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Recruiting Talent

SIMON BOARD
Department of Economics, UCLA
MORITZ MEYER-TER-VEHN
Department of Economics, UCLA
TOMASZ SADZIK
Department of Economics, UCLA

History TBD

We study a parsimonious model of a competitive labor market in which firms privately screen workers to identify talent. The equilibrium exhibits dispersion in wages and productivity; when talent is scarce, firms with superior screening skills post higher wages, attract better applicants, and recruit more talented workers. High-wage firms impose a compositional externality on low-wage firms, leading to equilibrium inefficiency: Welfare would be higher if low-skilled firms posted high wages and selected first. We also provide a micro-foundation for firms heterogeneous screening skills. When talented workers are better at screening (e.g. via superior referrals), a dynamic version of the economy converges to a unique steady state in which differences in talent, profits and screening skills persist forever.

Key words: Adverse selection, assortative matching, firm dynamics, labor market competition, productivity dispersion, recruitment, referrals, wage dispersion. JEL codes: D21, D82, M51.

1. INTRODUCTION

The success of most firms is built upon hundreds of individuals who take thousands of decisions, making it critical to identify and recruit the best talent. For example, the Netflix human resource manual states “One outstanding employee gets more done and costs less than two adequate employees. We endeavor to have only outstanding employees” (Hastings and McCord, 2009). Similarly, Google’s former head of human resources writes “Hiring is the single most important people activity in any organization. [...] Our greatest single constraint on growth has always, always been our ability to find great people” (Bock, 2015). Within economics, there is a large literature that measures the importance of employee talent, from top executives (Bertrand and Schoar, 2003) to blue-collar workers (Lazear, 2000), and from salespeople (Benson et al., 2019) to bureaucrats (Fenizia, 2022).

The market for talent is plagued by imperfect information.[1] As a result, firms carefully screen applicants, obtaining personal referrals and conducting interviews. In Behrenz’s

1. For example, economists have examined how measures of ability filter into wages (Farber and Gibbons (1996), Fredriksson et al. (2018)) and how certification affects wages and employment (Stanton and Thomas (2015), Pallais (2014)).

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(2001) survey, employers report that their most important source of information when hiring are methods of "private screening" in the form of interviews (41%) and personal contacts (25%), as compared to "public information" like references from past employers (21%), references from schools (5%) and the application (3%). Indeed, referrals account for over a third of US jobs (Holzer, 1987) and more than half the jobs at Google (Bock, 2015).

Firms differ in their skill at screening applicants. An exemplar is Google that is said to have "built the world's first self-replicating talent machine" by turning "every employee into a recruiter" (Bock, 2015). On the flip side, VW struggled to copy Tesla's battery operations because of "difficulty hiring qualified engineers in fields it knew very little about," where one recruiter noted "you can only become an expert if you do it yourself."[2]

We capture these ideas with a parsimonious model of a competitive labor market, where heterogeneously skilled firms privately screen workers (e.g. interviews, referrals). Equilibrium gives rise to dispersion in wages and productivity across firms; when talent is scarce, firms with superior screening skills complement them by posting high wages that attract better applicants. We show the market is inefficient in that equilibrium minimizes firms' aggregate effectiveness in sorting talented workers into the industry. Motivated by the Google and VW examples, we also endogenize firms' screening skills by considering a dynamic model in which firms with more talented employees are more skilled at recruiting. The economy converges to a steady state featuring persistent dispersion in talent, screening skills, and profits.

In Section 2, we introduce a static model of labor market competition in which firms have private information about workers' talent. Specifically, a continuum of firms competes for a continuum of workers who have high or low ability. Talented workers have positive value added, while untalented workers would be better employed outside the industry. Firms attract applicants by posting wages and then receive independent noisy signals about each applicant, with more skilled firms obtaining more accurate signals. The market is non-exclusive (workers can apply to all firms) and anonymous (firms do not know anything about their applicants, other than the fact they apply). We model this by assuming the highest-paying firm chooses from all workers while other firms hire from the remaining, adversely selected pool of workers. The applicant pool quality thus endogenously declines with the wage rank, giving rise to equilibrium dispersion of wages and productivity. Assuming that talent is scarce, we show that firm-worker matching is *positive assortative*. That is, firms with higher recruiting skills post higher wages, attract better applicants, and hire more talented recruits. Intuitively, skilled firms have a comparative advantage in hiring from a high-wage applicant pool with a balance of talented workers, rather than hiring from a low-wage pool in which few talented applicants remain.

The model speaks to important features of labor markets. In equilibrium, wages and productivity are dispersed across firms and positively correlated (e.g. Card et al. (2018)). The model describes how talent is allocated across firms and how wages differ from workers' marginal product (e.g. Card et al. (2013)).[3] It also generates mismatch, with

2. See "To Beat Tesla, Volkswagen Bets on Making Its Own EV Batteries," *Wall Street Journal*, 8th Aug 2021.

3. The typical way to identify worker talent is via the "AKM-decomposition" of wage dispersion into worker- and firm-effects (Abowd et al., 1999). However, this approach is problematic when firms

some untalented workers employed and other, talented workers excluded (e.g. Fredriksson et al. (2018)). Firms with better management pay higher wages, recruit better workers, and have higher productivity (e.g. Bender et al. (2018)). The model also predicts that an increase in screening skills (e.g. due to improved job tests and referrals) increases the dispersion of productivity and wages, as well as segregation, with talented workers increasingly working together, consistent with recent trends (e.g. Barth et al. (2016), Song et al. (2019)).

In Section 3, we examine the welfare consequences of the model. In contrast to classic matching models (e.g. Shapley and Shubik (1971), Becker (1973)), the positive assortative matching seen in equilibrium is inefficient. Intuitively, high-wage firms screen applicants first and exert a negative *compositional externality* on low-wage firms by extracting talent from the applicant pool. Positive assortative matching maximizes this externality since the high-wage firms are skilled at extracting talent; this outweighs the private gain from positive assortative matching. Indeed, we show that negative assortative matching, whereby low-skill firms offer high wages and screen first, minimizes the externality and optimally selects talent into the industry. Standard labor market policies (e.g. minimum wages, progressive taxes) do not help restore efficiency, but a wage cap can induce a random screening order and raise welfare.

In Section 4, we endogenize firms' recruiting skills in a dynamic version of the model based on the premise that firms with more talented employees are more skilled at recruiting.[4] This is seen in Gupta (2018), where more productive sales managers hire more productive salespeople who remain highly productive even after they switch teams. It is also consistent with Waldinger (2012, 2016)'s finding that the loss of star professors in Nazi Germany led to a permanent reduction in the quality of hires.

We show that persistent differences in firms' talent and recruiting skills arise endogenously in this dynamic model. The talent of each firm evolves as workers retire and today's recruits become tomorrow's recruiters. Assuming scarce talent, the unique equilibrium has talented firms post high wages, attract the best applicants, and hire talented recruits, reinforcing their initial advantage. If all firms start off with similar talent, the better endowed accumulate talent over time, while the worse endowed hire from poor, deteriorating applicant pools and lose talent. The economy converges to a steady state with persistent talent differentials, balancing two countervailing forces: imperfect screening which leads to mean reversion and equalizes firms, and positive assortative matching that amplifies differences across firms. While low-quality firms could in principle catch up by posting higher wages and hiring more talented workers, it is not profitable for them to do so. Thus, talent becomes a source of sustainable competitive advantage. Since steady state talent differences are sustained by firms' endogenous wage choices, they arise for any degree of correlation between productivity and recruiting skills, however weak.

imperfectly observe workers' ability. For example, in our model, identical firms can systematically pay different wages; differences between workers thus look like firm-effects.

4. There are two natural mechanisms for talented employees to be better at recruiting. First, they may provide better referrals, as is seen in the field experiments of Beaman and Magruder (2012) and Pallais and Sands (2016). Second, talented employees may be better at screening, as illustrated by the famous Dunning-Kruger effect: "The skills you need to produce a right answer are exactly the skills you need to recognize what a right answer is" ("The Anosognosic's Dilemma: Something's Wrong but You'll Never Know What It Is," *New York Times*, 10th June, 2010).

1.1. *Literature*

The static model of sequential screening is most closely related to Broecker (1990), Montgomery (1991), and Kurlat (2016). Broecker considers a version of our model with a finite number of homogeneous firms with unlimited vacancies. As in our model, adverse selection induces wage dispersion. In Montgomery's classic model of referrals, each talented employee refers one positively selected applicant; firms then make applicant-specific wage offers. In equilibrium, competition for referred workers induces wage dispersion as in Burdett and Judd (1983); firms that strike out on referred workers pay deterministic low wages in the non-referred market. Neither of these papers study our key questions of firm-worker matching and its inefficiency.

Like us, Kurlat (2016) studies non-exclusive markets where heterogeneous buyers screen sellers of uncertain quality. In Kurlat's financial markets, sellers' assets are *perfectly divisible* and, in his baseline model, buyers' signals about the assets are *nested*, in that a more informed buyer knows everything that a less informed buyer knows. In our labor market, workers look for one, *indivisible* job and firms' signals are (conditionally) *independent*.[5] These modeling assumptions are crucial. Our firms post different wages, low-wage firms are subject to adverse selection, and matching is positive assortative and inefficient; in contrast, Kurlat's buyers purchase at the same price, avoid adverse selection, and matching is negative assortative and efficient. When assets are divisible, sellers can predict how much they will sell on any given market, allowing them to simultaneously cross-list their assets across markets with no market facing an adversely selected left-over pool of assets. When signals are nested, high-skill firms can screen out all applicants who failed tests of low-skill firms and so do not mind hiring last. We elaborate on these differences in Appendix A and argue that, whenever goods exhibit some indivisibility and signals exhibit some independence, markets cannot clear simultaneously, inducing adverse selection and wage dispersion.

Adverse selection has also been introduced into models of random search (Lockwood (1991), Lauermann and Wolinsky (2016), Kaya and Kim (2018)) and directed search (Guerrieri et al. (2010), Li and Shimer (2019)). With random search, selective hiring dilutes the applicant pool at later firms, but firms do not control the order of applications. With directed search, firms do control their applicant pool by posting contract terms, but turned-down workers cannot apply to other firms. We combine firms' control over their applicants with sequential applications into a model of "directed sequential search" that abstracts from search frictions and focuses on information frictions.

Our model complements the wider literature on wage dispersion. In Burdett and Judd (1983) and Burdett and Mortensen (1998), dispersion derives from firms competing for more workers in an economy with search frictions, whereas our dispersion derives from firms competing for better workers in an economy with adverse selection. In many labor markets, the pertinent friction is in evaluating the quality of applicants, rather than finding them in the first place. Van Ours and Ridder (1992) find that "76% of all vacancies are filled by applicants who arrived during an application period that lasts for about 2 weeks", leading them to write that "vacancy durations should be interpreted as selection periods and not as search periods for applicants." This view is consistent

5. In the labor market context, nested signals capture public information such as reference letters, while independent signals capture private information such as personal referrals or interviews. Either modeling assumption seems plausible. Indeed, Kurlat and Scheuer (2021)'s labor market adaptation of Kurlat (2016) to Spencian signaling, studies nested signals for indivisible jobs.

with the extensive evidence of imperfect information in labor markets (e.g. Farber and Gibbons (1996)), the widespread use of referrals (e.g. Holzer (1987)), and the significant amount firms spend on screening candidates (e.g. Barron et al. (1985), Blatter et al. (2012)).[6]

Our heterogeneous-firm model contributes to the literature on firm-worker matching. Becker (1973) observes that more productive firms hire more talented workers when productivity is supermodular. In a dynamic model, Anderson and Smith (2010) and Anderson (2015) suppose agents match each period and evolve as a function of the match; they show that equilibrium is efficient, and derive sufficient conditions for matching to be positive assortative. In contrast to this literature, which focuses on complementary production, our paper focuses on asymmetric information in the hiring process, and shows that complementarities arise endogenously, with more skilled firms posting higher wages. Our model has different implications from Becker. On the normative side, we prove that equilibrium is inefficient; surprisingly, this inefficiency can prevail even with production complementarities (see Section 3.1). On the positive side, we examine how wage dispersion and mismatch depend on screening skills (see Section 2.3 and 3.2).

Our dynamic model provides a theory of firm evolution in which a firm's stock of talent constitutes a sustainable competitive advantage. Most closely related is Montgomery (1991)'s model of referrals in which wage offers are *applicant-specific* and so can depend on the quality of the referral. Each recruiter effectively acts like a separate firm, meaning talent and wages mean-revert over time and differences across firms erode. In our model, wage offers are *firm-specific*. This is consistent with the idea that firms reward employees according to a pay-scale that differs across firms (e.g. Netflix pays more than Disney).[7] In contrast to Montgomery, our dynamic model generates persistent differences in talent and wages; intuitively, high-talent firms pay high wages, so a talented worker hires better recruits if their colleagues are also talented (see Section 4.4).

More broadly, the dynamic model contributes to the firm dynamics literature in which competitive advantage stems from technology (Lucas and Prescott (1971), Hopenhayn (1992)), reputation (Jovanovic (1982)), or the stock of labor (Hopenhayn and Rogerson (1993)). By focusing on talent and recruiting, our paper provides a new channel through which firms can sustain a competitive advantage and gives rise to predictions concerning the inter- and intratemporal relationship between talent, productivity, and wages.

2. COMPETITIVE SCREENING

This section introduces our static model and characterizes equilibrium. Section 2.1 presents the model. Section 2.2 justifies the market clearing rule and describes its outcomes. Section 2.3 shows that wages are dispersed in equilibrium and provides sufficient conditions for positive assortative matching.

6. In recent years, online job search has further reduced search frictions: For example, Davis and Samaniego de la Parra (2024) report that 45% of job applications arrive in the first 48 hours and half the job postings are taken down in a week. The mean vacancy duration exceeds 40 days, so the average firm then spends over four weeks screening and selecting applicants.

7. This difference between firm-level wages and applicant-level wages is also what differentiates Burdett and Mortensen (1998) from Postel-Vinay and Robin (2002a, 2002b).

2.1. *Model*

A unit mass of firms, each with one vacancy, compete for a unit mass of workers. Workers differ in their talent θ, with proportion $\bar{q} \in (0,1)$ talented, $\theta = H = 1$, and the remainder untalented, $\theta = L = 0$. Firms select among applicants by administering a pass/fail test to each applicant. Talented workers pass all tests, while untalented workers are screened out with probability $p \in (0,1)$, independently across firms and workers. Firm screening skills p are distributed according to $F[\underline{p},\bar{p}]$, where $0 < \underline{p} \leq \bar{p} < 1$; for simplicity we assume throughout that the cdf F is invertible, except for a few clearly marked parts in the paper where we consider homogeneous screening skills.

The only strategic part of the model is that firms simultaneously choose wages w. Given wages, the labor market clears mechanically "top-to-bottom" with workers accepting their highest wage offer above their outside option. To operationalize this, suppose workers apply to firms from highest to lowest wage; if there is an atom in the wage distribution $G(w)$, assume that all workers break the tie in the same way, as if the firms were infinitesimally differentiated.[8] The highest firm screens applicants in a random order and hires the first who passes their test; the adversely selected remainder then apply to the "second" firm, and so on until all firms and workers are matched. We describe the outcome of this procedure in Section 2.2 and provide a micro-foundation in Appendix B.[9,10]

When a recruiter screens an applicant pool with expected talent q, proportion $1-(1-q)p$ of the applicants pass the test. Bayes' rule implies that the fraction of recruits who are talented equals

$$\lambda(q,p) := \frac{q}{1-(1-q)p}. \tag{2.1}$$

Expected talent $\lambda(q,p)$ increases in both the applicant quality q and the screening skill p.

Payoffs are as follows. Workers only care about wages and so accept any job paying more than their outside option, $\underline{w} \geq 0$. We call this outside option "unemployment", but it could be a job in a different industry.[11] Productivity is normalized to 1 for talented workers and 0 for untalented workers. Thus, when firm p posts wage w and attracts applicants q, its expected profits are

$$\pi := \lambda(q,p) - w. \tag{2.2}$$

We solve for Nash equilibrium in wages.

8. Formally, the wage quantile $x \in [0,1]$ of a firm with wage w is then drawn uniformly from $[\lim_{\epsilon \to 0} G(w-\epsilon), G(w)]$. Lemma 1, below, implies that this complication does not arise in equilibrium. This choice of tie-break is for simplicity; we discuss other tie-break rules in Appendix B.

9. The assumption that firms use binary, pass-fail tests and that talented workers pass the test with certainty is without loss. Indeed, consider a more general information structure with finitely many signals s that arise with probability $p^\theta(s)$. A firm that attracts any applicants attracts a continuum of them, and thus some applicant with the signal $\bar{s}$ that maximizes the odds-ratio $\bar{\ell} = p^H(\bar{s})/p^L(\bar{s})$. Recruit quality is then $\frac{q\bar{\ell}}{q\bar{\ell}+(1-q)}$ which collapses to (2.1) when $\bar{\ell} = p^H(\bar{s})/p^L(\bar{s}) = 1/(1-p)$ for $\bar{s} =$ "pass".

10. We can reinterpret firms' screening as referrals. Assume that each firm is connected to each talented worker via a referral with probability ϵ, and to each untalented worker with probability $\epsilon(1-p)$. The market clears top-to-bottom with each firm extending a wage offer to one of its referrals.

11. Alternatively, $\underline{w}$ could be an operating cost for the firm; under this formulation, equilibrium allocations and payoffs are the same while wages are shifted down by $\underline{w}$.

2.2. *Market Clearing*

Given firms' wages, our labor market clears mechanically "top-to-bottom". To analyze the quality of a firm's applicants as a function of its wage, suppose all firms post different wages, write $x \in [0,1]$ for the resulting wage quantiles, and $w(x), Q(x), P(x)$ for wage, applicant quality, and screening skills at wage quantile x. Wages $w(x) < \underline{w}$ do not attract applicants and result in zero profits.

The highest ranked firm faces applicant pool $Q(1) = \bar{q}$; thus, proportion $\lambda(Q(1), P(1))$ of its recruits are talented. Since firms select talented workers disproportionately, lower-ranked firms face an adversely selected applicant pool, meaning that $Q(x)$ falls as the firm rank x declines. Specifically, at rank x there is a total of $xQ(x)$ talented workers, of which firms $[x, x+dx]$ hire $\lambda(Q(x), P(x))dx$; hence $d[xQ(x)] = \lambda(Q(x), P(x))dx$. Rearranging, the talent pool evolves according to the *sequential screening equation*

$$Q_x(x) = \phi(Q(x), P(x), x) \quad \text{where} \quad \phi(q, p, x) := \frac{\lambda(q, p) - q}{x}. \tag{2.3}$$

Since screening is imperfect, some talent remains, $Q(x) > 0$, for all $x > 0$. However, firms pick over the applicants so many times that eventually no talent remains if every worker is employed, $Q(0) = 0$.[12]

Discussion. There are two potential concerns with the reduced-form top-to-bottom market clearing rule. First, it doesn't allow workers to "jump the queue" in an attempt to avoid adverse selection. For example, if there were only two firms with wages $w_1 > w_2$, a worker who applies to Firm 2 in "round 1" has higher expected talent than a worker who applies to Firm 2 in "round 2", after being rejected by Firm 1. Second, it doesn't allow workers to repeatedly apply to the same firm.

In Appendix B, we micro-found the reduced-form model as the equilibrium steady state of a dynamic application game with a large but finite number of long-lived firms and workers. Over time, old workers retire and newborn workers apply for the resulting job openings. We assume rejected workers can apply quickly to new job openings, so there is no timing incentive to "jump the queue". To address the above two concerns, we make two additional assumptions. First, firms do not observe applicants' histories, so there is no informational incentive to "jump the queue." Specifically, a worker who jumps over Firm 1 will reach Firm 2 at the same time as all the rejected workers from the "prior generation", and there is no way for the worker to prove she did not apply to Firm 1. Indeed, in many real-world labor markets, firms entertain applications on a continuous basis and treat those arriving on June 1st and June 2nd symmetrically. Second, we assume firms remember their own past applicants, allowing them to reject re-applicants who are more adversely selected than new applicants.

2.3. *Equilibrium Analysis*

In equilibrium, wages are distributed continuously. To see this, observe that if an atom of firms offered the same wage, then a firm could attract discretely better job applicants

12. To see $Q(x) > 0$ for $x > 0$, note that $(\log Q(x))_x = (\lambda(Q(x), P(x))/Q(x) - 1)/x = (1/(1-(1-Q(x))P(x)) - 1)/x$ is bounded for any fixed $x > 0$ since $\bar{p} < 1$. To see $Q(0) = 0$, note that $\underline{p} > 0$ implies that $(\log Q(x))_x$ is of order $1/x$, and so the integral $\log Q(x) = \log \bar{q} - \int_x^1 (\log Q)_x$ diverges as $x \to 0$, implying $\log Q(0) = -\infty$, or $Q(0) = 0$.

with a marginal wage raise. Similarly, there can be no gap $[w,w']$ in the wage distribution, since firms offering w' could attract the same applicants at the lower wage w. By the same argument, the lowest wage in the wage distribution equals the outside option $\underline{w}$, so the share of unemployed workers $\underline{x}\geq 0$ solves $w(\underline{x})=\underline{w}$. Since profits (2.2) rise in p, firms above some $\underline{p}=F^{-1}(\underline{x})$ enter the market with a wage above $\underline{w}$ while those below $\underline{p}$ exit (or, equivalently, post a wage below $\underline{w}$). To summarize:

Lemma 1. Equilibrium wages and productivity are continuously distributed with a minimum of $\underline{w}$. There is unemployment $\underline{x}>0$ if and only if $\underline{w}>0$. Firms $p\in[\underline{p},\bar{p}]$ enter.

Example 1 (Homogeneous Firms). Wage dispersion does not require heterogeneous screening skills, and also obtains when all firms have the same skill p. In this case, competition erases profits and wages coincide with expected productivity, $w(x)=\lambda(Q(x),p)$. The worst applicant quality $\underline{q}$ that firms are willing to consider is given by $\lambda(\underline{q},p)=\underline{w}$ and unemployment $\underline{x}$ solves $Q(\underline{x})=\underline{q}$.

In online Appendix E.1, we show that wage and productivity dispersion rises when screening skills p rise (say, due to improvements in algorithmic hiring or employment networks). Intuitively, the top firms extract more talented workers from the applicant pool, lowering the productivity and wages of low-wage firms. We also show that wage and productivity dispersion rises when average talent $\bar{q}$ falls (say, due to skill-biased technological change that gives rise to a few superstars). $\triangle$

We now characterize the equilibrium of our wage-posting game. We say there is *positive assortative matching* (PAM) between firms and applicants if $P(x)$ is increasing, meaning firms with high screening skills p attract applicant pools of high quality q. From Becker (1973), we know that equilibrium features PAM if skilled recruiters have a comparative advantage in screening applicants with higher expected talent, i.e. if expected recruit quality $\lambda(q,p)$ is supermodular, $\lambda_{qp}\geq 0$. Note the partial derivatives

$$\lambda_p(q,p)=\frac{q(1-q)}{(1-p(1-q))^2} \text{ and } \lambda_{qp}(q,p)=\frac{1-2\lambda(q,p)}{(1-p(1-q))^2}. \tag{2.4}$$

To ensure supermodularity, we thus assume that talent is *scarce*, in that a worker from the unselected pool who passes the most stringent test is more likely untalented than talented:

$$\lambda(\bar{q},\bar{p})\leq 1/2. \tag{2.5}$$

This is a joint condition on the distributions of talent $\bar{q}$ and screening skills $\bar{p}$; it is satisfied in industries with relatively few highly productive individuals, such as technology or sales.[13] This assumption is necessary for equilibrium sorting, but not for the welfare results in Section 3.

Theorem 1. *Assume scarce talent (2.5). Equilibrium exists and is unique. Matching is positive assortative.*

13. Writing about Google and Netflix, Bock (2015) says that "Only 10% of your applicants (at best) will be top performers," while Hastings and McCord (2009) say "In creative/inventive work, the best are 10 times better than average." Based on salespeople at 131 firms, Benson et al. (2019) confirm "a well-known heuristic [...] that the top 20% of the sales force is responsible for 80% of sales."

Proof. Given (2.4), scarce talent (2.5) implies that $\lambda_{qp}\geq 0$ for all $q\leq\bar{q},p\leq\bar{p}$, and matching is positive assortative. That is, higher skill firms post higher wages, and so $P(x)$ increases.

We can now construct the equilibrium. Given PAM, a firm's rank in the skill distribution equals its equilibrium wage rank, $F(p)=x$, and we can identify a firm by this rank x. The skill of the firm with wage-rank x is then given by $P(x)=F^{-1}(x)$, its applicant quality $Q(x)$ is determined by the sequential screening equation (2.3), and the recruit quality $\lambda(Q(x),P(x))$ by Bayes' rule, (2.1).

From this we can derive the entry threshold $\underline{p}$, wages, and profits. Denote the equilibrium wage required to attract applicants of quality q by $W(q)$. The marginal firm pays the outside option, so employment $\underline{x}$ is given by $\lambda(Q(\underline{x}),P(\underline{x}))=\underline{w}$, which determines the entry threshold $\underline{p}=F^{-1}(\underline{x})$. Wages are determined by firms' first-order conditions,

$$W_q(Q(x))=\lambda_q(Q(x),P(x)). \tag{2.6}$$

Finally, profits $\Pi(p)$ follow by the envelope condition

$$\Pi_p(P(x))=\lambda_p(Q(x),P(x)). \tag{2.7}$$

Intuitively, productivity $\lambda(q,p)$ depends on both the applicant pool quality and the screening ability; workers capture the marginal benefit of the former and firms capture the marginal benefit of the latter.

The so constructed wage profile $W(Q(x))$ is the only possible candidate for an equilibrium. To verify that the wages are indeed optimal, it suffices to note that marginal profits $\lambda_q(q,p)-W_q(q)$ single-cross in p and matching is positive assortative; hence the FOC (2.6) implies global optimality. ‖

Theorem 1 captures a natural complementarity in the recruiting function λ. Intuitively, recruiting skills do not matter if all applicants are either talented or untalented, but they do matter when applicant quality is intermediate. Since $Q(x)$ is bounded above by $\bar{q}$, our scarce-talent assumption (2.5) implies that skilled firms have a comparative advantage at screening better applicants. Thus, skilled firms pay high wages, attract high-quality applicants, recruit talented workers, and achieve high productivity and profits.[14]

Without the scarce talent assumption (2.5), PAM fails at the top of the distribution for $q=\bar{q}$ and $p=\bar{p}$. Intuitively, a firm with perfect screening skills $p=1$ does not face adverse selection and is happy to screen last. More generally, equation (2.4) implies that equilibrium matching must consist of upward-sloping "PAM-branches" in the lower-left of (q,p)-space where $\lambda<1/2$, and negative assortative, downward-sloping "NAM-branches" in the upper-right where $\lambda>1/2$. Online Appendix E.2 numerically computes, verifies, and plots simple instances of such equilibria for uniform recruiting skills $F(p)$ with single, non-overlapping PAM and NAM branches.

While we cannot fully characterize equilibria with abundant talent,[15] the insights from the "scarce talent" model are robust. First, allocations are continuous: Suppose we

14. The comparative advantage of skilled recruiters in screening high-quality applicant pools relies on our assumption that firms can screen applicants until one passes the test. In contrast, Li and Shimer (2019)'s directed search model assumes that each firm screens a single applicant. Multiplying (2.2) with the hiring probability yields $\tilde{\pi}=q-(1-(1-q)p)w$ with cross-partial $\partial^2\tilde{\pi}/\partial p\partial q=-w<0$. Intuitively skilled recruiters hire fewer untalented workers and thus have a comparative advantage at screening bad applicant pools with lots of untalented workers.

15. Characterizing equilibrium is a notoriously difficult problem: In the context of a classical matching problem with exogenous types, Anderson and Smith (2024) write "While perfect sorting does

are at the threshold, $\lambda(\bar{p},\bar{q})=1/2$; then a small increase in top skills to $\bar{p}+\epsilon$ induces a NAM branch for the highest firms $p\approx\bar{p}+\epsilon$, but firms $[\underline{p},\bar{p}]$ still have $\lambda(p,q)<1/2$ and are thus PAM. Second, when the outside option $\underline{w}$ is small, the PAM-branch is non-empty and, as we will see in Theorem 2, matching is inefficient.[16]

The benchmark model can be adapted in a number of ways. Online Appendix E.3 allows for correlation between firms' signals of a worker's talent. When talent is scarce and correlation is low, matching is positive assortative. Online Appendix E.4 extends the model to continuous types with linear hiring propensity. When types are exponentially distributed, matching is positive assortative.

3. WELFARE

A worker's productivity equals θ in our industry and $\underline{w}$ elsewhere. The social value of screening consists in sorting talented workers, $\theta=H$, into the industry and untalented workers, $\theta=L$, out of the industry. In other words, mismatch arises from unemployed talented workers, and employed untalented workers. Section 3.1 studies the welfare effect of firms' screening order while Section 3.2 studies the level of firms' screening skills.

3.1. *Screening Order*

Equilibria in matching models with transferable utility are typically efficient (e.g. Shapley and Shubik (1971), Becker (1973)). Surprisingly, this welfare theorem fails in our model. In particular, Theorem 2 shows that welfare is minimized by positive assortative matching and maximized by negative assortative matching (NAM). The key difference to standard matching models is the compositional externality: The applicant pool quality is endogenous and depends on the screening skill of higher-paying firms. Intuitively, high-skilled firms pick out more talented workers than low-skilled firms and introduce more adverse selection. This externality is maximized by PAM and minimized by NAM.[17] The following example illustrates the idea.

Example 2 (Two Types of Firms). Suppose mass η_H of firms are skilled with $p>0$, and mass η_L are unskilled with $p=0$. To abstract from entry, assume firms are on the short side of the market and the minimum wage $\underline{w}$ does not bind, so all firms enter. Under PAM, skilled firms first hire from the unselected pool $\bar{q}$; unskilled firms then hire from an adversely selected pool with quality $q<\bar{q}$. In contrast, under NAM, unskilled firms hire from the unselected pool $\bar{q}$; they do not change the quality of the talent pool, and so the skilled firms also hire from a pool of quality $\bar{q}$. Since the unskilled firm impose no compositional externality on the skilled firms, more talented workers are

not emerge in many economic settings, an impassable wall of mathematical complexity has stopped any general predictive matching theory." The model variant in our working paper (Board et al. (2017)) parametrized recruit talent λ as linear in recruiting skills, so the cross-partial of λ only depends on q; equilibrium screening skills are then a single-peaked function of the wage rank x when talent is abundant.

16. Conversely, for $\underline{w}>1/2$, profitable entry requires $\lambda>\underline{w}>1/2$, so equilibrium is NAM.

17. Compositional externalities of a different nature are seen in models of directed search. In Albrecht et al. (2023), workers applying to recent job postings exert an externality on later applicants if employers fail to remove their filled "phantom vacancies" from the market. In Athey and Ellison (2011)'s model of position auctions, high-quality advertisers serve more searchers than low-quality advertisers when winning first position in the ad auction; this order minimizes search costs, meaning equilibrium is efficient.

hired under NAM than under PAM, raising social surplus; formally this follows from the proof of Theorem 2. The welfare loss from incorrect sorting can be substantial: For example, $(\eta_H,\eta_L)=(0.09,0.90)$ firms screening with NAM has the same welfare as $(\eta'_H,\eta'_L)=(0.90,0.09)$ firms screening under PAM.[18] $\triangle$

To argue the inefficiency of the competitive equilibrium more generally, consider a planner who can direct all firms' entry and wage decisions but is subject to the same informational frictions.[19] Her problem is to choose employment $1-\underline{x}$ and a measure-preserving matching function, or equivalently *screening order*, $P:[\underline{x},1]\to[\underline{\underline{p}},\bar{p}]$ to maximize welfare[20]

$$S:=\int_{\underline{x}}^{1}[\lambda(Q(x),P(x))-\underline{w}]dx. \tag{3.8}$$

For example, in equilibrium, matching is positive assortative $P^{\text{PAM}}(x)=F^{-1}(x)$, with entry cutoff $\underline{x}$ satisfying $\lambda(Q(\underline{x}),P^{\text{PAM}}(\underline{x}))=\underline{w}$; assuming the same entry behavior, NAM is characterized by $P^{\text{NAM}}(x)=F^{-1}(\underline{x}+1-x)$. In contrast to the standard assignment model, the surplus of the x-ranked firm in the integrand of (3.8) depends on $P(x')$ for $x'>x$ via the applicant quality $Q(x)$.

We first argue that the planner wants the highest skilled firms to enter; hence the matching function P has range $[\underline{p},\bar{p}]$, where $\underline{p}=F^{-1}(\underline{x})$. Increasing skills $P(x)$ at wage rank x increases the surplus at that rank, $\lambda(Q(x),P(x))$; however, it reduces applicant quality $Q(\check{x})$ at lower ranks $\check{x}\in[\underline{x},x]$. This negative indirect effect diminishes but does not overturn the positive direct effect. Formally, equation (C.15) in the proof of Theorem 2 shows that the indirect effect scales down the direct effect by a factor

$$\nu:=\exp\left(-\int_{\underline{x}}^{x}\frac{\lambda_q(Q(\check{x}),P(\check{x}))}{\check{x}}d\check{x}\right). \tag{3.9}$$

Turning to the screening order P, we claim that surplus (3.8) increases whenever a low-skill firm p is promoted from wage rank x to $x'>x$ past firms with higher skills $P(\hat{x})>p$ for $\hat{x}\in[x,x']$. Thus PAM minimizes surplus, and NAM maximizes it. In a discrete analogue of our model, consider two firms with screening skills p and $p'=p+dp$ at adjacent wage ranks x and $x'=x+dx$ under PAM. The corresponding applicant quality equals $q=q'-\phi(q',p',x')dx$ and q', recalling the compositional externality ϕ from

18. *Proof:* With unemployment fixed at $\underline{x}=1-\eta_H-\eta_L$, welfare is decreasing in the talent remaining among the unemployed $Q(\underline{x})$. By (2.3), Q_x scales inversely with the number of remaining applicants x. Thus, mass η_H skilled recruiters screening at wage ranks $x\in[\underline{x},1-\eta_L]$ under NAM reduces applicant quality by the same amount as mass $\eta_H/(1-\eta_L)$ skilled recruiters at wage ranks $x\in[1-\eta_H/(1-\eta_L),1]$ under PAM. Thus, switching from PAM to NAM has the same welfare consequence as sticking to PAM while scaling up the number of skilled firms by a factor $1/(1-\eta_L)$.

19. In particular, the planner cannot communicate firms' test results to each other. With a continuum of firms and independent tests, allowing such communication would trivially solve any mismatch.

20. Throughout the paper we focus on pure matchings P. More generally, a mixed matching is a mass $1-F(\underline{\underline{p}})$ measure $M(p,x)$ on $[\underline{\underline{p}},\bar{p}]\times[\underline{x},1]$ with marginals $M_p\sim F[\underline{\underline{p}},\bar{p}]$ and $M_x\sim U[\underline{x},1]$, and sequential screening (3) becomes $Q_x(x)=\int_{\underline{\underline{p}}}^{\bar{p}}\phi(Q(x),p,x)dM(p|x)$. Theorem 1 shows that equilibrium matching is PAM, and hence pure. The proof of Theorem 2 extends to mixed matchings: Starting from any cdf $M(p,x)$, reshuffling firms to NAM increases surplus.

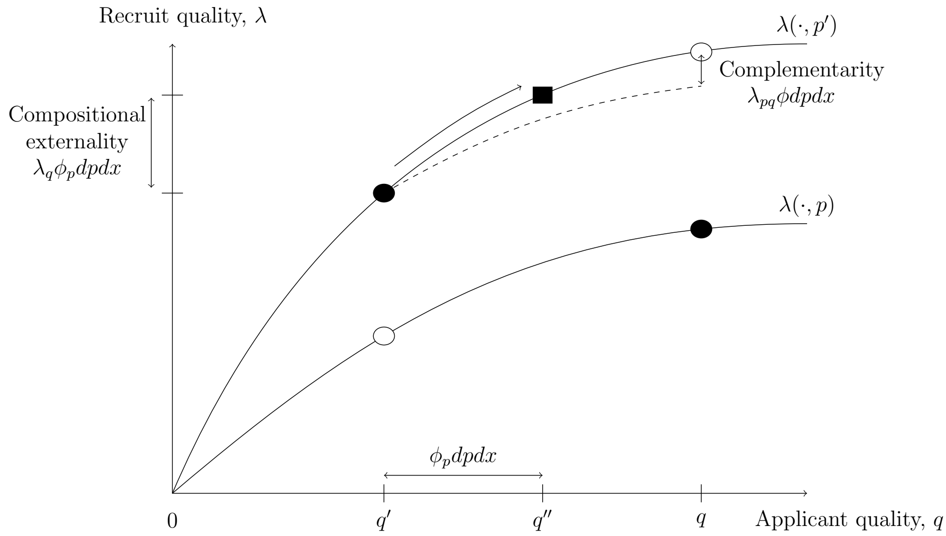

FIGURE 1

Recruit Quality with PAM and NAM. This figure illustrates the supermodularity and compositional externality that arise in Theorem 2. The empty dots represent PAM, the solid dots represent NAM if applicant quality were exogenous (the "Becker effect"), and the shift to the square comes from the endogeneity of the applicant pool (the "Akerlof effect").

(2.3). How does switching their screening order affect their joint recruited talent and hence surplus? First, the low-skill firm p now screens the better pool; since $\lambda(q,p)$ is supermodular, this lowers total surplus of the two firms by $\lambda_{qp}\phi dpdx$. In Figure 1 this is represented by the vertical shift from white circles representing PAM to the black circles representing NAM. If the pool quality was exogenous, as in Becker, this would be the end of the story. In our model, there is a second effect: firm p extracts $\phi_p dpdx$ less talent, increasing surplus by $\lambda_q\phi_p dpdx$. In Figure 1, this is represented by the applicant quality of firm p' rising from q' to q'', and the associated shift of the black circle to the black square. The next inequality shows that the second term outweighs the first, and so moving the low-skill firm p ahead of the high-skill firm p' raises surplus

$$\frac{\lambda_{qp}}{\lambda_q}=2\frac{1-q}{1-p(1-q)}-\frac{1}{1-p}<\frac{1-q}{1-p(1-q)}=\frac{\lambda_p}{\lambda}<\frac{\lambda_p}{\lambda}\frac{\lambda}{\lambda-q}=\frac{\phi_p}{\phi}. \tag{3.10}$$

In words, marginal recruiting success λ_q is less sensitive to p than absolute recruiting success λ, which in turn is less sensitive to p than the compositional externality ϕ. Thus, total surplus of the two firms p,p' is maximized by NAM; the effect on aggregate surplus (3.8) must be discounted by (3.9), but the sign remains unchanged.

The compositional externality overturns one of the most fundamental insights of the assignment model, namely the First Welfare Theorem.

Theorem 2. *For any level of unemployment $\underline{x}>0$, PAM minimizes surplus (3.8) and NAM maximizes surplus.*

Proof. See Appendix C.2. ||

From a policy perspective, it is hard to solve the inefficiency seen in equilibrium. In some rare cases, the planner might be able to directly implement negative assortative matching. For example, in the NFL "reverse" draft the lowest-ranked football teams pick first. To induce negative assortative matching indirectly through Pigouvian taxes on wages, the planner would need to charge higher wage-taxes to firms with higher screening skills p. If the planner cannot observe the skill of different firms but can restrict the set of admissible wages, her best policy (under scarce talent) is a single industry-level wage, inducing firms to select in a random order.[21] The same outcome can be achieved by a wage cap (see online Appendix F.1). More common labor market policies do not address the inefficiency. Unemployment benefits or minimum wages raise $\underline{w}$ but don't change the equilibrium screening order; these policies lower welfare since equilibrium entry is optimal given the screening order. Progressive taxes on wages or profits flatten net wages $w(x)$ without changing the screening order; this policy may also change the number of firms $\underline{x}$, and thus lower welfare.

The positive assortative matching that arises in equilibrium (Theorem 1) and its inefficiency (Theorem 2) are robust to natural generalizations of our model. First, consider a model with production complementarities, where a firm's revenue is multiplicative in firm-type and expected worker-type $h(p)\cdot\lambda(q,p)$, so high-type firms also have a higher marginal product. Since the (exogenous) productive-complementarity reinforces the (endogenous) screening-complementarity, equilibrium sorting remains positive assortative if talent is scarce (2.5). More surprisingly, for $h(p)=p$ the compositional externality overcomes both the productive complementarity and the screening complementarity, and PAM remains inefficient (see online Appendix F.2).

Second, the theorems are robust to screening costs. If screening costs κ per applicant and firms screen applicants until one passes their test, profit (2.2) becomes $\pi := \lambda(q,p) - \kappa/(1-p(1-q)) - w$. Since skilled firms are more selective, they incur higher screening costs and are more sensitive to applicant quality; thus equilibrium continues to exhibit PAM if talent is scarce (2.5). Moreover, this equilibrium continues to be inefficient, with the planner improving on PAM by swapping neighboring firms (see online Appendix F.3).

3.2. *Implications of the Compositional Externality*

In Section 3.1, we saw that the compositional externality generates inefficient sorting of firms. In this section we show that it impairs the market's ability to aggregate information and induces firms to over-invest in their screening skills.

Limits of Information Aggregation. In the model, each firm has independent signals about each worker. Intuition from multi-unit auctions might suggest a "wisdom of crowds" results, whereby the market perfectly aggregates information (e.g. Pesendorfer and Swinkels (1997)). In contrast to these common-value auctions, we now argue that our firms' screening of workers' independent talent is far less effective at aggregating

21. *Proof:* High-skill firms always offer weakly higher equilibrium wages than low-skill firms (by Theorem 1), while welfare increases with any promotion of low-skill over high-skilled firms (by Theorem 2).

information. For simplicity, focus on the case of homogeneous firms with screening skills p (Example 1). We quantify (the lack of) information aggregation in two ways:

First, imprecise signals are of no value to society. Formally, the impact of screening skills p on surplus $S(p)$ is initially zero, $S_p(0)=0$,[22] even though such skills improve the selection of individual firms, $\lambda_p(q,0)>0$. That is, a continuum of imprecise, independent signals is perfectly informative when aggregated by an auction for homogeneous goods, but perfectly uninformative when aggregated by our market clearing rule for heterogeneous workers. Intuitively, imprecise signals enable firms to weed out some untalented applicants, but workers get so many attempts at different firms that every worker passes one of the tests, resulting in no increase in social surplus. That is, the positive effect of marginal screening skills at high-wage firms is entirely offset by adverse selection at low-wage firms.

Second, social surplus is higher when firms have access to one public signal per worker that screens out bad workers with probability p (akin to Phelps (1972)), rather than each firm having a (conditionally) independent signal. To see this, note that with the public signal, posterior expected talent equals $\lambda(\bar{q},p)$ for the $1-(1-q)p$ applicants who pass the test, and 0 for the $(1-q)p$ applicants who fail the test. This distribution of expected talent is a mean-preserving spread of the distribution of expected talent with private signals, which consists of a continuous distribution from $\underline{w}$ to $\lambda(\bar{q},p)$ for employed workers and an atom at $\underline{q}$ for the unemployed. Since hiring decisions are efficient conditional on the available information, the superior information with public signals implies higher welfare. Intuitively, failing a public test black-lists an untalented job applicant, while failing a private test allows him to re-enter the pool, inducing adverse selection for other firms. Since firms earn zero profits, workers are better off, on average, if they each have a single, public, standardized test rather than lots of independent tests that give them "a second chance".

Investment Inefficiency. In the model, firms' screening skills p are exogenous. In practice, firms can raise their skills by improving their recruiting infrastructure (e.g. developing referral systems, training recruiters, engaging headhunters). If firms can choose their screening skill p at cost $c(p)$ (and talent is scarce), then wages and screening skills are dispersed and positively correlated across firms, with $p(x)$ solving $c'(p)=\lambda_p(Q(x),p)>0$ by (2.7). From society's perspective, firms over-invest in screening skills. Using equation (3.9), the marginal gain to society is $\nu\cdot\lambda_p(Q(x),p)$, where $1-\nu$ reflects the compositional externality on lower-ranked firms. In the extreme case when $\underline{w}=0$, we have full employment $\underline{x}=0$, the screening game is constant-sum, and all investment is wasteful. This overinvestment contrasts with the finding of efficient investment found in classic matching models (e.g. Cole et al. (2001)). Overinvestment can be important in practice: Bock (2015) urges companies to spend more on screening recruits, writing that "you can find a way to hire the very best, or you can hire average performers and try to turn them into the best. [...] At Google, we front-load our people investment. This means the majority of our time and money spent on people is invested in attracting, assessing, and cultivating new hires." Alas, not every firm can hire the best.

22. *Proof:* Assume that, generically, $\bar{q}\neq\underline{w}$. If $\bar{q}<\underline{w}$ then no-one is hired for small p and the result trivially follows. If $\bar{q}>\underline{w}$, then everyone is hired when $p=0$, and unemployment is $\underline{x}=\underline{X}(p)=0$. The result then follows because the integral $\int_{\underline{x}}^{x}\frac{\cdots}{\breve{x}}d\breve{x}$ in the discount factor (3.9) diverges.

4. ENDOGENOUS SCREENING SKILLS AND FIRM DYNAMICS

We now embed our static labor market into a model of firm dynamics to endogenize firms' screening skills and study the evolution of talent over time. The key premise is that firms with more talented employees are more skilled at recruiting. As discussed in footnote 4, this might happen because talented employees provide better referrals, or because they are better at identifying talented applicants. Firms thus desire talented workers both for the immediate increase in productivity, and for the benefit of having skilled recruiters in the future.

Section 4.1 describes the model. Section 4.2 solves for a firm's optimal wage path. Section 4.3 shows equilibrium is unique with high-talent firms offering high wages that complement their superior screening skills; they thus attract superior applicants, amplifying their talent advantage. Section 4.4 characterizes firm dynamics, and shows that the economy converges to a steady state that exhibits persistent dispersion in talent, screening skills, wages, and productivity. In equilibrium, the positive assortative firm-applicant matching offsets the regression to mediocrity that results from imperfect screening.

4.1. *Model*

Time $t \geq 0$ is continuous. There is a unit mass of firms, each with a unit mass of jobs. At time t, a firm is described by its proportion of talented workers $r(t)$; initially, the distribution of $r(0)$ is exogenous. At every instant $[t,t+dt]$, proportion αdt workers retire, leaving firms with vacancies. In the job market, there are then αdt open jobs and αdt applicants, of whom fraction $\bar{q}$ are talented. Analogous to the static model, firms compete for these applicants by posting lifetime wages $w(t)$.

Talented workers have an advantage in recruiting. We describe this relationship by a skill function $\psi(r)$ satisfying $\psi_r > 0$ and $\psi_{rr} \geq 0$. For example, if an employee with talent $\theta \in \{L,H\}$ has screening skill p^θ and firms ask random employees to act as recruiters, then the skill function is linear, $\psi(r) = p^L + r(p^H - p^L)$. As in Section 2, we assume that talent is scarce for all firms, which holds if

$$\lambda(\bar{q},\psi(1)) < 1/2. \tag{4.11}$$

The job market clears instantaneously, as in Section 2.1. If a firm with talent r posts wage w with rank x at time t, it attracts applicants with quality $Q(x,t)$ and hires recruits of quality $\lambda(Q(x,t),\psi(r))$. Writing $R(x,t)$ for the talent of the firm with wage rank x at time t, $Q(x,t)$ is determined by the sequential screening equation,

$$Q_x(x,t) = \phi(Q(x,t),\psi(R(x,t)),x) \text{ and } Q(1,t) = \bar{q}, \tag{4.12}$$

as in (2.3). In turn, the evolution of a given firm's talent $r(t)$ with wage rank $x(t)$ is given by the difference between its inflow λ and outflow r,

$$r_t(t) = \alpha\Big(\lambda(Q(x(t),t),\psi(r(t))) - r(t)\Big). \tag{4.13}$$

Turning to payoffs, workers maximize lifetime wages $w(t)$, while firms' revenue equals $r(t)$. To abstract from entry and exit, suppose that there is no outside option, $\underline{w} = 0$.

A firm's problem is to choose wages to maximize total discounted profits. Denoting the discount rate by $\beta > 0$, its value function is

$$V(r,s) = \max_{\{w(t)\}_{t\geq s}} \int_s^\infty e^{-\beta(t-s)}(r(t) - \alpha w(t))dt, \tag{4.14}$$

where $r(t)$ evolves according to (4.13) with initial condition $r(s) = r$.[23]

An equilibrium is given by a wage path $\{w(t)\}_{t\geq 0}$ for every firm, so that given the induced wage ranks $x(w,t)$ and applicant qualities $Q(x,t)$, every firm's wage path is optimal. We say an equilibrium is *essentially unique*, if the induced distribution over equilibrium trajectories $\{r(t)\}_{t\geq 0}$ is unique.[24]

Discussion. We assume that employment is for life. If firms identify and fire low types at a constant rate, the analysis would be qualitatively unchanged. The possibility of firing introduces a new term into the evolution of talent (4.13), and re-entry of fired workers into the applicant pool lowers $\bar{q}$. However, the firms' FOCs (4.16) are unchanged, meaning we still obtain positive assortative matching and steady state dispersion, as in Theorems 3-4 below.

As in Appendix B, we can micro-found the market clearing rule as the steady state of a game with a large number of firms and workers, where wages $w(x,t)$ vary over time. The corresponding version of Lemma 2 requires workers to be impatient (i.e., workers must have a different, larger discount rate β_w than firms β) in order to dissuade them from waiting for aggregate labor market conditions to change.

Unlike Section 3, there are no welfare margins in the dynamic model. In comparison to Section 3.1, the outside option is zero, $\underline{w} = 0$, so all workers are hired and the game is constant sum. And in comparison to Section 3.2, investments into screening skills are transfers to workers, rather than real economic costs.

4.2. *Firm's Problem*

First, we study a firm's optimal wage path $\{w(t)\}_{t\geq 0}$ for any given applicant function $Q(x(w,t),t)$ without imposing equilibrium restrictions on other firms. As in Section 2.3, it is convenient to write $W(q,t)$ for the wage required to attract applicants q at time t, and let the firm optimize directly over the applicant pools $\{q_t\}_{t\geq 0}$. After this change of variable, the firm's Bellman equation becomes

$$\beta V(r,t) = \max_q \{r - \alpha W(q,t) + \alpha(\lambda(q,\psi(r)) - r)V_r(r,t) + V_t(r,t)\}. \tag{4.15}$$

Firm value is determined by its flow profits plus appreciation due to talent acquisition and a secular trend. Assuming wages are differentiable, the first-order condition is

$$W_q(q,t) = \lambda_q(q,\psi(r))V_r(r,t). \tag{4.16}$$

Intuitively, the cost of attracting better applicants (the LHS) must balance the gains of a higher quality applicant pool which increases the recruit quality and thereby firm value

23. Note that the "wages" $w(t)$ are really one-time payments to each of the α newly hired employees at time t. More realistically, one could model worker compensation as constant flow wage $(\beta+\alpha)w(s)$ until retirement, but it simplifies the accounting to have firms incur these costs up-front.

24. This definition avoids two spurious notions of multiplicity. First, in continuous time, any firm's optimal strategy $\{w_t\}_{t\geq 0}$ can be unique only almost always. Second, if two or more firms are initially identical but then drift apart, only the distribution of trajectories can be determined uniquely.

(the RHS). Given scarce talent (4.11), the RHS of (4.16) is increasing in r, so firms with more talent have a higher marginal benefit from attracting better applicants, yielding positive assortative matching. Indeed, firms with more talent have higher marginal benefit from better applicants $\lambda_q(q,\psi(r))$ because of the supermodularity of λ, and such firms have a higher marginal value of talent $V_r(r,t)$ because $\psi(r)$ and $V(r,t)$ are convex in r (see Appendix D.1). Hence, wages are dynamic complements: An increase in today's wage raises tomorrow's talent, and thereby tomorrow's optimal wage.

To compute equilibrium wages from the first-order condition (4.16), we write $r(u)$ and $q(u)$ for the equilibrium trajectory of talent and applicant quality, and apply the envelope theorem and the law of motion of firm talent (4.13) to compute (see Appendix D.1 for details)

$$V_r(r(t),t) = \int_t^{\infty} e^{-\int_t^s \beta+\alpha\left(1-\lambda_p(q(u),\psi(r(u)))\psi_r(r(u))\right)du} ds. \tag{4.17}$$

Intuitively, the future benefit of better employees is discounted both at the interest rate β and the retirement rate α. But selective recruiting raises the persistence of firm talent or, equivalently, reduces the talent decay rate by a factor $1-\lambda_p\psi_r$.

4.3. *Equilibrium*

Given the single-firm analysis, it is straightforward to characterize equilibrium. Firms with more talent post higher wages and attract better applicants. More strongly, even if firms share the same talent $r(0)$ initially, they post different wages (as in the static model), recruit different types of workers, and diverge immediately (see Appendix D.2). Thus, in equilibrium, each firm is characterized by a rank x, which describes the firm's position in the talent, applicant, and wage distribution at all times $t>0$.

Equilibrium is then characterized in two steps

(1) Allocations. At time t, applicant quality $Q(x,t)$ is determined by sequential screening (4.12). The evolution of firm x's talent $R(x,t)$ is then given by the firm dynamics equation (4.13) with $x(t)\equiv x$.
(2) Payoffs. Firm x's marginal value of talent is determined by (4.17), with $q(u)= Q(x,u)$. Using this, wages $W(q,t)$ are given by the first-order condition (4.16), with $r=R(x,t)$, $q=Q(x,t)$, and $W(0,t)=\underline{w}=0$.

Given these wages, the FOC (4.16) implies global optimality because the net benefit of better applicants $\lambda_q(q,\psi(r))V_r(r,t)-W_q(q,t)$ single-crosses in r. Standard verification theorems then imply that the policy functions are indeed optimal. To summarize:

Theorem 3. *Assume talent is scarce (4.11). Equilibrium exists and is essentially unique. Firm-applicant matching is positive assortative and the distribution of talent has no atoms at any $t>0$.*

Proof. Only the claim about no atoms remains to be shown. See Appendix D.2. ‖

Thus, even if firms start off with identical talent, some post higher wages than others, attract better applicants, and hire better recruits. These firms accumulate talent, continue to pay high wages, and the distribution of talent disperses over time.

(a) Evolution of Recruits and Applicants **(b) Recruit Talent over Time**

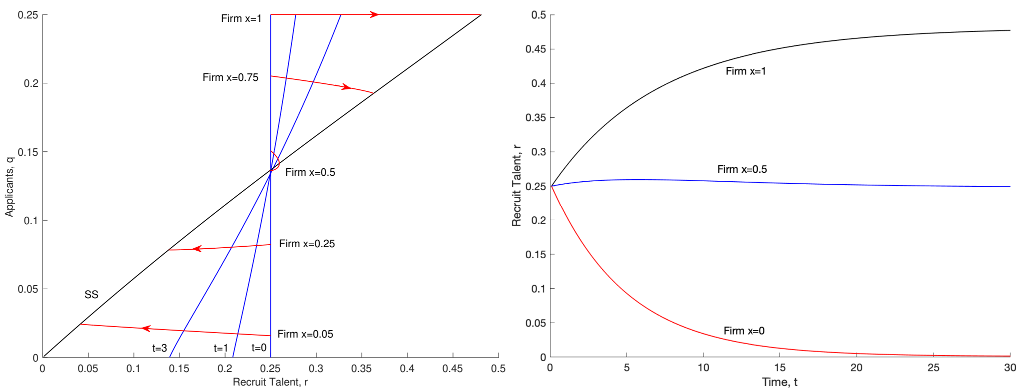

FIGURE 2

Equilibrium Dynamics of Applicant and Recruit Quality. All firms start with talent $\bar{q}=0.25$, and choose wages optimally as characterized in the text. The skill function is $\psi(r)=0.4+0.5r$ and (annual) turnover is $\alpha=0.2$.

Example 3 (Initially Homogeneous Firms). To understand the evolution of talent $R(x,t)$ and applicant quality $Q(x,t)$, consider Figure 2. At $t=0$, all firms employ average workers, with quality $\bar{q}=0.25$. In panel (a), the "vertical" lines represent the cross-sectional distribution of (r,q) at different times, while the "horizontal" lines represent the sample-paths of selected firms. The top-ranked firm recruits from the constant applicant pool $Q(1,t)=\bar{q}$, and so (4.13) implies that its talent grows monotonically and converges to a steady state. For lower-ranked firms, the dynamics are more subtle. For instance, firm $x=0.5$ initially improves as its recruits are more talented than its retirees. However, as higher-ranked firms become better at identifying talent, its applicant pool deteriorates and its quality eventually falls back. These time paths are shown in panel (b).

Turning to payoffs, note that firms earn zero lifetime value since they start homogeneous. The top firms initially lose money as they post high wages and invest in talent; this raises both their productivity and their screening skill, giving them an advantage in the labor market, and delivering a steady stream of profits. Over time, rents shift from workers to firms: early workers are paid more than their productivity as firms invest; later workers are paid less than their productivity as differentiation softens competition. $\triangle$

4.4. *Steady State Allocations*

Steady state talent among recruits and applicants $\{R^*(x),Q^*(x)\}$ is easily characterized. First, the talent of each firm's recruits and retirees balance,

$$\lambda(q,\psi(r))=r. \tag{4.18}$$

This has a unique fixed point, $r=\rho(q)$, since $\lambda(q,\psi(r))-r$ is convex (see Appendix D.1), positive at $r=0$, negative at $r=1$, and hence crosses zero exactly once. Naturally, firms with better applicants have higher talent; formally, $\rho(q)$ increases since $\lambda(q,\psi(r))-r$ rises in q and single-crosses from above in r.

Second, substituting $R^*(x)=\rho(Q(x))$ into the sequential screening equation (4.12), steady state applicant quality $Q^*(x)$ is given by

$$Q_x^*(x)=\phi(Q^*(x),\psi(\rho(Q^*(x))),x). \tag{4.19}$$

Together (4.18) and (4.19) pin down firm x's talent $R^*(x)$ and applicant quality $Q^*(x)$ in steady state. Differentiating and dropping arguments for legibility, steady state talent dispersion is given by

$$R_x^*=\rho_q Q_x^*=\frac{\lambda_q}{1-\lambda_p\psi_r}Q_x^*. \tag{4.20}$$

The economy converges to this steady state from any initial condition.

Theorem 4. *Assume talent is scarce (4.11). The steady state talent distribution $R^*(x)$ is unique and has no gaps or atoms. For any distribution of initial talent $r(0)$, firm x's equilibrium talent $R(x,t)$ converges to $R^*(x)$.*

Proof. To show convergence, Figure 2 suggests a "proof by induction." The top firm recruits from a pool of constant quality, so equation (4.13) implies that its talent converges to steady state. Then consider firm x close to 1. As the talent of higher firms converges, firm x's applicant pool converges, and then firm x's talent converges, too, by (4.13). See Appendix D.3 for details. ‖

In steady state, talent $R^*(x)$ is dispersed. If all firms were hiring from the same applicant pool q, talent differences from the steady state level $r-\rho(q)$ would decay exponentially. The link between talent and recruiting skills slows this decay but cannot stop it. Rather, the decay is halted by positive assortative matching: firms with skilled employees post higher wages and recruit from a better pool. As a result, the steady state supports permanent heterogeneity in firm quality, productivity and profits.

Talent thus generates a sustainable competitive advantage. One might wonder why a firm with untalented workers doesn't compete more aggressively in wages to build its talent over time. This strategy is feasible, but too expensive. High-talent firms have a higher marginal benefit from raising wages because recruiting skills and applicant quality are complements in the job market.

This logic relies on our assumption that firms have *firm-specific* wage policies. In contrast, Montgomery (1991)'s model of *applicant-specific* wage offers does not generate persistent dispersion.[25] If we interpret the skill function $\psi(r)$ as the firm sampling its current employees for referrals, our model predicts that a talented worker recruits better referrals if their colleagues are also talented. This is because high-talent firms pay high wages, and help convert the recommendation of the talented worker into a talented recruit. This does not happen in Montgomery, where each referral receives a separate wage offer.

25. Montgomery (1991) only has two periods. For an apples-to-apples comparison with our model, assume an infinite time horizon, continuous time, and that firms employ a unit mass of workers with aggregate talent $r(t)$ who retire at rate α. When a firm has a vacancy, a random current employee makes a referral. Write $\bar{\lambda}$ (resp. $\underline{\lambda}$) for the equilibrium expected talent of a recruit who was referred by a talented (resp. untalented) employee. Since wage offers are applicant specific, $\bar{\lambda}$ and $\underline{\lambda}$ do not depend on the talent of the firm's other employees. Talent evolution is then governed by $r_t=\alpha(\underline{\lambda}+(\bar{\lambda}-\underline{\lambda})r-r)$ and converges to $r^*=\underline{\lambda}/(1-\bar{\lambda}+\underline{\lambda})$, irrespective of initial talent.

4.5. *Steady State Payoffs*

The model generates predictions about how value is shared between workers and firms. In steady state the first-order condition (4.16) together with the marginal value of talent (4.17) imply that flow wages follow

$$(\beta+\alpha)W_q^* = \frac{(\beta+\alpha)\lambda_q}{\beta+\alpha\left(1-\lambda_p\psi_r\right)}. \tag{4.21}$$

Steady state flow profits $\Pi^*(q)$ in turn follow

$$\Pi_q^* = \rho_q - (\beta+\alpha)W_q^* = \frac{\beta\lambda_p\psi_r}{\left(1-\lambda_p\psi_r\right)\left(\beta+\alpha\left(1-\lambda_p\psi_r\right)\right)}. \tag{4.22}$$

Two comparative statics help illustrate the inner workings of the model. First, we note that *any* degree of correlation between productivity and recruiting skills $\psi_r > 0$ gives rise to non-vanishing talent dispersion. In the limit, as the direct talent-skill relationship ψ_r goes to zero, equation (4.20) shows that the strategic effect gives rise to dispersion $R_x^* = \lambda_q Q_x^*$. Despite this talent dispersion, equation (4.22) shows that steady state profits are zero since firms have weak incentives, as in Example 1. An increase in the importance of referrals in recruiting talent ψ_r then amplifies talent dispersion (4.20) and raises steady state profits (4.22). This arises because (i) high-paying firms fish out more of the talented workers from the applicant pool, and (ii) talent accumulates at the top firms, further raising their screening ability and the talent of their recruits.

Second, a rise in turnover α raises wages and shifts steady state rents from firms to workers. Indeed, a rise in turnover has no impact on the steady state distribution of talent $R^*(x)$, but raises the rate at which the economy converges to steady state. Equations (4.21) and (4.22) then imply that flow wages $(\beta+\alpha)W^*(Q^*(x))$ rise, and flow profits $\Pi^*(Q^*(x))$ fall for all firms x. Intuitively, when turnover is high, a firm's stock of talent quickly depletes and a low-talent firm can achieve almost the same profits as a high-talent firm by mimicking its wage policy; this intensifies competition and drives up wages.

Our analysis assumes revenue is linear and screening skills are weakly convex in talent r. Complementarities in production (in the form of convex revenue) reinforce equilibrium sorting and leaves steady state dispersion unchanged. Our results are also robust to a little concavity in revenue or screening since incentives for positive assortative matching are strict. But sufficient concavity induces negative assortative matching: For example, if 10% of talented employees suffice to guarantee good screening outcomes, then firms below the 10% threshold would bid aggressively to improve their screening.

5. CONCLUSION

In their survey on personnel economics, Oyer and Schaefer (2011) write "This literature has been very successful in generating models and empirical work about incentive systems [...] The literature has been been less successful at explaining how firms can find the right employees in the first place." By studying the equilibrium interaction of firms' recruiting strategies, we hope this paper takes a step in this direction.

The paper has three major contributions. First, it provides a simple model to understand a competitive labor market where firms have independent information about candidates' talent. Second, we show that compositional externalities induce inefficient equilibrium sorting. Third, we propose a new model of firm dynamics where similar firms diverge over time and the economy converges to a steady state with persistent dispersion in talent, wages, productivity, and profits.

The tractable nature of the model means it can be extended in a number of directions. First, it would be natural to allow firms to differ in their technology as well as their talent, and study how they combine to define firms' competitive advantages. For example, Waldinger (2016) argues the loss of human capital at German universities had a large, persistent effect on output, whereas the loss of physical capital had a small, temporary effect. Second, going beyond our one-dimensional model of talent, one might assume that firms also differ in the type of talent they seek to hire (e.g. generalists vs. specialists) which typically complements their broader organizational strategy. Third, we suppose that each worker is only on the job market once, but it would be natural to extend the model to allow for job-to-job transitions. Workers in low-wage jobs would then compete with new cohorts for high-wage jobs. Fourth, in many settings workers value peer quality or status as well as wages. For example, if all types of worker prefer to work at a firm with more talented colleagues, the allocation of talent is unchanged but wages are more compressed; moreover, firms have yet another reason to invest in talent, further transferring surplus from later generations to earlier ones.

The paper also raises empirical questions on the "production function of hiring". What makes some managers (or firms) better at recruiting than others? How does this skill interact with the firm's strategy (e.g. wages, advertising) in order to produce better recruits?

APPENDIX

A. COMPARISON WITH KURLAT (2016)

In this section we clarify the relationship between our model and that of Kurlat (2016) with "false positives". As discussed in Section 1.1, the key differences are that, in the language of our model, Kurlat considers perfectly divisible labor and perfectly nested signals.[26]

To appreciate these differences, we consider the effect of nested signals and divisible labor in our model. To reduce formalities, we try to keep these model variants as simple as possible. To this end, we suppose that the outside option $\underline{w}$ is close to 0 throughout.

A.1. *Nested Signals*

First consider a version of our model with nested, rather than independent signals. Assume there are two types of firms (skilled and unskilled). Signals are "nested" in that an untalented worker who passes the test of a skilled firm also passes the test of an unskilled firm. Workers thus fall into one of four categories: A) talented workers, who automatically pass both tests, B) untalented workers who pass both tests, C) untalented workers who pass the test of the unskilled firm but fail the test of the skilled firm, and D) untalented workers who fail both tests.[27]

26. Other differences between our model and Kurlat's are less relevant for the analysis. For instance, Kurlat also allows for patient sellers of good assets, who never trade; in our model this would correspond to additional talented workers with reservation wage $\underline{w}=1$. And Kurlat's buyers have a fixed monetary budget, so the desired number of assets depends on the price, whereas our buyers have unit demand (i.e. our firms want to hire one worker).

27. Category A corresponds to the "green assets" in Kurlat's table I, category C to "red assets," and category D to the "black assets."

With such perfectly nested signals, adverse selection and outbidding incentives are one-sided. Unskilled firms, who hire proportionally from categories A, B, and C, do not impose adverse selection on other firms. Skilled firms, who hire proportionally from categories A and B, but screen out C workers, impose a negative externality on unskilled firms. Thus, unskilled firms have an incentive to outbid skilled firms but not vice versa and neither type of firm has an incentive to outbid its own type.

The equilibrium wage distribution is thus degenerate, with all entering firms offering some wage w^* and workers endogenously breaking ties in favor of unskilled firms. The wage w^* is determined by the entry condition of unskilled firms. This equilibrium has negative assortative matching and efficient aggregate sorting, in contrast to the positive assortative matching and inefficient aggregate sorting of our model with independent signals. Note how the single-wage equilibrium relies on signals being perfectly nested: Whenever firms cannot perfectly screen out applicants who failed other firms' tests (as with imperfectly correlated signals in online Appendix E.3), firms have an incentive to outbid each other and equilibrium wages are dispersed.

A.2. *Simultaneous Markets for Divisible Labor*

Returning to independent signals, now consider a model with divisible labor, i.e. workers can rent themselves out by the second across many firms. As in Kurlat (2016), workers apply to all firms simultaneously and can offer their entire unit of labor to each firm, subject to the equilibrium aggregate budget constraint on their unit of labor. For simplicity, assume homogeneous firms with skill p.

In equilibrium, $1-(1-\bar{q})p$ firms enter and post the same wage $w^*=\lambda(\bar{q},p)$; workers offer all their time to each of these firms. At each entering firm, $1-(1-\bar{q})p$ applicants pass the test and each sells an infinitesimal share $1/(1-(1-\bar{q})p)$ of their time to the firm. High types pass all tests, and so sell all of their time. In contrast, low types pass each test with probability $1-p$, and thus share p of their time remains unsold. Perfectly anticipating rationing outcomes allows talented workers to exactly hit the budget constraint on their unit of labor.

This construction corresponds to the equilibrium in Kurlat (2016)'s online appendix with non-nested signals. The equilibrium is robust to any correlation structure between firms' signals. However, it relies on labor being perfectly divisible: In finite models (as in Appendix B) where each worker can only work for finitely many firms, the randomness of independent testing and rationing across a finite number of firms prevents markets from clearing simultaneously and gives rise to adverse selection.[28]

Our above example corresponds to a simple case of Kurlat's model where all workers apply simultaneously to all firms. The same points applies to Kurlat's general model with multiple markets with endogenous screening orders (e.g. top-to-bottom, or bottom-to-top), all of which clear simultaneously. With indivisible labor, markets cannot clear simultaneously since workers that are not hired in market A apply in market B. We must therefore specify a clearing order across markets, which means A and B are not really different markets, and renders moot the issue of endogenous clearing orders within markets.

A.3. *Summary*

The key differences in Kurlat's model are that labor is perfectly divisible and firms' signals are perfectly nested. Either one of these assumptions allows firms to avoid adverse selection, and equilibrium features a single wage and efficient aggregate sorting. But if labor is not perfectly divisible and signals are not perfectly nested, adverse selection emerges and equilibrium wages fan out. Our model assumes more strongly that labor is indivisible and signals are conditionally independent. Equilibrium sorting is then positive assortative (assuming scarce talent) and inefficient.

28. To be more concrete, assume indivisible labor and independent tie-breaking. In Kurlat's equilibrium, each firm's offer w^* is top-ranked by a random binomial number of applicants, so some firms are the top choice of no workers and have to fill their positions with an adversely selected applicant who did not get their top choice. The general point that non-exclusive markets for indivisible labor cannot clear truly simultaneously, and that workers with multiple offers choose the one with the highest wage, is familiar from Montgomery (1991) and the directed search literature (e.g. Galenianos and Kircher (2009)).

B. STEADY STATE JUSTIFICATION FOR MARKET CLEARING RULE

As discussed in Section 2.2, this appendix justifies the instantaneous, mechanical top-to-bottom market clearing rule that underlies the sequential screening equation (2.3).

Formally, consider a sequence of labor markets, indexed by the number $N \in \mathbb{N}$ of infinitely-lived firms, labeled by their quantiles $x \in X_N := \{1/N, ..., 1\} \in (0,1]$ in the screening skill distribution; each firm has K_N jobs. Time is discrete $t \in T_N := \{\delta_N, 2\delta_N, 3\delta_N, ...\} \subset \mathbb{R}_+$. Each period t, each firm loses one random worker (into retirement) and N new workers are born, whose talent is iid with $\Pr(\theta = H) = \bar{q}$.

At time $t = 0$, the firms $x \in X_N$ are endowed with K_N workers of arbitrary talent and simultaneously and publicly post (lifetime) wages w_x, that are fixed for the remainder of the game. At each time $t \in T_N$, each unmatched worker applies to a firm or exits the market. Each firm screens its applicants sequentially and hires one of them; firms cannot delay filling their jobs but have additional access to an infinite pool of "duds", i.e. applicants who are known to be untalented. If a worker applies multiple times to the same firm, their test result is the same each time. Hired workers stay with their firm until retirement.

The market is anonymous but with perfect recall. Applicants only know which firms they applied to in the past; they do not know their own type, why past firms rejected them, or the application choices of other workers. Firms only know their own past applicants and their test results; they do not know when a worker entered the market, which firms she applied to in the past, or her test results at the other firms.

Payoffs are as follows. As in the static model, workers get wage w when they join a firm and outside option $\underline{w}$ when they exit. Firms get $\theta - w$ when they hire a worker of quality θ. All payoffs are discounted at rate β.

We are interested in the continuum limit of these markets, $N \to \infty$. Fix real-time worker turnover α by setting $K_N \delta_N \equiv 1/\alpha$. Additionally, assume $N\delta_N \to 0$ so workers can apply to all jobs "quickly"; this implies $\delta_N \to 0$, $K_N \to \infty$, and $K_N/N \to \infty$. A sequence of strategy profiles is a *limit equilibrium* if there exists $(\epsilon_N) \gg 0$ with $\lim \epsilon_N = 0$, such that in market N all firms and all workers born at times $t \geq \epsilon_N$ are playing a ϵ_N-best response.

Define the following (sequence of) *baseline strategies.*

(1) Firm $x \in X_N$ chooses the equilibrium-wage $w(x)$ of the baseline, continuum model in equation (2.6).

(2) Workers apply to jobs with wages $w(x) \geq \underline{w}$ top-to-bottom (i.e. they apply to a firm with the highest wage, among firms they have not applied to in the past).[29]

(3) Each firm screens new applicants in random order and hires the first one to pass its test. If there is no such new applicant, the firm hires a dud.

Lemma 2. The baseline strategies are a limit equilibrium.

Proof. Consider items (1)-(3) of baseline strategies in reverse order, starting with a firm's problem at time $t > 0$ for any wage profile. Given workers' application strategies, the firm cannot affect its future applicants and optimally hires an applicant with the highest expected talent, as prescribed by its baseline strategy.

Turning to the workers' problem, choose $\epsilon_N > N\delta_N$ so every firm has its steady state number of applicants at $t \geq \epsilon_N$. The worker's chance of getting a job at any fixed firm is time-invariant. The only benefit of applying to low-wage firms first is to shorten the job-search, but the benefit of this time-value is upper-bounded by $1 - e^{-\beta N \delta_N} \to 0$.

Finally, consider wages. For quantile $x \in X_N$, let $Q_N(x)$ be expected applicant talent at rank x in market N. As grid size $1/N \to 0$, the law-of-motion for discrete-firm applicant talent $Q_N(x)$ approaches (3), and so its solution $Q_N(x)$ approximates $Q(x)$. Thus, the required wage for quality q applicants $W_N(q)$ approaches its continuum equilibrium level $W(q)$, so the wages are arbitrarily close to optimal for large N. $\|$

Lemma 2 provides a micro-foundation for our reduced-form static model. Underlying it are two key properties. First, workers don't wish to jump the queue. This follows since (i) the market is in steady state and firms don't know the age of applicants, so jumping the queue provides no information to firms,

29. If two or more firms post the same wages, workers break the tie in an arbitrary order, e.g. the same order, as in footnote 8, or independently across firms.

and (ii) workers can search quickly, so searching top-to-bottom is ϵ-optimal.[30] Second, workers don't wish to reapply. This follows since (i) firms recall past applicants and can reject them, and (ii) workers obtain the same test result when reapplying, so there is no advantage to passing the test twice.

One may have various concerns with the micro-founded model, the baseline strategies, and Lemma 2. One concern is that we use (limit) Nash equilibrium rather than subgame perfection; we do so because we do not know how to formalize "ϵ_N-sequential rationality" (e.g. Flesch and Predtetchinski (2016)) in a model where periods become short. Absent sequential rationality requirements, our baseline strategies simply have firms hire duds in the off-path event $\mathcal{E}$ where the only applicant to pass a firm's test is a re-applicant. To achieve the spirit of sequential rationality in this specific issue, one could instead have firms hire re-applicants in event $\mathcal{E}$. Re-applying remains suboptimal if $\log K_N/N \to 0$ so the probability of event $\mathcal{E}$ vanishes faster than δ_N.[31]

Another concern is equilibrium uniqueness; it is unclear how to establish this given the richness of the application subgame after $t=0$. Short of that, we can argue uniqueness of salient parts of the equilibrium. First, top-to-bottom is the essentially unique limit-optimal application order: The loss from applying to (and getting hired by) low-wage firms before trying one's chances at high-wage firms does not vanish with N. Second, given the baseline strategies in the application subgame after $t=0$, baseline wages constitute the unique limit Nash equilibrium for firms at $t=0$. Importantly, the fact that equilibrium wages are distributed smoothly without ties, Lemma 1, does not depend on the specific, uniform tie-break rule in footnote 8.

As in other directed search models, we assume that firms commit to posted wages at $t=0$. This is motivated by the idea that firms have "wage policies" (e.g. Netflix pays more than Disney). Whether firms would wish to renege depends on whether they can trick workers with the following bait-and-switch strategy: Post wage w and when a worker applies and passes the test, renege and make an exploding offer at a lower wage $w-\epsilon$; for small ϵ this still exceeds the worker's value of continued search. However, if the worker can defer the $w-\epsilon$ offer, and apply at the firms with intermediate wages $[w-\epsilon,w]$ before finally accepting, such reneging is equivalent to posting an initial wage of $w-\epsilon$ and hence unprofitable.

Other modeling assumptions are for simplicity. We assume each firm loses one worker a period, tying together firm size and period length, but one could let firms skip some periods or lose workers at a Poisson rate. We assume workers do not know their type or test outcomes, but the baseline strategy is still ϵ_N-optimal if they knew their type. We also assume a firm must fill a vacancy immediately, so hires a dud if no applicant passes the test, but one could allow them to delay in such an event, allowing us to remove the pool of duds from the model.

C. PROOF OF THEOREM 2

In this section we prove our main inefficiency result, Theorem 2. In online Appendix F we extend it to two model variants with production complementarities and screening costs, introduced in Section 3.1. To accommodate these variants, we first formulate a more general model where the surplus when a firm with skills p hires from an applicant pool with quality q is given by a general function $\omega(q,p)$. The baseline model corresponds to $\omega(q,p)=\lambda(q,p)-\underline{w}$, production complementarities correspond to $\omega(q,p)=h(p)\cdot\lambda(q,p)-\underline{w}$, and screening costs to $\omega(q,p)=\lambda(q,p)-\kappa/(1-p(1-q))-\underline{w}$.

We develop the apparatus to analyze aggregate surplus for general surplus functions $\omega(q,p)$ in Appendix C.1, and specialize to the baseline model and prove Theorem 2 in Appendix C.2.

C.1. *Aggregate Surplus for General Surplus Functions $\omega(q,p)$*

Fix the number of entering firms, and thereby the cutoff $\underline{x}$. Aggregate surplus under matching $P(\cdot)$ is given by

30. Note that the definition of "limit equilibrium" does not require the first few generations to play optimally; such agents may wish to "jump the queue" before the market reaches steady state. Fortunately, reaching steady state takes a vanishing amount of time.

31. *Proof:* Indeed, $\log K_N/N \to 0$ implies $\zeta^N/\delta_N \to 0$ for any $\zeta<1$. Then the chance of firm x having no new applicant who passed its test at any period in some time interval $[t,t']$ is upper-bounded by $((1-Q_N(x))P(x))^{Nx}(t'-t)/\delta_N$ which vanishes as $N\to\infty$.

$$S(P(\cdot))=\int_{\underline{x}}^{1}\omega(Q(x),P(x))dx. \tag{C.1}$$

Consider adding a new firm p at rank x. This has three effects on surplus. First, firm p hires from a applicant pool with quality $Q(x)$ and so generates surplus $\omega(Q(x),p)$. Second, firm p pushes out the marginal firm, $P(\underline{x})$. Third, it lowers the ranking of intermediate firms, $\check{x}<x$, thereby lowering their applicant quality, $Q(\check{x})$. To quantify the latter externality, note that firm p's hiring reduces applicant quality just below x by $\phi(Q(x),p,x)$, as defined by (2.3). For lower ranking firms $\check{x}$, this effect is mitigated in two ways. First, the reduction in applicant quality reduces talent extraction by intermediate firms $\hat{x}\in[\check{x},x]$; standard results on ODEs show that this reduces the effect on $Q(\check{x})$ by a factor $\exp\left(-\int_{\check{x}}^{x}\phi_q(Q(\hat{x}),P(\hat{x}),\hat{x})d\hat{x}\right)$.[32] Second, adding a firm at rank x pushes intermediate firms $\hat{x}\in[\check{x},x]$ to lower wage ranks and thereby reduces their externality by $-\phi_x(Q(\hat{x}),p(\hat{x}),\hat{x})$; this second effect in turn is reduced itself by the first effect. All told, firm p's total effect on applicant quality $Q(\check{x})$ is given by

$$\chi(\check{x};p,x,P(\cdot))=-\phi(Q(x),p,x)\exp\left(-\int_{\check{x}}^{x}\phi_q\right)-\int_{\check{x}}^{x}\left[\exp\left(-\int_{\check{x}}^{\hat{x}}\phi_q\right)(-\phi_x(Q(\hat{x}),P(\hat{x}),\hat{x}))\right]d\hat{x}, \tag{C.2}$$

where we dropped the arguments in the integrand $\phi_q=\phi_q(Q(\tilde{x}),P(\tilde{x}),\tilde{x})$ to enhance legibility. Putting these three effects together, firm p's (infinitesimal) net-contribution to surplus when assigned to wage rank x in matching $P(\cdot)$ is thus given by

$$s(p,x,P(\cdot))=\omega(Q(x),p)+\int_{\underline{x}}^{x}\chi(\check{x};p,x,P(\cdot))\omega_q(Q(\check{x}),P(\check{x}))d\check{x}-\omega(Q(\underline{x}),P(\underline{x})). \tag{C.3}$$

We wish to evaluate the effect of changing a matching function $P(\cdot)$ into a second matching function $P'(\cdot)$. To do this, suppose we move one firm p from rank $\underline{x}(p)$ to $\bar{x}(p)$ given matching $P(\cdot)$. The (infinitesimal) change in surplus equals

$$\int_{\underline{x}(p)}^{\bar{x}(p)}s_x(p,x,P(\cdot))dx.$$

More generally, let us sequentially move all firms $p\in[\underline{p},\bar{p}]$ in order of increasing p. The matching function changes over the course of this transformation, and we write $P^p(\cdot)$ for the matching function after firms $p'<p$ have been shifted; thus $P^{\underline{p}}(\cdot)=P(\cdot)$ and $P^{\bar{p}}(\cdot)=P'(\cdot)$. The aggregate change in surplus is given by

$$S(P'(\cdot))-S(P(\cdot))=\int_{\underline{p}}^{\bar{p}}\left[\int_{\underline{x}(p)}^{\bar{x}(p)}s_x(p,x,P^p(\cdot))dx\right]dF(p). \tag{C.4}$$

We first establish a general formula for the integrand in (C.4).

Lemma 3. The marginal surplus of moving firm p past wage rank x given matching $P(\cdot)$ with $P(x)=\hat{p}$ equals

$$s_x(p,x,P(\cdot))=\Big[\omega_q(Q(x),p)\phi(Q(x),\hat{p},x)-\omega_q(Q(x),\hat{p})\phi(Q(x),p,x)\Big] \tag{C.5}$$

$$-\Big[\phi_q(Q(x),p,x)\phi(Q(x),\hat{p},x)+\phi_x(Q(x),p,x)-\phi_q(Q(x),\hat{p},x)\phi(Q(x),p,x)-\phi_x(Q(x),\hat{p},x)\Big]\gamma,$$

where

$$\gamma:=\gamma(x,P(\cdot))=\int_{\underline{x}}^{x}\exp\left(-\int_{\check{x}}^{x}\phi_q\right)\omega_q(Q(\check{x}),P(\check{x}))d\check{x}. \tag{C.6}$$

To understand these equations, consider swapping p and $\hat{p}$, and let us drop the arguments $Q(x)$ and x to enhance legibility. The first term in the first line of (C.5) $\omega_q(p)\phi(\hat{p})$ is the increased surplus contribution of firm p when selecting before $\hat{p}$ from a pool with $dQ(x)=\phi(\hat{p})dx$ higher quality. Conversely, the

32. Indeed, Hartman (2002, Theorem 3.1) shows that the solution of the ODE $Q_x(x,\bar{q},p)=\phi(x,Q(x,\bar{q},p),p)$ with boundary condition $Q(1,\bar{q},p)=\bar{q}$ satisfies $Q_{x\bar{q}}=\phi_qQ_{\bar{q}}$.

second term $\omega_q(\hat{p})\phi(p)$ is $\hat{p}$'s decreased surplus when selecting after p. The square-bracketed term in the second line of (C.5) is the effect of switching p and $\hat{p}$'s screening order on residual pool quality $Q(x-dx)$. By screening applicants $q+\phi(\hat{p})dx$ from a pool of size $x+dx$ (rather than applicants q from pool size x), firm p's selection increases by $[\phi_q(p)\phi(\hat{p})+\phi_x(p)]dx$; conversely firm $\hat{p}$'s selection decreases by $[\phi_q(\hat{p})\phi(p)+\phi_x(\hat{p})]dx$. This change in applicant quality $dQ(x)$ affects revenue $\omega(Q(\check{x}),P(\check{x}))$ of all lower-ranking firms $\check{x}\in[\underline{x},x]$, where the exponential term in (C.6) equals $dQ(\check{x})/dQ(x)$ and captures differential selection by intermediate firms $\hat{x}\in[\check{x},x]$.

Clearly, switching two identical firms $p=\hat{p}=P(x)$ makes no difference; formally

$$s_x(P(x),x,P(\cdot))=0. \tag{C.7}$$

Proof. Differentiating (C.3)

$$\begin{aligned} s_x(p,x,P(\cdot)) &= \omega_q(Q(x),p)Q_x(x)+\chi(x;p,x,P(\cdot))\omega_q(Q(x),P(x)) \\ &\quad +\int_{\underline{x}}^{x}\chi_x(\check{x};p,x,P(\cdot))\omega_q(Q(\check{x}),P(\check{x}))d\check{x}. \end{aligned}$$

Since $Q_x(x)=\phi(Q(x),P(x),x)$ and $\chi(x;p,x,P(\cdot))=-\phi(Q(x),p,x)$, and recalling $P(x)=\hat{p}$, the first two terms correspond to the first line of (C.5).

As for the last term, $\chi_x(\check{x};p,x)$ equals

$$-\Big[\phi_q(Q(x),p,x)Q_x(x)+\phi_x(Q(x),p,x)-\phi(Q(x),p,x)\phi_q(Q(x),P(x),x)-\phi_x(Q(x),P(x),x)\Big]\exp\left(-\int_{\check{x}}^{x}\phi_q\right),$$

where the term is square-brackets equals the square-bracket term in the second line of (C.5), while integration over $\check{x}\in[\underline{x},x]$ yields $\int_{\underline{x}}^{x}\exp\left(-\int_{\check{x}}^{x}\phi_q\right)\omega_q(Q(\check{x}),P(\check{x}))d\check{x}=\gamma$. $\quad\|$

We cannot determine the sign of (C.5) in general. But at the lowest wage rank $x=\underline{x}$, the analysis simplifies because the integral domain in (C.6) collapses and so $\gamma=0$, yielding a necessary condition for optimality of PAM that is easy to check. We will check that this condition is violated for the model variants in Propositions 3 and 4, in online Appendix F.

Lemma 4. Assume that $1-\underline{x}$ firms $p\in[\underline{p},\bar{p}]$ enter. If

$$\frac{\omega_{qp}(Q(\underline{x}),\underline{p})}{\omega_q(Q(\underline{x}),\underline{p})}<\frac{\phi_p(Q(\underline{x}),\underline{p},\underline{x})}{\phi(Q(\underline{x}),\underline{p},\underline{x})} \tag{C.8}$$

then PAM does not maximize surplus.

Proof. Let us transform $P(\cdot)=P^{PAM}(\cdot)$ into another matching $P'(\cdot)$ with NAM for $x\in[\underline{x},\underline{x}+\epsilon]$ and PAM for $x\in(\underline{x}+\epsilon,1]$. Intuitively, if there were 10 firms with skill $p_1<\ldots<p_{10}$ then this would mean swapping p_1 and p_2, so firms are ranked $\{p_2,p_1,p_3,\ldots p_{10}\}$, from lowest to highest. Formally $P'(x)=F^{-1}(\underline{x}+\epsilon-(x-\underline{x}))$ for $x\in[\underline{x},\underline{x}+\epsilon]$, and $P'(x)=F^{-1}(x)$ for $x>\underline{x}+\epsilon$, where $\epsilon>0$ is small. We transform $P(\cdot)$ into $P'(\cdot)$ by shifting firms $p\in[\underline{p},F^{-1}(\underline{x}+\epsilon)]$ (in rising order of p) to their $P'(\cdot)$-wage rank $\bar{x}(p)=\underline{x}+\epsilon-[F(p)-F(\underline{p})]$. Since lower firms $p'<p$ have already been shifted to $\bar{x}(p')>\bar{x}(p)$ at firm p's "turn," firm p starts at rank $\underline{x}(p)=F(p)<\bar{x}(p)$ and is shifted exclusively past firms with higher screening skills $P^p(x)=F^{-1}[F(p)+x-\underline{x}]\geq p$, recalling the definition of the matching function $P^p(\cdot)$ at p's "turn."

We now argue that given (C.8) the net value of this transformation (C.4) is positive. Using (C.7), the integrand in (C.4) equals

$$s_x(p,x,P^p(\cdot))=s_x(P^p(x),x,P^p(\cdot))-\int_p^{P^p(x)}s_{xp}(\check{p},x,P^p(\cdot))d\check{p}=-\int_p^{P^p(x)}s_{xp}(\check{p},x,P^p(\cdot))d\check{p}. \tag{C.9}$$

To see that this is positive, differentiate (C.5) with respect to p, and evaluate for firm $\underline{p}$ at wage rank $\underline{x}$. The derivative of the second line is zero because $\gamma=0$ for $x=\underline{x}$. Thus,

$$s_{xp}(\underline{p},\underline{x},P^{PAM}(\cdot))=\omega_{qp}(Q(\underline{x}),\underline{p})\phi(Q(\underline{x}),\underline{p},\underline{x})-\omega_q(Q(\underline{x}),\underline{p})\phi_p(Q(\underline{x}),\underline{p},\underline{x}),$$

which is negative by (C.8). Next, $(\check{p},x,P^p(\cdot))$ converges to $(\underline{p},\underline{x},P^{PAM}(\cdot))$ for all $\check{p},p\in[\underline{p},F^{-1}(\underline{x}+\epsilon)]$ and $x\in[\underline{x},\underline{x}+\epsilon]$ as $\epsilon\to 0$, using for instance the topology of uniform convergence on matching functions $P(\cdot)$. Thus, the integrand of (C.9) is negative, and so the integral (C.4) is positive, so the transformation from $P(\cdot)$ to $P'(\cdot)$ raises welfare. $\quad\|$

C.2. *Proof of Theorem 2*

We now return to the baseline model where surplus is given by $\omega(q,p)=\lambda(q,p)-\underline{w}$. The comparison of the elasticities (3.10) implies (C.8), and so Lemma 4 already implies that PAM is inefficient. In particular, the proof of Lemma 4 shows that locally near the cutoff $\underline{x}$ shifting low-skill firms ahead of high-skill firms increases surplus.

Theorem 2 claims more strongly that surplus is minimized by PAM and maximized by NAM. We show this by arguing that for the baseline surplus function $\omega(q,p)=\lambda(q,p)-\underline{w}$ our local argument at the bottom of the wage distribution in fact holds globally.

Indeed, (C.5) simplifies considerably:

Lemma 5. When $\omega(q,p)=\lambda(q,p)-\underline{w}$, the marginal surplus of moving firm p past wage rank x given matching $P(\cdot)$ with $P(x)=\hat{p}$ equals

$$s_x(p,x,P(\cdot))=[\lambda_q(p)\phi(\hat{p})-\lambda_q(\hat{p})\phi(p)]\nu, \tag{C.10}$$

where $\nu:=1-\gamma/x=\exp\left(-\int_{\underline{x}}^{x}\frac{\lambda_q(Q(\breve{x}),P(\breve{x}))}{\breve{x}}d\breve{x}\right)\in(0,1)$.

The term in square-brackets corresponds to the incremental talent hired by firms p and $\hat{p}$ when the former is shifted ahead of the latter. The effect on aggregate surplus is scaled down by the factor ν from (3.9) since incremental talent hired by the marginal firms p and $\hat{p}$ reduces the talent hired by lower-ranking firms.

Proof. We will show that the marginal surplus (C.5) equals

$$s_x(p,x,P(\cdot))=\Big[\lambda_q(p)\phi(\hat{p})-\lambda_q(\hat{p})\phi(p)\Big]-\Big[\lambda_q(p)\phi(\hat{p})-\lambda_q(\hat{p})\phi(p)\Big]\frac{\gamma}{x} \tag{C.11}$$

and thus collapses to (C.10). Using the definition of $\omega(q,p)$, the first line in (C.5) equals the first square-bracket term in (C.11). Turning to the second line in (C.5) and recalling $\phi=\frac{\lambda-q}{x}$, elementary algebra implies

$$\phi_q(p)\phi(\hat{p})+\phi_x(p)-\phi_q(\hat{p})\phi(p)-\phi_x(\hat{p})=\frac{1}{x}[\lambda_q(p)\phi(\hat{p})-\lambda_q(\hat{p})\phi(p)]. \tag{C.12}$$

Multiplying by γ, the second line in (C.5) equals the second square-bracket term in (C.11).

To evaluate γ observe that

$$\exp\left(-\int_{\breve{x}}^{x}\phi_q\right)=\exp\left(-\int_{\breve{x}}^{x}\frac{\lambda_q(Q(\hat{x}),P(\hat{x}))-1}{\hat{x}}d\hat{x}\right)=\frac{x}{\breve{x}}\exp\left(-\int_{\breve{x}}^{x}\frac{\lambda_q(Q(\hat{x}),P(\hat{x}))}{\hat{x}}d\hat{x}\right).$$

Equation (C.6) thus simplifies to

$$\begin{aligned}\gamma&=x\int_{\underline{x}}^{x}\exp\left(-\int_{\breve{x}}^{x}\frac{\lambda_q(Q(\hat{x}),P(\hat{x}))}{\hat{x}}d\hat{x}\right)\frac{\lambda_q(Q(\breve{x}),P(\breve{x}))}{\breve{x}}d\breve{x}\\&=x\int_{\underline{x}}^{x}\frac{d}{d\breve{x}}\exp\left(-\int_{\breve{x}}^{x}\frac{\lambda_q(Q(\hat{x}),P(\hat{x}))}{\hat{x}}d\hat{x}\right)d\breve{x}=x\left[1-\exp\left(-\int_{\underline{x}}^{x}\frac{\lambda_q(Q(\breve{x}),P(\breve{x}))}{\breve{x}}d\breve{x}\right)\right],\end{aligned} \tag{C.13}$$

as required. ||

In the proof sketch in the body of the paper we argued that (C.10) is positive when $\hat{p}=p+dp$ by noting that $\lambda_q(p)\phi(p+dp)-\lambda_q(p+dp)\phi(p)=(\lambda_q\phi_p-\lambda_{qp}\phi)dp>0$, as shown in (3.10). To prove Theorem 2, we now generalize this local argument to show that transforming an arbitrary matching $P(\cdot)$ into $P'(\cdot)=P^{NAM}(\cdot)$ raises surplus. Intuitively, if there were 10 firms with skill $p_1<\ldots<p_{10}$, we would shift firm p_1 to the highest position, then shift firm p_2 to the second-highest position, and so on. Formally, we shift type-p firms in rising order of p to their NAM-rank $\bar{x}(p)=1-[F(p)-F(\underline{p})]$. At firm p's turn, i.e. in matching $P^p(\cdot)$, lower-skill firms $p'<p$ have already been shifted to their NAM rank in $\bar{x}(p')>\bar{x}(p)$, and so firm p starts at $\underline{x}(p)\le\bar{x}(p)$ and is shifted past higher-skill firms $P^p(x)\ge p$ for all $x\in[\underline{x}(p),\bar{x}(p)]$.

Recalling $s_x(P^p(x),x,P^p(\cdot))=0$ from (C.7), the marginal surplus along this transformation is given by

$$s_x(p,x,P^p(\cdot))=-\int_{p}^{P^p(x)}s_{xp}(\breve{p},x,P^p(\cdot))d\breve{p}. \tag{C.14}$$

To sign the integrand of this expression, we differentiate (C.10) with respect to p

$$s_{xp}(p,x,P^p(\cdot))=[\lambda_{qp}(p)\phi(\hat{p})-\phi_p(p)\lambda_q(\hat{p})](1-\gamma/x)<0.$$

Substituting back into (C.14) and recalling that $p<P^p(x)$, marginal surplus (C.14) is positive. Finally, integrating (C.14) over $p\in[\underline{p},\bar{p}]$ and $x\in[\underline{x}(p),\bar{x}(p)]$, we conclude that the aggregate change of surplus (C.4) from this transformation is positive, and NAM maximizes aggregate surplus. The analogue argument implies that PAM minimizes aggregate surplus.

Finally we establish that the planner indeed wants the firms with the highest screening skills to enter the market. To see this, differentiate firm p's contribution to surplus (C.3) with respect to p

$$\begin{aligned}
s_p(p,x,P(\cdot))&=\lambda_p(Q(x),p)+\int_{\underline{x}}^{x}\chi_p(\check{x};p,x,P(\cdot))\lambda_q(Q(\check{x}),P(\check{x}))d\check{x}\\
&=\lambda_p(Q(x),p)-\phi_p(Q(x),p,x)\int_{\underline{x}}^{x}\exp\left(-\int_{\check{x}}^{x}\phi_q\right)\lambda_q(Q(\check{x}),P(\check{x}))d\check{x}\\
&=\lambda_p(Q(x),p)-\left(\frac{\lambda_p(Q(x),p)}{x}\right)x\left[1-\exp\left(-\int_{\underline{x}}^{x}\frac{\lambda_q(Q(\check{x}),P(\check{x}))}{\check{x}}d\check{x}\right)\right]\\
&=\lambda_p(Q(x),p)\exp\left(-\int_{\underline{x}}^{x}\frac{\lambda_q(Q(\check{x}),P(\check{x}))}{\check{x}}d\check{x}\right)>0, \qquad \text{(C.15)}
\end{aligned}$$

where the second equality uses the definition of the externality χ in (C.2), and the third equality uses the definition of γ (C.6) and its evaluation in (C.13).

D. PROOFS FROM SECTION 4

D.1. *Derivation of Equation (4.17) and Proof that $V(r)$ is Convex*

The envelope theorem applied to (4.14) implies

$$V_r(r(t),t)=\int_t^\infty e^{-\beta(s-t)}\frac{\partial r(s)}{\partial r(t)}ds.$$

To compute the integrand, we write the solution of the talent evolution (4.13) as a function of its initial condition $r_s(s,\underline{r})=\zeta(r(s,\underline{r}),s)$ where $\zeta(r,s)=\alpha(\lambda(Q(s),\psi(r))-r)$ and $r(t,\underline{r})=\underline{r}$. As in footnote 32 we get $r_{s\underline{r}}=\zeta_r r_{\underline{r}}$ with boundary condition $r_{s\underline{r}}(t,\underline{r})=1$. Hence, (4.17) follows from

$$\frac{\partial r(s)}{\partial r(t)}=r_{\underline{r}}(s,r(t))=\exp\left(\int_t^s\zeta_r(r(u),u)du\right)=\exp\left(-\alpha\int_t^s[1-\lambda_p(Q(u),\psi(r(u)))\psi_r(r(u))]du\right).$$

To see that $V(r,t)$ is convex in r, we differentiate again to obtain

$$\begin{aligned}
V_{rr}(r(t),t)&=\int_t^\infty e^{-\beta(s-t)}\frac{\partial^2 r(s)}{\partial r(t)^2}ds\\
&=\int_t^\infty e^{-\beta(s-t)}\frac{\partial r(s)}{\partial r(t)}\alpha\int_t^s\left[\lambda_{pp}(Q(u),\psi(r(u)))\psi_r(r(u))^2+\lambda_p(Q(u),\psi(r(u)))\psi_{rr}(r(u))\right]\frac{\partial r(u)}{\partial r(t)}duds,
\end{aligned}$$

which is positive since $\lambda_p=\frac{q(1-q)}{(1-p(1-q))^2},\lambda_{pp}=\frac{2q(1-q)^2}{(1-p(1-q))^3},\psi_r,\psi_{rr}$ are all positive. For an intuition consider the random recruiter example where $\psi(r)=p^L+r(p^H-p^L)$. Intermediate levels of recruiting skills r have the draw back that the firm's wage must strike a compromise between the firm's low-skill and high-skill recruiters, while a firm with homogeneous recruiters, $r=0$ or 1, can choose the optimal wage for all.

D.2. *Proof of Theorem 3*

Here we complete the proof of Theorem 3 by arguing that if there is an atom of initially identical firms, these firms diverge immediately. Assume to the contrary, that at time $t>0$ an atom of firms has the same worker quality $r(t)$ and write $[x^0,x^1]$ for the talent-ranks of these firms. Since optimal wages rise in talent and hence talent differences never vanish, firms in the atom must have identical talent $r(s)$ for all $s<t$. At any time $s<t$ the wage distribution must be smooth by the arguments in Section 2.2. If firms in the atom post different wages, they drift apart. Hence, firms must mix as in footnote 20; in continuous-time we interpret mixing as distributional strategies, posting both high and low wages to attract good and bad applicants; they must thus be indifferent across a range of applicants $[q^0(s),q^1(s)]$ for all $s<t$. Thus, the first order condition (4.16) must hold with equality on $[q^0(s),q^1(s)]$ for all $s<t$ and the atom quality $r(s)$.

Such distributional strategies cannot be optimal because wages are dynamic complements: Consider a firm that deviates by always attracting the best applicants in the atom $q^1(s)$, rather than mixing over good and bad applicants; this firm's talent $r^1(s)$ strictly exceeds $r(s)$ at all $s>0$. Since the marginal benefit of attracting better applicants, the RHS of (4.16), strictly increases in r and (4.16) holds at $r(s)$, raising wages to attract the top applicants $q^1(s)$ is strictly profitable. This proves that initially identical firms diverge immediately.

D.3. *Proof of Theorem 4*

Here we show that firm x's talent $R(x,t)$ and applicant quality $Q(x,t)$ converge to their steady state levels $R^*(x)$ and $Q^*(x)$. For constant applicants $Q(x,t)\equiv q$, talent drifts towards $\rho(q)$. The complication is that firm x's applicant quality $Q(x,t)$ also changes over time, with $Q_x(x,t)$ given by (4.12) and $r_t(x,t)$ by (4.13). As discussed in Section 4.4, Figure 2 suggests a "proof by induction." The formal proof is more complicated because x is continuous; it proceeds by showing that the steady state satisfies a contraction property, and then applies the contraction mapping theorem over a small interval, akin to the proof of the Picard-Lindelöf theorem.

First, we establish a contraction property. Define the limits $\underline{Q}(x):=\liminf_t Q(x,t)$, $\bar{Q}(x):=\limsup_t Q(x,t)$, $\underline{r}(x):=\liminf_t R(x,t)$, and $\bar{R}(x):=\limsup_t R(x,t)$ and interpret (4.12) as an operator $\mathcal{Q}$, mapping firm quality functions $R(\cdot,t)$ into applicant quality functions $Q(\cdot,t)=\mathcal{Q}[R(\cdot,t)](\cdot)$. We claim that:

$$\mathcal{Q}[\rho(\bar{Q}(\cdot))](x)\le\underline{Q}(x)\le\bar{Q}(x)\le\mathcal{Q}[\rho(\underline{Q}(\cdot))](x). \tag{D.16}$$

To understand (D.16), first note that $\mathcal{Q}$ is antitone: If $R(x)\ge\hat{R}(x)$ for all x then $Q(x)=\mathcal{Q}(R(\cdot))(x)\le \mathcal{Q}(\hat{R}(\cdot))(x)=\hat{Q}(x)$, since $Q(1)=\hat{Q}(1)=\bar{q}$ and $\phi(q,\psi(r),x)$ rises in r. Intuitively, better recruiters introduce more adverse selection. Inequalities (D.16) then state that if applicant quality was equal to one of its limits, $\underline{Q}$ and $\bar{Q}$, and talent r was in steady state, the induced difference in applicant pools is larger than the original difference.

We prove (D.16) in two steps. First, since $R(x,t)$ drifts towards $\rho(Q(x,t))$, which is asymptotically bounded by $\rho(\underline{Q}(x))$ and $\rho(\bar{Q}(x))$, we have

$$\rho(\underline{Q}(x))\le\underline{R}(x)\le\bar{R}(x)\le\rho(\bar{Q}(x)) \tag{D.17}$$

for all x. Second,

$$\underline{Q}(x)=\lim_{t\to\infty}\inf_{t'>t}Q(x,t')=\lim_{t\to\infty}\inf_{t'>t}\{\mathcal{Q}[R(\cdot,t')](x)\}\ge\lim_{t\to\infty}\mathcal{Q}[\sup_{t'>t}\{R(\cdot,t')\}](x)=\mathcal{Q}[\bar{R}(\cdot)](x).$$

The first equality is the definition of the liminf, and the second the definition of the operator $\mathcal{Q}$. The inequality uses the antitonicity of $\mathcal{Q}$: Since $R(\hat{x},t')\le\sup_{t'>t}\{R(\hat{x},t')\}$ for all $t'>t$ and $\hat{x}$, we know that $\mathcal{Q}[R(\cdot,t')](x)$ (weakly) exceeds $\mathcal{Q}[\sup_{t'>t}\{R(\cdot,t')\}](x)$ for all $t'>t$ and x, and hence so does $\inf_{t'>t}\mathcal{Q}[R(\cdot,t')](x)$. The final equality uses the dominated convergence theorem to exchange the limit $t\to\infty$ and the operator $\mathcal{Q}$, as well as the definition of the limsup, $\bar{R}(x)=\lim_{t\to\infty}\sup_{t'>t}R(x,t')$. Together with the analogue argument that $\bar{Q}(x)\le\mathcal{Q}[\underline{R}(\cdot)](x)$, and applying the antitone operator $\mathcal{Q}$ to (D.17) we get

$$\mathcal{Q}[\rho(\bar{Q}(\cdot))](x)\le\mathcal{Q}[\bar{R}(\cdot)](x)\le\underline{Q}(x)\le\bar{Q}(x)\le\mathcal{Q}[\underline{R}(\cdot)](x)\le\mathcal{Q}[\rho(\underline{Q}(\cdot))](x),$$

establishing (D.16).

To complete the proof of convergence, suppose "inductively" that applicant and firm quality converge above some $\hat{x}\in(0,1]$, i.e. $\underline{Q}(x)=\bar{Q}(x)$, and hence $\underline{R}(x)=\bar{R}(x)$, for all $x\in(\hat{x},1]$. Fix ϵ, and let $\delta(\epsilon):=\max_{x\in[\hat{x}-\epsilon,\hat{x}]}\left|\bar{Q}(x)-\underline{Q}(x)\right|$. Since $\rho(q)$ is locally Lipschitz in q, and the RHS of the ODE (4.12) that defines $\mathcal{Q}[R(\cdot)](x)$ is locally Lipschitz in q and r, we get

$$\max_{x\in[\hat{x}-\epsilon,\hat{x}]}\left|\mathcal{Q}[\rho(\underline{Q}(\cdot))](x)-\mathcal{Q}[\rho(\bar{Q}(\cdot))](x)\right|\le K\epsilon\delta(\epsilon)<\delta(\epsilon)=\max_{x\in[\hat{x}-\epsilon,\hat{x}]}\left|\bar{Q}(x)-\underline{Q}(x)\right|$$

for some constant K and $\epsilon<1/K$, contradicting (D.16). Hence we must have $\underline{Q}(x)=\bar{Q}(x)$ and hence $\underline{R}(x)=\bar{R}(x)$, for all $x\in[\hat{x}-\epsilon,1]$, and thus for all $x\in[0,1]$.

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Data Availability

The code underlying Figure 2 is available on Zenodo at https://dx.doi.org/10.5281/zenodo.21385256.

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