark life-cycle model with overlapping generations to study the effects of demographic change. We derive two main theoretical results. First, in a small open economy, demographic change only affects macroeconomic aggregates by changing the age composition of the population. Given a demographic projection, these *compositional effects* can be calculated using data from a single cross section. Second, in an integrated world economy, the long-run effects of demographic change on wealth accumulation, interest rates, and global imbalances can be obtained by simply combining these compositional effects with macroeconomic aggregates, other cross-sectional statistics, and

[9] See, e.g., Eggertsson, Robbins and Wold (2021) on the effects of rising markups.

[10] See, e.g., Auclert, Malmberg, Rognlie and Straub (2025) for the effects of demographics on the supply of government debt.

[11] These forces include, on the demand side, falling TFP growth and rising inequality, and on the supply side, the rise of automation, intangible capital, housing, and markups. See, for instance, McGrattan and Prescott (2010), Rognlie (2015), Rachel and Smith (2017), Auclert and Rognlie (2018), Eggertsson et al. (2019), Straub (2019), Eggertsson et al. (2021), and Moll, Rachel and Restrepo (2022).

assumptions about two primitive elasticities.

2.1 Environment

Our environment is a world economy with overlapping generations of heterogeneous individuals. Time is discrete and runs from $t = 0$ to ∞, agents have perfect foresight, and capital markets are integrated. All assets share the same global return; other variables and parameters can vary across countries c.[12] We drop indices c unless there is risk of ambiguity.

Individuals. At each time t, a country has a population $N_t = \sum_j N_{jt}$ growing at rate $1 + n_t \equiv N_t / N_{t-1}$, with N_{jt} being the number of individuals of age j. Each individual faces an exogenous probability ϕ_j of surviving from age j to age $j+1$, so the probability of surviving from birth to age j is $\Phi_j \equiv \prod_{k=0}^{j-1} \phi_k$. The maximal lifespan is J, so that $\phi_J = 0$. We assume that this survival profile is constant over time, and that there is no migration (both assumptions are relaxed in section 4). Hence, the age distribution, $\pi_{jt} \equiv \frac{N_{jt}}{N_t}$, only varies over time due to changes in fertility and convergence dynamics.[13]

Individuals supply labor exogenously, face idiosyncratic income risk, and can partially self-insure and smooth income over their life cycle by saving in an actuarially fair annuitized asset, which has a purchase price equal to the survival probability ϕ_j and pays out $(1 + r_t)$ in case of survival.[14] Their effective labor supply is $\ell(z_j)$, where z_j is a stochastic process, and unless stated otherwise, all individual variables at age j are a function of the whole history of the idiosyncratic shocks z_j, which we denote z^j. The stochastic process z_j can be different across countries, and is arbitrary subject to being the same across cohorts, meaning that over time, individuals of the same age have identical distributions across histories regardless of their birth year.

Individuals with birth year k choose sequences of consumption $\mathfrak{c}_{jt}$ and assets $\mathfrak{a}_{j+1,t+1}$

[12] Appendix D.3 considers an extension that features a spread between safe and risky asset returns.

[13] Convergence dynamics for demographics are sometimes called "momentum". Appendix B.1 shows that, in most countries, fertility and momentum together account for the majority of population aging during their demographic transitions.

[14] One implication of this setup is that mortality does not show up in the Euler equation, since the utility shifter coming from survival is canceled by the price of the annuitized asset.

for all ages $j = 0, \ldots, J$ (with $t = j + k$) to solve the utility maximization problem

$$\begin{aligned}
&\max_{\{\mathfrak{c}_{jt}, \mathfrak{a}_{j+1,t+1}\}} \mathbb{E}_k \left[\sum_{j=0}^{J} \beta_j \Phi_j \frac{\mathfrak{c}_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right] \\
\text{s.t.} \quad & \mathfrak{c}_{jt} + \phi_j \mathfrak{a}_{j+1,t+1} \leq w_t \left((1-\tau)\ell_j(z_j) + tr(z^j) \right) + (1+r_t)\mathfrak{a}_{jt} \\
& \mathfrak{a}_{j+1,t+1} \geq -\bar{a} w_t,
\end{aligned} \tag{1}$$

where $\bar{a}$ is a borrowing constraint, w_t is the real wage per efficiency unit of labor at time t, r_t is the return on wealth, τ is the labor tax rate, and $tr(z^j)$ denotes transfers from the government, including wage-indexed social insurance and retirement transfers, for agents of age j with a history z^j.[15] The utility weight at age j is $\beta_j \Phi_j$, combining the survival probability Φ_j and an arbitrary age-specific utility shifter β_j. Deviations from exponential discounting ($\beta_j = \beta^j$ for some β) stand in for age-dependent factors that affect the marginal utility of consumption, such as health status or the presence of children. Hence, this model can capture many of the factors that the literature considers essential to understand savings: agents save for life-cycle reasons, for self-insurance reasons, to cover future health costs, and to provide for their children.[16]

The total wealth held by individuals of age j is the product of N_{jt} and the average wealth at age j, $a_{jt} \equiv \mathbb{E}\mathfrak{a}_{jt}$. Aggregate (private) wealth W_t is the sum across age groups:

$$W_t \equiv \sum_{j=0}^{J} N_{jt} a_{jt}. \tag{2}$$

Production. There is a single good used for private consumption, government consumption, and investment. Final output Y_t of this good is produced competitively from physical capital K_t and effective labor input L_t according to an aggregate production function F

$$Y_t = F(K_t, Z_t L_t),$$

where $Z_t \equiv Z_0(1+\gamma)^t$ captures labor-augmenting technological progress. We assume that F has constant returns to scale and diminishing returns to each factor. Effective labor input

[15] We use $\mathfrak{c}_{jt}$ and $\mathfrak{a}_{jt}$ to denote consumption and assets at the individual level, and will reserve ordinary letters for the cross-sectional averages $c_{jt} \equiv \mathbb{E}\mathfrak{c}_{jt}$ and $a_{jt} \equiv \mathbb{E}\mathfrak{a}_{jt}$ by age.

[16] We assume that children live with one of their parents, whose consumption at age j includes that of the children they care for. Formally, we set $\beta_j = \ell_j(z_j) = tr(z^j) = 0$ when $j < J^w$, for a J^w that denotes the start of working life independent from parents. Given this assumption, children do not consume or accumulate assets until age J^w.

L_t is a standard linear aggregator

$$L_t = \sum_{j=0}^{J} N_{jt} \bar{\ell}_j, \tag{3}$$

where $\bar{\ell}_j$ denotes average effective labor input per person of age j, capturing variation in experience and hours of work over the life cycle. Capital has a law of motion $K_{t+1} = (1-\delta)K_t + I_t$ where I_t is aggregate investment, and factor prices equal marginal products. The net rental rate of capital is $r_t = F_K\left(K_t/(Z_t L_t), 1\right) - \delta$, and the wage per efficiency unit of labor is $w_t = Z_t F_L\left(K_t/(Z_t L_t), 1\right)$.

We write $g_t \equiv Y_t/Y_{t-1} - 1$ for the growth rate of the economy. If r_t is constant and labor supply L_t grows at a constant rate n, then $g_t = (1+\gamma)(1+n) - 1$. Otherwise, g_t also reflects changes in capital intensity and in the labor force growth rate.

Government. The government purchases G_t goods, maintains a constant tax rate on labor income τ, gives individuals state-contingent transfers $tr(z^j)$ indexed to current wages w_t, and finances itself using a risk-free bond with real interest rate r_t. It faces the flow budget constraint

$$G_t + w_t \sum_{j=0}^{J} N_{jt} \mathbb{E}_j[tr(z^j)] + (1+r_t)B_t = \tau w_t \sum_{j=0}^{J} N_{jt} \bar{\ell}_j + B_{t+1}, \tag{4}$$

where a positive B_t denotes government borrowing. When demographic change disturbs the balance of aggregate tax receipts and expenditures, the government adjusts G_t to ensure that the debt-to-output ratio $\frac{B}{Y}$ remains fixed.

Equilibrium. Given demographics, government policy, an initial distribution of assets, and initial levels of bonds and capital across countries such that $F_K - \delta$ is equal to r_0 in each country, an *equilibrium* is a sequence of returns $\{r_t\}$ and country-level allocations such that, in each country, individuals optimize, firms optimize, and asset demand from individuals equals asset supply from firms and governments,

$$\sum_c W_t^c = \sum_c (K_t^c + B_t^c).$$

Dividing by world GDP Y_t, the above expression can be written as

$$\sum_c \frac{Y_t^c}{Y_t} \frac{W_t^c}{Y_t^c} = \sum_c \frac{Y_t^c}{Y_t} \left[\frac{K_t^c}{Y_t^c} + \frac{B_t^c}{Y_t^c} \right]. \tag{5}$$

Defining a country's net foreign asset position as the excess of wealth over capital and bonds, $NFA_t^c \equiv W_t^c - (K_t^c + B_t^c)$, (5) states that the average NFA-to-GDP ratio is zero, when countries are weighted by their GDP.

2.2 A small economy aging alone

We first study a small open economy undergoing demographic change, while all other countries have constant demographic parameters. In this case, the economy faces a global rate of return r which is exogenous and fixed—exogenous because the economy is small, and fixed since all other countries have fixed demography. This can be seen as the limit case when the economy has an arbitrarily small world GDP weight $\frac{Y_t^c}{Y_t}$, so that its demand and supply of assets do not affect the world equilibrium condition (5).[17] By studying this case, we can analyze how demographics affect macroeconomic aggregates *directly*, independent of any effects operating through equilibrium adjustments in returns r_t.

Focusing on wealth, our key finding is that demographic change does not affect the distribution of assets within age groups, only the distribution of people across age groups. Intuitively, the economy converges to a "balanced growth path by age", where each age group has a stable asset distribution growing at the same rate as technology.

Lemma 1. *In a small open economy facing a fixed interest rate r, the distribution of normalized asset holdings by age $\mathfrak{a}_{jt}/Z_t$ converges to an invariant distribution that depends on j and r, but not on the economy's age distribution. If normalized asset holdings start at this invariant distribution, there exists a time-invariant function $a_j(r)$ such that average asset holdings by age satisfy*

$$\frac{a_{jt}}{Z_t} = a_j(r), \quad \forall t. \tag{6}$$

Proof. See appendix B.2. □

The lemma follows since demographic change does not affect the parameters of individuals' life-cycle problems, once these problems are normalized by productivity. Hence, normalized savings decisions are identical as a function of age, income state, and asset holdings. Thus, beyond the initial cohorts, all cohorts have the same distribution of normalized assets. Further, if initial cohorts start at the invariant distribution, which we assume from now on, the result holds for each t, not only in the limit.

Given lemma 1, aggregate wealth per person satisfies

$$\frac{W_t}{N_t} = \sum_j \pi_{jt} a_{jt} = (1+\gamma)^t \sum_j \pi_{jt} a_{j0} \tag{7}$$

[17]To obtain a fixed interest rate, we assume that all other countries $c' \neq c$ are in demographic steady state given a set of mortality profiles $\phi_j^{c'}$ and a common growth rate of newborns n, where the constant growth rate ensures that countries preserve their relative size over time.

Wealth per person changes with the age composition π_{jt} of the population, and otherwise grows at the technological growth rate $1+\gamma$.

We next derive output per person. A constant global r implies a constant ratio of capital to effective labor $k(r)$, defined by $F_K(k(r),1) = r+\delta$. Aggregate output is then $Y_t = Z_t L_t F(k(r),1)$, where, from (3), aggregate effective labor is $L_t = N_t \sum_j \pi_{jt}\bar{\ell}_j$. Hence

$$\frac{Y_t}{N_t} = Z_t F(k(r),1)\sum_j \pi_{jt}\bar{\ell}_j = \frac{F(k(r),1)}{F_L(k(r),1)}(1+\gamma)^t \sum_j \pi_{jt}h_{j0} \tag{8}$$

where $h_{j0} = Z_0 F_L \bar{\ell}_j = w_0\bar{\ell}_j$ is equal to average labor earnings of individuals of age j, and we have used the fact that the initial wage is $w_0 = Z_0 F_L(k(r),1)$.

Taking the ratio of (7) and (8), we find that W_t/Y_t is proportional to the ratio of $\sum_j \pi_{jt}a_{j0}$ and $\sum_j \pi_{jt}h_{j0}$. The following proposition summarizes this result.

Proposition 1. *Consider a small open economy facing a constant r, with asset holdings starting at the invariant distribution associated with r. Then, wealth-to-GDP satisfies*

$$\frac{W_t}{Y_t} \propto \frac{\sum \pi_{jt}a_{j0}}{\sum \pi_{jt}h_{j0}} \qquad \forall t, \tag{9}$$

where $h_{j0} \equiv w_0\bar{\ell}_j$ is average pre-tax labor income by age, and $a_{j0} \equiv \mathbb{E}\mathfrak{a}_{j0}$ is average asset holdings by age, in year 0. The change in the wealth-to-GDP ratio is the same as the change in net foreign asset-to-GDP ratio: $W_t/Y_t - W_0/Y_0 = NFA_t/Y_t - NFA_0/Y_0$.

The proposition implies that all changes in W_t/Y_t reflect the changing age composition π_{jt} of the population, given fixed age profiles a_{j0} and h_{j0}. Equation (9) implies that the log change in wealth-to-GDP between year 0 and year t is given by

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_0}{Y_0}\right) = \log\left(\frac{\sum \pi_{jt}a_{j0}}{\sum \pi_{jt}h_{j0}}\right) - \log\left(\frac{\sum \pi_{j0}a_{j0}}{\sum \pi_{j0}h_{j0}}\right) \equiv \Delta_t^{comp}. \tag{10}$$

A key feature of (10) is that Δ_t^{comp} can be calculated from demographic projections and cross-sectional data alone, with demographic projections providing π_{jt} and cross-sectional data providing a_{j0} and h_{j0}. We call Δ_t^{comp} the *compositional effect* of aging on W_t/Y_t. Proposition 1 shows that, for a small open economy, this equals the log change in W_t/Y_t. The next section shows that Δ_t^{comp} also plays a key role in the integrated world economy.

2.3 Many countries aging together

Next, we analyze a world economy going through demographic change before converging to a balanced growth path where age structures and relative sizes of countries are stable. While the previous analysis assumed countries faced a constant interest rate $r_t = r$, interest rates now adjust period-by-period to clear the world asset market.

To analyze this case, we consider a first-order approximation of the world asset market clearing condition (5)

$$\sum_c \frac{W_0^c}{W_0} \Delta \log\left(\frac{W_t^c}{Y_t^c}\right) = \sum_c \frac{W_0^c}{W_0} \Delta \log\left(\frac{K_t^c + B_t^c}{Y_t^c}\right), \tag{11}$$

where $\Delta \log\left(\frac{W_t^c}{Y_t^c}\right) \equiv \log\left(\frac{W_t^c}{Y_t^c}\right) - \log\left(\frac{W_0^c}{Y_0^c}\right)$ and $\Delta \log\left(\frac{K_t^c+B_t^c}{Y_t^c}\right) \equiv \log\left(\frac{K_t^c+B_t^c}{Y_t^c}\right) - \log\left(\frac{K_0^c+B_0^c}{Y_0^c}\right)$ represent changes in log asset demand and supply relative to period 0.[18]

Absent any changes in interest rates, household asset profiles do not change, and the left of (11) is $\bar{\Delta}_t^{comp} \equiv \sum_c \frac{W_0^c}{W_0} \Delta_t^{comp,c}$ by proposition 1. In this sense, the compositional effect summarizes the full demographic "shock" to the world equilibrium. This shock causes a disequilibrium in the world asset market that must be resolved by adjustments in r_t.

Interest rate changes work through both asset supply and demand. For asset supply, the capital-output ratio only depends on contemporaneous r_t, implying $\Delta \log\left(\frac{K_t^c+B_t^c}{Y_t^c}\right) \simeq -(r_t - r_0)\epsilon^{s,c}$, where $\epsilon^{s,c} \equiv -\frac{\partial \log((K^c+B^c)/Y^c)}{\partial r} = \frac{\eta}{r_0+\delta} \frac{K_0^c}{W_0^c}$ is the semielasticity of asset supply, and η is the elasticity of substitution between capital and labor.[19] For asset demand, adjustment is more complex, potentially depending on the full sequence of $\{r_t\}$. However, long-run asset demand $\frac{W^c}{Y^c}$ only depends on the long-run interest rate r^{LR} and demographics. Writing $\epsilon^{d,c} \equiv \frac{\partial \log(W^c/Y^c)}{\partial r}$ for the semielasticity of this terminal asset demand evaluated at r_0,[20] the long-run interest rate change satisfies

$$\bar{\Delta}_{LR}^{comp} + \bar{\epsilon}^d \cdot (r_{LR} - r_0) \simeq -\bar{\epsilon}^s \cdot (r_{LR} - r_0), \tag{12}$$

[18] In this first-order approximation, the term involving $\Delta \frac{W_t^c}{W_t}$ drops out since we assume that initial net foreign asset positions $W_0^c - K_0^c - B_0^c$ are zero. In appendix D.1, we show how the formulas change in the case where initial NFAs are non-zero.

[19] This expression obtains due to our baseline assumption that B/Y is constant, as well as due to the absence of rents. If B/Y responds to r, then this adds an additional term to $\epsilon^{s,c}$. If fully capitalized rents are part of wealth then their value is proportional to $1/(r-g)$. As we show in appendix D.2, this adds a further term to $\epsilon^{s,c}$, and also adds a direct effect of population growth on asset supply. However, we abstract from endogenous adjustment of human capital as in Ludwig, Schelkle and Vogel (2012).

[20] Formally, $\epsilon^{d,c}$ is the derivative with respect to r of the balanced growth level of $\log W/Y$ in a small open economy with exogenous r, evaluated at the long-run steady-state age distribution and the initial r_0. This includes both the direct individual asset accumulation response to r, and the indirect response from the effect of r on wages. We discuss $\epsilon^{d,c}$ further in the next section.

where bars denote averages across countries using initial wealth shares $\omega^c \equiv W_0^c / W_0$. The following proposition summarizes our results and the $r_{LR} - r_0$ implied by (12).

Proposition 2. *Suppose that asset holdings start at the invariant distribution given r_0, that initial net foreign asset positions are zero, and that governments keep debt-to-GDP ratios constant. Then, the long-run change in the rate of return is, to first order,*

$$r_{LR} - r_0 \;\; \simeq \;\; -\frac{1}{\bar{\epsilon}^d + \bar{\epsilon}^s} \bar{\Delta}_{LR}^{comp}, \tag{13}$$

where $\bar{\epsilon}^s = \frac{\eta}{r_0+\delta} \frac{\bar{K}_0}{\bar{W}_0}$ is the average semielasticity of asset supply to r, and $\bar{\epsilon}^d$ is the average semielasticity of individual asset holdings to r. The wealth-weighted average log change in the wealth-to-GDP ratio is given by

$$\overline{\Delta_{LR} \log \left(\frac{W}{Y} \right)} \simeq \frac{\bar{\epsilon}^s}{\bar{\epsilon}^s + \bar{\epsilon}^d} \bar{\Delta}_{LR}^{comp} \tag{14}$$

Proof. See appendix B.3. □

The proposition shows how the excess asset demand from the compositional effect $\bar{\Delta}_{LR}^{comp}$ is absorbed. If $\bar{\epsilon}^s + \bar{\epsilon}^d$ is large, r falls little, because capital and assets are very sensitive to r. If $\frac{\bar{\epsilon}^s}{\bar{\epsilon}^s + \bar{\epsilon}^d}$ is large, wealth rises a lot, because a large share of the adjustment occurs through an increase in the capital stock rather than through a reduction in asset accumulation.

Beyond interest rates and wealth levels, our framework also speaks to global imbalances. To see why, note first that absent an adjustment in r, the net foreign asset position (NFA) of a country would increase one-for-one with its compositional effect. In equilibrium, r must fall to ensure that NFAs are zero on average, so the adjustment in r has to reduce average NFAs by the average compositional effect. Hence, the change in a country's NFA is determined by the difference between its compositional effect and the average compositional effect, subject to an additional adjustment when countries have heterogeneous semielasticities of asset demand and supply. The following proposition summarizes this argument.

Proposition 3. *Given the conditions of proposition 2, the long-run change in country c's net foreign asset position NFA^c satisfies*

$$\log \left(1 + \frac{\Delta_{LR} NFA^c / Y^c}{W_0^c / Y_0^c} \right) \simeq \Delta_{LR}^{comp,c} - \overline{\Delta}_{LR}^{comp} + \left(\epsilon^{d,c} + \epsilon^{s,c} - \left(\bar{\epsilon}^d + \bar{\epsilon}^s \right) \right) (r_{LR} - r_0) \tag{15}$$

Proof. See appendix B.3. □

Finally, since we have no direct way to predict the effect of demographics on long-run government debt targets, propositions 2 and 3 both assume a benchmark where each country keeps long-run debt-to-GDP constant. Appendix B.4 discusses alternative settings where debt-to-GDP changes in response to demographics. Two special cases stand out: when each country increases its debt-to-GDP target by the amount of its compositional effect, and when each country increases debt-to-GDP by the *average* world compositional effect. In the first case, there is no change in interest rates or net foreign assets, and each country's wealth increases by exactly its compositional effect. In the second case, the same conclusions hold for interest rates and world wealth, but net foreign assets in each country increase by the difference between its compositional effect and the global average, leaving the global imbalances predicted by proposition 3 intact.[21]

2.4 The asset demand semielasticity ϵ^d

Propositions 2 and 3 show that compositional effects determine aggregate outcomes given asset supply and demand semielasticities ϵ^s and ϵ^d.[22] As we have noted, the asset supply semielasticity ϵ^s is only a function of observables and of the elasticity of substitution between labor and capital η.

The asset demand semielasticity ϵ^d is more challenging to obtain. As noted by Saez and Stantcheva (2018), there is a "paucity of empirical estimates" for how long-run asset accumulation responds to changes in the rate of return.[23] Remarkably, however, in a special case of our model, it is possible to express ϵ^d only in terms of macroeconomic aggregates, the observed age profiles of assets and consumption, and the elasticities σ and η. The latter are standard macro parameters that have been studied by an extensive empirical literature.

This special case is a version of our model without income risk or borrowing constraints. To build intuition, we start by specializing further to the case where technology is Cobb-Douglas and $r = g$ in the initial steady state. Then, our results take the simple form:

$$\epsilon^d = \underbrace{\sigma \frac{C}{(1+g)W} \frac{\mathrm{Var} Age_c}{1+r}}_{\equiv \epsilon^d_{substitution}} + \underbrace{\frac{\mathbb{E} Age_c - \mathbb{E} Age_a}{1+r}}_{\equiv \epsilon^d_{income}}. \tag{16}$$

[21] This second case can be viewed as the limit of a specification where we make long-term debt-to-GDP highly responsive to interest rates, taking $\partial(B^c/Y^c)/\partial r$ uniformly to $-\infty$ across all countries.

[22] In this section we drop the country superscripts c for convenience. Subscripts c denote consumption.

[23] An elasticity of this kind is important in a variety of contexts, including for capital taxation (Feldstein 1978, Saez and Stantcheva 2018), the response of interest rates to automation (Moll et al. 2022), and the welfare implications of increasing the public debt (Aguiar, Amador and Arellano 2024). See section 3.2 for a discussion of empirical estimates.

Here, Age_a and Age_c are random variables that capture how asset holdings and consumption are distributed across different ages. The random variables range over ages j, with probabilities proportional to assets and consumption at each age.[24] Thus, $\text{Var}Age_c$ is large when consumption is spread out across different ages, and $\mathbb{E}Age_c - \mathbb{E}Age_a$ is positive if consumption, on average, occurs at higher ages than asset holdings do.

In appendix B.5, we derive equation (16), connecting it to the broader logic of life-cycle problems and the cross-sectional outcomes that they produce. The substitution effect $\sigma\epsilon^d_{substitution}$ scales with the elasticity of intertemporal substitution σ, and is proportional to $\text{Var}Age_c$ since there is more scope for intertemporal substitution if consumption is more spread out over the life cycle. The income effect ϵ^d_{income} reflects the fact that a higher r increases total income. The size of the increase is proportional to total wealth W and accrues at an average age of $\mathbb{E}Age_a$, and it is used to increase consumption by a uniform proportion across all ages, implying that the rise in consumption occurs at an average age of $\mathbb{E}Age_c$. Aggregate wealth increases if $\mathbb{E}Age_a$ is lower than $\mathbb{E}Age_c$, because then, on average, the extra interest income is saved before it is consumed.

For the more general case, there are two complications. First, when technology is not Cobb-Douglas, the labor share changes with r, introducing a new term. Second, our previous result relied on current values being the same as present values normalized by growth, which is no longer true when $r \neq g$. We have the following proposition.

Proposition 4. *Consider a small open economy with a steady-state population distribution π. If individuals face no income risk or borrowing constraints, the long-run semielasticity of the steady-state W/Y to the rate of return is given by*

$$\epsilon^d \equiv \frac{\partial \log(W/Y)}{\partial r} = \sigma\epsilon^d_{substitution} + \epsilon^d_{income} + (\eta - 1)\epsilon^d_{laborshare}. \tag{17}$$

When $r = g$, $\epsilon^d_{substitution}$ and ϵ^d_{income} are given by (16); when $r \neq g$ they are explicit functions of π_j, c_j and a_j given in the appendix. In either case, $\epsilon^d_{laborshare} \equiv \frac{(1-s_L)/s_L}{r+\delta}$ with $s_L \equiv \frac{wL}{Y}$.

Proof. See appendix B.5. □

Proposition 4 provides, to our knowledge, the first expression for the semielasticity of aggregate asset demand in a rich quantitative model as a function of measurable sufficient statistics. Earlier work has instead relied on numerical simulations (see, e.g., Summers 1981, Evans 1983, Cagetti 2001, Aguiar et al. 2024). While the literature has pointed out

[24] Formally, we define the probability mass of Age_a at each age j to be $\pi_j a_j / W$, the share of assets in the cross section held by people of age j, and likewise for Age_c. For the case $g = 0$, this is equivalent to defining the mass as the share of assets held at age j across the life cycle, but with the cross-sectional definition our result holds more generally.

that this elasticity can be affected by idiosyncratic income uncertainty, we show in section 4 that our formula still provides a close approximation in that context. Further, the results of proposition 4 are continuous at $r = g$, so that for small $r - g$, (16) is a good approximation to the actual $\epsilon^d_{substitution}$ and ϵ^d_{income}.[25]

3 Measurement and implications

This section uses the framework provided by propositions 1–4 to quantify the impact of demographics on macroeconomic aggregates. First, we combine demographic projections with representative household surveys to measure the compositional effect Δ_t^{comp} in 25 countries. Second, we use information on age profiles of consumption and wealth together with assumptions on η and σ to calculate the semielasticities of asset supply and demand to interest rates. Finally, we combine these results to forecast interest rates, wealth levels, and global imbalances until the end of the twenty-first century.

3.1 The compositional effect

Implementation. We take age distributions π_{jt} from the historical data and future projections of the 2019 United Nations World Population Prospects (reported in 5-year age buckets). For these projections, we consider three different scenarios, corresponding to the UN's baseline projection as well as their "high" and "low" fertility scenarios.[26]

For the age profiles of labor income and wealth, we use representative household surveys. We use labor income data from the Luxembourg Income Study (LIS), which provides harmonized labor surveys for a wide range of countries; we use wealth data from a collection of wealth surveys such as the US Survey of Consumer Finances (SCF) and the European Household Finance and Consumption Survey (HFCS). Our exercise starts in 2016 and the surveys are from this year whenever possible; otherwise, we use the closest available year. See appendix table A.1 for a complete list of data sources and survey years.

For labor income, h_{j0} is the 2016 average pretax labor income of individuals of age j. We calculate it by dividing the total labor income earned by individuals of age j—including wages, salaries, bonuses, fringe benefits, and self-employment income before social security and labor income taxes—by the number of individuals of age j.

[25] $\epsilon^d_{laborshare}$ tends to be small enough that for η close to 1, its contribution is insignificant.

[26] Note that in addition to fertility, these age distributions reflect changing mortality, which is outside the formal scope of section 2 and matters for both compositional and non-compositional reasons. We use the full projected age distributions to calculate the compositional effect here, and will show in the richer model of section 4 that the non-compositional effects of changing mortality are relatively small.

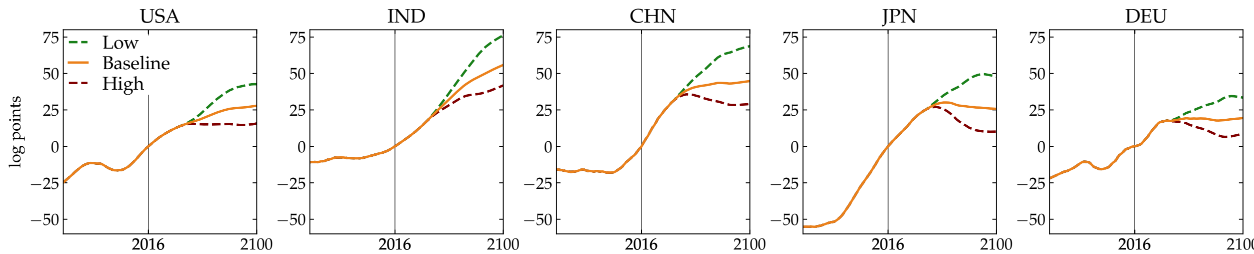

Figure 2: Compositional effect of demographics, 1950 to 2100

Notes: This figure depicts the evolution of the compositional effect of demographic change on wealth-to-GDP, calculated using equation (10) for t =1950 to 2100, reported in log points (100 log). The base year is 2016 (vertical line). The solid orange line corresponds to the medium fertility scenario from the UN, the dashed green line to the low fertility scenario, and the dashed red line to the high fertility scenario.

For assets, a_{j0} is the 2016 average individual net worth of individuals of age j. We measure it as total assets net of liabilities, with housing[27] and defined contribution pension wealth included as assets, and mortgages included as liabilities. For the United States, we also add age-specific estimates of the funded component of the empirically important private defined benefit (DB) pension plans (Sabelhaus and Volz 2019). We map household wealth from the surveys to individuals by splitting wealth equally across the head of household, the spouse, and any other household members who are at least as old as the head.[28]

We use these demographic projections and age profiles of asset and labor income to construct the compositional effect from 1950 to 2100 for the twenty-five countries in our sample. Recall that in the model, Δ^c_{comp} corresponds to the log change in W/Y that a country would experience if it were a small open economy facing a fixed r.

Results. The results from this calculation are displayed in figure 2 for five major countries, with additional countries in figure A.3. Between 1950 and 2016, the compositional effect is positive everywhere, equal to 23 log points on average, and 25 in the United States. To provide context for these magnitudes, we note that the actual log change in W/Y over this period—which reflects other forces as well as general equilibrium adjustments—was 59 log points for the average country with available data, and 32 in the US (see table A.4).

Looking ahead from 2016 to 2100, the effect remains positive, is even larger on average,

[27] Barring any direct feedback from the age distribution to the household problem via, say, land prices, proposition 1 remains true even if households partly accumulate assets in the form of housing. However, the presence of land does have general equilibrium implications by affecting the elasticity of asset supply, as we show in appendix D.2.

[28] Appendix C.3 shows that the results are robust to using different splitting rules, or to constructing income and wealth at the household level, and combining this with demographic projections for the age distribution of the heads of households.

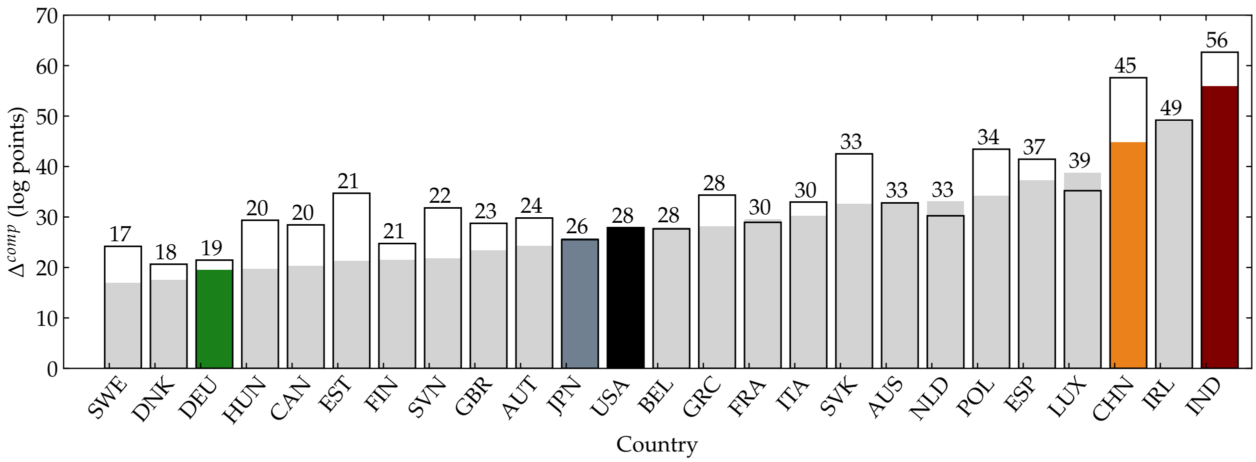

Figure 3: Compositional effect and contribution from demographics alone, 2016-2100

Notes: The solid bars show, for each country, the compositional effect on wealth-to-GDP between 2016 and 2100 in log points, calculated using equation (10), and also reported on top of the bars. These values correspond to the end point of Figure 2. The transparent bars calculate Δ^{comp} using the US age profiles a_{j0} and h_{j0}, but country-specific age distributions π_{jt}.

and is heterogeneous across countries, ranging from 17 log points in Sweden to 45 in China and 56 in India, with 28 log points in the United States. In the high fertility scenario, the effect is reduced by a younger population: it is brought down to 29 log points in China and to 16 in the United States; in contrast, the low fertility scenario sees even sharper aging, and the effect swells to 69 log points in China and 43 in the United States.

Figure 3 provides more detail on the heterogeneity across countries, with solid bars displaying the compositional effect to 2100 for the main population scenario. In principle, this cross-country heterogeneity could reflect either differences in demographic evolution or differences in the age profiles of assets and labor income. While both matter, the former is the main factor: countries with large effects are those whose demographic transitions are later and faster. The transparent bars in figure 3 illustrate this point by recomputing compositional effects while counterfactually assuming that all countries have the same asset and income profile as the United States. While the levels are slightly higher, meaning that US age profiles imply bigger compositional effects, the cross-country heterogeneity is similar.[29] In appendix C.4, we consider a related exercise, recalculating the average wealth-weighted compositional effect $\bar{\Delta}^{comp}$ when we randomly reshuffle age-wealth and age-labor profiles across countries, holding fixed their demographic projections. The resulting distribution of $\bar{\Delta}^{comp}$ is tight and centered close to the actual $\bar{\Delta}^{comp}$ we calculate in the data.

[29]By contrast, appendix figure A.5 shows that countries tend to experience similar compositional effects if they are all assumed to experience US demographics.

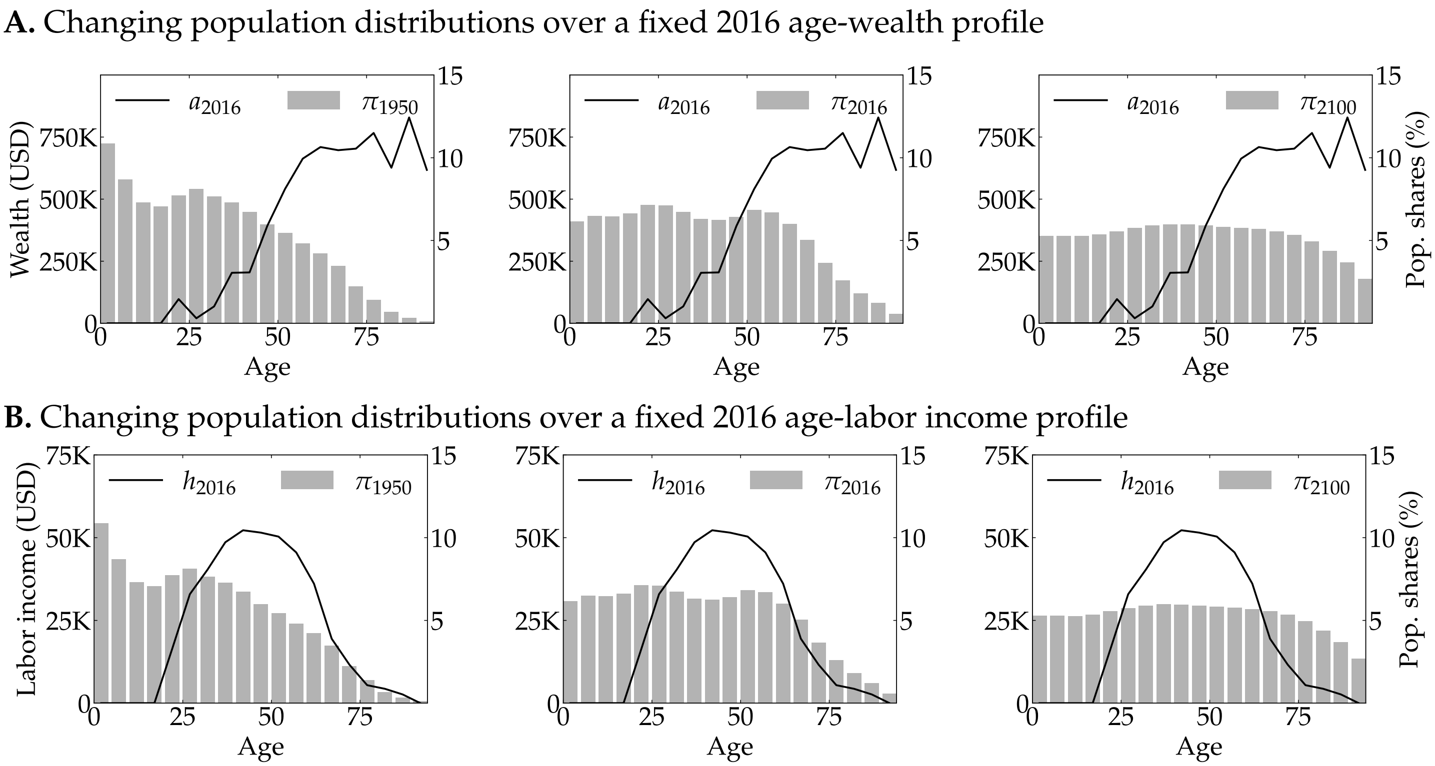

Figure 4: US age-wealth and labor income profiles with population age distributions

Notes: The solid lines in Panel A show the 2016 US age-wealth profiles from the SCF, expressed in current USD. The solid lines in panel B show the 2016 age-income profile from the LIS (CPS), expressed in current USD. Bars represent age distributions: 1950 age distribution in the left panels, 2016 age distribution in the middle panels, and 2100 age distribution in the right panels.

Unpacking the compositional effect: the case of the United States. The compositional effect reflects the interaction between population aging and the shapes of the wealth and income profiles. To help explain the magnitudes that we find, we study the case of the United States in greater detail.

The main mechanisms are summarized in figure 4. The grey bars show the evolution of the population distribution, starting young in 1950 and growing progressively older over time. In the figure, this population evolution is superimposed with the 2016 profiles of assets and labor income, with panel A illustrating how demographic change pushes up assets by moving individuals into high asset ages, and panel B illustrating how demographic change first pushes up aggregate labor income as the baby boomers reach middle age—the so-called "demographic dividend" (Bloom et al., 2003)—and later pushes down aggregate labor income as more individuals reach old age.

The total compositional effect in (10) can be expressed in terms of a shift in assets and

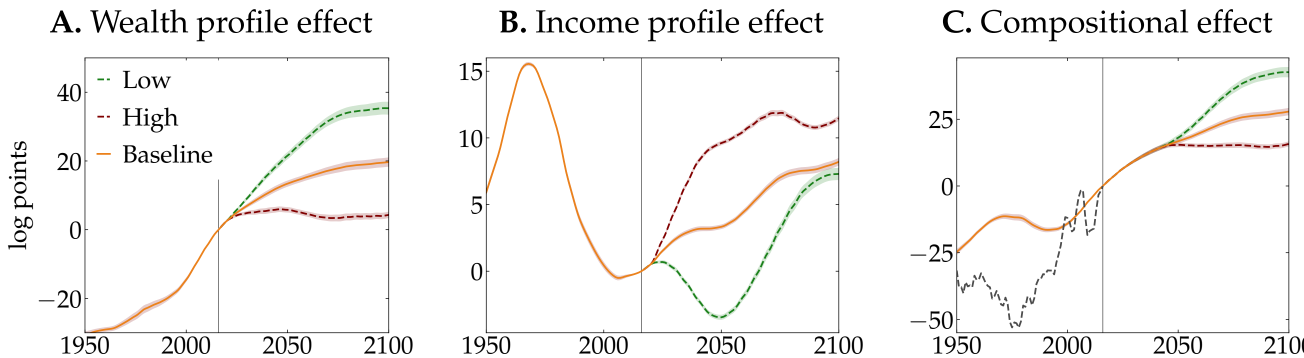

Figure 5: Compositional effects for W and Y: United States 1950-2100

Notes: This figure shows the evolution of the two terms in equation (18). Panel A presents the contribution from the wealth profile, $\Delta_t^{comp,a}$. Panel B presents the contribution from the labor income profile, $\Delta_t^{comp,h}$. Panel C presents the overall compositional effect Δ_t^{comp} from equation (10), which is equal to the sum of panel A and panel B by equation (18), overlaid with historical data from the WID. In all graphs, the solid orange line corresponds to the baseline fertility scenario, the dashed green to the low fertility scenario, and the dashed red line to the high fertility scenario of the 2019 UN World Population Prospects. A bootstrapped 95% confidence interval is computed by resampling observations 10,000 times with replacement.

labor supply:

$$\Delta_t^{comp} = \underbrace{\log\left(\frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{j0} a_{j0}}\right)}_{\Delta_t^{comp,a}} + \underbrace{\left(-\log\left(\frac{\sum_j \pi_{jt} h_{j0}}{\sum_j \pi_{j0} h_{j0}}\right)\right)}_{\Delta_t^{comp,h}}. \tag{18}$$

$\Delta_t^{comp,a}$ is positive if the share of people in high asset ages increases, and $\Delta_t^{comp,h}$ is positive if the share of people in high labor income ages *decreases*. Since old people hold relatively more assets and work relatively less, aging eventually makes both terms positive.

Figure 5 displays the evolution of $\Delta_t^{comp,a}$ and $\Delta_t^{comp,h}$. Panel A shows that $\Delta_t^{comp,a}$ is positive throughout the sample period. The trend flattens towards the end of the 21st century as aging becomes concentrated in very old ages where asset accumulation ceases. However, the trend never reverses, due to the well-known fact that asset decumulation in old age is quite limited. A large literature has debated the extent to which this limited decumulation reflects life-cycle forces, late-in-life-risks, or bequest motives (see, e.g., Abel 2001, Ameriks and Zeldes 2004, De Nardi, French and Jones 2010, De Nardi, French, Jones and McGee 2023). Our model allows for these forces in a reduced-form way by allowing β_j to vary arbitrarily with age.[30]

Panel B shows $\Delta_t^{comp,h}$ falling between 1970 and 2010 and then increasing throughout

[30] A high β_j in old age implies that individuals must continue to hold assets in old age to finance high late-in-life expenditures, which we interpret as a reduced form for late-in-life health costs or bequests. When we formally model bequests in section 4, we find that the compositional effect remains dominant.

the rest of the 21st century, eventually contributing 8 log points to the compositional effect. This non-monotonic pattern is due to the so-called "demographic dividend". The literature on this topic has shown that, as the population distribution moves across the hump-shaped profile of labor earnings, there is initially an output increase followed by a decline (Bloom, Canning and Sevilla 2003; Cutler et al. 1990). Our findings complement this literature by connecting the output effect of demographics to an inverted effect on the wealth-to-GDP ratio. Quantitatively, this effect contributes a quarter of the full increase in Δ_t^{comp} for the United States between 2016 and 2100.

Our results relate to earlier findings by Poterba (2001), who used a shift-share analysis with population projections until 2050 and data from the 1983–1995 waves of the SCF to conclude that $\Delta_t^{comp,a}$ (which he called "projected asset demand") would be stable beyond 2020. He used this result to argue that an asset market meltdown was unlikely. In contrast to Poterba, we find a substantial increase in $\Delta_t^{comp,a}$ throughout the remainder of the twenty-first century, reflecting our use of later SCF waves, and, more importantly, population projections with narrower age bins. In addition, Poterba's analysis abstracted from the labor supply term $\Delta_t^{comp,h}$, which we find is not trivial.

For other countries, the logic behind Δ^{comp} is broadly similar to that for the United States. In an online appendix,[31] we reproduce Figures 4 and 5 for all twenty-five countries in our sample. While each country has its own peculiarity—for instance, the timing of the demographic dividend is very uneven—in all of them, aging pushes individuals into higher-asset, lower-income age groups after 2050.

Robustness to base year and construction of age profiles. Our calculations use a single cross section of asset and labor income profiles. This is consistent with the model, where age profiles are stable over time and grow at a constant rate γ. Given this feature, any cross section will imply the same compositional effect, and cross-sectional estimates of a_{j0} and h_{j0} will agree with estimates of age effects from a time-age-cohort decomposition of repeated cross sections, provided that growth loads on time rather than on cohort effects.

In practice, this property is not satisfied exactly: the age profiles of wealth and labor income vary in shape over time. In particular, one worry is that 2016 profiles are not representative due to transient one-time factors (say, large capital gains since 1980, as in Bauluz and Meyer 2024).

Appendix C.3 examines the robustness of our results to using different base years for labor income and asset profiles. Focusing on the US, we use twelve waves of the LIS (since 1974) for labor income and 21 waves of the SCF (since 1958) for wealth.[32] Calculating

[31] Available at http://web.stanford.edu/~aauclert/demowealth21_country_appendix.pdf

[32] Prior to 1989, we use the SCF+ data developed by Kuhn, Schularick and Steins (2020), a reweighted and

Δ^{comp} between 2016 and 2100 for all 252 possible combinations, we find that numbers are similar for all post-1989 waves, and somewhat smaller—between 1/2 and 2/3 as big—for earlier SCF waves, consistent with capital gains having increased elderly wealth in later periods. We also show that our results are robust to using age profiles from a time-age-cohort decomposition as well as to different methods of allocating household wealth to individuals.

3.2 Asset supply and demand semielasticities

We now turn to calculating the semielasticities of asset supply and demand using the formulas in propositions 2 and 4.

Asset supply semielasticity $\bar{\epsilon}^s$. The global asset supply semielasticity captures the response of the capital-output ratio to the required rate of return. Proposition 2 provides a closed-form solution for this semielasticity,

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \frac{\bar{K}_0}{\bar{W}_0}, \tag{19}$$

showing that $\bar{\epsilon}^s$ is proportional to the initial global capital-wealth ratio $\frac{\bar{K}_0}{\bar{W}_0}$, the inverse of the user cost of capital $r_0 + \delta$, and the elasticity of substitution between capital and labor η. To ensure that asset supply equals total wealth, we define the global value of capital as total wealth net of government bonds, implying $\frac{\bar{K}_0}{\bar{W}_0} = 0.79$.[33] We obtain $r_0 + \delta = 9.5\%$ when calibrating r_0 using the method from appendix A.1, and taking average depreciation δ of private fixed assets from the BEA Fixed Asset Accounts (see the calibration in section 4 for details). Given these numbers, $\bar{\epsilon}^s$ is between 4.1 and 12.4 for η in a typical range from 0.5 to 1.5; it is equal to 8.3 with a Cobb-Douglas aggregate production function ($\eta = 1$).

Asset demand semielasticity $\bar{\epsilon}^d$. The semielasticity $\bar{\epsilon}^d$ captures how much global asset accumulation responds to changes in r in the long-run. Proposition 4 expresses $\epsilon^{d,c}$ in each country c as a function of cross-sectional observables, the elasticity of intertemporal substitution (EIS) σ, and capital-labor substitution η. With common σ and η across countries, we obtain

$$\bar{\epsilon}^d = \sigma \cdot \bar{\epsilon}^d_{substitution} + \bar{\epsilon}^d_{income} + (\eta - 1) \cdot \bar{\epsilon}^d_{laborshare}, \tag{20}$$

harmonized version of SCF designed to maximize compatibility with post-1989 data.

[33] Implicitly, the assumption is that any deviation between measured assets and wealth reflects undermeasured capital. Another way to explain such differences is to have land or capitalized markups. This case is discussed in appendix section D.2.

where bars denote cross-country averages weighted by initial wealth levels.

Implementing the formulas for $\epsilon^{d,c}$ in proposition 4, we obtain $\bar{\epsilon}^d_{substitution} = 43.7$, $\bar{\epsilon}^d_{income} = -0.6$ and $\bar{\epsilon}^d_{laborshare} = 5.8$. The dominant and positive substitution effect means that $\bar{\epsilon}^d$ is positive unless σ is extremely low.[34] When σ has a reasonable value of 0.5 and the production function is Cobb-Douglas, $\bar{\epsilon}^d = 21.2$. This means that a decrease in r by one percentage point would reduce households' desired wealth relative to GDP by around 21%.

The formulas in proposition 4 rely on the distribution of consumption and wealth across different age groups j. We construct these distributions by weighting the wealth and consumption profiles by the long-run (2100) population distribution. The wealth profiles by age are obtained as in section 3, and the consumption profiles are backed out from the household budget constraint (1) given the age profiles of wealth and income.[35]

To get a sense for the drivers of income and substitution effects, equation (16) is helpful.[36] The income effect is approximately the gap between average ages of consumption and asset holdings. Since consumption occurs slightly earlier in life than asset accumulation, there is a small negative income effect. For the substitution effect, we multiply C/W (roughly 1/8) by the variance of the age of consumption. If consumption is distributed uniformly from ages 20 to 85, this variance is approximately $(85-20)^2/12 \approx 352$. This back-of-the-envelope calculation suggests $\bar{\epsilon}^d_{substitution} \approx (85-20)^2/(12 \cdot 8) = 44$, close to our actual result of 43.7.

Comparison to existing empirical estimates. Several studies have examined how asset accumulation responds to capital taxes. Moll et al. (2022) survey this literature and find that ϵ^d ranges from 1.25 to 35.[37] This range aligns with equation (20) for reasonable σ and η values. No empirical estimates are negative, in line with our finding that substitution effects dominate income effects.

[34] For example, with a Cobb-Douglas production function, $\bar{\epsilon}^d$ is positive as long as σ exceeds 0.014.

[35] We use this indirect procedure, rather than consumption surveys directly, since the latter tend to be less comprehensive than wealth surveys and are not available for all countries in our study. The implied distributions are presented in appendix figure A.7. Note that this procedure effectively counts any bequests as part of consumption.

[36] Our exact implementation uses slightly different equations allowing for $r \neq g$, but in practice $r - g$ is sufficiently small to make equation (16) a good quantitative guide.

[37] This literature includes Kleven and Schultz (2014), Zoutman (2018), Jakobsen, Jakobsen, Kleven and Zucman (2020), and Brülhart, Gruber, Krapf and Schmidheiny (2022). While Moll et al. (2022) focus on $\partial \log W/\partial r$, this should equal our $\epsilon^d = \partial \log(W/Y)/\partial r$ since the underlying micro experiments in this literature (mostly wealth taxes) likely did not create differential changes in Y across groups.

Table 1: Change in world interest rate and wealth-to-GDP

	A. $r_{LR} - r_0$			**B.** $\overline{\Delta_{LR} \log\left(\frac{W}{Y}\right)}$		
		σ			σ	
η	0.25	0.50	1.00	0.25	0.50	1.00
0.60	-2.45	-1.33	-0.69	12.2	6.6	3.4
1.00	-1.71	**-1.07**	-0.62	14.1	**8.9**	5.1
1.25	-1.43	-0.96	-0.58	14.8	9.9	6.0

Notes: This table presents predictions for the change in the total return on wealth (r) and the wealth-weighted change in log wealth-to-GDP ($\log(W/Y)$) between 2016 ($t = 0$) and 2100 ($t = LR$) using our sufficient statistic methodology. Columns vary the assumption on the elasticity of intertemporal substitution σ, rows vary the assumption on the elasticity of capital-labor substitution η. Central estimates are in bold. r is expressed in percentage points, and wealth in percent ($100 \cdot \log$).

3.3 General equilibrium implications

We now combine our compositional effects and semielasticities with propositions 2 and 3 to obtain long-run general equilibrium changes. Here, we define the long run as 2100.

The rate of return and wealth-to-GDP ratios. Proposition 2 shows that long-run changes in the rate of return and average wealth levels are functions of $\bar{\Delta}^{comp}_{LR}$, $\bar{\epsilon}^s$, and $\bar{\epsilon}^d$.

We calculate $\bar{\Delta}^{comp}_{LR} \equiv \sum_c \omega^c \Delta^{comp,c}_{2100} = 31.7$ log points by taking each country's compositional effects until 2100 from section 3.1, averaged using 2016 wealth levels. Equations (19) and (20) in section 3.2 express $\bar{\epsilon}^s$ and $\bar{\epsilon}^d$ in terms of capital-labor substitutability η and the elasticity of intertemporal substitution σ. Our central estimate uses canonical values of $\eta = 1$ and $\sigma = 0.5$. Given the uncertainty surrounding the value of these parameters, however, we also consider a collection of lower and higher values. For the EIS, we consider a low value of $\sigma = 0.25$ and a high value of $\sigma = 1$, spanning the range typically considered in the macroeconomics literature (see, e.g., Havránek 2015). For capital-labor substitution, we consider a low value $\eta = 0.6$ taken from Oberfield and Raval (2021), and a high value $\eta = 1.25$ taken from Karabarbounis and Neiman (2014).[38]

Table 1 presents our results. The left-hand panel shows the changes in the rate of return, calculated using equation (13), while the right-hand panel shows the average change in log wealth-to-GDP in percent, calculated using equation (14).

We find that the equilibrium return r unambiguously falls in response to demographic change, refuting the "great demographic reversal" hypothesis (Goodhart and Pradhan, 2020). This result follows because $\bar{\Delta}^{comp}_{LR}$ and $\bar{\epsilon}^s + \bar{\epsilon}^d$ are both positive for any plausible

[38] A recent meta-analysis (Gechert, Havranek, Irsova and Kolcunova, 2022) suggests an even lower $\eta = 0.3$, which would imply a larger decline in r and smaller rise in W/Y.

combination of σ and η. Intuitively, the compositional effect increases net asset demand, and if $\bar{\epsilon}^s + \bar{\epsilon}^d > 0$, then a fall in r is required to equalize the world's supply and demand of assets.[39] In our central scenario, r falls by $31.7/(21.2 + 8.3) = 1.07$ percentage points by the end of the twenty-first century; the fall is larger when σ or η are small, since this limits the responsiveness of asset supply and demand to falling returns.[40]

For our central scenario, average wealth-to-GDP increases by 8.9 log points (corresponding to an increase from 456% to 498% of world GDP). While substantial, this increase is smaller than the average compositional effect of 31.7 log points, since the equilibrium response from the compositional effect is multiplied by a factor of $\bar{\epsilon}^s/(\bar{\epsilon}^s + \bar{\epsilon}^d) \simeq 1/3$, which is the share of adjustment occurring through increases in investment rather than through reductions in asset accumulation. Intuitively, whenever $\bar{\epsilon}^d > 0$, the general equilibrium response is smaller than the compositional effect, since households accumulate fewer assets as interest rates fall. Wealth responses are larger when investment is elastic relative to accumulation; that is, when η is large relative to σ.

Our finding of sizable but not radical increases in wealth-to-GDP ratios lies between the predictions by Piketty (2014) and Krusell and Smith (2015): Piketty argues that a lower population growth rate will push up W/Y dramatically,[41] while Krusell and Smith argue that the prediction of constant W/Y from the representative-agent model is more consistent with empirical responses of savings rates to changes in the growth rate.

Robustness to risky assets and rents. The results presented so far have assumed the existence of a single, global world interest rate. In appendix D.3, we relax this assumption, instead letting households invest in both "safe" and "risky" assets with different returns, with government bonds making up the supply of world safe assets and capital the supply of world risky assets. We show that, for a range of plausible values of the elasticity of the risky asset share to the risk premium, the effect on the average interest rate remains quantitatively close to the findings in table 1. In this extended model, demographic change pushes up the risk premium, as older households reduce the risk in their portfolios and push up the net demand for safe assets (similar to Kopecky and Taylor 2022). This effect, however, is of only moderate size, with the risk premium rising by 0.37pp in our central

[39] In section 5, we explain why thinking of equilibrium in terms of flows rather than stocks can lead one to miss this conclusion.

[40] Using numerical simulations, Papetti (2021a) presents similar comparative statics. Appendix G.2 shows that the functional form implied by our sufficient statistic formulas fits his results very well. Our calculations also assume that all countries have a zero initial NFA; appendix D.1 relaxes this assumption and finds quantitatively similar results.

[41] Piketty (2014) uses the identity $W/Y = s/g$ with a stable savings rate s. Given a long-run fall in g from 1.5% to 1% induced by slower population growth, this implies a log increase in W/Y of $100 \cdot \log(1.5) = 40$ log points, far larger than our 8.9 log points.

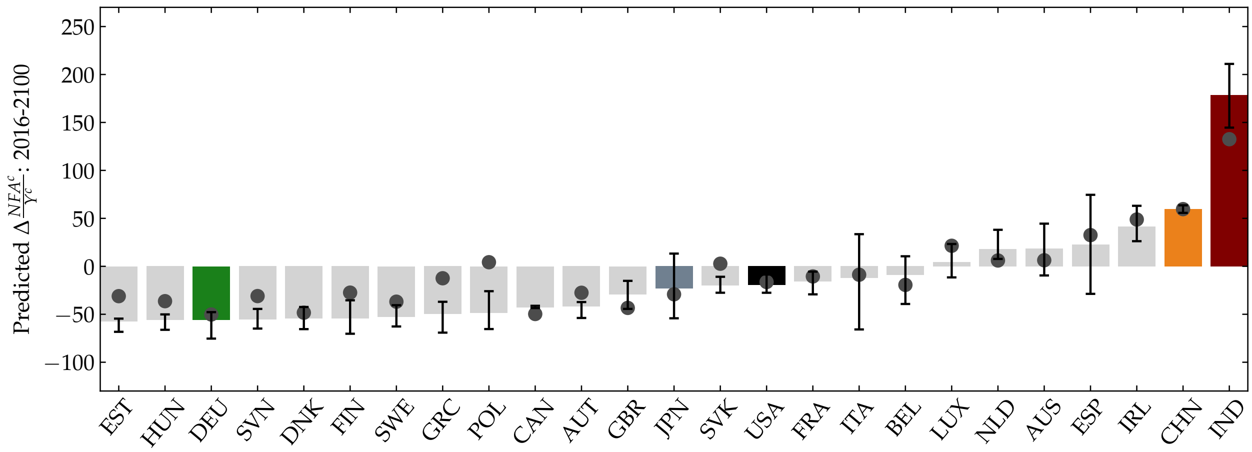

Figure 6: Long-run NFAs under alternative assumptions for σ and η

Notes: This figure presents predictions for $NFAs$ using our sufficient statistic methodology. The solid bars report $\Delta_{LR} NFA^c/Y^c$ calculated by applying equation (15), assuming $\sigma = 0.5$ and $\eta = 1$. The confidence intervals correspond to the maximum and the minimum value obtained from this formula across all possible combinations of σ and η considered in Table 1. The dots correspond to the demeaned compositional effect, $\Delta_{LR}^{comp,c} - \overline{\Delta}_{LR}^{comp}$, the first term in equation (15), which is independent of σ and η.

case, and it is largely orthogonal to the effect on the average return, which now falls by 1.12pp rather than 1.07pp.

Our results also assume that capital is the only privately supplied asset, with no role for capitalized rents. Appendix D.2 allows for rents originating with either markups or land, but finds little change in our central predictions. While capitalizing rents raises the semielasticity of asset supply to returns, this is roughly offset by lower population growth reducing asset supply through a lower value of future rents.

Global imbalances. Next, we turn to the evolution of net foreign asset positions. Figure 6 shows the changes between 2016 and 2100 predicted by the formula in proposition 3. The bars display the main results, which feature a large divergence in NFA positions, with India experiencing an increase of 179 percentage points, China an increase of 60 percentage points, and Germany a decrease of 56 percentage points.

The divergence in NFAs mainly reflects the substantial heterogeneity in compositional effects found in section 3.1. By proposition 3, this heterogeneity affects global imbalances through the demeaned compositional effects $\Delta_{LR}^{comp,c} - \bar{\Delta}_{LR}^{comp}$, whose direct implications for NFAs (assuming no heterogeneity in ϵ^s and ϵ^d) are plotted as circles in figure 6. The demeaned compositional effects broadly mirror the predicted changes in NFAs.

Compared to the results on r and W/Y, the results on global imbalances are less sensitive to the value of the elasticities η and σ. As proposition 3 shows, semielasticities only affect global imbalances insofar as they differ across countries. Since changing η and σ primarily moves semielasticities in parallel across countries, they have a relatively limited effect on the differences across countries. In the figure, the confidence bands show the minimum and the maximum prediction as η and σ parameters are varied in the range considered above. With a few exceptions, these bands are quite tight.

The importance of demeaned compositional effects suggests a *dynamic* projection for NFAs that simply uses the demeaned compositional effect at each point in time:[42]

$$\Delta \frac{NFA_t^c}{Y_t^c} \simeq \frac{W_0^c}{Y_0^c}\left(e^{\left(\Delta_t^{comp,c} - \bar{\Delta}_t^{comp}\right)} - 1\right) \tag{21}$$

Panel A of figure 7 implements this calculation. The solid lines show global imbalances until today for the five large economies discussed in the introduction, and the dashed lines show the projections from equation (21). In the next few decades, we expect to see a widening of existing global imbalances: China's net foreign assets will rise substantially, while those of the US will decline. Although these trends flatten mid-century, the second half of the 21st century features a conspicuous rise in India's net foreign assets, offset partly by a decline in Germany and Japan, whose demographic transitions at that point are nearly complete. These results arise from the heterogeneity in compositional effects that we documented in section 3.1, with China and India having very large $\Delta_t^{comp,c}$ relative to the world average.

3.4 Historical fit

We now explore how well our framework explains historical patterns in returns, wealth, and NFAs. To do so, we conduct a historical exercise that is symmetric to the forward-looking exercise in the previous section: we take the formulas for changes from the base year 2016 to the "long run" of 2100, but replace 2100 age distributions with the age distributions at time t, for each t from 2016 going back to 1950. Below is a high-level summary, with appendix C.6 providing details.

For r and W/Y, figure A.9 shows that in our central case with $\sigma = 0.5$ and $\eta = 1$, demographics explain half of the historical interest rate decline (112 of 215 basis points) and 15% of the wealth increase (7 of 47 log points) since 1950.[43] With $\sigma = 0.25$ and $\eta = 1.25$, these shares rise to 65% and 30% respectively.

[42] Appendix figure A.8 instead applies equation (15) at each point, taking into account the interest rate adjustment and the heterogeneity in elasticities across countries.

[43] For wealth, we focus on the subset of countries with available wealth data in 1950: US, UK, France, Germany, the Netherlands, Spain, and Sweden.

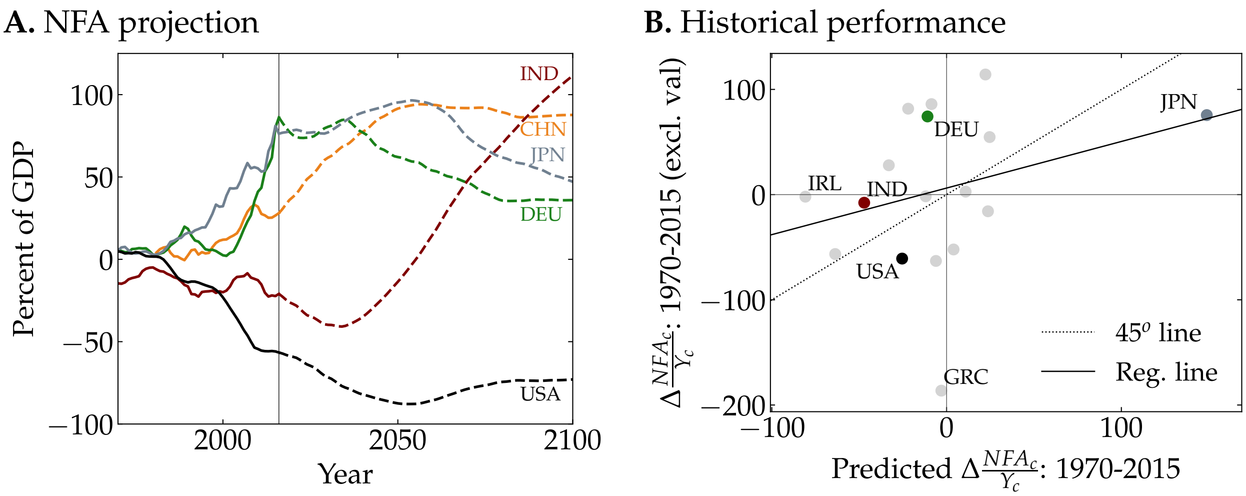

Figure 7: Using the demeaned compositional effect to project NFAs

Notes: Panel A projects NFA-to-GDP ratios between 2016 and 2100 from demeaned compositional effects, using equation (21). Solid lines are historical developments in valuation-free NFAs from figure 1. Panel B shows the projection from the demeaned compositional effect (x-axis) against the actual change in valuation-free NFAs between 1970 and 2015 (y-axis) for the 17 countries for which the data is available (source: Lane and Milesi-Ferretti 2018). The dotted line is a 45^o line, the solid line is the ordinary least squares regression line, with slope 0.44.

These findings suggest an important role for demographics but also for other forces, especially in determining W/Y. This pattern is consistent with a literature stressing positive shifts in both asset demand (falling productivity growth, inequality) and asset supply (automation, debt, markups) during this era,[44] which all tended to raise W/Y but had offsetting effects on r. Implementing a simple demand-supply accounting framework, we infer that there have been positive shifts in both asset demand and supply, with demographics accounting for 30% of the asset demand shift in our central case.

Next, we compare the NFAs predicted from equation (21) with observed historical changes. Since our model lacks valuation effects, we use a valuation-free measure of historical NFAs constructed by cumulating current accounts (as in, e.g., Gourinchas and Rey 2007). Panel B in figure 7 shows the raw correlation between predicted and actual changes in NFAs between 1970 and 2015. In this simple exercise, the regression coefficient is 0.44 with a standard error of 0.38. While the result does not directly reject our theory, since the theoretically predicted slope of '1' is inside the confidence interval, it does not provide strong support for it either. On the other hand, in appendix table A.7, we show that in a shorter sample 1993–2015, which allows us to include six additional countries, the coefficient estimate is statistically significant and close to one: 1.19 with a standard error of 0.45.

[44]See the references in footnote 11.

Two clear outliers in figure 7B are Japan and Greece, which have both experienced large increases in government debt over the 1970–2015 period. This matters, since in our theory, higher government debt reduces the NFA one-for-one in the long run.[45] In table A.7, we show that a regression of the valuation-free NFAs on the compositional effect and the change in debt to GDP delivers coefficients that are close to the theoretical benchmarks of $+1$ for the compositional effect and -1 for debt/GDP and are statistically significant, with similar results holding in the short sample. Statistical significance there, however, is somewhat sensitive to removing prominent outliers, especially in the long sample. We conclude that historical changes in NFAs are generally consistent with the predictions from our model, but that there is not enough data for a decisive verdict.

4 The compositional effect in a quantitative model

In our sufficient statistic analysis so far, we predicted equilibrium outcomes from a small set of parameters and data moments. The underlying model in section 2 was rich in some respects, but it also abstracted from a number of forces that the literature has found to be important to explain savings: bequest motives, changing mortality, and changes in government taxes, transfers, and retirement policy.

In this section, we extend the baseline model to incorporate these features. We simulate the model numerically, and study how well the sufficient statistic analysis holds up. We find that it remains an excellent guide for predicting changes in rates of return, both qualitatively and quantitatively. The main exception is when the fiscal adjustment in response to an aging population is one-sided, falling either entirely on tax increases or benefit cuts.

4.1 Extending the model

The basic setup is the same as in section 2. Below we outline the main new features, and provide details in appendix E.1.

We continue to omit the country superscript c unless there is a risk of ambiguity. For the production sector, we now assume that F is a CES production function with elasticity η. We make two modifications to the specification of demographics: survival rates ϕ_{jt} can vary over time, and there is an exogenous number of migrants M_{jt} of age j at time t.

[45]Concretely, since $\frac{NFA^c}{Y^c} = \frac{W^c}{Y^c} - (\frac{K^c}{Y^c} + \frac{B^c}{Y^c})$, a relative change in $\frac{B^c}{Y^c}$ across countries implies an equal and opposite relative change in $\frac{NFA^c}{Y^c}$, holding fixed the interest rate and any policies that affect wealth accumulation (for example, if debt moves due to changes in government consumption). This carries through to general equilibrium if we abstract from heterogeneity in $\epsilon^{s,c}$ and $\epsilon^{d,c}$, as in (21). Intuitively, rising debt acts similarly to the compositional effect, but in the opposite direction.

To allow for a longer working life, we introduce a time-varying retirement policy. We also introduce bequests governed by non-homothetic preferences, which help explain asset inequality and the limited decumulation of assets at old ages. We remove annuitization given the limited share of wealth that is annuitized in practice; individuals now self-insure against mortality risks, with assets remaining at death given as bequests. Last, we assume that there is intergenerational transmission of ability. These are all standard features of quantitative OLG models (see, e.g., De Nardi 2004). We obtain the following individual problem:

$$\max \mathbb{E}_k \sum_{j=0}^{J} \beta_j \Phi_{jt} \left[\frac{\mathfrak{c}_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \Upsilon Z_t^{\nu-\frac{1}{\sigma}} \left(1-\phi_{jt}\right) \frac{\left(\mathfrak{a}_{j+1,t+1}\right)^{1-\nu}}{1-\nu} \right] \tag{22}$$

$$\text{s.t.} \quad \mathfrak{c}_{jt} + \mathfrak{a}_{j+1,t+1} \leq w_t \left((1-\tau_t)\ell_{jt}(z_j)(1-\rho_{jt}) + tr_{jt}(z_j) \right) + (1+r_t)[\mathfrak{a}_{j,t} + b^r_{jt}(z_j)] \tag{23}$$

$$\mathfrak{a}_{j+1,t+1} \geq -\bar{a} Z_t.$$

Compared to the setup in section 2, there are now bequest preferences with curvature $\nu \leq \frac{1}{\sigma}$ that allow for non-homotheticity. These are scaled with a constant Υ, mortality risk $1-\phi_{jt}$, and a term $Z_t^{\nu-\frac{1}{\sigma}}$ that makes non-homotheticity consistent with balanced growth. In the budget constraint, $b^r_{jt}(z_j)$ denotes bequests received. The factor $\rho_{jt} \in [0,1]$ specifies retirement policy as the fraction of nonworking time for individuals of age j at time t.

The individual state z_j consists of a permanent component θ, which is Markov across generations, and a transient component ε_j, which is Markov across years, both normalized to have mean 1. Total labor supply is the product of these two components and a deterministic age profile: $\ell_{jt}(z_j) = \theta\varepsilon_j\bar{\ell}_j$. Bequests received $b^r_{jt}(z_j)$ are obtained from pooling all bequests from parents of each type θ, distributing them across ages j in proportion to a fixed factor F_j, and across types θ' in proportion to the intergenerational transition matrix of types $\Pi(\theta'|\theta)$.

For government policy, we assume that transfers reflect the social security system and are given by $tr_{jt}(z_j) = \rho_{jt}\theta d_t$, where d_t denotes the time-varying replacement rate. The government policy consists of a sequence of retirement policies $\{\rho_{jt}\}$ and a fiscal rule that targets an eventually converging sequence of government debt $\{\frac{B_t}{Y_t}\}$, where the debt sequence is obtained by dynamically adjusting replacement rates d_t, taxes τ_t and consumption G_t.

4.2 Asset demand and supply in the extended model

Unlike in the baseline model of section 2, demographic change in our extended model affects individual asset accumulation and labor supply even for a fixed r by generating variation over time in received bequests $b^r_{jt}(\theta)$, survival rates ϕ_{jt}, tax and benefit policy $\{\tau_t, d_t\}$, and retirement policy ρ_{jt}. These changes create non-compositional effects on the wealth-to-GDP ratio, and imply that propositions 2 and 3 no longer hold, since these propositions relied on the compositional effect summarizing all effects of demographics.

However, the asset demand and supply framework underpinning these propositions still applies to the extended model, provided that we replace the compositional effect $\Delta_t^{comp,c}$ with the more general notion of a "small open economy" effect $\Delta_t^{soe,c}$. This effect captures the full shift in the asset demand curve at a fixed interest rate, with $\Delta^{soe} - \Delta^{comp} \neq 0$ indicating non-compositional effects on asset demand.

Proposition 5. *If the wealth holdings of agents start in a steady-state distribution given r_0 and π_0^c, then proposition 2 and 3 hold in the extended model, with $\Delta^{comp,c}$ replaced by $\Delta^{soe,c}$, where $\Delta_t^{soe,c}$ is defined as the log change in the wealth-to-GDP ratio between 0 and t in a small open economy equilibrium with a constant rate of return r_0.*

Proof. See appendix E.2. □

Proposition 5 provides a general framework for interpreting the effects of demographics. In appendix G.1, we use this framework to analyze the findings in Eggertsson et al. (2019) (EMR) and Gagnon et al. (2021) (GJLS), two recent papers that find very different effects of demographics on the real interest rate from 1970 to 2015. EMR's higher estimate is explained primarily by a compositional effect that is much larger than we calculate in the data, driven both by a steeper age-wealth profile and by a larger change in the age distribution.[46]

Next, we calibrate our model and interpret the results through the lens of proposition 5.

4.3 Calibration

We calibrate a world economy consisting of the same 25 economies studied in section 3. To obtain parameters for each country, we calibrate a steady-state version of our model to 2016 data. Starting from this steady state, we then simulate the model from 2016 onward given demographic projections.

Steady-state calibration procedure Appendix E.3 spells out the steady-state version of our model, which for the most part is standard.[47] The main calibration parameters and

[46] EMR's lower semielasticities, especially a low ϵ^s, also play some role.

[47] The main non-standard element is a counterfactual flow of migrants, which we introduce to ensure that the steady state implied by the 2016 birth and death rates can exactly match the observed age distribution in

results are displayed in table 2. For parameters that are common across countries, the "All" column displays the world value. Country-specific parameters have a c-superscript, and the US values are displayed for illustration. Below we summarize the main elements of the calibration, with some supplemental information in appendix E.4.

The real rate of return r is the 2016 value from figure 1 in the introduction, with the calculation described in appendix A.1. For the wealth-to-GDP ratio W^c/Y^c, we use the same data as in section 3. We use data from the IMF to obtain country-specific debt levels B^c/Y^c and net foreign asset positions NFA^c/Y^c, adjusted to ensure that $\sum_c NFA^c = 0$. The capital-output ratio is obtained residually as $K^c/Y^c = W^c/Y^c - B^c/Y^c - NFA^c/Y^c$.[48]

On the production side, we set the elasticity of substitution between labor and capital to unity, $\eta = 1$. Countries have a common labor-augmenting growth rate γ calibrated to the average growth rate in output per effective labor unit Y_t^c/L_t^c between 2000 and 2016. The common depreciation rate is calibrated from average depreciation of US private fixed assets in 2016, obtained from the BEA fixed asset accounts. Given these parameters, we obtain the investment to output ratio and the labor share in each country from K^c/Y^c, as well as the country-specific growth rate $g^c \equiv (1+n^c)(1+\gamma) - 1$.

For government, we calibrate retirement policy ρ_j^c to match the decline of labor income after the retirement age $J^{r,c}$. Formally, we set $\rho_j^c = 0$ for $j < J^{r,c}$ and $\rho_j^c = 1 - h_j^c/h_{J^{r,c}-1}^c$ for $j \geq J^{r,c}$, where h_j^c is gross labor income as measured in section 3. The retirement age is obtained from the OECD, adjusted to ensure that labor income is not lower than benefit income.[49] We define the income tax rate τ using OECD data on the average tax wedge on personal earnings. Transfers capture the social security system, and satisfy $tr^c(z_j) = \rho_j^c \theta \overline{d}^c$, where we calibrate the replacement rate $\overline{d}^c$ by targeting country-specific benefit-to-GDP ratios net of taxes from the OECD Social Expenditure Database. Government consumption G^c/Y^c is set to satisfy the government budget constraint, given B^c/Y^c.

For the income process, the deterministic component of labor supply $(1-\rho_j^c)\ell_j^c$ is set proportionally to h_j^c. For the idiosyncratic term z, the log transient component follows an AR(1) process over the life cycle, and the log permanent component follows an AR(1)

2016. This method is similar to the one used in Penn Wharton Budget Model (2019), and is one way to address a generic problem in the calibration of steady-state demographic models, which is that observed mortality and population shares are generally inconsistent with a stationary population distribution. This adjustment is only needed in the steady state: to simulate the dynamics after 2016, we use the migration flows given in demographic projections.

[48]Note that the implied K/Y for the US is high relative to standard measures of capital stock. As mentioned in section 3.2, our methodology implicitly assumes that unmeasured capital accounts for this gap. See appendix D.2 for an alternative assumption using markups.

[49]Our main source is the OECD's data on "effective age of labor market exit" from the OECD Pensions at a Glance guide. In seven countries, the age provided by the OECD implies that labor market exit happens after the age at which aggregate labor income falls below implied benefit income. In those cases, we define the latter age as the date of labor market exit. See appendix E.4 for details.

Table 2: Calibration parameters

Parameter	Description	US	All	Source
Demographics				
J^w, J	Initial and terminal ages		$20, 95$	
n^c	Population growth rate	0.5%		UN World Population Prospects
π_j^c	Population distribution			—
ϕ_j^c	Survival probabilities			—
Returns and assets				
r	Real return on wealth		3.8%	Appendix A.1
W^c/Y^c	Total wealth over GDP	440%		WID
B^c/Y^c	Debt over GDP	107%		IMF
NFA^c/Y^c	Net foreign assets	−36%		IMF
K^c/Y^c	Capital over GDP	369%		$\frac{W^c}{Y^c} - \frac{B^c}{Y^c} - \frac{NFA^c}{Y^c}$
Production side				
I^c/Y^c	Investment over GDP	30%		$\frac{K^c}{Y^c}(\delta + g^c)$
$1 - s_L^c$	Capital share	35%		$(r + \delta)\frac{K^c}{Y^c}$
δ	Depreciation rate		5.70%	$\frac{\delta K}{K}$ (BEA private fixed asset tables)
γ	Technology growth		1.99%	Average 2000-16 growth in $\frac{Y_t}{\sum N_{jt} h_{j0}}$
η	K/L elasticity of subst.		1	Standard
Government policy				
$J^{r,c}$	Retirement age	62		See text
G^c/Y^c	Gov. cons. over GDP	13%		Government budget
$\overline{d}^c$	Social security benefits	92%		Benefits-to-GDP from OECD (target)
τ^c	Labor tax rate	32%		Balanced total budget
Income process				
χ_ϵ	Idiosyncratic persistence		0.91	Auclert and Rognlie (2018)
v_ϵ	Idiosyncratic std. dev.		0.91	Auclert and Rognlie (2018)
χ_θ	Intergenerational persist.		0.677	De Nardi (2004)
v_θ	Intergenerational std. dev.		0.61	De Nardi (2004)
$\bar{a}$	Borrowing limit		0	
Preferences				
σ	EIS		0.5	Standard
$\bar{\beta}^c$	Discount factor	0.9655		See text
ξ^c	Discount factor curvature	0.00071		See text
Υ	Bequests scaling factor		78.5	See text
ν	Bequest curvature		1.92	See text

process across generations. The parameters of these processes are taken from Auclert and Rognlie (2018) and De Nardi (2004). We assume that the distribution of bequests received across ages F_j is common across countries, and we match it to the age distribution of bequests received in the U.S. Survey of Consumer Finances.

The remaining parameters are the elasticity of intertemporal substitution σ, the time preference profile β_j, and the weight and curvature on bequests (Υ, ν). We assume that parameters σ, Υ, and ν are common across countries, and set σ to 0.5, in line with the central case of section 3. To match country-specific age-wealth profiles, we allow the level shifters β_j to vary across countries according to a quadratic formula, $\log \beta_j^c = j \times \log \bar{\beta}^c + \xi^c (j - 40)^2$, where $\xi^c = 0$ corresponds to exponential discounting. To discipline the common ν, we calibrate it jointly with $\bar{\beta}^{US}$, ξ^{US}, and Υ to minimize the squared distance to the US profile of wealth by age and the distribution of bequests, subject to the constraint of precisely matching the US aggregate wealth-to-GDP ratio.[50] For all other countries, we set $\bar{\beta}^c$ and ξ^c to fit the profile of wealth by age, again subject to the constraint of exactly matching the wealth-to-GDP ratio.

Appendix E.4 presents our calibration outcomes. The model fits the data's labor and wealth profiles well. This implies in particular that the model's long-run compositional effects $\Delta^{comp,c}$ are always within a couple of log points of that of the data (see table A.12).

4.4 Simulations and results

The steady-state calibration pins down the individual parameters, the production parameters, and the initial state of all economies. To study the effect of demographic change, we feed in paths for demographic variables from the UN World Population Prospects for 2016 to 2100, assuming a smooth transition to a long-run world demographic steady state from 2100 onwards.[51] We are interested in how wealth levels, rates of return, and net foreign asset positions evolve, and how this evolution relates to our findings from section 3.

Formally, we assume that the world economy has reached a stationary equilibrium in 2300 and we solve for the transition between 2016 and 2300. We hold preferences and the aggregate production function constant, but let government policy instruments change over time as aging creates fiscal shortfalls that need to be compensated. In our main specification, we assume that the retirement schedule ρ_{jt} in all countries shifts upward by one month per year over the first 60 years of the simulation (in line with CBO's projection for the US), and that the government operates a fiscal rule that keeps the debt-to-output ratio constant by relying equally on tax increases, benefit cuts, and government consumption

[50] Appendix E.4 reports the bequest distribution against the data from Hurd and Smith (2002).

[51] See appendix E.5 for details about our assumptions for the demographic transition after 2100.

Table 3: Extended model vs. baseline sufficient statistic results: 2016–2100

	Δr	$\overline{\Delta \log \frac{W}{Y}}$	$\bar{\Delta}^{comp}$	$\bar{\Delta}^{soe}$	$\bar{\epsilon}^d$	$\bar{\epsilon}^s$
Extended model	-1.24	12.3	32.2	33.6	19.7	8.3
Sufficient statistic analysis	-1.07	8.9	31.7	31.7	21.2	8.3
From sufficient statistic to extended model						
+ Drop annuitization, add bequests	-1.16	12.8	32.1	32.1	17.1	8.3
+ Adjust bequests received	-1.36	15.2	32.1	42.2	23.2	8.3
+ Add income risk	-1.41	15.5	32.2	38.3	18.2	8.3
+ Change perceived mortality	-1.43	14.6	32.2	40.5	20.7	8.3
+ Increase retirement age	-1.25	12.8	32.2	35.0	20.4	8.3
+ Change taxes and transfers (= extended)	-1.24	12.3	32.2	33.6	19.7	8.3
Alternative fiscal rules						
Only lower expenditures	-1.25	12.8	32.2	35.0	20.4	8.3
Only higher taxes	-0.96	8.3	32.2	23.2	17.4	8.3
Only lower benefits	-1.49	15.4	32.2	42.7	20.7	8.3

Notes: Δr, $\overline{\Delta \log \frac{W}{Y}}$, $\bar{\Delta}^{comp}$, and $\bar{\Delta}^{soe}$ denote the changes in the model simulation between 2016 and 2100, with Δr reported in percentage points and the other three reported in percent ($100 \cdot \log$).

reductions.

Table 3 reports the simulation results for Δr and $\overline{\Delta \log W/Y}$, together with the corresponding average compositional effect $\bar{\Delta}^{comp}$, the average small open economy effect $\bar{\Delta}^{soe}$, and the average asset demand and supply semielasticities $\bar{\epsilon}^d$ and $\bar{\epsilon}^s$.[52] We present the results from our extended model on the first line and the sufficient statistic analysis for the same σ and η on the second line as a point of comparison.

Changes in r. The interest rate decline in the extended model is $\Delta r = -1.24$pp, mildly larger than -1.07pp in the sufficient statistic analysis. The larger decline is partly due to the first-order approximation from proposition 5 under-predicting the true change: it gives $\Delta r = -\frac{\bar{\Delta}^{soe}}{\bar{\epsilon}^d + \bar{\epsilon}^s} = -1.20$pp. The remaining difference reflects the combined effect of $\bar{\Delta}^{soe}$ being slightly larger than the data compositional effect $\bar{\Delta}^{comp}$ (33.6 vs. 31.7), and the extended model having a slightly lower $\bar{\epsilon}^d$ (19.7 vs. 21.2).[53]

[52]Here, $\bar{\Delta}^{comp}$ is calculated as in section 3, and we construct $\bar{\Delta}^{soe}$ by simulating the model for each country given a fixed r_0. For each country, the semielasticities $\epsilon^{d,c}$ and $\epsilon^{s,c}$ are obtained by perturbing r at a small open economy steady state constructed with 2100 demographics, and calculating the effect on steady-state W/Y and K/Y.

[53]The $\bar{\epsilon}^s$ is identical since it is a function of external parameters and moments that are targeted in the calibration. The extended model's $\bar{\Delta}^{soe}$ is higher in part because the model has a slightly higher $\bar{\Delta}^{comp}$ than in the data (32.2 vs. 31.7); the two numbers are close because the model targets a least-squares fit to empirical age-wealth profiles, but not identical because the model does not precisely hit those profiles.

The subsequent lines decompose the extended model results in terms of successive modifications of the baseline model that underlies the sufficient statistic results. The first four modifications all make the decline in r larger, while the last two make it smaller.

We first modify the baseline model minimally by removing annuitization and introducing bequest utility as in (22), with bequests given at death.[54] Still, all household problem inputs remain fixed at 2016 levels during the transition: bequest amounts stay constant despite rising old-to-young ratios; government maintains fixed taxes, transfers, and retirement ages; and households' savings decisions ignore falling mortality rates.[55] Under these assumptions, $\bar{\Delta}^{comp} = \bar{\Delta}^{soe}$, as asset profiles remain constant for fixed interest rates.

The next line adjusts bequests received to match bequests given throughout the transition, not just in steady state. This significantly raises $\bar{\Delta}^{soe}$ through a "bequest concentration" effect, as bequests are distributed among fewer young people. This change also increases the $\bar{\epsilon}^{d}$, as savings responses to r persist across generations. Adding income risk on the next line reduces both $\bar{\Delta}^{soe}$ and, more substantially, $\bar{\epsilon}^{d}$. This occurs because income risk induces a buffer-stock behavior that shortens horizons, leading to faster spend-downs of inheritances and smaller responses to r. These three changes combined result in rates falling more steeply than in the sufficient statistic analysis: -1.41pp versus -1.07pp.

The next step is to have individuals perceive the true declining path of mortality when making their savings decisions, rather than having them assume a fixed 2016 mortality profile. This increases $\bar{\Delta}^{soe}$, since individuals save more for the now-higher probability of old age, but also increases $\bar{\epsilon}^{d}$, since a longer horizon expands the scope for intertemporal substitution. The net effect is a slightly larger fall in r, at -1.43pp rather than -1.41pp.

The small effect of mortality might be surprising, especially in light of standard life-cycle models, which often find that mortality declines are the primary demographic driver of falling interest rates (Krueger and Ludwig, 2007, Carvalho et al., 2016, Bielecki, Brzoza-Brzezina and Kolasa, 2020, Papetti, 2021a). In part, this is because the mortality effect documented in those papers generally includes the effect of lower mortality increasing the share of old people, which is compositional and already included in our results. It is also, however, because our model has a relatively muted response of savings to longevity.

The standard life-cycle story on mortality and savings is that assets are held to finance consumption during old age—and when old age becomes longer, people hold more assets. But life-cycle models struggle to match the limited decumulation of assets in old age (De Nardi, French and Jones 2016), and when we calibrate our model to hit the empirical

[54]This change includes adopting the extended model's calibration approach, using bequest utility parameters and a quadratic log discount factor path instead of arbitrary age-by-age discount factors as in the baseline.

[55]Formally, we assume that the government confiscates a share of bequests to keep age- and type-specific bequest receipts constant over the transition, adjusting government consumption to balance its budget.

wealth-by-age profile, the calibration leans heavily on bequest motives.[56] Even after the mortality decline, people still die and leave bequests with probability one, and pushing that date later in life does not dramatically change the incentive to save. In appendix E.6, we show that perceived mortality can play a larger role if there is no bequest motive and households only save for consumption purposes, but that this comes at the cost of a poor fit to the age-wealth profile.

The next adjustment is a 5-year increase in the retirement age, included as a reduced form way to capture individual and policy responses to longevity. This change pushes out the age at which households stop working, and instead receive benefits, by five years. The effect is to shrink the decline in r substantially, from -1.43pp to -1.25pp. This is because later retirement attenuates the decline in labor supply from an aging population, causing the denominator of wealth-to-GDP to fall by less, so that $\bar{\Delta}^{soe}$ is smaller.

Finally, we let the government raise taxes and cut retirement benefits to close the deficits from population aging, rather than just cutting G, giving us the full extended model. This last step does little to Δr. However, this is not because fiscal adjustment is irrelevant—indeed, a one-sided adjustment can matter a great deal. We show this in the final three lines of the table, where we modify the extended model by assuming that the government closes deficits along the transition entirely through lower consumption G, higher taxes τ or cuts to social security $\bar{d}$, rather than relying on all three equally. The first case is identical to the model two lines above, prior to adjusting taxes and transfers. When adjustment takes place only through higher taxes, however, workers have fewer resources to save, and $\bar{\Delta}^{soe}$ is far smaller, leading to a smaller Δr. When adjustment takes place only through lower social security benefits, workers are forced to save more for retirement, and the opposite happens.[57]

Changes in W/Y. Consistent with the formula $\overline{\Delta \log \frac{W}{Y}} \approx -\bar{\epsilon}^s \Delta r$ from proposition 5, the variation in $\overline{\Delta \log \frac{W}{Y}}$ across the rows of table 3 closely parallels the variation in Δr, showing that the two are driven by similar mechanisms. Similarly, the larger Δr in the model also translates into a larger $\overline{\Delta \log \frac{W}{Y}}$, explaining a third of the difference between the model and sufficient statistic analysis. Most of the remaining difference comes from there being a negative correlation between projected population growth and initial NFAs. Appendix D.1 augments the sufficient statistic analysis to allow for initial NFA positions, and explains how this correlation increases $\overline{\Delta \log \frac{W}{Y}}$, contributing an extra 1.4 log points.

[56]These bequest motives play a similar role to medical expenditures required near the end of life, also emphasized by a large literature including De Nardi et al. (2010).

[57]The importance of fiscal adjustment choices for macroeconomic outcomes has been discussed in the pension reform literature (see, for example, Feldstein 1974, Auerbach and Kotlikoff 1987, and Kitao 2014).

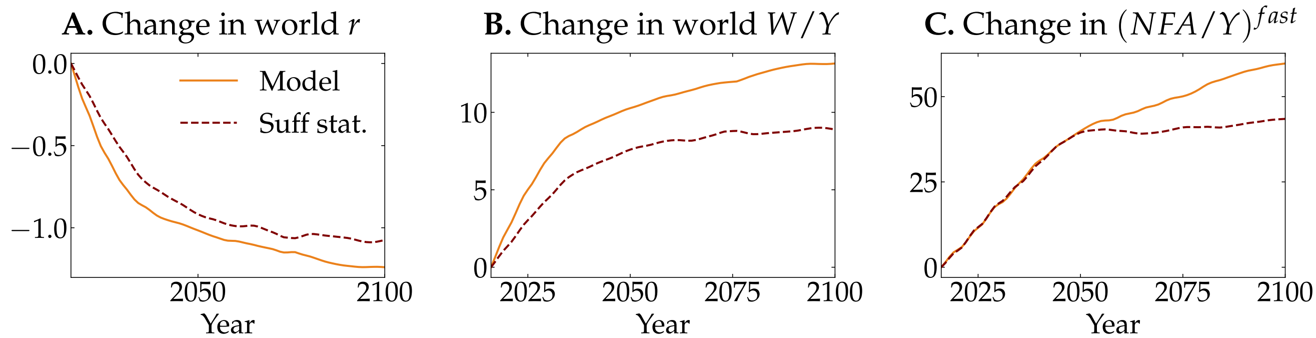

Figure 8: Transition dynamics for rates of return, wealth & NFAs in fast-aging countries

Notes: This figure presents the model change in world interest rate and wealth-to-GDP between 2016 and 2100. Solid lines correspond to simulations from our extended model, dashed lines to the sufficient statistic formulas $\Delta r = -\frac{\bar{\Delta}_t^{comp}}{\bar{\epsilon}^d + \bar{\epsilon}^s}$ and $\frac{W_0}{Y_0}\overline{\Delta \log W/Y} = \frac{W_0}{Y_0}\frac{\bar{\epsilon}^s}{\bar{\epsilon}^d + \bar{\epsilon}^s}\bar{\Delta}_t^{comp}$. The rate of return r is in pp, $\log W/Y$ is in log points, and NFA in percent of GDP.

Transition dynamics and changes to net foreign asset positions. We display the full transition paths of world r and W/Y in figure 8. To test how well the long-run sufficient statistic formulas in propositions 2–3 work at different horizons, we apply them at each date t through 2100, combining the time-varying compositional effects $\bar{\Delta}_t^{comp}$ with the semielasticities $\bar{\epsilon}^d$ and $\bar{\epsilon}^s$, with $\bar{\epsilon}^d$ recalculated using the age distribution at t.

For r and W/Y, the fit is similar throughout the transition: the change in world r is consistently a bit smaller in the sufficient statistic analysis than in the full model, and the change in average W/Y is consistently smaller by a larger proportion, in line with table 3.

Next, panel C illustrates the dynamics of global imbalances by plotting the change in the combined NFA/Y of "fast-aging" countries, defined as those with an above-median compositional effect. The sufficient statistic line is based on the demeaned compositional effect $\Delta_t^{comp,c} - \bar{\Delta}_t^{comp}$, as in figure 7. We see that it captures the first half of the transition in the extended model very well, but it falls behind the model in the later 21st century. Appendix figure A.15 further plots the relationship between compositional and extended model changes in NFA until 2100 across all countries, showing that NFA predictions based on Δ^{comp} are strongly correlated to model NFAs, and that predictions based on demeaned Δ^{soe}, which accounts for non-compositional effects (but not heterogeneity in ϵ^s and ϵ^d), are almost identical to model NFAs.[58]

[58]The extended model's larger global imbalances in the second half of the 21st century arise from a large non-compositional effect $\Delta^{soe} - \Delta^{comp}$ in India, which ages rapidly during that period.

5 Demographic change and savings rates

So far, we have analyzed demographics through the lens of stocks: wealth, capital, and net foreign asset positions. An alternative perspective is to focus on flows: savings, investment, and the current account.

The flow perspective has a long tradition in the literature on aging.[59] One key observation in this literature is that the savings rate is hump-shaped in age, so that as the population continues to age, the aggregate savings rate eventually declines. Observers have made various macroeconomic predictions based on this effect: that aging will raise interest rates (see the Summers quote in the introduction or Lane 2020), decrease standards of living by impairing capital accumulation (Bloom, Canning and Fink 2010), or exert inflationary pressure as the number of consumers increases relative to the number of producers (Goodhart and Pradhan 2020). Using a shift-share on savings, Mian et al. (2021) argued that demographics were unimportant for interest rate trends in the post-war United States.

These predictions are not borne out in our analysis. Instead, we find that aging unambiguously lowers the real interest rate, thereby increasing capital intensity and output.[60] A lower real interest rate also implies *less* inflationary pressure in any standard model in which this pressure is captured by the natural interest rate.[61]

To unpack this apparent contradiction, we return to our baseline model of section 2. We first show that this model also predicts a negative effect of aging on savings rates going forward, in line with the literature discussed above. To do this, we note that the aggregate net private savings rate in a small open economy satisfies

$$\frac{S_t}{Y_t} \propto \frac{\sum_j \pi_{jt} s_{j0}}{\sum_j \pi_{jt} h_{j0}}, \tag{24}$$

where $s_{jt} = \phi_j a_{j+1,t+1} - a_{j,t}$ is average net personal savings by age at date t, which equals total net income from capital, labor, and transfers minus consumption (see appendix F for a proof). Equation (24) shows that holding r constant, changes in the aggregate savings rate are purely determined by compositional forces, just like with wealth-to-GDP.[62] Figure 9

[59] See, e.g., Summers and Carroll (1987), Auerbach and Kotlikoff (1990), Bosworth et al. (1991), Higgins (1998), Lane (2020), and Mian et al. (2021).

[60] GDP per person may still decline overall if the workforce composition effect overwhelms capital deepening, but this is a separate channel that does not go through savings, as is clear from equation (8).

[61] That is, in a version of our model with nominal rigidities, if monetary policy does not fully accommodate the natural rate decline by lowering the intercept of its Taylor rule, actual inflation will decline.

[62] While we could in principle perform this calculation using measured savings rates by age, we prefer instead to calculate (24) using cross-sectional profiles of assets and income alone. This avoids the amplification of measurement error that stems from taking the difference between two large quantities, disposable income and consumption, that are themselves observed with error.

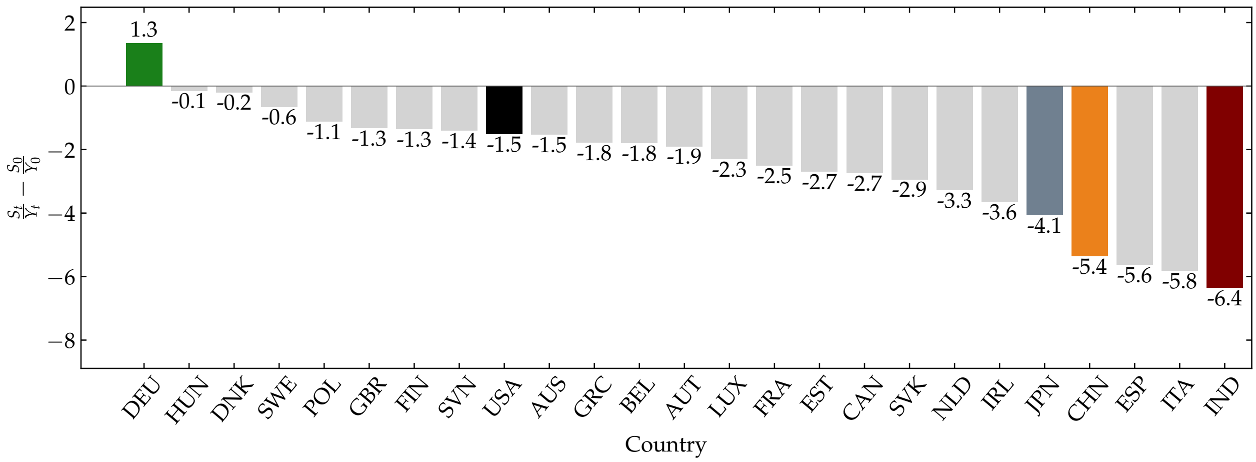

Figure 9: Compositional effect on savings, 2016-2100

Notes: Each bar shows the value of the implied change in the savings-to-GDP ratio from the compositional effect between 2016 and 2100 across countries, reported in pp. Appendix F details the calculation.

shows the resulting compositional effects on savings rates through 2100. These are indeed negative in all countries except for Germany.[63]

In equilibrium, net saving in any period must equal the increase in the stock of assets. On a balanced growth path where K, B, and Y all grow at rate g, we therefore have

$$\frac{S}{Y} = g\frac{K}{Y} + g\frac{B}{Y}, \tag{25}$$

where the left is net savings, and the right represents net investment gK/Y and net public borrowing gB/Y (see appendix F for a derivation). Since $W/Y = K/Y + B/Y$, dividing by g gives the identity $W/Y = (S/Y)/g$ emphasized by Piketty and Zucman (2014).

Panel A of figure 10 illustrates (25). The compositional effect from figure 9 implies a leftward shift in the savings curve. At first glance, this might seem to imply an increase in r, as represented by the hollow circle. However, since demographic change lowers the population growth rate and therefore g, the other curve also shifts left, and the overall effect is a decline in r. This effect is missed by Summers, Lane (2020), and Mian et al. (2021).

To understand this result, it is helpful to consider panel B, which depicts the same situation in the space of stocks. Here, only the asset demand curve shifts—to the right—and the unambiguous implication is a decline in r.[64] But the curves in panel A are identical to

[63] Figure A.17 shows, however, that in many countries, the effect was positive prior to 2016. This gives some support to the common view that aging of baby boomers has pushed up savings in recent decades.

[64] For given r, standard neoclassical theory implies that K/Y is not affected by demographic change. In our model, B/Y is also constant. This is subject to debate, but we note that the effect of demographic change on B/Y could take either sign: if lawmakers hold deficits gB/Y constant, B/Y will rise, but if they hold net payments $(r-g)B/Y$ constant, it will fall.

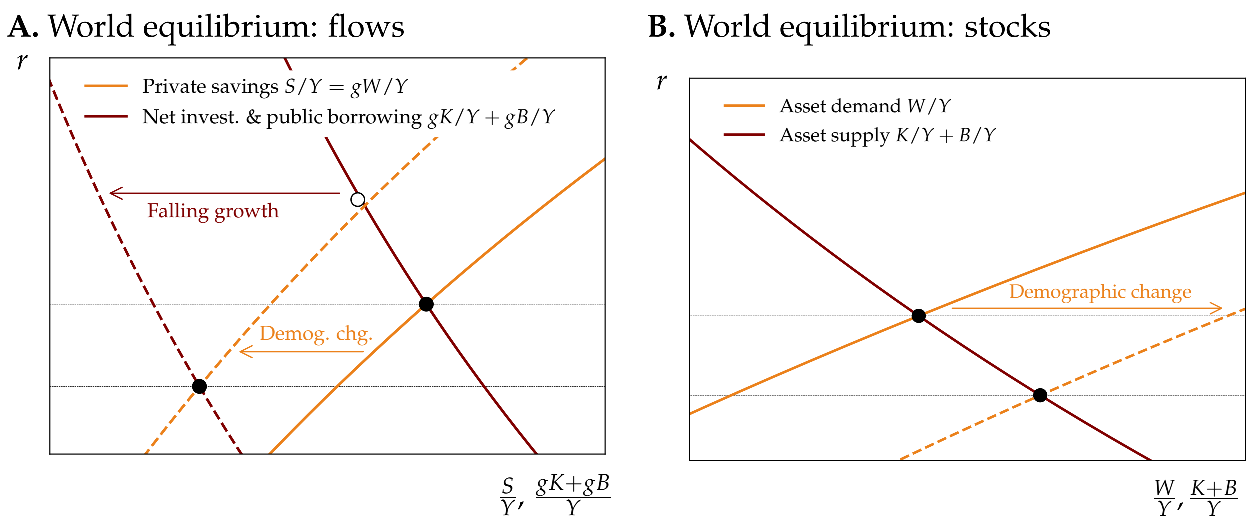

Figure 10: World asset market equilibrium

Notes: This figure represents asset market equilibrium in flow space (panel A) and in stock space (panel B). The growth rate g converts Panel B into Panel A. At given r, demographic change raises W/Y and lowers g.

panel B, just both multiplied by g.[65] Hence, although both curves in panel A shift left, the net investment curve shifts left by more, producing the same decline in r as in panel B.[66]

We conclude that the "flow" view of equilibrium in panel A is in principle just as valid as the "stock" view of equilibrium in panel B, but *only* if we remember the effect of g on net investment. Ignoring this effect in the context of demographic change, which can significantly push down long-run g, may give the wrong sign for the change in r. This example illustrates the value of using a fully specified GE model to guide sufficient statistic analysis.

6 Conclusion

We project out the compositional effect of aging on the wealth-to-GDP ratio of 25 countries until the end of the twenty-first century. This effect is positive, large and heterogeneous across countries. According to our model, this will lead to capital deepening everywhere, falling real interest rates, and rising net foreign asset positions in India and China financed by declining asset positions in the United States.

[65] Net savings-to-GDP is $S_t/Y_t = (W_{t+1} - W_t)/Y_t$. In steady state, this is $S/Y = g(W/Y)$, since $W_{t+1} = (1+g)W_t$. Similarly, net investment-to-GDP is $g(K/Y)$ and net public borrowing-to-GDP is $g(B/Y)$.

[66] Goodhart and Pradhan (2020) acknowledge that investment can also fall in response to demographics, but argue that savings will fall faster than investment. An important part of their argument is that the labor scarcity caused by aging will trigger a rise in labor-saving investment demand. In our model, the capital-labor ratio does increase in equilibrium, but only because the real interest rate falls.

Data availability statement

Data and code replicating the tables, figures, and other results in this paper can be found on Zenodo at the following link: https://doi.org/10.5281/zenodo.19010088.

References

Abel, Andrew B., "Will Bequests Attenuate the Predicted Meltdown in Stock Prices When Baby Boomers Retire?," *Review of Economics and Statistics*, November 2001, *83* (4), 589–595.

—, "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security," *Econometrica*, March 2003, *71* (2), 551–578.

Acemoglu, Daron and Pascual Restrepo, "Secular Stagnation? The Effect of Aging on Economic Growth in the Age of Automation," *American Economic Review Papers and Proceedings*, May 2017, *107* (5), 174–179.

Aguiar, Mark, Manuel Amador, and Cristina Arellano, "Micro Risks and (Robust) Pareto Improving Policies," *American Economic Review*, 2024, *forthcoming.*

Ameriks, John and Stephen P. Zeldes, "How Do Household Portfolio Shares Vary with Age?," *Manuscript*, 2004.

Antunes, Antonio and Valerio Ercolani, "Intergenerational Wealth Inequality: The Role of Demographics," *Banco de Portugal Working Paper*, February 2020.

Attanasio, Orazio P., Sagiri Kitao, and Giovanni L. Violante, "Quantifying the Effects of the Demographic Transition in Developing Economies," *Advances in Macroeconomics*, January 2006, *6* (1), 1–44.

Attanasio, Orazio, Sagiri Kitao, and Giovanni L. Violante, "Global Demographic Trends and Social Security Reform," *Journal of Monetary Economics*, January 2007, *54* (1), 144–198.

Auclert, Adrien and Matthew Rognlie, "Inequality and Aggregate Demand," Working Paper 24280, National Bureau of Economic Research, February 2018.

—, Hannes Malmberg, Matthew Rognlie, and Ludwig Straub, "The Race Between Asset Supply and Asset Demand," *Proceedings of the Jackson Hole Economic Policy Symposium*, November 2025.

Auerbach, Alan J. and Laurence J. Kotlikoff, *Dynamic Fiscal Policy*, Cambridge: Cambridge University Press, April 1987.

— and —, "Demographics, Fiscal Policy, and U.S. Saving in the 1980s and Beyond," *Tax Policy and the Economy*, 1990, *4*, 73–101.

Backus, David, Thomas Cooley, and Espen Henriksen, "Demography and Low-Frequency Capital Flows," *Journal of International Economics*, April 2014, *92*, S94–S102.

Bauluz, Luis and Timothy Meyer, "The Wealth of Generations," *World Inequality Lab working paper no 2024/04*, January 2024.

Bielecki, Marcin, Michał Brzoza-Brzezina, and Marcin Kolasa, "Demographics and the Natural Interest Rate in the Euro Area," *European Economic Review*, October 2020, *129*, 103535.

Bloom, David E., David Canning, and Günther Fink, "Implications of Population Ageing for Economic Growth," *Oxford Review of Economic Policy*, December 2010, *26* (4), 583–612.

—, —, and Jaypee Sevilla, *The Demographic Dividend: A New Perspective on the Economic Consequences of Population Change*, RAND Corporation, February 2003.

Bonfatti, Andrea, Selahattin İmrohoroğlu, and Sagiri Kitao, "Japan and the Allocation Puzzle in an Aging World," *Journal of Economic Dynamics and Control*, December 2022, *145*, 104544.

Bornstein, Gideon, "Entry and Profits in an Aging Economy: The Role of Consumer Inertia," *Manuscript*, November 2021.

Börsch-Supan, Axel, Alexander Ludwig, and Joachim Winter, "Ageing, Pension Reform and Capital Flows: A Multi-Country Simulation Model," *Economica*, June 2006, *73* (292), 625–658.

Bosworth, Barry, Gary Burtless, and John Sabelhaus, "The Decline in Saving: Evidence from Household Surveys," *Brookings Papers on Economic Activity*, 1991, *1991* (1), 183–256.

Brülhart, Marius, Jonathan Gruber, Matthias Krapf, and Kurt Schmidheiny, "Behavioral Responses to Wealth Taxes: Evidence from Switzerland," *American Economic Journal: Economic Policy*, November 2022, *14* (4), 111–150.

Cagetti, Marco, "Interest Elasticity in a Life-Cycle Model with Precautionary Savings," *American Economic Review Papers and Proceedings*, May 2001, *91* (2), 418–421.

Carvalho, Carlos, Andrea Ferrero, and Fernanda Nechio, "Demographics and Real Interest Rates: Inspecting the Mechanism," *European Economic Review*, September 2016, *88*, 208–226.

Cutler, David M., James M. Poterba, Louise M. Sheiner, and Lawrence H. Summers, "An Aging Society: Opportunity or Challenge?," *Brookings Papers on Economic Activity*, 1990, *1990* (1), 1–73.

Domeij, David and Martin Flodén, "Population Aging and International Capital Flows," *International Economic Review*, July 2006, *47* (3), 1013–1032.

Eggertsson, Gauti B., Jacob A. Robbins, and Ella Getz Wold, "Kaldor and Piketty's Facts: The Rise of Monopoly Power in the United States," *Journal of Monetary Economics*, November 2021, *124*, S19–S38.

—**, Neil R. Mehrotra, and Jacob A. Robbins**, "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, January 2019, *11* (1), 1–48.

Evans, Owen J., "Tax Policy, the Interest Elasticity of Saving, and Capital Accumulation: Numerical Analysis of Theoretical Models," *American Economic Review*, June 1983, *73* (3), 398–410.

Feldstein, Martin, "Social Security, Induced Retirement, and Aggregate Capital Accumulation," *Journal of Political Economy*, September 1974, *82* (5), 905–926.

—, "The Welfare Cost of Capital Income Taxation," *Journal of Political Economy*, April 1978, *86* (2), S29–S51.

Gagnon, Etienne, Benjamin K. Johannsen, and David López-Salido, "Understanding the New Normal: The Role of Demographics," *IMF Economic Review*, March 2021.

Geanakoplos, John, Michael Magill, and Martine Quinzii, "Demography and the Long-Run Predictability of the Stock Market," *Brookings Papers on Economic Activity*, 2004, *2004* (1), 241–307.

Gechert, Sebastian, Tomas Havranek, Zuzana Irsova, and Dominika Kolcunova, "Measuring Capital-Labor Substitution: The Importance of Method Choices and Publication Bias," *Review of Economic Dynamics*, 2022, *45*, 55–82.

Gomme, Paul, B Ravikumar, and Peter Rupert, "Secular stagnation and returns on capital," *Federal Reserve Bank of St. Louis Economic Synopses*, 2015, (19).

Goodhart, Charles and Manoj Pradhan, *The Great Demographic Reversal: Ageing Societies, Waning Inequality, and an Inflation Revival*, 1st ed., Cham, Switzerland: Palgrave Macmillan, August 2020.

Gourinchas, Pierre-Olivier and Hélène Rey, "International Financial Adjustment," *Journal of Political Economy*, August 2007, *115* (4), 665–703.

Havránek, Tomáš, "Measuring Intertemporal Substitution: The Importance of Method Choices and Selective Reporting," *Journal of the European Economic Association*, December 2015, *13* (6), 1180–1204.

Henriksen, Espen R., "A Demographic Explanation of U.S. and Japanese Current Account Behavior," *Manuscript*, December 2002.

Higgins, Matthew, "Demography, National Savings, and International Capital Flows," *International Economic Review*, 1998, *39* (2), 343–369.

Hurd, Michael and James P. Smith, "Expected Bequests and Their Distribution," Working Paper 9142, National Bureau of Economic Research, September 2002.

İmrohoroğlu, Ayşe, Selahattin İmrohoroğlu, and Douglas H. Joines, "A Life Cycle Analysis of Social Security," *Economic Theory*, February 1995, *6* (1), 83–114.

Jaimovich, Nir and Henry E. Siu, "The Young, the Old, and the Restless: Demographics and Business Cycle Volatility," *American Economic Review*, June 2009, *99* (3), 804–826.

Jakobsen, Katrine, Kristian Jakobsen, Henrik Kleven, and Gabriel Zucman, "Wealth Taxation and Wealth Accumulation: Theory and Evidence From Denmark," *Quarterly Journal of Economics*, February 2020, *135* (1), 329–388.

Jones, Callum, "Aging, Secular Stagnation, and the Business Cycle," *The Review of Economics and Statistics*, November 2023, *105* (6), 1580–1595.

Jones, Charles I., "The End of Economic Growth? Unintended Consequences of a Declining Population," *American Economic Review*, November 2022, *112* (11), 3489–3527.

Karabarbounis, Loukas and Brent Neiman, "The Global Decline of the Labor Share," *Quarterly Journal of Economics*, February 2014, *129* (1), 61–103.

Kitao, Sagiri, "Sustainable Social Security: Four Options," *Review of Economic Dynamics*, October 2014, *17* (4), 756–779.

Kleven, Henrik and Esben Schultz, "Estimating Taxable Income Responses Using Danish Tax Reforms," *American Economic Journal: Economic Policy*, November 2014, *6* (4), 271–301.

Kopecky, Joseph and Alan M. Taylor, "The Savings Glut of the Old: Population Aging, the Risk Premium, and the Murder-Suicide of the Rentier," Working Paper 29944, National Bureau of Economic Research April 2022.

Kotlikoff, Laurence J., Kent Smetters, and Jan Walliser, "Privatizing Social Security in the United States—Comparing the Options," *Review of Economic Dynamics*, July 1999, *2* (3), 532–574.

Krueger, Dirk and Alexander Ludwig, "On the Consequences of Demographic Change for Rates of Returns to Capital, and the Distribution of Wealth and Welfare," *Journal of Monetary Economics*, January 2007, *54* (1), 49–87.

Krusell, Per and Anthony A. Smith, "Is Piketty's "Second Law of Capitalism" Fundamental?," *Journal of Political Economy*, August 2015, *123* (4), 725–748.

Kuhn, Moritz, Moritz Schularick, and Ulrike I. Steins, "Income and Wealth Inequality in America, 1949–2016," *Journal of Political Economy*, March 2020, *128* (9), 3469–3519.

Lane, Philip R and Gian Maria Milesi-Ferretti, "The external wealth of nations revisited: international financial integration in the aftermath of the global financial crisis," *IMF Economic Review*, 2018, *66* (1), 189–222.

Lane, Phillip, "International Inflation Co-Movements," *Speech at the Inflation: Drivers and Dynamics 2020 Online Conference*, May 2020, *Federal Reserve Bank of Cleveland/European Central Bank.*

Lee, Ronald Demos and Andrew Mason, *Population Aging and the Generational Economy: A Global Perspective*, Edward Elgar Publishing, 2011.

Lisack, Noemie, Rana Sajedi, and Gregory Thwaites, "Population Aging and the Macroeconomy," *International Journal of Central Banking*, June 2021, *17* (2).

Ludwig, Alexander, Thomas Schelkle, and Edgar Vogel, "Demographic Change, Human Capital and Welfare," *Review of Economic Dynamics*, January 2012, *15* (1), 94–107.

Maestas, Nicole, Kathleen J. Mullen, and David Powell, "The Effect of Population Aging on Economic Growth, the Labor Force, and Productivity," *American Economic Journal: Macroeconomics*, April 2023, *15* (2), 306–332.

Mankiw, N. Gregory and David N. Weil, "The Baby Boom, the Baby Bust, and the Housing Market," *Regional Science and Urban Economics*, May 1989, *19* (2), 235–258.

McGrattan, Ellen R. and Edward C. Prescott, "Unmeasured Investment and the Puzzling US Boom in the 1990s," *American Economic Journal: Macroeconomics*, October 2010, *2* (4), 88–123.

Mian, Atif R., Ludwig Straub, and Amir Sufi, "What Explains the Decline in r*? Rising Income Inequality Versus Demographic Shifts," *Jackson Hole Conference Proceedings*, 2021, *2021-104.*

Moll, Benjamin, Lukasz Rachel, and Pascual Restrepo, "Uneven Growth: Automation's Impact on Income and Wealth Inequality," *Econometrica*, 2022, *90* (6), 2645–2683.

Nardi, Mariacristina De, "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, 2004, *71* (3), 743–768.

—, Eric French, and John B. Jones, "Why Do the Elderly Save? The Role of Medical Expenses," *Journal of Political Economy*, February 2010, *118* (1), 39–75.

—, —, and John Bailey Jones, "Savings After Retirement: A Survey," *Annual Review of Economics*, October 2016, *8* (Volume 8, 2016), 177–204.

—, —, —, and Rory McGee, "Why Do Couples and Singles Save during Retirement? Household Heterogeneity and its Aggregate Implications," Working Paper 28828, National Bureau of Economic Research, July 2023.

—, Selahattin Imrohoroğlu, and Thomas J. Sargent, "Saving and Pension Reform in General Equilibrium Models," *Oxford Review of Economic Policy*, March 2001, *17* (1), 20–39.

Oberfield, Ezra and Devesh Raval, "Micro Data and Macro Technology," *Econometrica*, 2021, *89* (2), 703–732.

Papetti, Andrea, "Demographics and the Natural Real Interest Rate: Historical and Projected Paths for the Euro Area," *Journal of Economic Dynamics and Control*, November 2021, *132*, 104209.

—, "Population Aging, Relative Prices and Capital Flows Across the Globe," *Bank of Italy Working Paper No. 1333*, April 2021.

Penn Wharton Budget Model, "Penn Wharton Budget Model: Dynamic OLG Model," *Technical Report*, May 2019.

Peruffo, Marcel and Josef Platzer, "Secular Drivers of the Natural Rate of Interest in the United States: A Quantitative Evaluation," *Manuscript*, August 2024.

Peters, Michael and Conor Walsh, "Population Growth and Firm-Product Dynamics," *Manuscript*, June 2022.

Piketty, Thomas, *Capital in the Twenty-First Century*, 1st ed., Cambridge, MA: Belknap Press, March 2014.

— and Gabriel Zucman, "Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010," *Quarterly Journal of Economics*, August 2014, *129* (3), 1255–1310.

Poterba, James M., "Demographic Structure and Asset Returns," *Review of Economics and Statistics*, November 2001, *83* (4), 565–584.

Rachel, Lukasz and Lawrence H. Summers, "On Secular Stagnation in the Industrialized World," *Brookings Papers on Economic Activity*, 2019, pp. 1–54.

— and Thomas D. Smith, "Are Low Real Interest Rates Here to Stay?," *International Journal of Central Banking*, September 2017, *13* (3), 1–42.

Rognlie, Matthew, "Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity?," *Brookings Papers on Economic Activity*, 2015, pp. 1–54.

Rubin, Willa, "The Life and Possible Death of Low Interest Rates: Interview with Olivier Blanchard and Larry Summers [Planet Money Podcast]," April 2023.

Sabelhaus, John and Alice Henriques Volz, "Are Disappearing Employer Pensions Contributing to Rising Wealth Inequality?," *FEDS Notes No 2019-02-01*, February 2019.

Saez, Emmanuel and Stefanie Stantcheva, "A Simpler Theory of Optimal Capital Taxation," *Journal of Public Economics*, June 2018, *162*, 120–142.

Sposi, Michael, "Demographics and the Evolution of Global Imbalances," *Journal of Monetary Economics*, March 2022, *126*, 1–14.

Straub, Ludwig, "Consumption, Savings, and the Distribution of Permanent Income," *Manuscript*, June 2019.

Summers, Lawrence H., "Capital Taxation and Accumulation in a Life Cycle Growth Model," *American Economic Review*, September 1981, *71* (4), 533–544.

— **and Christopher D. Carroll**, "Why is U.S. National Saving so Low?," *Brookings Papers on Economic Activity*, 1987, *1987* (2), 607–642.

Zoutman, Floris T., "The Elasticity of Taxable Wealth: Evidence from the Netherlands," *Manuscript*, November 2018.