Capital Unemployment*

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Abstract

This paper studies the unemployment of physical capital—defined as idle units searching to be traded—and its macroeconomic implications. I provide evidence documenting that capital unemployment is large, volatile, and increases during economic downturns. I construct a capital-accumulation model that explains these patterns, with trading frictions in capital markets, which give rise to equilibrium capital unemployment, and financial shocks, which lead to large fluctuations in trading probabilities and capital unemployment. Using the model, I show that trading frictions and capital unemployment matter for aggregate dynamics. First, unemployed capital affects aggregate investment dynamics: Downturns characterized by large increases in unemployed capital are followed by investment slumps, because the economy tends to recover by absorbing existing unemployed capital rather than by producing new capital goods. Second, capital unemployment constitutes a propagation mechanism from financial shocks to economic activity, which shows up at the aggregate level as measured total factor productivity.

JEL Classification: E22, E23, E32

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1 Introduction

> *When it was first finished, the Empire State Building sat empty. The Great Depression hit the country hard. Thousands of companies closed down and millions of people were out of work. No company wanted the office space. People started calling it the "Empty State Building."*
>
> – C. Willis and D. Friedman, *Building the Empire State*, 1998

Capital accumulation plays a central role in macroeconomics for both business-cycle dynamics and long-run growth. A standard assumption in capital-accumulation models is that capital trading occurs in Walrasian markets, with prices ensuring that buyers' capital demand absorbs capital listed by sellers. Yet empirical studies on physical capital markets using microlevel data document patterns consistent with trading frictions and long times between the moment a unit of capital is listed and the time it is sold (see, for example, Ramey and Shapiro, 2001; Gavazza, 2011). This raises a key question: How do frictions in the trading of physical capital matter for aggregate dynamics?

Motivated by this question, in this paper I study the unemployment of physical capital—defined as idle units searching to be traded—and its macroeconomic implications. I first document that in the data, capital unemployment is large, volatile, and increases during economic downturns. I construct a capital-accumulation model that explains these data patterns, with trading frictions in capital markets, which give rise to equilibrium unemployment, and financial shocks, which lead to large fluctuations in trading probabilities and capital unemployment. Using the model, I then show that trading frictions and capital unemployment matter for aggregate dynamics. First, the stock of unemployed capital affects aggregate investment dynamics: downturns characterized by large increases in unemployed capital are followed by investment slumps because the economy tends to recover by absorbing existing unemployed capital rather than by producing new capital goods. Second, fluctuations of capital unemployment are a quantitatively important propagation mechanism from financial shocks to economic activity, which shows up at the aggregate level as measured total factor productivity.

The paper begins by measuring capital unemployment and documenting its main empirical patterns. At the aggregate level, I construct a historical measure of the capital unemployment rate for the U.S. since 1980 using data from major real estate firms on vacant nonresidential space. These data show that capital unemployment is a relevant phenomenon, with a historical average of 11%, and that it tends to be more pronounced during economic downturns. For example, across multiple U.S. recessions, capital unemployment reached levels 20–30% above its mean and took several years to recover. I document similar patterns in European economies during the Euro crisis. I complement this aggregate characterization with a new restricted-access dataset of online postings of unemployed capital structures for Spain. These data suggest that unemployed capital units remain listed for several months (8 on average) and, while listed, receive time-varying search activity measured by the online visits each property receives. Moreover, aggregate search activity is higher in periods of low unemployment than in periods of high unemployment, akin to a Beveridge curve.

I then construct a model that explains the patterns of capital unemployment observed in the data. The environment is that of a stochastic neoclassical growth model in which households produce capital and firms employ it in production, with two additional ingredients. The first is that capital is traded in a non-Walrasian market with search frictions, which give rise to equilibrium capital unemployment. Search is directed, meaning that sellers choose the price at which to post their capital units and buyers choose the price at which to search; this structure, which has been used to model other frictional markets (see, for example, Shimer, 1996; Moen, 1997; Menzio and Shi, 2011; Wright, Kircher, Julien, and Guerrieri, 2021, and references therein), is consistent with that observed in capital markets data. The second ingredient is financial frictions that affect firms' capital purchases, which I introduce by considering frictional intermediaries that mediate financing between nonfinancial firms and households (as, for example, in Gertler and Kiyotaki, 2010). The model features technology and financial shocks, where the latter primarily affect intermediaries' net worth and the severity of financial frictions.

In a quantitative version of the model with trading and financial frictions, I show that financial shocks generate volatile and countercyclical fluctuations of capital unemployment, aligned with those observed in the data. Negative financial shocks lead to increases in intermediaries' marginal cost of external finance, which contract the supply of funds for firms and increase their cost of capital; this leads to declines in firms' capital purchases and search activity, which decreases the probability that capital sellers find buyers and increases capital unemployment. The large effects of financial shocks on capital unemployment are not observed in response to technology shocks. This is because, as in the standard neoclassical stochastic model, fluctuations in total factor productivity have only a quantitatively small effect on firms' discount factor and the shadow price of employed capital, which in turn implies small fluctuations in firms' search effort and trading probabilities. In fact, a version of the model with only shocks to total factor productivity predicts fluctuations of capital unemployment one order of magnitude smaller than those observed in the data (a result reminiscent of that found in Shimer, 2005, for the labor market).

I highlight two macroeconomic implications of incorporating capital trading frictions and capital unemployment. First, capital unemployment affects aggregate investment dynamics. When a downturn is characterized by a large increase in capital unemployment, it is then followed by an investment slump: large levels of capital unemployment decrease the probability of selling capital, which reduces the incentives to produce capital goods; the economy then recovers from the downturn by absorbing existing capital stock instead of producing new capital units. I show that this mechanism can help explain an important part of the investment slump following the U.S. Great Recession, which is puzzling from the perspective of standard capital-accumulation models (see, for example, Kydland and Zarazaga, 2012; Hall, 2014). I provide empirical evidence that supports the relationship between capital unemployment and investment rates predicted by the model using data on capital structures across U.S. regions. In particular, I show that Metropolitan Statistical Areas with a greater capital-unemployment rate at the trough of the Great Recession were regions with lower investment rates during the recovery, and that this relationship is quantitatively aligned with that predicted by the model. Since fluctuations in capital unemployment are mostly driven by financial shocks, this model prediction is also consistent with the finding that financial crises tend to be followed by persistent slumps (see, for instance, Cerra and Saxena, 2008; Reinhart and Rogoff, 2009).

A second macroeconomic implication of capital trading frictions is that capital unemployment constitutes a sizable channel through which financial shocks propagate to economic activity. In a real business-cycle model with only financial frictions to finance investment, these shocks typically have a small effect on output (less than 0.1% change in output for a one-standard-deviation financial shock) because the capital stock, which is the input in the production function, exhibits only modest fluctuations. In the presence of trading frictions, where the input in production is employed capital, the output response to financial shocks is several times larger (with a peak effect of 0.8% for a one-standard-deviation financial shock). Because capital-unemployment fluctuations show up at the aggregate level as measured total factor productivity, this mechanism could explain part of the large fluctuations in measured productivity observed in financial crises and great depressions (Kehoe and Prescott, 2007; Chari, Kehoe, and McGrattan, 2007; Calvo, Colombi, Coricelli, and Ottonello, 2026).

It is worth highlighting that the interaction between capital-trading frictions and financial shocks plays a central role in making these frictions relevant for macroeconomic dynamics. The first paper to study search frictions in the physical-capital market, Kurmann and Petrosky-Nadeau (2007), did so in an environment without financial frictions and found that capital-market search frictions are not quantitatively important for the business cycle. Absent financial shocks, the same result obtains in my model: the small fluctuations in capital unemployment implied by the model have modest macroeconomic implications. Introducing financial frictions and shocks that primarily affect the severity of these frictions generates large fluctuations in capital unemployment, similar to those observed in the data, and leads to quantitatively significant effects on output and investment. This result is consistent with the logic in Hall (2017), who shows that discount-factor shocks can generate realistic business-cycle dynamics in search-and-matching models of the labor market.

Related literature. This paper is related to several bodies of research. The first is the literature on investment dynamics, which includes early work on capital-adjustment costs (e.g., Lucas, 1967; Brainard and Tobin, 1968); models with fixed costs (e.g., Caballero, Engel, and Haltiwanger, 1995; Khan and Thomas, 2008; Caballero, 1999, and references therein) and with investment-adjustment costs (e.g., Christiano, Eichenbaum, and Evans, 2005; Eberly, Rebelo, and Vincent, 2012); and models with irreversibilities (e.g., Bertola and Caballero, 1994; Abel and Eberly, 1996; Veracierto, 2002). I contribute to this literature by linking investment dynamics to trading frictions in capital markets.

Second, the paper contributes to the literature on unemployment and non-Walrasian markets (surveyed, for example, in Rogerson, Shimer, and Wright, 2005). Empirically, it shows that several facts documented for labor market flows and unemployment (e.g., Davis, Haltiwanger, and Schuh, 1998; Mortensen and Pissarides, 1999a; Shimer, 2010, and references therein) are also observed in capital markets and for capital unemployment. Theoretically, it shows a new mechanism in a search-and-matching framework related to the fact that, unlike the labor force, physical capital is produced and this production is affected by capital unemployment. As an asset, the search frictions studied here are also related to those of financial assets (for a recent survey, see Lagos, Rocheteau, and Wright, 2015) and housing markets (see, for example, Wheaton, 1990; Krainer, 2001; Caplin and Leahy, 2011; Piazzesi, Schneider, and Stroebel, 2020). Complementary to this literature, I focus on assets used as input in production.

Third, the paper is related to the literatures on misallocation (e.g., Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008) and capital reallocation (e.g., Ramey and Shapiro, 1998a,b; Eisfeldt and Rampini, 2006). Capital unemployment can be viewed as an extreme margin affecting the allocation of capital, by which some units in the economy remain idle while searching to be traded. I contribute to this literature by documenting that this allocation margin exhibits substantial business-cycle variation and can be linked to trading frictions in reallocation flows between unemployed and employed capital.[1,2] Related to this, since capital unemployment reflects a form of unutilized capital, the model is also linked to models of endogenous capital utilization (e.g., Greenwood, Hercowitz, and Huffman, 1988; Cooley, Hansen, and Prescott, 1995; Gilchrist and Williams, 2000). The paper shows how fluctuations in the share of idle capital in the economy can result from capital-market trading frictions.[3]

Finally, the paper contributes to the literature on the effects of financial shocks in the macroeconomy. The literature has analyzed different channels through which shocks that primarily affect the severity of financial frictions influence the aggregate economy, including working capital financing (e.g., Neumeyer and Perri, 2005; Jermann and Quadrini, 2012; Mendoza, 2010); default risk (e.g., Arellano, Bai, and Kehoe, 2019); aggregate demand in models with nominal rigidities (e.g., Bernanke, Gertler, and Gilchrist, 1999; Christiano, Motto, and Rostagno, 2014); and misallocation in heterogeneous-firm models (e.g., Kiyotaki and Moore, 1997; Khan and Thomas, 2013); see Schwartzman (2012) for a detailed discussion of these channels. The paper shows a new channel for the transmission of financial shocks through their effect on capital unemployment in models with trading frictions in capital markets.

Road map. The rest of the paper is organized as follows. Section 2 discusses the concept of capital unemployment, its measurement, and its main empirical patterns. Section 3 presents the

[1] The empirical literature on misallocation has traditionally focused on how capital is distributed across active establishments and firms (see, for example, Bartelsman, Haltiwanger, and Scarpetta, 2013; Midrigan and Xu, 2014; Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez, 2017; Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry, 2018; David and Venkateswaran, 2019; Kehrig and Vincent, 2019; Hopenhayn, 2014, and references therein). By contrast, the study of capital unemployment highlights a complementary margin affecting the allocation of capital, arising between active firms and other agents in the economy that hold capital but are not actively producing (e.g., unemployed capital from exiting firms, which is usually not included in establishment–based measures of misallocation). The allocation margin I study builds on the approach in Gavazza (2016), who uses business–aircraft data to examine the welfare implications of trading frictions in the allocation of assets.

[2] Cao and Shi (2014); Ottonello (2014); and Wright, Xiao, and Zhu (2018, 2020) study the implications of search frictions for capital reallocation among production firms. In this paper's framework, capital reallocation corresponds to transitions within employed capital (or "on-the-job search" in the labor literature). For other related studies on capital reallocation, see Cui (2013); Lanteri (2018); and Li and Whited (2015). See also Eisfeldt and Shi (2018) for a recent review of this literature.

[3] Two relevant papers that link utilization to search frictions are Michaillat and Saez (2015), who develop a theory of unemployment fluctuations in which search frictions in the labor and product markets interact; and Bai, Rios-Rull, and Storesletten (2019), who develop a theory with search frictions in product markets in which demand has a productive role.

model. Section 4 calibrates the model and compares its predictions for capital unemployment with data counterparts. Section 5 studies the macroeconomic implications of capital unemployment, and Section 6 concludes.

2 Capital Unemployment: Concept, Measurement, and Empirical Patterns

In this section, I define the concept of capital unemployment and discuss its measurement. I then document key facts about capital unemployment and connect unemployment to frictions in capital markets. I also discuss complementary empirical findings in the literature.

2.1 Definitions

I consider a classification for capital stocks similar to the one used in the labor market of employed, unemployed, and inactive workers (e.g., by the U.S. Bureau of Labor Statistics). For this classification, it is useful to consider a capital unit as the measure of analysis, in the same way a worker is the unit of analysis in the labor-market classification. Formally, assuming that capital goods are indivisible (i.e., are available to trade in integer quantities only) and denoting by J_t^{k} the set of capital goods in the economy in a given period, a capital unit is an element of J_t^{k}. Capital units in a given period can be classified as follows:

***Definition* 1** (Employed, unemployed, and inactive capital)**.** *Consider a given unit of physical capital in a given time period. The unit is said to be:*

i. Employed: if it has been used for the production of goods and services within the period;

ii. Unemployed: if it has not been used for production within the period, and there has been active search to trade (sell or rent) the unit; or

iii. Inactive: if it has not been used for production within the period, and there has been no active search to trade the unit.

For expositional simplicity, I focus here on an economy with homogeneous capital. Appendix A.1 extends these definitions to multiple types of capital. Denote by $s_t(j) \in \{\mathrm{e}, \mathrm{u}, \mathrm{n}\}$ the state of capital unit j in period t: $s_t(j) = \mathrm{e}$ if it is employed, $s_t(j) = \mathrm{u}$ if unemployed, and $s_t(j) = \mathrm{n}$ if inactive. The stock of capital is given by $K_t \equiv K_t^{\mathrm{e}} + K_t^{\mathrm{u}} + K_t^{\mathrm{n}}$, where $K_t^{\mathrm{u}} \equiv \sum_{j \in J_t^{\mathrm{k}}} \mathbb{I}\{s_t(j) = \mathrm{u}\}$ is the stock of unemployed capital in period t; $K_t^{\mathrm{e}} \equiv \sum_{j \in J_t^{\mathrm{k}}} \mathbb{I}\{s_t(j) = \mathrm{e}\}$ the stock of employed capital; $K_t^{\mathrm{n}} \equiv \sum_{j \in J_t^{\mathrm{k}}} \mathbb{I}\{s_t(j) = \mathrm{n}\}$ the stock of inactive capital; and $\mathbb{I}\{\}$ the indicator function. I define the capital-unemployment rate in this economy as $u_t^{\mathrm{k}} \equiv \frac{K_t^{\mathrm{u}}}{\tilde{K}_t}$, where $\tilde{K}_t \equiv K_t^{\mathrm{e}} + K_t^{\mathrm{u}}$ denotes "active capital;" and I define the non-employment rate of capital as $n_t^{\mathrm{k}} \equiv \frac{K_t^{\mathrm{u}} + K_t^{\mathrm{n}}}{K_t}$.

2.2 Measurement and data description

My main measurement for capital unemployment comes from data on nonresidential structures. These data have two main advantages: (i) they provide systematic historical information that can be mapped to Definition 1 and that allows me to consistently study the dynamics of capital unemployment over a long period of time; and (ii) they represent a large share of the capital stock. Section 2.3 discusses related empirical evidence for equipment.

Aggregate data To measure capital unemployment at the aggregate level, I use data from `CBRE` and `REIS`, two major real estate firms that collect historical data at the national level for the U.S. on the vacancy rate of commercial nonresidential properties. These are defined as the share of office, retail, and industrial space that, at any point in time, is vacant or unoccupied and marketed available for occupancy by tenants. Data Appendix A.2 discusses the details of data collection and the data's representativeness. Using these data, I construct an aggregate measure of capital unemployment for structures ($u_{\mathrm{s},t}^{\mathrm{k}}$) for the period 1980–2015 as a weighted average of the different types of properties: $u_{\mathrm{s},t}^{\mathrm{k}} = \omega_{\mathtt{off},t} u_{\mathtt{off},t}^{\mathrm{k}} + \omega_{\mathtt{ret},t} u_{\mathtt{ret},t}^{\mathrm{k}} + \omega_{\mathtt{ind},t} u_{\mathtt{ind},t}^{\mathrm{k}}$, where $u_{\mathtt{off},t}^{\mathrm{k}}$, $u_{\mathtt{ret},t}^{\mathrm{k}}$, and $u_{\mathtt{ind},t}^{\mathrm{k}}$ denote, respectively, the vacancy rates of office, retail, and industrial space, and $\omega_{\mathtt{off},t}$, $\omega_{\mathtt{ret},t}$, and $\omega_{\mathtt{ind},t}$ denote weights for each of these properties in period t. For the weights, I use current-cost shares of different types of assets from BEA data (for details, see Appendix A.2). Relative to other work using vacancy rates, to my knowledge, this is the first to document business-cycle moments of aggregate data for the U.S. (summarized in Table 1).[4] I also obtain aggregate data on capital unemployment in European economies (Greece, Ireland, Portugal, Spain, and the U.K.) from `Jones Lang LaSalle`, another major real estate firm.

In addition to the aggregate behavior for the U.S. economy, this dataset contains disaggregated information for Metropolitan Statistical Areas (MSAs). I use this regional variation later in the paper to relate capital unemployment to investment rates across regions during the last recession.

[4] Vacancy rates have been extensively used in the urban economics literature to characterize the real estate market (see, for example, DiPasquale and Wheaton, 1996). In addition, Kurmann and Petrosky-Nadeau (2007) use vacancy rates to illustrate the presence of search frictions in the physical capital market.

To construct investment rates, I use the information on the capital stock (or "inventory") reported for each type of property, weighted using the same procedure as for the capital-unemployment rate.

Micro-level data. I complement the aggregate capital-unemployment data with two new microlevel datasets. The first is a dataset of unemployed units listed online for trade, which allows me to measure the duration of capital unemployment in Section 2.3 and document empirical patterns that characterize frictions in capital markets linked to unemployment in Section 2.4. These data were obtained from `Idealista`, one of Europe's main online real estate intermediaries. The website contains listings of vacant office and retail space for rent or sale. The dataset is an unbalanced panel with more than 10 million monthly observations for 1.2 million listings, containing all listings posted on the website for Spain during 2005–2018. It includes a property identifier, which allows me to measure the number of months a unit remains listed. For a small subset of listings (4%), the dataset includes a verification by the owner that the listing exited the platform because the property was rented or sold, allowing a precise interpretation of the number of months listed as unemployment duration. For each property, the dataset contains its exact location, classified into the three geographic levels used in Spain: municipality, district, and neighborhood. Appendix Table B.1 lists the 50 municipalities with the most listings. Overall, the dataset covers more than 3,000 municipalities. Appendix Tables B.2 and B.3 detail the districts and neighborhoods contained in the two largest municipalities, Madrid and Barcelona. The dataset also contains characteristics of each capital unit (e.g., size, condition), detailed in Appendix Table B.4, as well as the listing price and the number of clicks each listing receives per month, which I use as a measure of search activity.

The second dataset tracks the historical activity of individual commercial space. These data were obtained from `EIXOS`, which specializes in collecting geographic data on commercial real estate in several cities worldwide. The dataset covers the city of Barcelona and reports the exact street-level locations of commercial space. At different points in time, it provides information on whether a store is empty or operating and, in the latter case, the business name at each location. The dataset contains over 71,000 locations with records of their business activity spanning 2009 to 2019, as well as more than 10,000 business transitions (i.e., changes in the business operating the store). This dataset allows me to measure the non-employment rate n_t^{k}, including the share of unemployed and inactive units. It also allows me to document capital specificity, measured by the frequency with which stores that change business repeat their industry.

In terms of the external validity of the Spanish data, the next section shows that aggregate capital-unemployment data in Spain displays similarities, both in terms of average levels and dynamics in the last recession, to those of the U.S. and other European economies. This suggests that Spain has external relevance for studying the dynamics of physical capital markets.

2.3 Empirical patterns

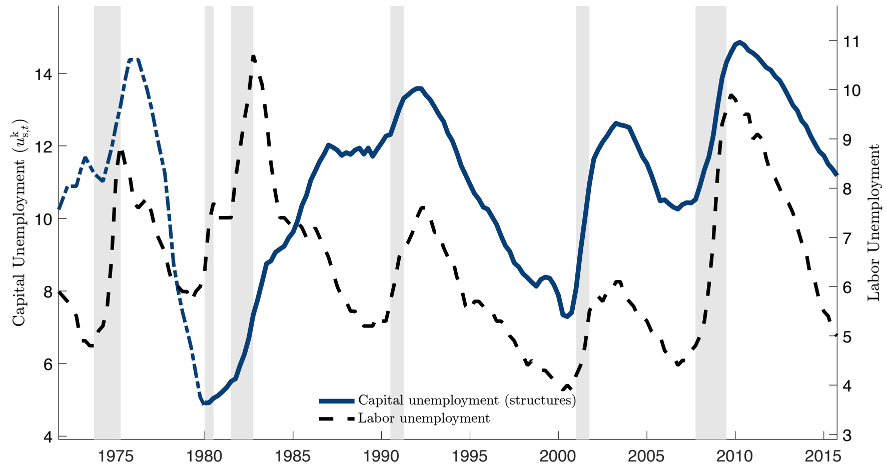
Level of unemployment Figure 1 depicts the constructed capital unemployment rate for structures in the U.S. since 1980. It also presents data based only on office space for the 1970s, with the labor unemployment rate as a benchmark. Key moments are reported in Table 1. These data show that mean capital unemployment over 1980–2015 is 10.9%, which is relatively large compared, for instance, with that of labor unemployment (6.4% during this period). Appendix Table B.5 shows that mean unemployment is also large in European economies, including Greece, Ireland, Portugal, Spain, and the U.K., with an average in these economies of 11%.[5]

Cyclicality of unemployment Table 1 shows that capital unemployment is volatile and persistent. Its standard deviation is 2.4% in the raw data and 1.2% at business-cycle frequencies (detrending data using the HP filter), which is similar to that of labor unemployment. Its autocorrelation is 0.9 for raw data and 0.7 at business-cycle frequencies. These fluctuations exhibit a systematic comovement with the business cycle, with a correlation with aggregate output at business-cycle frequencies of around -0.5. To illustrate this, Figure 1 shows that capital unemployment increased during the last five recession episodes (shaded gray in the figure), and tends to decrease during expansions, except during the second half of the 1980s—a period identified in the urban economics literature as a boom in the construction of nonresidential structures (see, for example, Wheaton, 1999).

Regarding the different components of capital unemployment, Table 1 reports second moments for the growth rate of unemployed and employed capital and the total capital stock (represented here by "active capital": employed plus unemployed). It first shows that movements in the rate of capital unemployment are driven by changes in unemployed capital stock, since the growth rate of the unemployed capital stock is five times more volatile than that of the employed capital stock in raw data (and eight times at business-cycle frequencies). It also indicates that the growth rate of

[5]In the case of Barcelona, I can also measure the non-employment rate of capital n_t^{k} from the `EIXOS` dataset, corresponding to the share of empty retail stores, which indicates an average for the period 2009-2014 of 14.6%. In this period, the capital unemployment rate of office space in Barcelona is 13.5%. Although not strictly similar in terms of coverage, this suggests comparable figures for these two variables.

Figure 1: Capital-Unemployment Rate for Structures in the U.S.



Note: Data expressed in percentages. Capital unemployment for U.S. structures for the period 1980–2015 is constructed based on vacancy rates of office, retail, and industrial units. Data from CBRE and REIS. For the period 1972–1980, capital-unemployment data (double-dashed line) include only office space. Labor unemployment refers to the civilian unemployment rate; data from the Federal Reserve of St. Louis. Shadow areas denote NBER recession dates. For details on data construction, see Section 2.2 and Appendix A.2.

Table 1
Moments of Capital Unemployment and Aggregate Flows.

	Mean	Standard dev.		Autocorrelation		Corr. with
		Raw	*HP*	*Raw*	*HP*	GDP *(HP)*
Capital Structures						
Capital unemployment rate ($u^{\rm k}_{\rm s}$)	10.9%	2.4%	1.2%	0.91	0.72	-0.52
Unemployed capital growth ($g_{K^{\rm u}_{\rm s}}$)	4.3%	11.6%	8.4%	0.57	0.23	-0.16
Employed capital growth ($g_{K^{\rm e}_{\rm s}}$)	2.4%	2.2%	1.0%	0.87	0.40	0.49
Total capital growth ($g_{\tilde{K}_{\rm s}}$)	2.5%	2.5%	0.8%	0.92	0.35	0.35
Other variables						
Labor unemployment	6.4%	1.6%	1.1%	0.81	0.62	-0.91

Note: Moments were computed using annual data for the period 1980–2015. Capital unemployment, unemployed capital growth, employed capital growth, and total capital growth (i.e.,“active capital” growth) were constructed based on vacancy rates and the inventory of office, retail, and industrial units. Data from CBRE and REIS. Labor unemployment refers to the civilian unemployment rate. Data from the Federal Reserve of St. Louis. Columns labeled *Raw* refers to moments of raw data. Columns labeled *HP* refer to moments of detrended data using the HP filter. Correlation with GDP refers to the correlation of each variable of interest with the cyclical component of real GDP per working-age population. Data from the Bureau of Economic Analysis. For details on data construction, see Section 2.2 and Appendix A.2.

the unemployed capital is countercyclical, while that of the employed capital and total capital is procyclical.

The aggregate pattern documented in Figure 1 during recession episodes is also observed across

U.S. regions and in European economies. Appendix Figure B.1 shows that in the last two U.S. recessions prior to the pandemic, the increase in capital unemployment was generalized across MSAs. Appendix Figure B.2 also shows that the increase in capital unemployment during the Great Recession was also observed in European economies. Appendix Table B.5 shows that capital unemployment is volatile, persistent, and countercyclical in these economies, with second moments comparable to those presented in Table 1 for the U.S. In the case of Barcelona, Figure B.3 shows that the non-employment rate of capital n_t^{k} also sharply increases during the crisis.

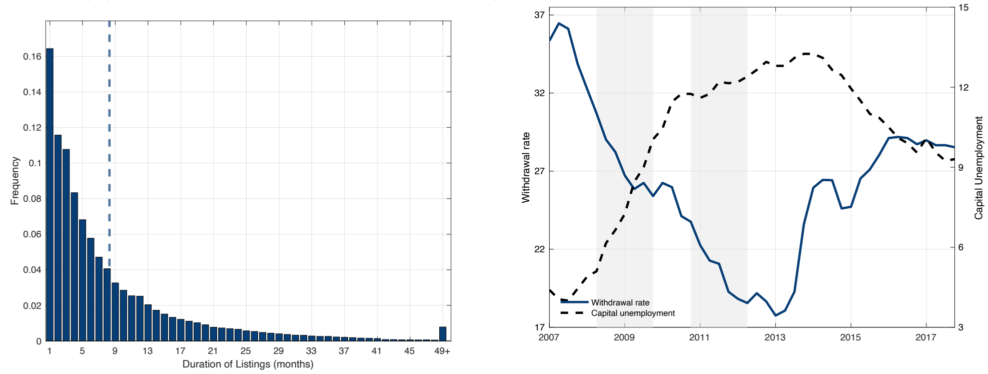
Unemployment duration A defining characteristic of labor unemployment is that workers tend to search for multiple periods before finding a job. A similar pattern can be observed for unemployed capital units listed for trade. Panel (a) of Figure 2 shows the distribution of listing durations in `Idealista` for Spain, with an average of 8 months for a listed unemployed capital unit. Appendix Figure B.4 shows the distribution of listing durations for properties in which the owner confirmed to `Idealista` that the reason the listing exited the platform was renting or selling the property, with a longer average duration of 11 months. Appendix Table B.6 reports summary statistics of the distribution of listing duration and shows that similar patterns are observed in different types of units: office, retail, sale, and rent. Overall, these data show a considerable duration of unemployment for capital.

The data also show that listings withdrawn from the platform (proxying for the probability of trading unemployed capital) are procyclical. This is illustrated in Panel (b) of Figure 2, which depicts, together with the capital-unemployment rate, the evolution of the 4-quarter moving average of the listing-withdrawal rate, defined as the number of listings exiting the site in a given quarter over the total number of listings in that quarter. From 2007 to 2012, when the Spanish economy was in a severe recession and capital unemployment was increasing, the listing-withdrawal rate fell by half, from 36% to 17% per quarter, suggesting greater difficulty in matching unemployed capital during bad times. This occurs despite a significant contraction in posted prices; Appendix Figure B.5 shows that capital prices declined by more than 50% on average during this period. Panel (b) of Figure 2 also shows that after 2013, following the Spanish recovery, the listing-withdrawal rate increased and the capital-unemployment rate declined.

Although the paper's main focus is on the average duration and its cyclical properties, the cross-section of duration exhibits rich patterns that are worth mentioning. For example, Appendix Figure B.6 shows that properties that have been listed for fewer months tend to exhibit higher withdrawal

Figure 2: Capital Unemployment Duration

(a) Duration of Listings (b) Withdrawal and Unemployment Rates



Note: Panel (a) shows the distribution of the duration of unemployed-capital listings, referring to the number of months a unit was listed on the online platform `Idealista` in Spain. The dashed vertical line denotes the average duration, 8.3 months. See Appendix Table B.6 for a set of summary statistics of this distribution and Appendix Figure B.4 for the distribution of the duration of listings with confirmed trades. Panel (b) shows the 4-quarter moving average of the listing-withdrawal rate, defined as the number of listings that exit the platform in a given quarter over the total number of listings in that quarter, and capital unemployment, both expressed in percent. Capital unemployment refers to the office-space vacancy rate for Barcelona and Madrid; data from `Jones Lang LaSalle`. For comparability, the withdrawal rate refers to the same type of property as capital unemployment (i.e., office space for rent). Results are similar if the withdrawal rate for all listings is considered instead. For details on the data, see Sections 2.2 and 2.3. Shadow areas in Panel (b) denote *Spanish Business Cycle Committee* recession dates.

rates than properties that have been listed for more months. These patterns are consistent with the presence of capital specificity, by which recently listed units have a different trading probability than those that have already been listed for several periods (e.g., in stock-flow models, such as those studied in Taylor, 1995; Coles and Muthoo, 1998; Coles and Smith, 1998; Ebrahimy and Shimer, 2010).[6] I further discuss the possible presence of trading frictions and specificity in Section 2.4.

Evidence for equipment and whole capital stock Several empirical studies suggest that the evidence documented in this section also applies to capital equipment or the whole capital stock. Using equipment-level data from closing aerospace plants, Ramey and Shapiro (2001) document that selling equipment typically takes several years, consistent with high unemployment duration. Regarding evidence on both types of capital, Becker, Haltiwanger, Jarmin, Klimek, and Wilson (2006), using U.S. Census data, show that firms' acquisitions of used capital in a given

[6] As argued in the labor market literature, this empirical pattern can not only reflect differences in the duration dependence in the trading probability of individual capital units but also changes in the composition of capital units with different expected duration. Appendix Figure B.7 (reproduced here for convenience) shows that, when splitting the sample of properties by their initial price or search intensity (measured by the number of clicks), the withdrawal rate is also downward sloping in duration within each group. A more formal decomposition of how individual property characteristics contribute to the relationship between withdrawal rates and duration could be an interesting avenue for future work.

year amount, on average, to only 64% of the capital of establishments that exited in the previous year. Consistent with this, Harris and Drinkwater (2000), using plant-level records covering the whole U.K. manufacturing sector, show that approximately 30% of the aggregate capital stock corresponds to plants that have closed. Overall, therefore, these studies suggest that the phenomenon of capital unemployment also applies to other types of capital.

2.4 Unemployment and frictions in capital markets

The fact that individual capital units are posted for several periods before being traded is consistent with a non-Walrasian market for capital. This section discusses capital market frictions that can be linked with this market dynamic.

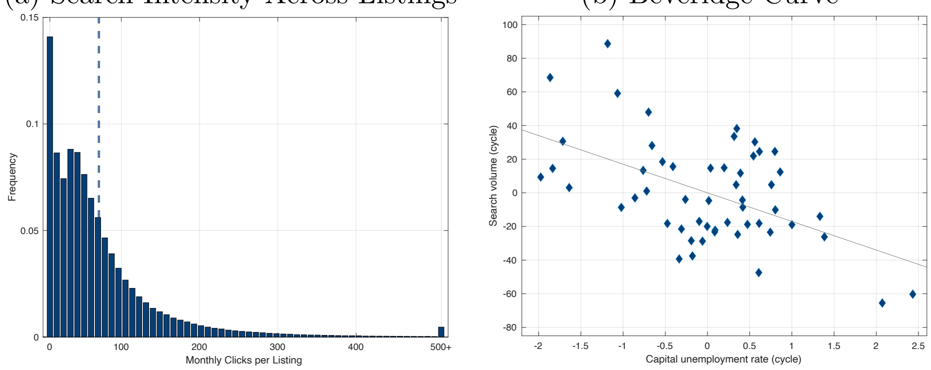
Search-and-matching frictions Gavazza (2011) shows that the microlevel patterns of trading and prices in the commercial aircraft market are aligned with the predictions of models with search frictions. Consistent with this, I use the microlevel data on online postings to document that the empirical patterns in the market for structures are aligned with general predictions of search-and-matching models.

First, in markets with search frictions, potential buyers tend to dedicate time or effort to inspecting possible units to trade. The number of clicks received by unemployed capital listings on the online platform is consistent with this behavior. Panel (a) of Figure 3 reports the distribution of clicks received by each unit while listed online. On average, each post receives roughly 60 clicks per month. Appendix Table B.7 reports summary statistics for the distribution of clicks, which show similar behavior for office and retail space and for units for sale and rent. Appendix Table B.8 shows that the number of clicks received by individual properties is linked to their trading probabilities. In particular, for both the full sample of listings and the subsample with verified trades, the probability that a unit exits the platform in a given month is positively associated with the number of clicks received in the previous month: within a neighborhood, period, and property type, listings receiving one standard deviation more clicks than the mean tend to have 1 to 2 percentage points higher probability of exiting, which is a 13% to 17% higher probability than the mean listing (see Appendix Table C.1 for summary statistics). In addition, at the aggregate level, search activity measured by clicks tends to be higher in periods of low capital unemployment and lower in periods of high unemployment. Panel (b) of Figure 3 shows a negative correlation of around -0.5 between the cyclical component of aggregate search activity and capital unemployment, similar to a *Beveridge curve* in the labor market.

Second, a key prediction of search models is that market tightness, defined as the ratio of buyers to sellers in a market, is linked to trading probabilities (see, for example, Rogerson, Shimer, and Wright, 2005). In particular, under standard assumptions for matching technologies, markets with higher market tightness tend to have, all else equal, higher trading probabilities for sellers. Appendix C.1 exploits the geographical and temporal variation of market tightness and trading probability across unemployed-capital online listings and provides descriptive evidence consistent with this relationship: Higher market tightness in the neighborhood and month in which a property is listed is associated with a higher probability of trading. Appendix C.1 also shows that the link between market tightness and trading probabilities is observed only for units of comparable size; this is consistent with the presence of capital specificity, which I discuss next.

FIGURE 3: SEARCH ACTIVITY IN CAPITAL MARKETS.

(a) Search Intensity Across Listings (b) Beveridge Curve



Note: Panel (a) reports the distribution of search intensity per announcement on the online platform `Idealista` in Spain, defined as the number of clicks per month each listing received. The dashed vertical line denotes the average search intensity per announcement: 59 clicks per month. See Table B.7 for a set of summary statistics for this distribution. Panel (b) shows the cyclical component of search volume (defined as the total number of clicks received by all listings) and capital unemployment. Capital unemployment refers to the office-space vacancy rate for Barcelona and Madrid; data from `Jones Lang LaSalle`. Search volume is expressed in log deviations from trend and capital unemployment in deviations from trend, both in percent. Variables were detrended using the HP filter. For details on the data, see Sections 2.2 and 2.4.

Capital specificity A possible reason behind search frictions in capital markets is the presence of capital specificity (studied, for example, by Caballero and Hammour, 1998; Altig, Christiano, Eichenbaum, and Linde, 2011). Kermani and Ma (2023), using liquidation recovery rates, report a high degree of capital specificity, with the average liquidation value of a firm's plant, property, and equipment being around 33% of its book value.[7] Specificity can lead to search frictions if it requires buyers to dedicate time and resources to search across available capital units and evaluate which units better suit their needs.

I use the microlevel data on structures to further explore capital specificity in this market. I refer here to specificity as a property of capital units by which some units are more suitable for some businesses than for others. For instance, structures would exhibit specificity if entrepreneurs interested in opening a restaurant have requirements for commercial space, such as the size and spatial distribution of the unit or the number and size of windows, that differ from those of entrepreneurs seeking to open another type of business (e.g., a supermarket or a fitness center). To investigate this type of specificity, I use the `EIXOS` data described in Section 2.2, which track the history of commercial brands operating at individual business locations. Using these data, Appendix C.2 shows that stores that experience a change in their operating business exhibit a substantive frequency of repeating their economic activity (at the 6-digit NAICS, with 51 industries in the dataset). These patterns are consistent with an economy with specificity, in which certain capital units are more suitable for certain economic activities than others, leading to a higher probability of repeating activity when there is a change in the operating business.

Mismatch unemployment Motivated by the presence of specificity, I also investigate the role of mismatch in driving capital unemployment. In labor markets, mismatch between workers' attributes and job openings' requirements and locations is an important explanation for unemployment, complementary to search frictions (see Shimer, 2007, and references therein). In Appendix C.4, I quantify the importance of mismatch for capital unemployment following the method developed by Şahin, Song, Topa, and Violante (2014), which measures the extent to which labor unemployment can be explained by a mismatch between the markets in which vacancies are created and those in which workers are searching for a job, using data on vacancies and unemployment across different geographic areas and sectors.

I implement this method for capital markets using the dataset of online listings for Spain. Appendix Figure C.1 shows that although mismatch increased during the contraction in the Euro crisis, this has only a modest effect on the level of mismatch unemployment. This exercise suggests that although there is evidence of mismatch in capital markets, fluctuations in capital unemploy-

[7] For cases that report recovery rates for equipment and fixed assets separately, the mean recovery rate is 33% for equipment and 43% for structures, which suggests that specificity characterizes both forms of capital, and potentially more so for used equipment than for used structures. I thank Amir Kermani and Yueran Ma for providing me with a split of this estimate.

ment are likely driven more by changes in economic conditions that affect the aggregate purchasing activity of capital buyers than by a mismatch between the markets in which buyers search to invest and those in which unemployed capital is located.

3 A Model of Business Cycles with Capital Unemployment

In this section I construct a framework to study the fluctuations of capital unemployment and their macroeconomic implications. The environment is that of the stochastic neoclassical growth model with two main additional ingredients: trading frictions in capital markets and financial frictions that affect firms' investments. The roles of key assumptions and alternative model variants are discussed in Section 4.

3.1 Environment

Time is discrete and infinite. At the beginning of each period $t \geq 0$, a stochastic event, $s_t \in S$, is realized. Let $s^t = [s_0, s_1, .., s_t]$ denote the history of events until time t, with a given initial realization s_0. There are final goods, capital goods, and labor services. Final goods are perishable, and capital goods depreciate at a constant rate $\delta \in (0, 1)$ at the start of the period. The economy is populated by a representative household, nonfinancial firms, and financial intermediaries.

Preferences and technology The representative household is composed of a continuum of individuals. Every period, a fraction $1 - \nu$ of them have the skill to produce final goods, and a fraction ν have the skill to work in search activities.[8] The household's preferences over consumption and hours worked are described by the lifetime utility function $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [(1-\nu)u_t(c_{\mathrm{yt}}(s^t), h_{\mathrm{yt}}(s^t)) + \nu u_t(c_{\mathrm{st}}(s^t), h_{\mathrm{st}}(s^t))]\gamma_n^t$, where $c_{\mathrm{yt}}(s^t)$ and $h_{\mathrm{yt}}(s^t)$ denote per capita consumption and hours worked for household members working in the production of final goods in state history s^t, and $c_{\mathrm{st}}(s^t)$ and $h_{\mathrm{st}}(s^t)$ those of household members working in search activities, with identical allocations within each skill group in each period and state; $\mathbb{E}_t$ denotes the expectation conditional on the state history s^t; β is the subjective discount factor; $u_t : \mathbb{R}^2_+ \to \mathbb{R}$ is a continuous and differentiable function, increasing in the first argument and decreasing in the second argument; and $\gamma_n \geq 1$ denotes the

[8] The assumption of skill specialization within household members captures an environment in which searching for capital to match business ideas requires specific skills that cannot be frictionlessly acquired by workers who usually perform other work activities over the business cycle. Section 4 shows that some degree of specialization of search activities is important to match the procyclicality of the selling probability observed in the data.

gross population growth rate within the representative household.[9] At the end of each period, the household has access to a linear technology to produce new capital goods using final goods.

A continuum of identical nonfinancial firms with measure one have access to a technology to produce final goods employing matched capital and labor as inputs, given by $y_t(s^t) = A_t(s^t) f(k^{\mathrm{e}}_{t+1}(s^t), \gamma^t \ell_t(s^t))$, where $k^{\mathrm{e}}_{t+1}(s^t)$ denotes the stock of employed capital used in production in period t, after matching and trading have taken place in state s^t; $\ell_t(s^t)$ denotes labor input; $A_t(s^t)$ denotes an exogenous productivity factor that affects the production technology and is a function of the random variable s^t; $\gamma > 1$ denotes the constant gross rate of labor-augmenting technical progress; and $f : \mathbb{R}^2_+ \to \mathbb{R}_+$ is a differentiable function, increasing in both arguments, concave, and homogeneous of degree one, that satisfies the Inada conditions.[10] Along with the realization of aggregate shocks, every period each unit of a firm's capital used in production in the previous period receives, independently across units and over time, a match-specific shock with probability ψ that renders the capital unit unusable for that particular firm in production. As this shock is firm-specific and other firms can use these capital units for production, firms can trade their unmatched capital with other firms. This match-specific shock can be mapped to the separation flows of capital units from their operating firms observed in the data, such as the exit of establishments.

Trading frictions The model's first main ingredient is the assumption that markets for capital goods are decentralized and characterized by trading frictions. In particular, I assume that sellers choose at what price, q, to post their units and buyers must dedicate hours of work to search and match with capital units and can direct their search toward a submarket with a specific price. These hours worked reflect the time and resources firms dedicate to finding capital units that match specific business ideas they might have and can be used in production (e.g., the search for a location, as discussed in Section 2.4). The assumed structure for the capital market is similar to that used in competitive-search models of the labor market (see, for example, Shimer, 1996; Moen, 1997; Menzio and Shi, 2011) and is consistent with that documented in the data in Section 2: Owners of unemployed capital structures post their units to sell, including their prices, and potential buyers

[9]Preferences are indexed by time because the quantitative model uses the preferences introduced by Greenwood, Hercowitz, and Huffman (1988), which, in economies with growth, require the utility from leisure to rise with technological progress to remain consistent with a balanced-growth path (see, for example Neumeyer and Perri, 2005). I include population and technology growth in the model to better match the investment rates observed in the data, which are a sizable flow for employed and unemployed capital.

[10]The notation $k^{\mathrm{e}}_{t+1}(s^t)$ reflects that this stock is both used in production in state s^t and carried forward as the state variable into period $t + 1$. As further described below, newly produced capital only enters the unemployed stock in the following period, so that capital produced in period t cannot be used in production until period $t + 1$, leading to a time-to-build assumption akin to that in the standard neoclassical model.

search the set of posted units. Since the focus is on trading frictions in capital markets, the rest of the markets in the economy are assumed to be Walrasian. I consider an extension with frictional labor markets in Section 4.

In capital markets, after shocks are realized, the flow of new matches in submarket q is determined by the constant-returns-to-scale meeting technology $\mathcal{M}(k_t^{\mathrm{s}}(q,s^t),\gamma^t v_t(q,s^t))$, where $k_t^{\mathrm{s}}(q,s^t)$ and $v_t(q,s^t)$ denote, respectively, capital posted by sellers at the price q and hours worked by buyers searching to purchase capital at price q.[11] The labor-augmenting technology in the matching technology is necessary for a balanced growth path, given a labor-augmenting technology in the production of final goods. In each active submarket q, the market tightness, denoted $\theta_t(q,s^t)\equiv\frac{\gamma^t v_t(q,s^t)}{k_t^{\mathrm{s}}(q,s^t)}$, is defined as the ratio between buyers' hours worked, adjusted by technical progress, and the mass of capital posted by sellers.[12] Visiting submarket q in period t, sellers face a probability $p(\theta_t(q,s^t))$ of selling capital and buyers match a mass of capital $\mu_t(\theta_t(q,s^t))$ per hour of search.

The presence of trading frictions gives rise to unemployed capital—which is held by both households, who produce new capital to sell—and firms, who hold unmatched used capital they cannot use in production that could be employed by other firms. The evolution of their stock of unemployed capital (abstracting, without loss of generality, from agents seeking to sell capital units at multiple prices or from the choice to not sell capital units) is given by

$$k_{\mathrm{H}t+1}^{\mathrm{u}}(s^t)=(1-p(\theta_t(q_{\mathrm{H}t}^{\mathrm{u}}(s^t),s^t)))(1-\delta)k_{\mathrm{H}t}^{\mathrm{u}}(s^{t-1})+i_t(s^t), \tag{1}$$

$$k_{\mathrm{F}t+1}^{\mathrm{u}}(s^t)=(1-p(\theta_t(q_{\mathrm{F}t}^{\mathrm{u}}(s^t),s^t)))(1-\delta)k_{\mathrm{F}t}^{\mathrm{u}}(s^{t-1})+\psi(1-\delta)k_t^{\mathrm{e}}(s^{t-1}), \tag{2}$$

where $k_{\mathrm{H}t}^{\mathrm{u}}(s^{t-1})$ and $k_{\mathrm{F}t}^{\mathrm{u}}(s^{t-1})$ are households' and firms' stocks of unemployed capital at the beginning of period t; $i_t(s^t)$ denotes new capital goods produced by households; and $p(\theta_t(q_{\mathrm{H},t}^{\mathrm{u}}(s^t),s^t))$ and $p(\theta_t(q_{\mathrm{F},t}^{\mathrm{u}}(s^t),s^t))$ are the probability of selling unemployed capital given their choices of listing price $q_{\mathrm{H},t}^{\mathrm{u}}(s^t)$ and $q_{\mathrm{F},t}^{\mathrm{u}}(s^t)$. Equation (2) embeds the assumption that capital units that become unmatched in a given period become available for trade only in the following period.

Firms' evolution of employed capital (abstracting from firms' selling their employed capital,

[11]In the quantitative analysis in Section 4, the model allows new and used capital goods to trade in markets with different meeting technologies. This distinction has not yet been introduced in the current section, where, for expositional clarity, I maintain the assumption of a single meeting technology for all capital goods. The full version of the model in which new and used capital trade in separate markets with different meeting technologies is presented in Appendix D.3.1.

[12]Following the directed search literature (e.g., Moen, 1997; Menzio and Shi, 2011; Wright, Kircher, Julien, and Guerrieri, 2021), in submarkets that are not visited by any seller, $\theta_t(q,s^t)$ is an out-of-equilibrium conjecture that helps determine the equilibrium.

which does not occur in the equilibrium considered) is, in turn, given by

$$k^{\mathrm{e}}_{t+1}(s^t) = (1-\delta)(1-\psi)k^{\mathrm{e}}_t(s^{t-1}) + \int \mu_t(\theta_t(q,s^t))v_t(q,s^t)M_t(\mathrm{d}q,s^t), \tag{3}$$

where $v_t(q,s^t)$ denotes the hours of work dedicated to searching for capital listed at a price of q, which yields a mass $\mu_t(\theta_t(q,s^t))\,v_t(q,s^t)$ of matched capital; and $M_t(\cdot,s^t)$ is a measure on active submarkets.[13]

Financial frictions The second main ingredient of the model is the presence of financial frictions in the financing of firms' investments (their role is discussed in detail in Section 4). I introduce financial frictions to the model by considering frictional intermediaries that mediate in the process of financing between nonfinancial firms and households. In particular, the economy features a set of financial intermediaries indexed by ι that are owned by households and, each period, invest in financial claims on nonfinancial firms $a_{\iota t+1}(s^t)$ using their net worth $n_{\iota t}(s^t)$ (further specified below) and external finance $x_{\iota t}(s^t)$. Intermediaries face a cost of raising external finance from households equal to $\mathbb{I}_{\{x_{\iota t}(s^t)>0\}}\mathcal{C}(x_{\iota t}(s^t), n_{\iota t}(s^t))$ units of final goods per unit raised. As in the quantitative corporate-finance literature (e.g., Gomes, 2001; Hennessy and Whited, 2007), these costs are designed to capture flotation and other costs associated with raising external finance. To make the model's resource constraint comparable to that of the neoclassical framework, I assume these costs are rebated to households as lump-sum transfers. Furthermore, I follow the parameterization of these costs used to model financial intermediaries in Morelli, Ottonello, and Perez (2022), with $\mathcal{C}(x,n) = \phi\frac{x}{n}$, which generates aggregation across intermediaries. The parameter $\phi \geq 0$ governs the marginal cost of raising external finance faced by intermediaries, with $\phi = 0$ corresponding to a frictionless case that is isomorphic to an economy in which households have direct holdings of firms' equity.[14]

Finally, to ensure that intermediaries do not outgrow their financial frictions, I assume that they exit with an exogenous probability (see, for example, Gertler and Kiyotaki, 2010). To introduce "financial shocks" to the model (i.e., shocks that affect the severity of financial frictions), I assume that this probability, denoted $\zeta_t(s^t)$, is stochastic, representing the second source of aggregate risk

[13] In equilibria in which only one submarket is active, $M_t(\mathrm{d}q,s^t)$ is a unit point mass at that specific q.

[14] It is worth mentioning that in spite of trading and financial frictions, the assumed ownership structure in the model creates a structure similar to that of the "large-family assumption" used in frictional asset and labor markets (see, for example, Shi, 1997; Shimer, 2010), which brings the model closer to a frictionless representative-agent benchmark.

in the economy.[15] As will be discussed in Section 4, these types of shocks play a central role in accounting for the fluctuations of capital unemployment observed in the data. Each period, a mass of $\overline{\zeta}$ new intermediaries enter the economy, endowed with net worth $\overline{n}_t \equiv \overline{n}\gamma^t\gamma_n^t$, financed by households via a lump-sum transfer.

3.2 Optimization

Firms I set up agents' problems recursively. The individual state variables are the firm's stocks of employed and unemployed capital at the beginning of a period, represented by the vector $\mathbf{k} \equiv [k_{\mathrm{e}}, k_{\mathrm{u}}]$. The equity value of a firm $V_{\mathrm{F}}(\mathbf{k}, s^t)$ solves the Bellman equation:

$$
\begin{aligned}
V_{\mathrm{F}}(\mathbf{k}, s^t) = & \max_{k'_{\mathrm{e}}, k'_{\mathrm{u}}, \ell, \{v(q)\geq 0\}, q_{\mathrm{u}}} div + \mathbb{E}_t[m_{t+1}(s^{t+1})V_{\mathrm{F}}(\mathbf{k}', s^{t+1})] \qquad \text{(P1)} \\
& \text{s.t. } div + \int (q\mu_t(\theta_t(q, s^t)) + w_{\mathrm{st}}(s^t))v(q)M_t(\mathrm{d}q, s^t) = A_t(s^t)f(k'_{\mathrm{e}}, \gamma^t\ell) - w_{\mathrm{y}t}(s^t)\ell + q_{\mathrm{u}}p(\theta_t(q_{\mathrm{u}}, s^t))(1-\delta)k_{\mathrm{u}}, \\
& k'_{\mathrm{e}} = (1-\delta)(1-\psi)k_{\mathrm{e}} + \int \mu_t(\theta_t(q, s^t))v(q)M_t(\mathrm{d}q, s^t), \\
& k'_{\mathrm{u}} = (1 - p(\theta_t(q_{\mathrm{u}}, s^t)))(1-\delta)k_{\mathrm{u}} + \psi(1-\delta)k_{\mathrm{e}},
\end{aligned}
$$

where div are dividends transferred from firms to shareholders in period t; $w_{\mathrm{st}}(s^t)$ and $w_{\mathrm{y}t}(s^t)$ are the wage rates in search activities and in the production of final goods; and $m_{t+1}(s^{t+1})$ is intermediaries' stochastic discount factor, derived below. The first restriction of Problem (P1) is the firm's flow-of-funds constraint. Its right-hand side contains the sources of funds, which include net revenues from production and selling unemployed capital units. Its left-hand side contains the uses of funds, which include dividends and the cost of purchasing capital in the decentralized market. The latter has two components: the total cost of capital purchased, $\int q\mu_t(\theta_t(q, s^t))v(q)M_t(\mathrm{d}q, s^t)$, and the cost of hiring workers to search and match with capital in the decentralized market $\int w_{\mathrm{st}}(s^t)v(q)M_t(\mathrm{d}q, s^t)$.[16] Let $\widehat{div}_{\mathrm{F}}(\mathbf{k}, s^t)$, $\hat{k}^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t)$, $\hat{k}^{\mathrm{u}}_{\mathrm{F}}(\mathbf{k}, s^t)$, $\hat{\ell}_{\mathrm{F}}(\mathbf{k}, s^t)$, $\{\hat{v}_{\mathrm{F}}(q, \mathbf{k}, s^t)\}$, and $\hat{q}^{\mathrm{u}}_{\mathrm{F}}(\mathbf{k}, s^t)$ denote the policy functions associated with Problem P1; $\hat{\theta}^{\mathrm{u}}_{\mathrm{F}}(\mathbf{k}, s^t) \equiv \theta_t(\hat{q}^{\mathrm{u}}_{\mathrm{F}}(\mathbf{k}, s^t), s^t)$ is the market tightness faced by firms when selling unemployed capital, determined by their choice of prices in listing their units and

[15] Jermann and Quadrini (2012) and Christiano, Motto, and Rostagno (2014) provide model-based evidence that financial shocks represent an important source of fluctuations in the U.S. economy. Gilchrist and Zakrajšek (2012) present empirical evidence consistent with this view. The modeling choice for financial shocks as fluctuations of intermediaries' exit captures the narrative from the history of crises in which the exit of financial institutions plays a central role (see, for example, Reinhart and Rogoff, 2009; Gertler and Gilchrist, 2018, for the U.S. Great Recession).

[16] Although in equilibrium only one submarket may be active, it is convenient to formulate the firm's problem in terms of a search-effort schedule across all price-indexed submarkets, $\{v_t(q, s^t)\}$. This formulation allows me to derive more clearly the free-entry condition at each potential submarket, which, as discussed later, plays a central role in determining the equilibrium market-tightness function $\theta_t(q, s^t)$.

the given market-tightness function; and $\hat{\mathbf{k}}_{\mathrm{F}}(\mathbf{k}, s^t) \equiv [\hat{k}^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t), \hat{k}^{\mathrm{u}}_{\mathrm{F}}(\mathbf{k}, s^t)]$ firms' vector of capital holdings.

Firms' optimal choice of hours worked in the production of final goods ℓ is standard and equates the marginal product of labor to the wage in these activities: $A_t(s^t)\gamma^t f_\ell(\mathbf{k}, s^t) = w_{\mathrm{y}t}(s^t)$, where $f_\ell(\mathbf{k}, s^t) \equiv f_2(\hat{k}^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t), \gamma^t \hat{\ell}_{\mathrm{F}}(\mathbf{k}, s^t))$. Firms' optimal choice of employed capital, k'_{e}, is characterized by the Euler equation

$$Q^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t) = A_t(s^t) f_k(\mathbf{k}, s^t) + \mathbb{E}_t[m_{t+1}(s^{t+1})(1-\delta)((1-\psi)Q^{\mathrm{e}}_{\mathrm{F}}(\hat{\mathbf{k}}_{\mathrm{F}}(\mathbf{k}, s^t), s^{t+1}) + \psi Q^{\mathrm{u}}_{\mathrm{F}}(\hat{\mathbf{k}}_{\mathrm{F}}(\mathbf{k}, s^t), s^{t+1}))], \quad (4)$$

where $Q^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t)$ and $Q^{\mathrm{u}}_{\mathrm{F}}(\mathbf{k}, s^t)$ denote, respectively, the Lagrange multipliers associated with the laws of motion for employed and unemployed capital and $f_k(\mathbf{k}, s^t) \equiv f_1(\hat{k}^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t), \gamma^t \hat{\ell}_{\mathrm{F}}(\mathbf{k}, s^t))$. Equation (4) expresses that firms that accumulate employed capital equate its shadow price in terms of final goods, $Q^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t)$, to its expected marginal benefit, composed of the marginal product of employed capital and the discounted expected shadow price of the next period's capital. The latter takes into account that with probability ψ, capital is hit with a match-specific shock and must be sold as unemployed capital.

Firms accumulate employed capital by purchasing it in the decentralized market, which requires hours of work to search. A finite choice for $v(q, s^t) \geq 0$ requires that, for all q, $\mu_t\left(\theta_t(q, s^t)\right) Q^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t) \leq \mu_t\left(\theta_t(q, s^t)\right) q + w_{\mathrm{s}t}(s^t)$, together with the complementary slackness condition

$$\hat{v}_{\mathrm{F}}(q, \mathbf{k}, s^t)\left(w_{\mathrm{s}t}(s^t) + \mu_t\left(\theta_t(q, s^t)\right) q - \mu_t\left(\theta_t(q, s^t)\right) Q^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t)\right) = 0. \quad (5)$$

These conditions express that the shadow price of matched capital, $Q^{\mathrm{e}}_{\mathrm{F}}(\mathbf{k}, s^t)$, cannot exceed the marginal cost of purchasing capital, $\left(q + \frac{w_{\mathrm{s}t}(s^t)}{\mu_t(\theta_t(q, s^t))}\right)$; and that firms are willing to direct positive search effort only to submarkets in which this marginal cost equals its shadow price. Given that submarkets differ in their price q, firms are indifferent across markets to which they direct a positive search effort only if higher prices are offset by rates at which workers find a match, $\mu_t\left(\theta_t(q, s^t)\right)$. This is represented graphically in Panel (a) of Appendix Figure E.1, which depicts isocost curves for buyers in the plane (q, θ) for a given wage w_{s}. Curves to the southwest imply a lower cost per unit of capital and are preferred by firms buying capital.

Finally, firms' optimal choice of unemployed capital, k'_{u}, is characterized by the Euler equation

$$\begin{aligned}Q^{\mathrm{u}}_{\mathrm{F}}(\mathbf{k},s^t) =&\mathbb{E}_t\Big[m_{t+1}(s^{t+1})(1-\delta)\Big(p(\hat{\theta}^{\mathrm{u}}_{\mathrm{F}}(\hat{\mathbf{k}}_{\mathrm{F}}(\mathbf{k},s^t),s^{t+1}))\hat{q}^{\mathrm{u}}_{\mathrm{F}}(\hat{\mathbf{k}}_{\mathrm{F}}(\mathbf{k},s^t),s^{t+1})\\ &+(1-p(\hat{\theta}^{\mathrm{u}}_{\mathrm{F}}(\hat{\mathbf{k}}_{\mathrm{F}}(\mathbf{k},s^t),s^{t+1})))Q^{\mathrm{u}}_{\mathrm{F}}(\hat{\mathbf{k}}_{\mathrm{F}}(\mathbf{k},s^t),s^{t+1})\Big)\Big],\end{aligned} \tag{6}$$

which expresses that firms equate the shadow price of unemployed capital in terms of final goods, $Q^{\mathrm{u}}_{\mathrm{F}}(\mathbf{k},s^t)$, to the expected discounted marginal revenue of unemployed capital, linked to the price chosen to post unemployed units and its associated selling probability. Given that submarkets differ in their price, firms will only be indifferent across submarkets if those with lower price have associated a higher probability of selling capital, $p(\theta)$. This is represented graphically in Panel (b) of Appendix Figure E.1, which depicts isorevenue curves for sellers in the plane (q,θ). Curves to the northeast imply a higher revenue per unit of capital and are preferred by firms selling unemployed capital. In equilibrium, discussed in the next subsection, firms selling capital take as given the buyers' isocost curves implied by their search optimality condition and choose the price that maximizes revenue along this locus, represented in Panel (c) of Appendix Figure E.1. At this point, the marginal rates of substitution between θ and q are the same for buyers and sellers.

Households The individual state variable of a household is its stock of unemployed capital at the beginning of a period. Its recursive problem is given by

$$V_{\mathrm{H}}(k_{\mathrm{u}},s^t)=\max_{c_{\mathrm{y}},c_{\mathrm{s}},h_{\mathrm{y}},h_{\mathrm{s}},i,k'_{\mathrm{u}},q_{\mathrm{u}}}[(1-\nu)u_t(c_{\mathrm{y}},h_{\mathrm{y}})+\nu u_t(c_{\mathrm{s}},h_{\mathrm{s}})]\gamma^t_n+\beta\mathbb{E}_t[V_{\mathrm{H}}(k'_{\mathrm{u}},s^{t+1})] \tag{P2}$$

$$\begin{aligned}\text{s.t. } &[(1-\nu)c_{\mathrm{y}}+\nu c_{\mathrm{s}}]\gamma^t_n+i=[(1-\nu)w_{\mathrm{y}t}(s^t)h_{\mathrm{y}}+\nu w_{\mathrm{s}t}(s^t)h_{\mathrm{s}}]\gamma^t_n+q_{\mathrm{u}}p(\theta_t(q_{\mathrm{u}},s^t))(1-\delta)k_{\mathrm{u}}+T_t(s^t),\\ &k'_{\mathrm{u}}=(1-p(\theta_t(q_{\mathrm{u}},s^t)))(1-\delta)k_{\mathrm{u}}+i,\end{aligned}$$

where $T_t(s^t)$ denotes net transfers received by households in state s^t (including transfers from exiting intermediaries, net of the recapitalization of new entrants, and the rebate of intermediaries' issuance costs), which is an equilibrium object taken as given by households. The first restriction of Problem P2 is the household's budget constraint, with its left-hand side representing the uses of income: consumption, and investment; and its right-hand side the sources of income: labor income, capital sales revenues, and transfers from the ownership of intermediaries. Let $\hat{c}^{\mathrm{y}}_{\mathrm{H}}(k_{\mathrm{u}},s^t)$, $\hat{c}^{\mathrm{s}}_{\mathrm{H}}(k_{\mathrm{u}},s^t)$, $\hat{h}^{\mathrm{y}}_{\mathrm{H}}(k_{\mathrm{u}},s^t)$, $\hat{h}^{\mathrm{s}}_{\mathrm{H}}(k_{\mathrm{u}},s^t)$, $\hat{i}_{\mathrm{H}}(k_{\mathrm{u}},s^t)$, $\hat{k}^{\mathrm{u}}_{\mathrm{H}}(k_{\mathrm{u}},s^t)$, and $\hat{q}^{\mathrm{u}}_{\mathrm{H}}(k_{\mathrm{u}},s^t)$ denote the policy functions associated with Problem P2; $\hat{c}_{\mathrm{H}}(k_{\mathrm{u}},s^t)\equiv[(1-\nu)\hat{c}^{\mathrm{y}}_{\mathrm{H}}(k_{\mathrm{u}},s^t)+\nu\hat{c}^{\mathrm{s}}_{\mathrm{H}}(k_{\mathrm{u}},s^t)]\gamma^t_n$ households' total consumption;

and $\hat{\theta}^{\mathrm{u}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t) \equiv \theta_t(\hat{q}^{\mathrm{u}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t), s^t)$ their choice of market tightness when selling capital, determined by their choice of list prices for their units and the given market tightness function.

Households optimally equate the marginal utility of consumption across members working in different activities (i.e., $u_c(k_{\mathrm{u}}, s^t) \equiv u_{1t}(\hat{c}^{\mathrm{y}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t), \hat{h}^{\mathrm{y}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t)) = u_{1t}(\hat{c}^{\mathrm{s}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t), \hat{h}^{\mathrm{s}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t))$) and exhibit standard labor supply choices, equalizing the marginal rate of substitution between leisure and consumption to the wage in each activity (i.e., $-\frac{u_{2t}(\hat{c}^{j}_{\mathrm{H}}(k_{\mathrm{u}}, s^t), \hat{h}^{j}_{\mathrm{H}}(k_{\mathrm{u}}, s^t))}{u_{1t}(\hat{c}^{j}_{\mathrm{H}}(k_{\mathrm{u}}, s^t), \hat{h}^{j}_{\mathrm{H}}(k_{\mathrm{u}}, s^t))} = w_{jt}(s^t)$ for $j \in \{\mathrm{s}, \mathrm{y}\}$). In addition, households' optimal capital accumulation is characterized by the Euler equation

$$1 = \mathbb{E}_t\left[\lambda_{\mathrm{H}t+1}(k_{\mathrm{u}}, s^{t+1})(1-\delta)\left(p(\hat{\theta}^{\mathrm{u}}_{\mathrm{H}}(\hat{k}^{\mathrm{u}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t), s^{t+1}))\hat{q}^{\mathrm{u}}_{\mathrm{H}}(\hat{k}^{\mathrm{u}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t), s^{t+1}) + (1 - p(\hat{\theta}^{\mathrm{u}}_{\mathrm{H}}(\hat{k}^{\mathrm{u}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t), s^{t+1})))\right)\right], \tag{7}$$

where $\lambda_{\mathrm{H}t+1}(k_{\mathrm{u}}, s^{t+1}) \equiv \beta\frac{u_c(\hat{k}^{\mathrm{u}}_{\mathrm{H}}(k_{\mathrm{u}}, s^t), s^{t+1})}{u_c(k_{\mathrm{u}}, s^t)}$. A household's Euler equation (7) is similar to a firm's Euler equation for unemployed capital (6), with two main differences. First, because households can transform final goods into new capital one-for-one, the shadow price in final-goods units of accumulating unemployed capital (i.e., households' analog to $Q^{\mathrm{u}}_{\mathrm{F}}(\mathbf{k}, s^t)$) is equal to one. Second, firms' discount factor differs from that of households because of households' indirect ownership of nonfinancial firms through frictional financial intermediaries, as further discussed below. Therefore, although households and firms face a similar trade-off in the choice of the price in listing their capital units for sale (described above and illustrated in Appendix Figure E.1), their choice under this trade-off will typically feature a different choice of submarket.

Financial intermediaries Intermediaries maximize the lifetime discounted payouts transferred to households, $\max \mathbb{E}_t \sum_{r=0}^{\infty} \Lambda_{t,t+r}(s^{t+r})\pi_{\iota t+r}(s^{t+r})$, where $\pi_{\iota t}(s^t)$ denotes the net payments of intermediary ι to households in state s^t, and $\Lambda_{t,t+r}(s^{t+r})$ is households' stochastic discount factor, taken as given by intermediaries and further discussed below. At the beginning of each period, the amount of final goods a financial intermediary obtains from their investments in nonfinancial firms' claims, or net worth, is given by

$$n_{\iota t}(s^t) = (Div_t(s^t) + P_t(s^t))a_{\iota t}(s^{t-1}), \tag{8}$$

where $Div_t(s^t)$ and $P_t(s^t)$ denote, respectively, dividend and price per claim on nonfinancial firms in state s^t, which are equilibrium objects taken as given by intermediaries. Before exit shocks are realized, the equity value of a financial intermediary with net worth n solves the following Bellman equation:

$$V_{\mathrm{I}}(n, s^t) = \zeta_t(s^t)n + (1-\zeta_t(s^t)) \max_{x,a'} \left\{\pi + \mathbb{E}_t[\Lambda_{t,t+1}(s^{t+1})V_{\mathrm{I}}(\hat{n}(a', s^{t+1}), s^{t+1})]\right\} \tag{P3}$$

$$\begin{aligned} \text{s.t. } & P_t(s^t)a' = n + x, \\ & \pi = -x(1 + \mathbb{I}_{\{x>0\}}\mathcal{C}(x,n)), \\ & \hat{n}(a', s^{t+1}) = (Div_{t+1}(s^{t+1}) + P_{t+1}(s^{t+1}))a'. \end{aligned}$$

Problem (P3) expresses that, with probability $\zeta_t(s^t)$, an intermediary exits and transfers its net worth to households, while with probability $1-\zeta_t(s^t)$ it continues its operations. Conditional on survival, the intermediary chooses (x, a') to maximize its value. Newly entering intermediaries solve the same problem as surviving incumbents, evaluated at their entry net worth. The first constraint is the flow-of-funds constraint: the intermediary's purchase of nonfinancial-firm claims, $P_t(s^t)a'$, must equal the sum of its net worth n and the external finance raised, x. The second constraint reflects the intermediary's financial friction, described in the previous section, which implies a cost $\mathcal{C}(x,n)$ per unit of external finance x. Let $\hat{x}_{\mathrm{I}}(n, s^t)$ and $\hat{a}_{\mathrm{I}}(n, s^t)$ denote the policy functions associated with Problem (P3), and let $R_{\mathrm{F}t+1}(s^{t+1}) \equiv \frac{Div_{t+1}(s^{t+1})+P_{t+1}(s^{t+1})}{P_t(s^t)}$ denote the return on a nonfinancial firm claim.

I restrict attention to equilibria in which nonfinancial firms' claims are large enough so that intermediaries optimally choose to raise external finance in every state.[17] In such equilibria, Lemma D.1 shows that intermediaries' value function is linear in their net worth. The linearity of intermediaries' value functions allows for aggregation across intermediaries in the sense that only aggregate variables matter for equilibrium. To see this, note that the optimal choice of external finance for all intermediaries is characterized by

$$1 + 2\phi\mathcal{X}_t(s^t) = \mathbb{E}_t[\Lambda_{t,t+1}(s^{t+1})\overline{V}_{\mathrm{I}}(s^{t+1})R_{\mathrm{F}t+1}(s^{t+1})], \tag{9}$$

where $\mathcal{X}_t(s^t)$ is intermediaries' optimal external finance relative to their net worth, which is the same for all active intermediaries (for more details, see the proof of Lemma D.1). Condition (9) expresses that the intermediary optimally raises external finance to equate its marginal cost to the expected risk-adjusted returns on nonfinancial-firm claims. Condition (9) also implies that there

[17] Lemma D.1 involves conditioning on the path of equilibrium objects that are taken as given by intermediaries; in the quantitative analysis (Section 4), I focus on parameterizations for which these conditions are always satisfied.

exist an intermediaries' stochastic discount factor that prices claims on nonfinancial firms, i.e., $\mathbb{E}_t[m_{t+1}(s^{t+1})R_{\mathrm{F}t+1}(s^{t+1})] = 1$; this is given by $m_{t+1}(s^{t+1}) = \frac{\Lambda_{t,t+1}(s^{t+1})\overline{V}_{\mathrm{I}}(s^{t+1})}{1+2\phi\mathcal{X}_t(s^t)}$, which indicates that states with a higher marginal cost of external finance are associated with a lower stochastic discount factor, all else equal. In addition, aggregating the intermediaries' flow-of-funds constraint, transfers, and net worth evolution across active intermediaries (surviving incumbents and entrants), one obtains conditions that link intermediaries' aggregate purchase of assets, $\mathcal{A}_{t+1}(s^t) \equiv \int \hat{a}_{\mathrm{I}}(n_{\iota t}(s^t), s^t)\, \mathrm{d}\iota$, their transfers to households $\Pi_t(s^t)$, and their aggregate active net worth $N_t(s^t) \equiv \int n_{\iota t}(s^t)\, \mathrm{d}\iota$ to the financial shocks $\zeta_t(s^t)$ and other aggregate variables:

$$P_t(s^t)\mathcal{A}_{t+1}(s^t) = N_t(s^t)(1 + \mathcal{X}_t(s^t)), \tag{10}$$

$$\Pi_t(s^t) = \zeta_t(s^t)(Div_t(s^t) + P_t(s^t))\mathcal{A}_t(s^{t-1}) - \mathcal{X}_t(s^t)N_t(s^t) - \overline{\zeta}\,\overline{n}_t, \tag{11}$$

$$N_t(s^t) = (1 - \zeta_t(s^t))(Div_t(s^t) + P_t(s^t))\mathcal{A}_t(s^{t-1}) + \overline{\zeta}\,\overline{n}_t. \tag{12}$$

Therefore, in defining the equilibrium in the next section, one can consider only aggregate variables for intermediaries—namely, the path for $\{\overline{V}_{\mathrm{I}}(s^t), \mathcal{X}_t(s^t), \Pi_t(s^t), \mathcal{A}_{t+1}(s^t), N_t(s^t), m_{t+1}(s^{t+1})\}$.

3.3 Equilibrium

Let $K^{\mathrm{e}}_t(s^{t-1})$ denote the aggregate employed capital held by firms at the beginning of period t and $K^{\mathrm{u}}_{\mathrm{F}t}(s^{t-1})$ and $K^{\mathrm{u}}_{\mathrm{H}t}(s^{t-1})$ the stocks of aggregate unemployed capital held by firms and households at the beginning of period t. The equilibrium is formally defined in Appendix D.1. Using this definition, I derive key model aggregates. First, aggregating the capital-accumulation constraints of unemployed capital held by households and firms, (1) and (2), together with their policy functions, I obtain the law of motion of aggregate unemployed capital:

$$K^{\mathrm{u}}_{t+1}(s^t) = (1 - p^{\mathrm{u}}_t(s^t))(1-\delta)K^{\mathrm{u}}_t(s^{t-1}) + \psi(1-\delta)K^{\mathrm{e}}_t(s^{t-1}) + I_t(s^t), \tag{13}$$

where $K^{\mathrm{u}}_{t+1}(s^t) \equiv K^{\mathrm{u}}_{\mathrm{H}t+1}(s^t) + K^{\mathrm{u}}_{\mathrm{F}t+1}(s^t)$ denotes equilibrium aggregate unemployed capital, $I_t(s^t) \equiv \hat{i}_{\mathrm{H}}(K^{\mathrm{u}}_{\mathrm{H}t}(s^{t-1}), s^t)$ equilibrium aggregate investment, and $p^{\mathrm{u}}_t(s^t) \equiv p^{\mathrm{u}}_{\mathrm{F}t}(s^t)\frac{K^{\mathrm{u}}_{\mathrm{F}t}(s^{t-1})}{K^{\mathrm{u}}_t(s^{t-1})} + p^{\mathrm{u}}_{\mathrm{H}t}(s^t)\frac{K^{\mathrm{u}}_{\mathrm{H}t}(s^{t-1})}{K^{\mathrm{u}}_t(s^{t-1})}$ the equilibrium average trading probability of unemployed capital, where $p^{\mathrm{u}}_{\mathrm{F}t}(s^t) \equiv p(\hat{\theta}^{\mathrm{u}}_{\mathrm{F}}([K^{\mathrm{e}}_t(s^{t-1}), K^{\mathrm{u}}_{\mathrm{F}t}(s^{t-1})], s^t))$ is the equilibrium trading probability of firms and $p^{\mathrm{u}}_{\mathrm{H}t}(s^t) \equiv p(\hat{\theta}^{\mathrm{u}}_{\mathrm{H}}(K^{\mathrm{u}}_{\mathrm{H}t}(s^{t-1}), s^t))$ is that of households. Using (3) and equilibrium policy functions, the law of motion of aggregate employed capital can, in turn, be expressed by

$$K^{\mathrm{e}}_{t+1}(s^t) = (1-\psi)(1-\delta)K^{\mathrm{e}}_t(s^{t-1}) + (1-\delta)p^{\mathrm{u}}_t(s^t)K^{\mathrm{u}}_t(s^{t-1}), \tag{14}$$

which uses the fact that in equilibrium, the flow of capital purchased by buyers in a submarket equals the capital sold by sellers in that submarket: $\mu_t(\theta^{\mathrm{u}}_{jt}(s^t))\hat{v}_{\mathrm{F}}(q^{\mathrm{u}}_{jt}(s^t), [K^{\mathrm{e}}_t(s^{t-1}), K^{\mathrm{u}}_{\mathrm{F}t}(s^{t-1})], s^t) = p^{\mathrm{u}}_{jt}(s^t)(1-\delta)K^{\mathrm{u}}_{jt}(s^{t-1})$ for $j \in \{\mathrm{F}, \mathrm{H}\}$, where $q^{\mathrm{u}}_{\mathrm{F}t}(s^t) \equiv \hat{q}^{\mathrm{u}}_{\mathrm{F}}([K^{\mathrm{e}}_t(s^{t-1}), K^{\mathrm{u}}_{\mathrm{F}t}(s^{t-1})], s^t)$ and $q^{\mathrm{u}}_{\mathrm{H}t}(s^t) \equiv \hat{q}^{\mathrm{u}}_{\mathrm{H}}(K^{\mathrm{u}}_{\mathrm{H}t}(s^{t-1}), s^t)$ denote firms' and households' equilibrium choices of submarkets, and $\theta^{\mathrm{u}}_{\mathrm{F}t}(s^t) \equiv \theta_t(q^{\mathrm{u}}_{\mathrm{F}t}(s^t), s^t)$ and $\theta^{\mathrm{u}}_{\mathrm{H}t}(s^t) \equiv \theta_t(q^{\mathrm{u}}_{\mathrm{H}t}(s^t), s^t)$ the associated equilibrium market tightness.

Laws of motion (13) and (14) show how trading probabilities' fluctuations play a central role for unemployed- and employed-capital fluctuations. To characterize the equilibrium trading probabilities, it is useful to note that firms' indifference condition to purchase capital (5) determines the equilibrium market tightness function: For all $q < Q^{\mathrm{e}}_{\mathrm{F}t}(s^t)$,

$$\theta_t(q, s^t) = \mu_t^{-1}\left(\frac{w_{\mathrm{st}}(s^t)}{Q^{\mathrm{e}}_{\mathrm{F}t}(s^t) - q}\right) \tag{15}$$

and $\theta_t(q, s^t) = 0$ for all $q \geq Q^{\mathrm{e}}_{\mathrm{F}t}(s^t)$, where $Q^{\mathrm{e}}_{\mathrm{F}t}(s^t) = Q^{\mathrm{e}}_{\mathrm{F}}([K^{\mathrm{e}}_t(s^{t-1}), K^{\mathrm{u}}_{\mathrm{F}t}(s^{t-1})], s^t)$ is the equilibrium shadow price of employed capital. Using this condition and under the additional assumption of a Cobb-Douglas meeting technology (which is frequently used in the search literature for labor markets; e.g., Mortensen and Pissarides, 1999b; Shimer, 2010, and will be used in the quantitative analysis in the next section), one can characterize the equilibrium listed prices and trading probabilities of unemployed capital.

***Lemma* 1.** *If $\mathcal{M}(k, \gamma^t v) = \min\{\overline{m}k^{\eta}(\gamma^t v)^{1-\eta}, k\}$, with $\eta \in (0,1)$, $\overline{m} > 0$, an equilibrium with $p^{\mathrm{u}}_{jt}(s^t) < 1$ for $j \in \{\mathrm{F}, \mathrm{H}\}$ is characterized by the following capital prices and trading probabilities:*

$$\hat{q}^{\mathrm{u}}_{jt}(s^t) = \eta Q^{\mathrm{e}}_{\mathrm{F}t}(s^t) + (1-\eta)Q^{\mathrm{u}}_{jt}(s^t), \tag{16}$$

$$p^{\mathrm{u}}_{jt}(s^t) = \overline{m}\left(w_{\mathrm{st}}(s^t)^{-1}\left(\overline{m}\gamma^t\right)(1-\eta)\left(Q^{\mathrm{e}}_{\mathrm{F}t}(s^t) - Q^{\mathrm{u}}_{jt}(s^t)\right)\right)^{\frac{1-\eta}{\eta}}, \tag{17}$$

where $Q^{\mathrm{u}}_{\mathrm{F}t}(s^t) \equiv Q^{\mathrm{u}}_{\mathrm{F}}([K^{\mathrm{e}}_t(s^{t-1}), K^{\mathrm{u}}_{\mathrm{F}t}(s^{t-1})], s^t)$ denotes firms' equilibrium shadow prices for unemployed capital and $Q^{\mathrm{u}}_{\mathrm{H}t}(s^t) = 1$ denotes those for households for all s^t.

Proof. See Appendix D.2

■

Lemma 1 shows two results. First, it expresses that the price at which unemployed capital is traded in equilibrium is a weighted average of the shadow price of employed and unemployed capital, with the weights determined by the matching elasticity η. As shown in Panel (c) of Appendix Figure E.1, at this point the marginal rates of substitution between market tightness and capital price are the same for buyers and sellers. Second, given the cost of search activities $w_{\mathrm{st}}(s^t)$, the selling probability is linked to the difference between the shadow prices of employed and unemployed capital. A larger gap generates a larger difference between the shadow price of employed capital and the listed price of unemployed capital and leads to greater capital demand and search activity from buyers, with higher trading probabilities for sellers. As shown in detail in the next section, this implies that fluctuations in the trading probability, which are critical for capital-unemployment fluctuations, will be tightly linked to fluctuations in the shadow price of employed capital, which are largely influenced by financial shocks.

Finally, combining (13) and (14), I obtain the law of motion of the aggregate capital stock of the standard neoclassical model:

$$K_{t+1}(s^t) = (1-\delta)K_t(s^{t-1}) + I_t(s^t), \tag{18}$$

where $K_{t+1}(s^t) \equiv K^{\mathrm{e}}_{t+1}(s^t) + K^{\mathrm{u}}_{t+1}(s^t)$ is the economy's total capital stock. Using the flow-of-funds constraints of firms and intermediaries together with households' budget constraints, one obtains the economy's resource constraint:

$$C_t(s^t) + I_t(s^t) = Y_t(s^t), \tag{19}$$

where $Y_t(s^t) \equiv A_t(s^t) f(K^{\mathrm{e}}_{t+1}(s^t), \gamma^t \hat{\ell}_{\mathrm{F}t}(K_{\mathrm{F}t}(s^{t-1}), s^t))$ is aggregate output, with $K_{\mathrm{F}t}(s^{t-1}) \equiv [K^{\mathrm{e}}_t(s^{t-1}), K^{\mathrm{u}}_{\mathrm{F}t}(s^{t-1})]$, and $C_t(s^t) \equiv \hat{c}_{\mathrm{H}}(K^{\mathrm{u}}_{\mathrm{H}t}(s^{t-1}), s^t)$ is aggregate consumption.

4 Accounting for Capital Unemployment Fluctuations

4.1 Calibration

I calibrate the model to match key moments in the U.S. economy for the period 1980–2015 with a time unit of one quarter. The model has three groups of parameters, detailed in Table 2 and described next.

Neoclassical block The first group of parameters is shared with the neoclassical stochastic-growth model and is set to standard values from the literature. For households' preferences, I assume period utility $u_t(c,h) = \log\left(c - \chi\gamma^t \frac{1}{1+\xi} h^{1+\xi}\right)$, as in Greenwood, Hercowitz, and Huffman (1988), and set the Frisch elasticity of labor supply, $1/\xi$, to one.[18] I set the disutility of labor, χ, to target steady-state hours worked $h_{\mathrm{y}} = 1/3$, and the subjective discount factor to $\beta = 0.994$, which corresponds to a steady-state annual real interest rate of 4%. For firms' technologies, I assume a Cobb-Douglas production function $f_t(k, \gamma^t h) = k^\alpha \left(\gamma^t h\right)^{1-\alpha}$ and set the capital share to $\alpha = 0.35$ (consistent with Fernald, 2014). I set the depreciation rate to $\delta = 0.021$, corresponding to an 8% annual rate for nonresidential capital (BEA, Fixed Asset tables); the growth rate of technical progress to $\gamma = 1.004$, consistent with an annual technology growth rate of 1.6% observed from 1980 to 2015 (BEA); and population growth to $\gamma_n = 1.0027$, consistent with an annual 1.1% growth rate of the working-age population (ages 15–64, data from the Federal Reserve Bank of St. Louis and OECD). For TFP, I assume a log-AR(1) process $\log A_{t+1} = \rho_A \log A_t + \varepsilon^A_{t+1}$, where $\varepsilon^A_{t+1} \sim N(0, \sigma^2_A)$. As further described below, I calibrate ρ_A and σ_A jointly with other parameters to match key business cycle moments (primarily the volatility and persistence of output), implying $\rho_A = 0.95$ and $\sigma_A = 0.006$, consistent with standard values in the business cycle literature.

Trading frictions The second group of parameters is related to trading frictions in the capital market and is the most novel part of the calibration. For the separation rate from match-specific shocks, I set $\psi = 0.008$, which corresponds to the 3.2% average exit rate of U.S. establishments, obtained as a weighted average of exit rates for establishments of different sizes reported by the U.S. Census Bureau; see Appendix A.2 for details. For the matching technology, I allow for the possibility that new and used capital goods have different matching efficiencies, with $\mathcal{M}_j(k, \gamma^t v) = \min\{\overline{m}_j k^\eta (\gamma^t v)^{1-\eta}, k\}$ with $\overline{m}_j > 0$ for $j \in \{\text{new}, \text{used}\}$. In an alternative parameterization in the next section, I also consider the case in which all capital has the same matching efficiency.[19] To calibrate $\overline{m}_{\text{new}}$ and $\overline{m}_{\text{used}}$, I target a steady-state ratio of matching flows from used capital to exit flows, $\left(\frac{p(\theta_{\text{used,ss}}) K^{\text{u}}_{\text{used,ss}}}{\psi K^{\text{e}}_{\text{ss}}}\right)$, of 64%, consistent with the average reported by Becker, Haltiwanger,

[18]The preferences proposed by Greenwood, Hercowitz, and Huffman (1988) eliminate the wealth effect on labor supply and are widely used in models with financial shocks to generate business-cycle patterns aligned with the data (see, for example, Mendoza, 2010). In the class of models considered, I use these preferences so the model generates procyclical consumption, aligned with its data counterpart (as reported in Table 3). With separable preferences, the model with financial shocks generates counterfactual countercyclical consumption, even without trading frictions. For details on the model without trading frictions, see Section 5.2 and Appendix F.

[19]In Section 3, for expositional simplicity, I described the baseline model with a single matching technology. For full details on the baseline model with heterogeneous matching technology, see Appendix D.3.1.

Table 2
Baseline Parameter Calibration

Parameter	Description	Value
Neoclassical block		
β	Discount Factor	0.994
$1/\xi$	Frisch elasticity of labor supply	1
α	Share of capital	0.35
δ	Depreciation rate	0.021
γ	Technology growth	1.004
γ_n	Population growth	1.003
ρ_A	Persistence of TFP	0.95
σ_A	SD of innovations to TFP	0.006
Trading frictions in capital markets		
ψ	Separation rate from match-specific shocks	0.008
η	Curvature matching technology	0.50
$\overline{m}_{\text{used}}$	Efficiency matching technology, used capital	0.58
$\overline{m}_{\text{new}}$	—, new capital	5.62
Financial frictions		
$\overline{\zeta}$	Average exit rate intermediaries	0.070
$\overline{n}$	Net worth new intermediaries	2.31
ϕ	Intermediaries' cost of external finance	0.80
σ_ζ	SD of innovations to financial shocks	0.101

Notes: This table reports parameters used in the baseline calibration of the model. The disutility of labor and share of households working in search activities have been set to $\chi = 6.2$ and $\nu = 0.04$ to target $h_{\text{y}} = h_{\text{s}} = 1/3$ for households' steady-state per capita hours worked in the production of final goods and search activities.

Jarmin, Klimek, and Wilson (2006) using U.S. Census data. This target, together with the evolution of employed and unemployed capital along the balanced-growth path, implies $\overline{m}_{\text{new}} = 5.62$ and $\overline{m}_{\text{used}} = 0.58$, consistent with the idea that used capital faces greater trading frictions. For the elasticity of the matching function, I consider the symmetric case $\eta = 0.5$ as a benchmark in the baseline calibration (used, for example, by Shimer, 2010, in labor markets), and study how results vary with alternative values in Appendix D.5. Finally, for the share of households working in search activities, I set $\nu = 0.0374$ to target the same steady-state per capita hours worked as for workers in final-good production, $h_{\text{s}} = 1/3$.[20] The calibration of the benchmark model implies an untargeted average capital-unemployment rate of 12.8%.[21] Given the average unemployment rate of 10.9% reported in Section 2 for structures, this is a plausible value. Section 4.2 also reports predictions for

[20]This value of ν is of a similar order of magnitude as the average share of workers in real estate and machinery leasing (1.6% for 1990–2017, BLS). Under the assumed segmentation of search activities and preferences, ν primarily affects predicted moments for total hours worked and the relative consumption of each type of worker.

[21]Using (14), along the balanced-growth path, the unemployment rate of capital is

$$u^{\text{k}}_{\text{ss}} = \frac{\psi(1-\delta) + \gamma\gamma_n - 1 + \delta}{\gamma\gamma_n - (1-\delta)(1-p_{\text{ss}}) + \psi(1-\delta)},$$

where p_{ss} denotes the steady-state average matching probability of unemployed capital.

a version of the model that distinguishes structures from equipment, detailed in Appendix D.3.2, and shows results similar to those of the benchmark calibration.

Financial frictions The third group of parameters governs financial frictions. Following Gertler and Kiyotaki (2010), I target an average annual excess return of nonfinancial firms over the risk-free rate of 100 basis points, which implies $\overline{n} = 2.3$. For financial shocks, I assume an i.i.d. shock to the log survival rate with standard deviation σ_ζ to focus on the model's endogenous propagation mechanism. I also report results for persistent shocks, which yield similar but more prolonged dynamics. I jointly calibrate the aggregate shock parameters in the baseline model, $\{\sigma_A, \rho_A, \sigma_\zeta\}$, to match the persistence and volatility of output and the volatility of nonfinancial firms' market value in the data (see Appendix A.2.2 for data sources).[22] Given these parameters, the degree of financial frictions is governed by intermediaries' marginal cost of external finance, ϕ, and their average exit rate, $\overline{\zeta}$. I calibrate these parameters to match the volatility of investment and the market value of intermediaries' net worth, obtaining the values reported in Table 2.[23]

Business cycles Table 3 compares the predictions of the calibrated model for standard business-cycle moments and their data counterparts for the U.S. economy. By design, the model predicts a volatility of output, investment, and the market value of nonfinancial firms, which are the moments targeted in the calibration, broadly aligned with data counterparts. The volatility of intermediaries' market value, also targeted in the calibration, is above that of real variables but below the level observed in the data. In terms of untargeted moments, the model predicts volatilities for consumption and hours worked close to those observed in the data and a positive correlation for these variables, investment, and the market value of intermediaries and nonfinancial firms with output, as observed in the data. Therefore, the analysis in the next sections consists of studying the fluctuations of capital unemployment in an economy that exhibits business-cycle patterns broadly consistent with the data. The next section focuses on the fluctuations of capital unemployment in this economy, and Section 5 on the macroeconomic implications of capital unemployment, entering in more detail into the model's transitional dynamics and propagation of aggregate shocks.

[22] In the calibration, I constrain ρ_A to be no greater than 0.95, the value used in King and Rebelo (1999). The calibrated value attains this bound, resulting in an autocorrelation of output of 0.58, below the 0.87 observed in the data.

[23] The calibrated value of the exit rate is larger than in Gertler and Kiyotaki (2010), but close to that in Morelli, Ottonello, and Perez (2022), which uses a modeling structure similar to that in this paper. A sufficiently high value of this parameter is needed under the assumed model structure to study equilibria in which intermediaries do not outgrow their financial frictions. As is standard in this literature, this parameter can have a broader interpretation than an exit rate, also capturing payouts from non-exiting intermediaries to shareholders.

Table 3
Business-Cycle Moments

	Data	**Model**
Volatility		
Output, $\sigma(Y)^*$	1.3%	1.4%
Investment, $\sigma(I)^*$	4.7%	3.4%
Consumption, $\sigma(C)$	1.1%	0.9%
Hours worked, $\sigma(H)$	1.8%	1.5%
Nonfinancial firms' market value, $\sigma(V_F)^*$	10.7%	9.7%
Financial intermediaries' market value, $\sigma(V_I)^*$	14.8%	6.5%
Correlation with output		
Investment, $\rho(I, Y)$	0.74	0.91
Consumption, $\rho(C, Y)$	0.89	0.81
Hours worked, $\rho(H, Y)$	0.87	0.80
Nonfinancial firms' market value, $\rho(V_F, Y)$	0.52	0.55
Financial intermediaries' market value, $\rho(V_I, Y)$	0.37	0.55

Notes: This table reports U.S. business-cycle moments and their model counterparts. Moments with (*) have been targeted in the calibration. The column *Data* reports moments computed using quarterly data for the period 1980–2015, detrended using the HP filter with a smoothing parameter of 1,600. See Appendix A.2.2 for details on data sources. The column *Model* reports moments from the model-simulated data, using the baseline model presented in Section 3 with parameterization from Table 2, detrended using the HP filter, and averaged over 500 simulations of the same length as the data.

4.2 Capital unemployment fluctuations in the model and the data

Baseline model Table 4 shows that the baseline model predicts fluctuations in capital unemployment aligned with those documented in Section 2.3. In the U.S. data, the capital-unemployment rate of structures is volatile and countercyclical, with a volatility of 7% and a −0.55 correlation with output. The baseline model likewise predicts volatile and countercyclical capital unemployment, with volatility between 6% and 11% and a correlation with output between −0.18 and −0.55, depending on the matching-technology assumption. These predictions are similar to those of a model that distinguishes structures from equipment, detailed in Appendix D.3.2, where the predictions for the capital-unemployment rate of structures are more directly comparable to the data. Table 4 also reports that the model generates fluctuations in capital prices aligned with those observed for U.S. structures (see Appendix A.2.2 for data sources). For trading probabilities, although no systematic U.S. data are available, the model predictions in Table 4 are consistent with the evidence for Spanish structures described in Section 2.3, where trading probabilities (proxied by listing-withdrawal rates) exhibit a volatility of 23% and a 48% correlation with output.

To further study the model's predicted fluctuations of capital unemployment, Figure 4 reports

Table 4

Capital Unemployment Fluctuations: Model and Data

	Volatility			**Correlation with output**		
	Capital unemployment $\sigma(u)$	Capital price $\sigma(q)$	Trading probability $\sigma(p(\theta_{\mathrm{u}}))$	Capital unemployment $\rho(u, Y)$	Capital price $\rho(q, Y)$	Trading probability $\rho(p(\theta_{\mathrm{u}}), Y)$
A. Data (U.S., structures)	7%	9%		-0.55	0.76	
B. Baseline model						
with homogeneous capital						
and heterogeneous matching technology	11%	6%	44%	-0.55	0.50	0.45
and single matching technology	6%	6%	16%	-0.18	0.22	0.25
with structures and equipment						
moments for total capital	8%	6%	25%	-0.45	0.49	0.41
moments for structures	5%	6%	12%	-0.33	0.49	0.51
moments for equipment	20%	7%	50%	-0.44	0.47	0.42
C. Alternative model assumptions						
i. Financial frictions						
No financial frictions ($\phi = 0$)	0.4%	0.1%	1%	0.95	0.90	0.86
No financial shocks ($\sigma_\zeta = 0$)	0.4%	0.1%	1%	0.93	0.99	0.97
Capital producer	4%	0%	5%	0.04	0.26	0.24
ii. Aggregate shocks						
Shocks to technology new capital	7%	19%	15%	0.12	0.03	0.09
Shocks to matching technology	7%	3%	27%	-0.38	-0.28	0.34
iii. Labor markets						
No segmentation for search activities	14%	5%	56%	-0.04	-0.20	-0.27
Frictional labor markets	12%	7%	50%	-0.61	0.51	0.44

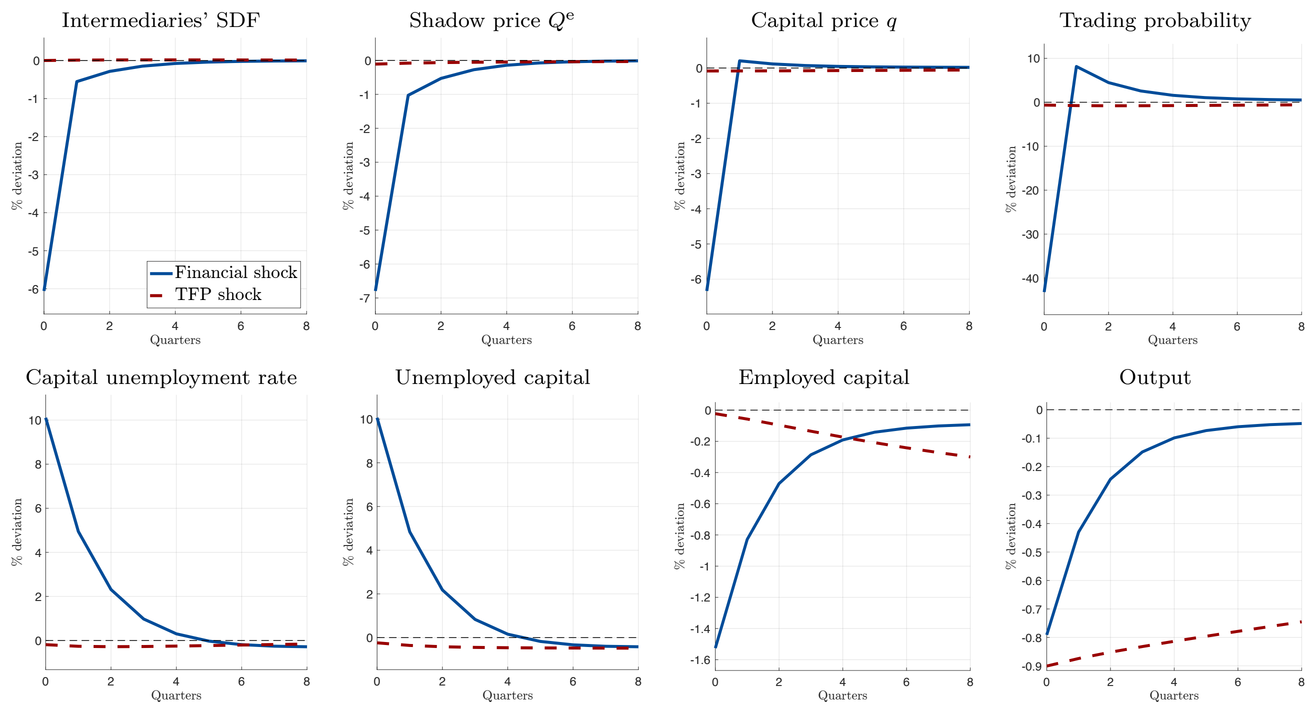
Notes: This table reports business-cycle moments of U.S. capital-market variables and their model counterparts. Panel A report moments for the capital-unemployment rate and prices of structures, using quarterly data for the period 1980–2015 and detrended in logs using the HP filter with a smoothing parameter of 1,600. See Section 2 and Appendix A.2.2 for details on data sources. Panels B and C report moments from the model-simulated data for different model specifications, detrended in logs using the HP filter, and averaged over 500 simulations of the same length as the data. Panel B, *Baseline model*, reports the results for the model presented in Section 3, together with the extensions in Appendix D.3. The row *and heterogeneous matching technology* refers to the parameterization from Table 2. The row *and single matching technology* refers to a parameterization in which the trade of new and used capital goods is assumed to have the same matching efficiency, $\overline{m}_{\text{new}} = \overline{m}_{\text{used}} = 2.7$, with the rest of the parameter values the same as in Table 2. The row *with structures and equipment* corresponds to the baseline model with separate structures and equipment, detailed in Appendix D.3.2. Panel C, *Alternative model assumptions*, reports the results for the models presented in Appendix D.4, as well as some alternative parameterizations. The rows *No financial frictions* and *No financial shocks* correspond, respectively, to a parameterization of the baseline model with $\phi = 0$ and $\sigma_\zeta = 0$ and the other parameters as in Table 2. The rows *Shocks to technology new capital* and *Shocks to matching technology* refer to versions of the model with no financial shocks ($\sigma_\zeta = 0$) and, respectively, investment-specific technology shocks and shocks to the efficiency of the matching technology, described in Appendices D.4.1 and D.4.2. In both models, these aggregate shocks are assumed to be i.i.d. and their volatility is set to target the volatility of the capital-unemployment rate observed in the data. Appendix Figures E.3 and E.4 show similar model responses when these shocks follow persistent processes.The row *Capital producers* corresponds to a version of the model in which capital producers face the same degree of financial frictions as final–goods producers, described in Appendix D.4.3. The row *No segmentation for search activities* corresponds to a version of the model with a unified labor market for work activities in the production of final goods and search, described in Section D.4.4. The row *Frictional labor markets* corresponds to a version of the model with search frictions in the labor market, described in Appendix D.4.5.

the responses of capital-market variables to macroeconomic shocks. The solid blue lines show the responses to a contractionary financial shock (an increase in ζ), which reduces the total net worth available for intermediaries to invest in firms. Because intermediaries face financial frictions to recapitalize, the negative financial shock increases their marginal cost of external finance and lowers their stochastic discount factor m; this contracts the supply of funds for firms and raises their cost of capital, reducing the shadow price of employed capital Q^{e}, as well as firms' capital purchases, the stock of employed capital, and output. The decline in firms' capital demand and search effort then reduces the trading probability faced by capital sellers, $p(\theta^{\mathrm{u}})$, and increases the stock of unemployed capital. As a result, the negative financial shock generates a sizable rise in unemployed capital together with an output contraction, as typically observed during the recession episodes documented in Section 2.3. Appendix Figure E.2 shows that persistent financial shocks yield similar results with more long-lived effects on capital unemployment.

The dotted red lines in Figure 4 show that, in contrast to financial shocks, changes in TFP have a modest effect on capital unemployment, one order of magnitude smaller than the response to financial shocks. A key difference between these shocks is that TFP shocks have only a minor impact on intermediaries' stochastic discount factor and therefore do not induce sizable movements in trading probabilities. Although TFP shocks also affect trading probabilities through other channels, such as the marginal product of employed capital, Figure 4 shows that these effects do not translate into large movements in market tightness and trading probabilities. Moreover, since contractionary TFP shocks reduce the production of new capital, their overall effect is a decline in the stock of unemployed capital and in the capital-unemployment rate, which is at odds with the patterns observed in the data during bad times.

The differential response of capital unemployment to the two types of shocks discussed above indicates that financial frictions play a key role in explaining the large and countercyclical fluctuations predicted by the model. To further highlight this point, Panel C of Table 4 reports the predictions of a version of the model with no financial frictions ($\phi = 0$) and one with no financial shocks ($\sigma_\zeta = 0$). In both cases the model predicts a volatility for capital unemployment and capital prices one order of magnitude smaller than that observed in the data. This is consistent with the well-known finding that the canonical real-business-cycle model generates small fluctuations in asset prices relative to the data (e.g., Mehra and Prescott, 1985). These small fluctuations in asset values, in turn, are linked to small fluctuations in trading probabilities and capital unemployment. The financial frictions and shocks in the model can therefore be seen as a way to introduce realistic movements in asset prices, following the literature on intermediary-based asset pricing (see He and Krishnamurthy, 2018, for a recent survey), which in turn lead to large fluctuations in trading probabilities and in patterns for capital unemployment aligned with the data.

Figure 4: Response of Capital Unemployment to Contractionary Macroeconomic Shocks



Note: This figure shows the impulse responses of intermediaries' expected discount factor $\mathbb{E}[m']$, firms' shadow price of employed capital Q^{e}, capital price q, the trading probability $p(\theta^{u})$, the capital-unemployment rate u^{k}, the stock of unemployed capital K^{u}, the stock of employed capital K^{e}, and output Y under the baseline model, presented in Section 3, with the parameterization of Table 2. The solid blue lines show the responses to a one-standard-deviation contractionary financial shock. In the baseline model, this is an i.i.d. shock. Appendix Figure E.2 reports the responses to a persistent financial shock. The dotted red lines show the responses to a negative one-standard-deviation TFP shock. The horizontal axis displays quarters after the shock. Impulse responses are expressed in percentage deviations from the steady state.

Additional Exercises Panel C of Table 4 presents a set of additional exercises to discuss the roles of other assumptions in the baseline model. The first set of exercises examines whether types of shocks other than those in the baseline model can also generate realistic movements in capital unemployment and prices. In the environment considered, natural candidates are shocks to the technology to produce new capital goods (e.g., Greenwood, Hercowitz, and Huffman, 1988; Justiniano, Primiceri, and Tambalotti, 2010) and shocks to the efficiency of the matching technology (e.g., Andolfatto, 1996, in the labor market literature), detailed in Appendices D.4.1 and D.4.2. Table 4 reports moments of capital-market variables for a version of the model with no financial shocks and introduces alternative technology shocks calibrated to match the volatility of capital unemployment observed in the data.[24] Results indicate that neither of the two alternative technology shocks generates patterns of capital unemployment and prices consistent with the data. Shocks to the technology to produce new capital generate procyclical capital unemployment. As illustrated in Appendix Figure E.3, an increase in the efficiency of producing new capital goods has a large effect on new capital production, and an expansionary shock raises the stock of unemployed capital units together with output.[25] In contrast, shocks to the efficiency of the matching technology generate a counterfactual countercyclical behavior of capital prices. As shown in Appendix Figure E.4, an increase in matching efficiency lowers the shadow price of employed capital, pushing down capital prices during an expansion.[26] These results suggest that, in a real-business-cycle model, the joint patterns of fluctuations in capital unemployment and prices generated by financial shocks cannot be easily replicated by other types of shocks.

The second set of exercises studies the role of the baseline model's assumptions about the labor market. First, the baseline model assumes a segmentation between work activities in the production of final goods and search activities. Table 4 shows that a version of the model in which labor is perfectly substitutable across these activities leads to a counterfactual acyclical capital unemployment. The reason is that the decline in search effort following a negative financial shock pushes down wages, which, in the absence of segmentation, increases hours worked and economic activity in final-good production (see Figure E.5). This suggests that some degree of specialization in search activities is important to account for a volatile and procyclical trading probability. Second, the baseline model assumes a Walrasian labor market. Table 4 shows that the main predictions of the model for capital unemployment are preserved when the model is extended to incorporate frictional labor markets (for details, see Appendix D.4.5).

The third exercise examines the model structure for financial frictions. The baseline model follows a common structure in the macro–finance literature in which financial frictions affect the fi-

[24]For each shock, I consider two parameterizations: one in which the shock is i.i.d. and one in which it follows an AR(1) process with an autoregressive coefficient of 0.7. Table 4 reports the i.i.d. case, but similar conclusions are obtained for persistent shocks. Appendix Figures E.3 and E.4 report impulse responses to the two shocks under both persistence assumptions.

[25]The procyclical pattern for capital unemployment predicted by this shock is consistent with that observed in the United States during the second half of the 1980s, which, as noted in Section 2.3, is identified in the urban economics literature as a boom in the construction of nonresidential structures. This suggests that shocks of this type may have played an important role during that period. However, as documented in Section 2.3, the remaining U.S. sample is characterized by countercyclical capital unemployment.

[26]Relatedly, one could consider shocks to firms' marginal efficiency of investment, i.e., affecting firms' efficiency for transforming new matches into employed capital. In this case, an increase in firms' marginal efficiency of investment raises capital prices (reflecting a higher valuation of matches by entrepreneurs) but lowers the shadow price of employed capital and the market value of nonfinancial firms.

nancing of final-goods producers' investment (e.g., Bernanke, Gertler, and Gilchrist, 1999). Table 4 shows that an alternative version of the model in which capital producers face the same degree of financial frictions as final–goods producers (detailed in Appendix D.4.3) generates counterfactual acyclical capital unemployment and a volatility of capital prices that is substantially below that observed in the data. The reason is that, in this setting, a negative financial shock contracts the supply of capital goods, which reduces the stock of unemployed capital and dampens the decline in the capital price. This suggests that the presence of stronger financial frictions faced by final–goods producers relative to capital–goods producers can be important for accounting for key capital–market patterns.

Finally, Appendix D.5 shows how the model predictions of the baseline model vary under alternative parameterizations of trading frictions in capital markets. Overall, the results show that the model with trading frictions in capital markets and financial frictions generates volatile and countercyclical capital unemployment for a wide range of plausible parameter values.

5 Macroeconomic Implications of Capital Unemployment

I now use the calibrated model to study the macroeconomic implications of capital unemployment. Section 5.1 focuses on transitional dynamics and shows how the level of capital unemployment affects subsequent investment dynamics. Section 5.2 focuses on responses to shocks and shows how fluctuations in capital unemployment constitute a sizable propagation mechanism from financial shocks to economic activity. Section 5.3 uses these results to discuss the relationship between the capital-unemployment model and traditional models of variable capital utilization.

5.1 Capital unemployment and persistent investment slumps

Transitional dynamics Figure 5 shows how the capital-unemployment rate affects the transitional dynamics predicted by the model. Panel (a) considers an initial situation in which the total capital stock is at its steady-state level and the capital-unemployment rate is above its steady-state level. The transition exhibits a nonmonotonic path for the capital stock, which declines initially and then converges back to its steady-state level. These dynamics reflect two opposing forces. On the one hand, below-steady-state employed capital implies a high marginal product of capital, which induces firms to accumulate employed capital. On the other hand, above-steady-state capital unemployment leads to a low probability of selling capital, which reduces the incentives to produce new capital. Initially, the latter force dominates, and the absorption of unemployed capital crowds out investment. As unemployed capital is absorbed into employment, the selling probability recovers, and so do the incentives to produce capital, causing the capital stock to converge back to its steady-state level.[27]

I then consider an initial situation in which the total capital stock is below its steady-state level, which in the neoclassical model would be associated with high investment rates and a monotonic convergence toward the steady state. Panel (b) of Figure 5 shows that, in the capital-unemployment model, the convergence of the capital stock depends on the initial level of capital unemployment. When the initial unemployment rate is at its steady-state level, the capital stock exhibits monotonic transitional dynamics, similar to the standard neoclassical pattern. Panel (a) of Appendix Figure E.6 shows that these dynamics are quantitatively close to those of a neoclassical model with a similar parameterization (see Appendix F.1 for details). In contrast, when initial capital unemployment is large enough, the dampening effect of capital unemployment on the production of capital goods can generate nonmonotonic transitional dynamics: Even though the total capital stock starts below its steady-state level, the economy initially experiences investment below steady state (an *investment slump*) and investment increases toward steady state only once a substantial part of the above-steady-state unemployed capital has been absorbed. I next show how this mechanism can help explain the investment dynamics observed after the U.S. Great Recession.

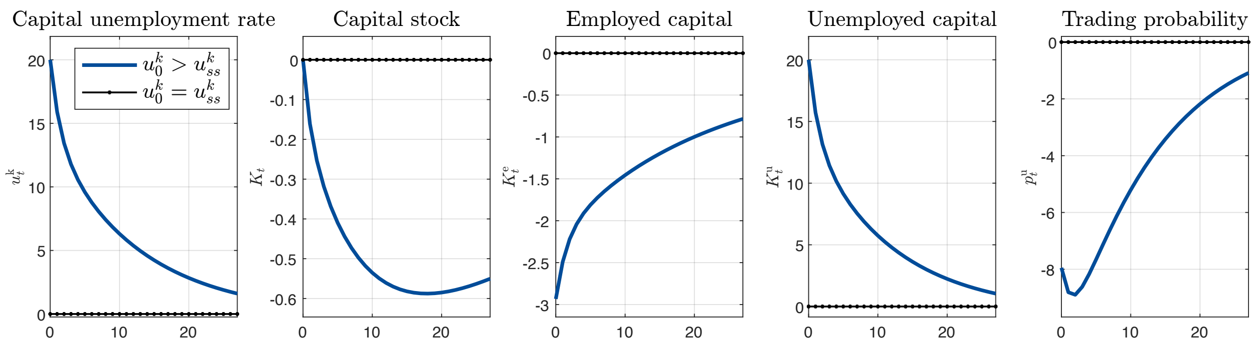
An application to the U.S. Great Recession Panel (a) of Figure 6 shows that a persistent investment slump followed the U.S. Great Recession: 3 years after the recession trough, the capital stock continued to fall relative to its trend, to levels of 4% below trend by 2013. Several researchers have argued that this pattern of recovery is puzzling from the perspective of standard capital-accumulation models, in which the capital stock's being below its trend generates a strong force in the economy for capital accumulation (see, for example, Kydland and Zarazaga, 2016; Hall, 2014; Gutiérrez and Philippon, 2016).

Panel (b) of Figure 6 shows that the U.S. Great Recession was also followed by a sizable increase in capital unemployment, reaching levels 35% above its mean by 2010. Motivated by the effect of capital unemployment on the transitional dynamics of the capital stock, I use the model to assess its role in driving the observed capital-accumulation patterns. The exercise proceeds in

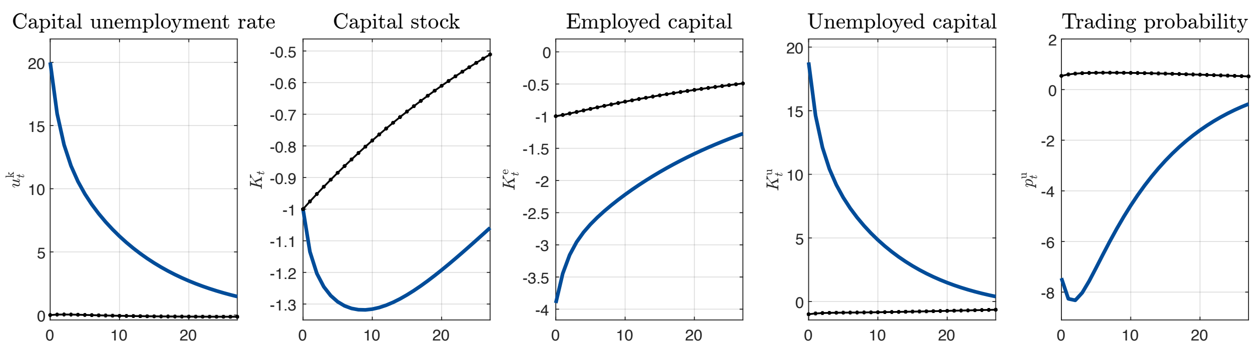
[27] Appendix G presents a version of the model with a closed-form solution for the accumulation of employed and unemployed capital in which the relationship between capital unemployment and investment rates can be further characterized analytically.

FIGURE 5: CAPITAL UNEMPLOYMENT AND TRANSITIONAL DYNAMICS.

(a) Initial total capital stock at steady-state level



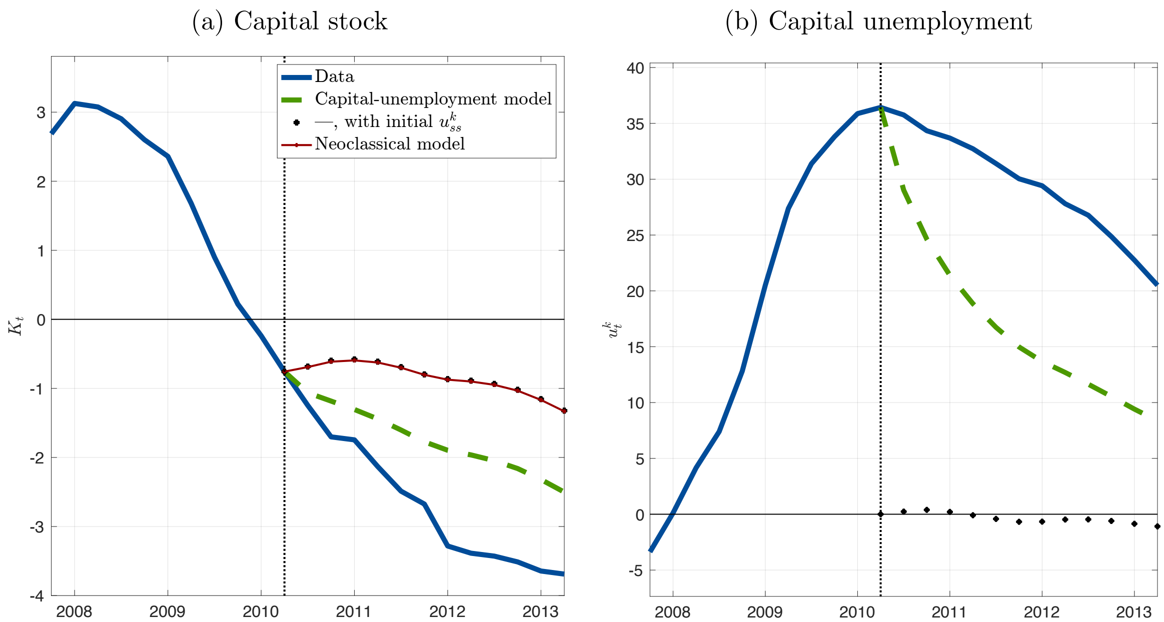
(b) Initial total capital stock below steady-state level



Note: Variables are expressed as percentages relative to their steady-state values. Transitional dynamics computed using the policy function of the capital-accumulation model presented in Sections 3 and 4, with $A_t = 1$ and $\zeta_t = \overline{\zeta}$ for all t. Panel (a) presents simulations that start from the capital-stock steady-state level at $t = 0$ and two alternative values of the initial capital-unemployment rate: The dotted black line starts from the steady-state capital-unemployment rate, and the solid blue line starts from a capital-unemployment rate 20% above its steady state. Panel (b) presents the same exercises but for simulations that start from a level of the capital stock 1% below its steady-state level at $t = 0$. The horizontal axis displays quarters after the initial state.

two steps. First, I use the parameterized model's policy functions to compute its predictions for the recovery from the Great Recession, starting from the observed level of the capital stock and capital unemployment in 2010.II, which corresponds to the peak level of capital unemployment, and under an estimated sequence of shocks the economy experienced during this period (for details, see Appendix D.6). Second, I compute the counterfactual recovery predicted by the model starting from the same capital stock and sequence of shocks but beginning from the steady-state level of capital unemployment rather than its observed level in 2010.II. As a benchmark, I also compute the recovery predicted by a neoclassical model with a similar parameterization, detailed in Appendix F.1, starting from the same observed capital stock and the same estimated sequence of productivity shocks.

Figure 6: Capital Unemployment and Recovery from the U.S. Great Recession



Note: Panel (a) reports data on quarterly U.S. nonresidential capital stock, expressed as percentages relative to a log-quadratic trend. Data source: Fernald (2014). Panel (b) reports data on the capital-unemployment rate, detailed in Section 2, expressed as percentages relative to its mean. The dashed green lines labeled *Capital-unemployment model* show model predictions using the policy functions of the baseline parameterized capital-unemployment model described in Sections 3 and 4, starting from the observed level of the capital stock and capital unemployment in 2010.II and the extracted sequence of TFP shocks depicted in Appendix Figure D.4. The dotted black line labeled *—, with initial* $u^{\mathrm{k}}_{\mathrm{ss}}$, refers to the same exercise, but starting from the steady-state capital unemployment rate. The marked red line refers to the same exercise but in the neoclassical model, detailed in Appendix F.1, starting from the observed level of the capital stock and using the extracted sequence of TFP shocks depicted in Panel (a) of Appendix Figure D.4. For details on these exercises, see the main text and Appendix D.6.

The results of this exercise are shown in Figure 6. Panel (a) shows the recovery predicted by the neoclassical model, depicted by the marked red line. The model predicts a declining path for the capital stock after the recession trough, driven by the slowdown in TFP the U.S. economy experienced during the recovery phase of the Great Recession, which reduces investment incentives (see Panel (a) of Appendix Figure D.4). However, by the end of the period considered, the neoclassical model predicts a deviation of the capital stock from trend of about a third of that observed in the data, consistent with the view that the post–Great Recession investment slump is puzzling from a neoclassical perspective. The dashed green line in Panel (a) shows that the capital-unemployment model with only productivity shocks accounts for a larger share of this slump, predicting a path for the capital stock closer to the data. The black dotted line shows that if the economy had started from the steady-state unemployment rate of capital, the capital-unemployment model would predict a path close to that of the neoclassical model. Through the lens of the model, almost half of the gap between the data and the neoclassical model in the deviation of the capital stock from trend following the recovery from the U.S. Great Recession can therefore be explained by the dampening effect of the large amount of unemployed capital the economy accumulated during the recession.[28] Panel (b) of Figure 6 shows that the capital-unemployment model with only productivity shocks also accounts for almost half of the persistence in the unemployment rate of capital.

Supportive evidence Using regional U.S. data, I provide descriptive evidence supporting the relationship between capital unemployment and investment predicted by the model's transitional dynamics. I show that regions with a higher level of capital unemployment at the Great Recession trough experienced lower investment rates throughout the recovery. For this exercise, I use the regional data described in Section 2, which provide measures of capital-unemployment and investment rates for structures across MSAs. I construct two variables. First, for each MSA, I measure the level of capital unemployment in 2010.II, which corresponds to the peak level of capital unemployment at the national level (henceforth referred to as the trough of the Great Recession in this exercise), relative to its historical mean. Second, for each MSA, I measure the average investment rate for the subsequent period (2010–2014) relative to its historical mean. Section 2 and Appendix A.2 provide details on how these variables are constructed.[29]

[28] Appendix Figure D.5 shows that, if one additionally extracts financial shocks from the observed path of capital unemployment, the capital-unemployment model predicts an investment slump closer to that observed during the recovery from the U.S. Great Recession. For more details, see Appendix D.6.

[29] In this analysis, I focus on office space, for which I can measure the historical mean using data for all regions since the 1990s. For retail and industrial space, data for all MSAs start in the 2000s. As an alternative, I use the 2007

Appendix Figure B.10 depicts the relationship between capital unemployment and investment recovery from the Great Recession, which indicates that MSAs with higher capital unemployment at the trough tended to experience lower investment rates during the subsequent recovery, with a correlation of -0.52. Appendix Table D.2 shows that this negative relationship is statistically significant and holds after controlling for other cyclical variables, namely, the deviation of the capital stock from trend and labor unemployment at the crisis trough. To obtain a model counterpart of this relationship, in Appendix D.6.2 I extend the model to include different regions and simulate the predicted path for investment rates in each region starting from its observed capital-unemployment rate. Appendix Table D.2 shows that the slope of the relationship between investment rates and capital unemployment obtained from the model-simulated data lies within the confidence intervals estimated in the data.

5.2 Capital unemployment as a propagation mechanism for financial shocks

A second dimension in which trading frictions in capital markets matter for aggregate dynamics is that fluctuations in capital unemployment constitute a sizable channel through which financial shocks propagate to economic activity. I show this by comparing the aggregate responses to financial shocks in the capital-unemployment model to those of a similar model with Walrasian capital markets (see Appendix F.2 for details on this model and its parameterization). The dashed green lines in Figure 7 show the aggregate response to a one-standard-deviation negative financial shock in the model with Walrasian capital markets. Panel (a) shows that financial shocks have only a modest effect on output in this model, with a peak effect of 0.06%, which is one order of magnitude smaller than observed output volatility. The reason is that although aggregate investment exhibits a large response to the shock (with a peak effect of 4%), the capital stock, which is the input in the production function in this model, displays only a modest response (see Panels (b) and (c)).

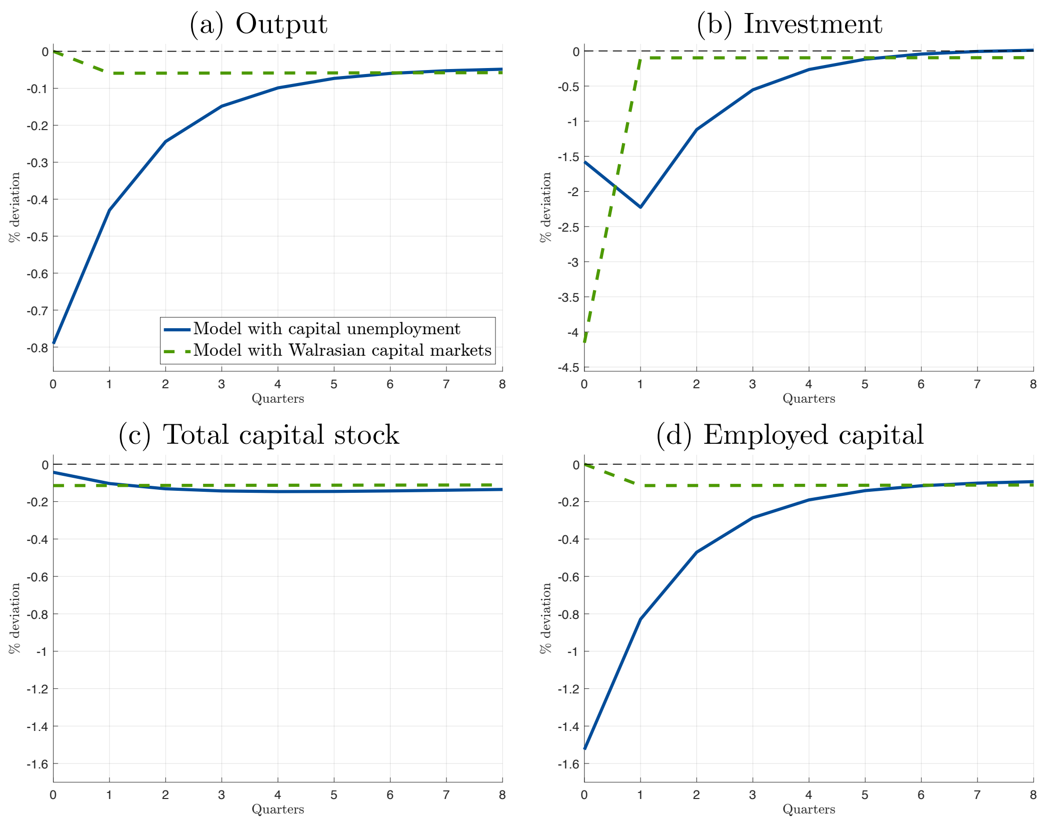
The solid blue line in Panel (a) of Figure 7 shows that the model's output response with trading frictions in capital markets is one order of magnitude larger than in the model with Walrasian capital markets, with a peak effect of 0.8%. Although the peak responses of aggregate investment and the capital stock to financial shocks are of similar order of magnitude in both models, the capital-unemployment model features a sizable response of employed capital relative to the total capital stock, a response that is absent in the model without trading frictions, where employed capital coincides with the total capital stock. This occurs because, as discussed in Section 4 and

pre–Great Recession level instead of the historical mean and obtain similar results for these types of properties.

illustrated in Figure 4, negative financial shocks reduce intermediaries' stochastic discount factors, contracting the supply of funds for firms and increasing their cost of capital, which lowers capital demand and search effort and reduces the selling probability of unemployed capital. Therefore, trading frictions in the capital market can generate large responses of economic activity to financial shocks, transmitted through fluctuations in the capital-unemployment rate.

It is worth highlighting that, as shown in Appendix Figure E.7, trading frictions in the capital market do not lead to substantially larger output responses to TFP shocks. This is because, as discussed in Section 4, TFP shocks have only small effects on capital unemployment because of their small impact on the shadow price of employed capital and trading probabilities.

Figure 7: Financial Shocks and Trading Frictions in Capital Markets



Note: Impulse responses of output (Y), investment (I), total capital stock (K), and employed capital (K^{e}) to a one-standard-deviation contractionary financial shock. Responses labeled *Model with capital unemployment* are those under the baseline model presented in Section 3 with the parameterization of Table 2. Responses labeled *Model with Walrasian capital markets* correspond to those in the version of the model with no trading frictions in capital markets, detailed in Appendix F.2. In this model, all capital is employed, so Panel (d) reports K_t, the input in the production function. The horizontal axis displays quarters after the shock. Impulse responses are expressed in percentage deviations from the steady state.

5.3 Capital unemployment and variable capital utilization

I now extend the capital-unemployment model to include an intensive margin of variable utilization for employed capital, a common ingredient in business-cycle models. To introduce this margin, I assume that firms' technology for producing final goods is $y_t(s^t) = A_t(s^t)f(k^{\mathrm{e}}_{t+1}(s^t)\epsilon_t(s^t), \gamma^t \ell_t(s^t))$, where $\epsilon_t(s^t)$ denotes the utilization rate of employed capital in state s^t, normalized to one in steady state. Following Christiano, Eichenbaum, and Evans (2005), higher utilization entails a resource cost $a(\epsilon_t)$ per unit of employed capital, with $a(1) = 0$, a'(1)¿0, and $\xi_\epsilon \equiv a''(1)/a'(1)$ governing the curvature of the utilization cost function. To carry out the exercises in the previous sections in this extended model, I incorporate this form of utilization in both the capital-unemployment model and the benchmark models without trading frictions, setting $\xi_\epsilon = 5.4$ as estimated in Justiniano, Primiceri, and Tambalotti (2011).

The extended model features two forms of utilization: that resulting from capital unemployment and that resulting from the variable utilization of employed capital. To make their link explicit, one can use firms' production function and express equilibrium aggregate output as

$$Y_t(s^t) = A_t(s^t)f(K_{t+1}(s^t)\mathcal{E}_t(s^t), \gamma^t \ell_t(s^t)), \tag{20}$$

where $\mathcal{E}_t(s^t) = (1 - u^{\mathrm{k}}_{t+1}(s^t))\epsilon_t(s^t)$ is total aggregate utilization. Equation (20) indicates that, all else equal, variations in capital unemployment and in the variable utilization of employed capital lead to identical variations in gross output. However, I highlight two dimensions in which distinguishing between these two forms of utilization matters for business cycles.

First, economies with different compositions of aggregate utilization $\mathcal{E}_t(s^t)$, between unemployed capital and the variable utilization of employed capital, will typically exhibit different subsequent investment dynamics. On the one hand, unemployed capital is a state variable that, as shown in Section 5.1, affects the transitional dynamics of the capital stock. Appendix Figure E.8 shows that these predictions are similar in the extended model with capital unemployment and variable utilization of employed capital. On the other hand, the variable utilization of employed capital is a control variable; as illustrated in Panel (b) of Appendix Figure E.6, simply introducing this margin does not affect the capital stock's monotonic convergence in the neoclassical model (see, for example, Calvo, 1975).

Second, the two forms of utilization respond differently to different types of shocks. Appendix Figure E.9 shows this by decomposing the responses of total aggregate utilization $\mathcal{E}_t(s^t)$ to shocks.

Results indicate that the response of total utilization to TFP shocks is dominated by the response of variable utilization of employed capital, while its response to financial shocks is dominated by the response of capital unemployment. This occurs both because capital unemployment responds substantially more to financial shocks than to TFP shocks (as discussed in Section 4) and because variable utilization of employed capital responds more to TFP shocks than to financial shocks. In fact, contractionary financial shocks lead to increases in utilization because the decline in employed capital raises the marginal benefit of utilization and its optimal level.[30] The differential responses of capital unemployment and variable utilization of employed capital imply that, as shown in Appendix Figure E.10, capital unemployment constitutes a sizable channel through which financial shocks propagate to economic activity, a channel that is absent without trading frictions in capital markets, as discussed in Section 5.2 for the baseline model.

From these exercises, I conclude that capital unemployment is complementary to the traditional concept of variable capital utilization of employed capital. Consistent with the view that these capture different dimensions of utilization, Appendix B shows that in the last four recession episodes before the pandemic, capital unemployment displays patterns that differ significantly from those of traditional utilization measures (represented with data from Fernald, 2014). This suggests that business-cycle models can benefit from incorporating both forms of capital utilization.

6 Conclusions

In this paper, I document that unemployment, a widely studied phenomenon in the labor market, is also observed in the market for physical capital. Capital unemployment is large, volatile, and increases during bad times. I show that these fluctuations can largely be understood as the combination of trading frictions, which give rise to unemployed capital, and financial shocks, which generate substantive variations in trading probabilities. Fluctuations in capital unemployment constitute a powerful propagation mechanism for financial shocks, operating through time-varying shifts in the allocation of capital between producing agents and holders of idle units. These fluctuations also have implications for investment dynamics, leading to persistent investment slumps following down-

[30] I also consider an alternative formulation in which utilization affects the depreciation rate of employed capital, as in Greenwood, Hercowitz, and Huffman (1988), where $\hat{\delta}(\epsilon) = \frac{1}{\omega}\epsilon^{\omega}$, with $\omega > 1$ implying that higher utilization leads to faster depreciation. As in their work, I calibrate ω to match the average depreciation rate observed in the data, given the other parameter values. This version yields similar transitional dynamics and a larger increase in utilization following contractionary financial shocks, as the decline in intermediaries' stochastic discount factor makes firms more willing to depreciate capital more rapidly (see Schwartzman, 2012, for a detailed discussion of this mechanism and possible modifications).

turns. These implications are consistent with the patterns observed following financial crises, such as the U.S. Great Recession.

Although the paper's focus has been positive, the macroeconomic implications of trading frictions suggest interesting questions for normative analysis. For instance, combining the search frictions considered in this paper with asymmetric information could allow scope for policies related to asset purchases and subsidy programs (as shown in Guerrieri and Shimer, 2014). In addition, given the interaction between capital unemployment and investment, it would be interesting to study unemployment's relationship to monetary policy. These extensions are planned for future research.

Data Availability Statement

The data and code underlying this article are available on Zenodo, at https://doi.org/10.5281/zenodo.20098361.

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