

# Liquidity Traps, Prudential Policies and International Spillovers\*

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May 2026

## Abstract

We investigate optimal monetary and macroprudential policies in an open economy with aggregate demand externalities and an occasionally binding zero lower bound constraint. Our analysis highlights that the optimal policy balances output stabilization and capital flow management. When macroprudential policy is available, monetary policy stabilizes the output gap. By contrast, when macroprudential policy is not available, monetary policy is used prudentially. However, contrary to a widespread view, raising the interest rate is not necessarily the optimal prudential policy. Finally, we show that international spillovers operate through the world real rate, but macroprudential policies provide insulation from the adverse effects of foreign policies.

**Keywords:** Capital flows, monetary and macroprudential policies, liquidity traps, international spillovers

**JEL Classifications:** E21, E23, E43, E44, E52, E62, F32

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\*For useful comments and suggestions, we thank the editor and two anonymous referees, as well as Mick Devereux, Luca Fornaro, Rishabh Kirpalani, Nir Jaimovich, Karlye Stedman, and seminar participants at Berkeley, Chicago Fed, Cleveland Fed, the Federal Reserve Board, Iowa State, the IMF, Minneapolis Fed, Michigan, UC Santa Cruz, University of Virginia, Wisconsin, the Bank of Canada Workshop on Monetary Policy, and the NBER IFM and ME Summer Institute Meetings. The views expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

# 1 Introduction

The macroeconomic landscape has undergone a profound transformation in the past 15 years, marked by the recurring presence of the zero lower bound on interest rates. In response, central banks around the world have turned to alternative policies to reduce their vulnerability. Notably, the adoption of capital flow management policies, aimed at stabilizing capital flows, has emerged as a new pillar of the traditional macroeconomic toolkit. However, our understanding of how these policies should be integrated with other macro policies, particularly monetary policy, remains limited.

This paper aims to address this gap by providing a theoretical and quantitative framework to answer key questions: How should central banks optimally use monetary and capital flow management policies? How does the availability of capital flow management alter the normal conduct of monetary policy? Is there a prudential role for monetary policy? Should capital flow management primarily be used *ex-ante* or also *ex-post*?

We consider a dynamic open economy model with incomplete markets and nominal rigidities where the central bank faces an occasionally binding zero lower bound constraint on nominal interest rates. When an adverse shock turns the natural rate negative, the central bank becomes constrained by the zero lower bound, and the economy undergoes a recession. A central feature of the model is the presence of a non-tradable sector, which implies that changes in domestic income and capital flows alter the demand for these goods. In this environment, we examine policy constellations where the central bank uses monetary policy or macroprudential policy in the form of capital controls, or a combination of both.

Consider first the case when the central bank can use both monetary policy and capital controls. We show that in this case, it is optimal to use monetary policy to stabilize output at the efficient level when the zero lower bound (ZLB) is not binding, and to use capital flow taxes to manage capital flows. The central bank taxes capital inflows when there is a positive probability of a liquidity trap in the next period and potentially subsidizes capital inflows when the economy is already in a liquidity trap. The scope for taxes on capital inflows originates from an aggregate demand externality (Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016). When the central bank cannot reduce rates below the ZLB, a higher stock of external debt lowers aggregate demand for consumption and reduces employment in the economy. Because households do not internalize the adverse effect of their individual demand on overall employment, they borrow too much in good times, when the ZLB is not binding, and this exacerbates the liquidity trap *ex post*. When the

central bank is already at the ZLB, the externality may potentially reverse sign. To the extent that the severity of the recession in a liquidity trap is more severe than the expected recession tomorrow, households borrow too little in a liquidity trap and this calls for a subsidy on capital inflows.

Monetary policy and capital controls pursue distinct objectives: monetary policy aims to stabilize output, while capital controls focus on stabilizing capital flows. Despite these separate goals, there is a significant interplay between the two. For instance, when the central bank imposes higher taxes on capital inflows, a more expansionary monetary policy becomes necessary to achieve optimal output levels.

Consider next the case where macroprudential policy is not available. We show that in this case, monetary policy plays a dual role. On the one hand, it manages aggregate demand to close the output gap, following the traditional macro-stabilization role: it loosens monetary policy and depreciates the exchange rate in a recession, and tightens monetary policy and appreciates the exchange rate in a boom. At the same time, monetary policy serves a *prudential role* when confronting the risk of a liquidity trap. In particular, monetary policy seeks to reduce capital inflows during good times to make the economy less vulnerable to a liquidity trap. These two policy objectives for monetary policy imply that monetary policy faces an intertemporal tradeoff: it needs to balance the current output gap and stabilizing capital flows (so as to close future output gaps).

Contrary to a widespread policy view, we show that raising the interest rate is not necessarily the optimal prudential intervention. Given aggregate income, an increase in the nominal interest rate leads to a decline in consumption and borrowing through an intertemporal substitution effect, as consistent with the conventional policy view. However, the reduction in demand reduces output in general equilibrium and leads to a higher need for borrowing to smooth consumption. We show that if the elasticity of substitution between tradables and non-tradables is higher than the intertemporal elasticity, a rise in the interest rate is counterproductive because it inefficiently increases the level of borrowing.

Furthermore, we also show that while the macroprudential tax on debt is positive only if the zero lower bound is likely to bind in the following period, monetary policy is used prudentially as long as the zero lower bound is foreseen to bind in some distant future. This is because a binding zero lower bound at any period  $t$  necessarily generates deviations from the output gap at  $t - 1$ . Iterating backward implies the need for a prudential monetary intervention today. The key lesson is that because monetary policy is a blunter instrument, it must be used more preemptively than macroprudential policy.

We show that a calibrated version of the model can match reasonably well episodes of

liquidity traps in advanced economies as well as key untargeted unconditional moments. Our quantitative evaluation underscores that optimal macroprudential policy can substantially improve macroeconomic stabilization and alleviate the costs of liquidity traps. In the absence of macroprudential policy, employment falls about 3.3% during a liquidity trap, and the unconditional welfare cost of liquidity traps is 0.37% of permanent consumption. With macroprudential policy, the fall in employment is only 1% and the welfare cost is reduced to 0.04%. In terms of policies, we find that the ex-ante prudential tax on inflows is 0.1%, while the tax during liquidity traps is  $-0.02\%$  on average. We also find that while liquidity traps are less frequent and less severe with macroprudential policy, perhaps surprisingly, they tend to last longer.

Our final set of results is concerned with international spillovers. We show that foreign policies affect a small open economy's welfare only insofar as they influence the world real interest rate. Specifically, when foreign countries' policies lead to a reduction in the world real interest rate, the domestic economy's welfare falls when it is vulnerable to a liquidity trap in the future *and* macroprudential policy is not available. This is because the reduction in the real rate exacerbates the overborrowing problem emerging from the aggregate demand externality. We also argue that these spillovers can open the door to currency wars, necessitating monetary policy cooperation. However, our analysis shows that macroprudential policies can insulate the domestic economy from monetary policy spillovers and eliminate the need for coordination, in line with the arguments raised by [Blanchard \(2021\)](#).

**Related literature.** Our paper relates to several strands of the literature. First, our paper belongs to the literature on aggregate demand externalities that emerge from nominal rigidities and constraints on monetary policy, such as fixed exchange rates or the zero lower bound ([Schmitt-Grohé and Uribe, 2016](#); [Farhi and Werning, 2016](#); [Korinek and Simsek, 2016](#)). These papers focus on the optimal macroprudential policy, given an exogenous monetary policy, or deal with jointly optimal monetary and macroprudential policy. A distinct contribution of our paper is to characterize how the availability of macroprudential policy, or lack thereof, affects the optimal monetary policy and to trace international spillovers and their welfare implications. Specifically, we demonstrate how macroprudential policy can insulate an economy from international policy spillovers and avoid a currency war.<sup>1</sup>

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<sup>1</sup>[Farhi and Werning \(2020\)](#) is another recent paper that examines monetary and macroprudential policy interactions, but in the context of a two-period closed economy model with behavioral features. See also [Coulibaly \(2023\)](#) and [Basu et al. \(2020\)](#) for models of optimal policies featuring pecuniary externalities.

Our paper is also related to the vast literature on the zero lower bound. Eggertsson and Woodford (2003) highlight the importance of forward guidance by which a promise of monetary stimulus in the future can help the central bank stimulate aggregate demand during a liquidity trap. In our open economy model, we show how the central bank can reduce ex-ante the probability of a liquidity trap by taxing capital inflows and relax ex-post its severity by subsidizing capital inflows.<sup>2</sup>

In the open economy literature, a key theme is the extent to which liquidity traps are transmitted across countries. Some notable papers addressing this theme are Caballero, Farhi and Gourinchas (2021), Eggertsson, Mehrotra, Singh and Summers (2016), and Fornaro and Romei (2019).<sup>3</sup> In particular, Fornaro and Romei (2019) demonstrate that capital account policies may lead to a global paradox of thrift. When countries away from a liquidity trap implement macroprudential policies, they reduce the world real interest rate and tighten the zero lower bound constraint of those countries in a liquidity trap. Although these malign general equilibrium implications are present in our setup, our framework also features a countervailing force by which the reduction in the risk-free rate pushes up household borrowing and thereby stimulates demand.

Egorov and Mukhin (2023) and Fanelli (2023) also study the interaction between optimal monetary policy and optimal capital controls. Egorov and Mukhin (2023) consider a setup with dollar currency pricing and a general production and asset structure. They show that although monetary policy cannot achieve full insularity, capital controls are not desirable, because they fail to affect external aggregate demand. Fanelli (2023) considers an environment with home and foreign currency portfolios and incomplete markets. He shows how the optimal monetary policy under commitment balances a shock absorber role and an insurance role of exchange rates. Moreover, he finds that the central bank optimally taxes inflows, but it applies the same tax, irrespective of the currency, to a second order. Our analysis focuses on an environment with inflows in foreign currency and highlights the prudential role of taxes on inflows to reduce the vulnerability to a binding zero lower bound.

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Several other studies consider monetary and macroprudential interactions but do not characterize optimal policies (e.g., Aoki, Benigno and Kiyotaki, 2016; Van der Ghote, 2021; Rubio and Yao, 2020; Ferrero, Harrison and Nelson, 2024). Collard, Dellas, Diba and Loisel (2017) study joint optimal monetary and macroprudential policy in the context of a moral hazard externality.

<sup>2</sup>We solve for the optimal discretionary policy, but we show that the scope for these capital flow management policies is preserved under commitment.

<sup>3</sup>Other open economy studies include Cook and Devereux (2013); Devereux and Yetman (2014); Acharya and Bengui (2018); Jeanne (2009); Benigno and Romei (2014); Fornaro (2018); Corsetti, Mavroeidi, Thwaites and Wolf (2019a); Corsetti, Mueller and Kuester (2019b); Kollmann (2021); and Amador, Bianchi, Bocola and Perri (2020). Notable closed economy studies also include Krugman (1998), and Werning (2011).

**Outline.** Section 2 presents the model. Section 3 studies optimal monetary and macroprudential policy. Section 4 presents the quantitative analysis. Section 5 analyzes international spillovers. Section 6 concludes.

## 2 Model

We consider a two-sector dynamic small open economy with nominal rigidities and an occasionally binding zero lower bound constraint. In this section, we describe the competitive equilibrium in this economy, given central bank policies.

### 2.1 Households

The small open economy is populated by a continuum of identical households of measure one. Their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \prod_{k=0}^{t-1} \delta_k \right) U(c_t, h_t), \quad (1)$$

where  $U$  is a strictly concave function that is increasing in consumption,  $c$ , and decreasing in labor,  $h$ . The discount factor between  $t$  and  $t + 1$  is given by  $\beta\delta_t$ , where  $\delta_t$  is a shock assumed to follow a first-order Markov process.

The consumption good  $c_t$  is a linearly homogeneous aggregator of tradable and non-tradable consumption goods—respectively,  $c_t^T$  and  $c_t^N$ .<sup>4</sup>

$$c_t = c(c_t^T, c_t^N).$$

We will use  $u(c_t^T, c_t^N, h_t)$  to denote the utility as a function of the two consumption goods and labor, and define the intertemporal elasticity of substitution and the elasticity of substitution between tradables and non-tradables as

$$\sigma_t \equiv -\frac{U'(c_t)}{c_t U''(c_t)}, \quad \text{and} \quad \gamma_t \equiv \frac{d \log(c_t^N / c_t^T)}{d \log(u_T(c_t^T, c_t^N, h_t) / u_N(c_t^T, c_t^N, h_t))'}$$

where  $u_T$  and  $u_N$  represent the marginal utility of tradable and non-tradable consumption, respectively.

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<sup>4</sup>The linearly homogeneous function for consumption assumes implicitly that both tradable and non-tradable goods are normal. As we will see, this assumption is important for the mechanism.

**Budget constraint.** In each period  $t$ , households supply  $h_t$  units of labor and are endowed with  $y_t^T$  units of tradable goods. We assume that  $y_t^T$  is stochastic and follows a first-order Markov process. Households receive a wage rate,  $W_t$ , collect profits,  $\phi_t^N$ , and consume tradables and non-tradables at prices  $P_t^T$  and  $P_t^N$ . All these variables are expressed in terms of domestic currency, which serves as the numeraire.

Households can save/borrow by trading one-period, non-state-contingent bonds. We denote respectively by  $b_{t+1}^*$  and  $b_{t+1}$  the holdings of a bond that pays a gross return  $R_t^*$  in units of tradables and  $R_t$  in units of domestic currency. The government controls the nominal rate  $R_t$  and can tax both bonds at a rate  $\tau_t$ . When  $\tau_t > 0$ , households face a tax on borrowing and a subsidy on savings. Conversely, when  $\tau_t < 0$ , households face a subsidy on borrowing and a tax on savings. The tax revenues are rebated by the government using lump-sum transfers  $T_t$ . The budget constraint of the representative household is therefore given by

$$P_t^N c_t^N + P_t^T c_t^T + \frac{1}{1 + \tau_t} \left[ \frac{b_{t+1}}{R_t} + P_t^T \frac{b_{t+1}^*}{R_t^*} \right] = \phi_t^N + W_t h_t + P_t^T (y_t^T + T_t) + b_t + P_t^T b_t^*. \quad (2)$$

Households are also subject to a no-Ponzi-game constraint of the form

$$\lim_{j \rightarrow \infty} \mathbb{E}_t \left[ \frac{b_{t+j+1}}{P_{t+j}^T \prod_{s=1}^j R_s} + \frac{b_{t+j+1}^*}{\prod_{s=1}^j R_s^*} \right] \geq 0. \quad (3)$$

**Optimality conditions.** We use  $\{x_t\}$  to refer to the sequence  $\{x_t\}_{t=0}^{\infty}$  for some variable  $x$ . The households' problem consists of choosing  $\{c_t^N, c_t^T, h_t, b_{t+1}, b_{t+1}^*\}$  to maximize the expected present discounted value of utility (1), subject to (2) and (3).

The first-order conditions for consumption and labor yield

$$\frac{W_t}{P_t^N} = - \frac{u_h(c_t^T, c_t^N, h_t)}{u_N(c_t^T, c_t^N, h_t)}, \quad (4)$$

$$\frac{P_t^N}{P_t^T} = \frac{u_N(c_t^T, c_t^N, h_t)}{u_T(c_t^T, c_t^N, h_t)}. \quad (5)$$

Condition (4) is the labor supply condition, which equates the marginal rate of substitution between leisure and non-tradable consumption with the wage in units of non-tradable consumption. Condition (5) equates the marginal rate of substitution between tradables and non-tradables to the relative price.

The first-order conditions for the real and nominal bond holdings yield

$$u_T(c_t^T, c_t^N, h_t) = \beta \delta_t R_t^* (1 + \tau_t) \mathbb{E}_t \left[ u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1}) \right], \quad (6)$$

$$\frac{u_T(c_t^T, c_t^N, h_t)}{P_t^T} = \beta \delta_t R_t (1 + \tau_t) \mathbb{E}_t \left[ \frac{u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1})}{P_{t+1}^T} \right]. \quad (7)$$

The two intertemporal Euler equations equate the marginal utility benefit from saving in nominal or real bonds to the marginal utility costs of cutting tradable consumption today to purchase the bonds.

## 2.2 Firms

The non-tradable good is produced by a continuum of firms in a perfectly competitive market. Each firm produces a non-tradable good according to a strictly increasing and concave production technology given by  $y_t^N = A_t F(n_t)$ , where  $n_t$  denotes the labor employed. Firms' profits are given by

$$\phi_t^N = P_t^N A_t F(n_t) - W_t n_t, \quad (8)$$

We assume that the price of non-tradables is perfectly rigid,  $P_t^N = \bar{P}^N$ , and that firms produce goods to satisfy households' demand. This implies that labor demand in equilibrium is given by  $n_t = F^{-1}\left(\frac{c_t^N}{A_t}\right)$ . We extend the model to allow for partial price adjustments in our quantitative analysis.

## 2.3 Government

The government sets a nominal interest rate  $R_t \geq 1$  and a tax on all forms of bond issuances  $\tau_t$ . As is common in the literature, this tax can be interpreted as a capital control or as a macroprudential policy (see e.g., [Bianchi, 2011](#); [Schmitt-Grohé and Uribe, 2016](#); and [Fornaro and Romei, 2019](#)). The tax is assumed to be rebated lump-sum to households, an assumption that is without loss of generality given that Ricardian equivalence holds.<sup>5</sup> That is, the government's budget constraint is

$$T_t = -\frac{\tau_t}{1 + \tau_t} \left[ \frac{b_{t+1}}{P_t^T R_t} + \frac{b_{t+1}^*}{R_t^*} \right]. \quad (9)$$

<sup>5</sup>We abstract from other so-called unconventional fiscal policies that can relax the zero lower bound (see e.g. [Correia, Farhi, Nicolini and Teles, 2013](#)). We also abstract from differential taxes on domestic and foreign currency bonds (see e.g. [Acharya and Bengui, 2018](#)). As long as there are some limitations on the use of these policies (either political or economic), the first-best cannot be implemented, and our key results would remain.

## 2.4 Prices, Interest Parity, and Exchange Rates

We assume that the law of one price holds for the tradable good, that is,  $P_t^T = e_t P_t^{T*}$ , where  $e_t$  is the nominal exchange rate defined as the price of the foreign currency in terms of the domestic currency, and  $P_t^{T*}$  is the price of the tradable good denominated in foreign currency. Using the Euler equations for the real bond (6) and the domestic currency bond (7), and the law of one price, we obtain the risk-adjusted uncovered interest parity condition:

$$R_t^* \mathbb{E}_t u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1}) = R_t \mathbb{E}_t \left[ u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1}) \frac{P_t^{T*}}{P_{t+1}^{T*}} \frac{e_t}{e_{t+1}} \right]. \quad (10)$$

Condition (10) equates the marginal utility benefit of investing in a real bond and the domestic currency bond.<sup>6</sup>

## 2.5 Competitive Equilibrium

Market clearing for labor requires that the units of labor supplied by households equal the aggregate labor demand by firms:

$$h_t = n_t. \quad (11)$$

Market clearing for the non-tradable good requires that output be equal to non-tradable consumption:

$$y_t^N = c_t^N. \quad (12)$$

We assume that the bond denominated in domestic currency is traded only domestically. We make this assumption to abstract from portfolio problems and from the possibility of inflating away external debt.<sup>7</sup> Market clearing therefore implies

$$b_{t+1} = 0. \quad (13)$$

Combining the budget constraints of households, firms, and the central bank, as well as market clearing conditions, we arrive at the resource constraint for tradables, or the

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<sup>6</sup>With a constant  $P_t^{T*}$ , a first-order approximation of (10) yields the familiar UIP condition equating the differences in returns between a domestic currency bond and a foreign currency bond to the expected depreciation rate.

<sup>7</sup>See [Fanelli \(2023\)](#) for an interesting study of optimal monetary policy with nominal external debt and incomplete markets. In his model, the government can commit to future policies and use monetary policy to improve risk-sharing in addition to the standard objectives.

balance-of-payment condition:

$$c_t^T - y_t^T = b_t^* - \frac{b_{t+1}^*}{R_t^*}, \quad (14)$$

which says that the trade balance must be financed with net bond issuances.

An equilibrium, given government policies, is defined as follows.

**Definition 1.** Given an initial condition  $b_0^*$ , exogenous process  $\{y_t^T, \delta_t, R_t^*, A_t, P_t^{T*}\}$ , a rigid price  $\bar{P}^N$ , and government policies  $\{R_t, \tau_t\}$ , a competitive equilibrium is a stochastic sequence of prices  $\{e_t, P_t^T, P_t^N, W_t\}$  and allocations  $\{c_t^T, c_t^N, b_{t+1}^*, n_t, h_t\}$  such that

- (i) households optimize, hence (2)-(7) hold;
- (ii) firms choose hours to meet demand; that is,  $A_t F(h_t) = c_t^N$ ;
- (iii) labor market clears (11) and the domestic currency bond is in zero net supply (13);
- (iv) the government budget constraint (9) is satisfied;
- (v) the law of one price holds,  $P_t^T = e_t P_t^{T*}$ , and  $P_t^N = \bar{P}^N$ ;
- (vi) the resource constraint for tradables (14) holds.

## 2.6 First-Best Allocation

We conclude the description of the model by presenting the first-best allocation. We consider a benevolent social planner of the small open economy who chooses allocations subject to a resource constraint. The planner's problem can be written as

$$\begin{aligned} \max_{\{b_{t+1}^*, c_t^N, c_t^T\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \prod_{k=0}^{t-1} \delta_k \right) u \left( c_t^T, c_t^N, F^{-1} \left( \frac{c_t^N}{A_t} \right) \right), \quad (15) \\ \text{subject to} \\ c_t^T = y_t^T + b_t^* - \frac{b_{t+1}^*}{R_t^*}. \end{aligned}$$

The first-best allocation equates the value of one additional employed unit of labor to the marginal cost of leisure:

$$A_t F'(h_t) u_N(c_t^T, c_t^N, h_t) = -u_h(c_t^T, c_t^N, h_t), \quad (16)$$

and the marginal utility of current consumption to the marginal utility benefit of saving:

$$u_T(c_t^T, c_t^N, h_t) = \beta \delta_t R_t^* \mathbb{E}_t \left[ u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1}) \right]. \quad (17)$$

It should be clear that the allocations in a competitive equilibrium with flexible prices would coincide with the first-best. This can be seen by noting that if firms could adjust prices, we would have  $A_t F'(h_t) = W_t / P_t^N$ , which, combined with households' labor supply decision (4), would yield (16).<sup>8</sup> Moreover, as we will see, with sticky prices, a government that can choose monetary policy without any constraints would choose to replicate the flexible price allocation and hence implement the first-best allocation. We note that New Keynesian open economy models often feature monopolistic competition and terms of trade externalities, which create an additional wedge between competitive equilibrium with nominal rigidities and flexible price equilibria. Our framework allows us to focus squarely on aggregate demand management considerations.

The departure of the competitive equilibrium allocations from the first-best can be conveniently summarized in the labor wedge, defined below:

$$\psi_t \equiv \left[ 1 + \frac{u_h(c_t^T, c_t^N, h_t)}{A_t F'(h_t) u_N(c_t^T, c_t^N, h_t)} \right] u_T(c_t^T, c_t^N, h_t). \quad (18)$$

By (16),  $\psi_t = 0$  when production is efficient. A positive labor wedge,  $\psi_t > 0$ , reflects a recession, whereas a negative labor wedge,  $\psi_t < 0$ , reflects overheating.

### 3 Optimal Monetary and Macroprudential Policies

In this section, we investigate optimal monetary and macroprudential policies. To shed light on the policy interactions, we proceed to first study optimal macroprudential policy given monetary policy, then we study joint optimal monetary and macroprudential policy, and finally we study optimal monetary policy in the absence of macroprudential policy.

#### 3.1 Macroprudential Policy

We consider a generic monetary policy that depends on the history of all shocks. An advantage of this formulation is that we are able to provide a general characterization of macroprudential policy encompassing multiple monetary policy regimes. This will set the stage for analyzing the interactions between optimal monetary and macroprudential policies.

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<sup>8</sup>In addition, notice that (17) coincides with households' optimality (6) when  $\tau_t = 0$ .

Under an arbitrary monetary policy, the production of non-tradable goods is, in general, inefficient. For example, given the sticky price  $\bar{P}^N$ , a low exchange rate implies a high relative price for non-tradables, and hence a low demand for non-tradable goods, which leads firms to reduce production and opens a positive labor wedge.

Given an exogenous sequence of  $\{e_t\}$ , the government chooses the state-contingent tax on debt  $\{\tau_t\}$  that maximizes private agents' welfare among the set of competitive equilibria. The problem can be written as

$$\begin{aligned} \max_{\{b_{t+1}^*, c_t^N, c_t^T\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \prod_{k=0}^{t-1} \delta_k \right) u \left( c_t^T, c_t^N, F^{-1} \left( \frac{c_t^N}{A_t} \right) \right), \quad (19) \\ \text{subject to} \\ c_t^T = y_t^T + b_t^* - \frac{b_{t+1}^*}{R_t^*}, \\ \frac{\bar{P}^N}{e_t P_t^{T*}} = \frac{u_N(c_t^T, c_t^N, F^{-1}(\frac{c_t^N}{A_t}))}{u_T(c_t^T, c_t^N, F^{-1}(\frac{c_t^N}{A_t}))}. \end{aligned}$$

The last constraint in problem (19) reflects that allocations must be such that households are optimally choosing their expenditures between tradables and non-tradables, given the relative price. Totally differentiating this equation, we arrive at

$$d \log(c_t^N) = d \log(c_t^T) + \gamma_t d \log(P_t^T) \quad (20)$$

Equation (20) highlights how given a relative price of non-tradables, higher tradable consumption must be associated with higher non-tradable consumption to the extent that both tradables and non-tradables are normal goods. Moreover, the increase in the demand for non-tradables implies that firms must respond by raising employment. This general equilibrium effect is key for the characterization of the optimal macroprudential tax presented below.

**Proposition 1** (Optimal macroprudential policy). *Consider an exogenous exchange rate policy  $\{e_t\}$ . The optimal tax on borrowing (19) satisfies*

$$\tau_t = \frac{1}{\beta \delta_t R_t^* \mathbb{E}_t u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1})} \left\{ -\frac{1-\tilde{\omega}_t}{\tilde{\omega}_t} \psi_t + \beta \delta_t R_t^* \mathbb{E}_t \left[ \frac{1-\tilde{\omega}_{t+1}}{\tilde{\omega}_{t+1}} \psi_{t+1} \right] \right\}, \quad (21)$$

where  $\psi_t$  is the labor wedge, defined in (18), and  $\tilde{\omega}_t \equiv P_t^T c_t^T / (P_t^T c_t^T + \bar{P}^N c_t^N)$ .

*Proof.* In Appendix A.1. □

Equation (21) provides an analytical characterization of the optimal tax that emerges to correct the aggregate demand externality at work in the model. When households make savings decisions, they do not internalize that by redirecting consumption over time, this affects employment allocations in general equilibrium. In particular, when households save more today, they push down demand and employment today and increase them in the future. Therefore, changes in aggregate household decisions can move employment closer or further away from the efficient level.

These results are related to the aggregate demand externality in [Schmitt-Grohé and Uribe \(2016\)](#) and [Farhi and Werning \(2012\)](#). Our analytical characterization highlights that the sign of  $\tau_t$  is in principle ambiguous and depends in particular on the relative importance of the labor wedge in periods  $t$  and  $t + 1$ . If the economy is currently at full employment and the expected (risk-adjusted) labor wedge is positive, a positive tax on debt is optimal to induce households to internalize that an increase in one unit of savings today leads to an increase in aggregate demand tomorrow, stimulating employment and reducing the labor wedge. When the labor wedge today and tomorrow are both positive, the central bank trades off the marginal benefits from stimulating future demand and easing the recession tomorrow with the marginal costs from reducing current demand and deepening the recession today. Finally, if today's labor wedge is negative, taxing borrowing and postponing consumption helps to reduce overheating.

Next, we turn to analyze the interaction between monetary policy and macroprudential policy, which is our main focus.

## 3.2 Joint Monetary and Macroprudential Policies

We now consider a central bank that jointly conducts macroprudential and monetary policy. The central bank chooses the tax on inflows,  $\tau$ , and the nominal rate,  $R$ , to maximize households' welfare. Importantly, the central bank is subject to a zero lower bound that restricts its ability to achieve the first-best allocations.

In contrast to the previous section, here, the optimal policy for the central bank is subject to a time inconsistency problem, common in environments with a zero lower bound (e.g., [Eggertsson and Woodford, 2003](#)). We examine the optimal policy without commitment, which we see as the one that is most relevant practically.<sup>9</sup> In particular, we study a Markov perfect equilibrium in which the policies of the central bank at each point in time depend on the relevant payoff states. We use  $s \equiv \{y^T, \delta, A, R^*, P^{T*}\}$  to denote

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<sup>9</sup>We discuss the problem under commitment in Section 3.4.

the current realizations of exogenous shocks, and  $\{\mathcal{E}(b^*, s), \mathcal{C}^T(b^*, s), \mathcal{C}^N(b^*, s), \mathcal{H}(b^*, s)\}$  to denote the stationary policy functions for the nominal exchange rate, tradable and non-tradable consumption, and employment followed by future central banks.

Using  $V(b^*, s)$  to denote the value function for the central bank, we can write the recursive problem as follows:

$$V(b^*, s) = \max_{R, e, b', c^N, c^T, \tau} u\left(c^T, c^N, F^{-1}\left(\frac{c^N}{A}\right)\right) + \beta\delta \mathbb{E}_{s'|s} V(b^*, s'), \quad (22)$$

subject to

$$c^T = y^T + b^* - \frac{b^*'}{R^*}, \quad (22a)$$

$$\frac{\bar{P}^N}{eP^{T*}} = \frac{u_N(c^T, c^N, F^{-1}(\frac{c^N}{A}))}{u_T(c^T, c^N, F^{-1}(\frac{c^N}{A}))}, \quad (22b)$$

$$R^* = R \mathbb{E}_{s'|s} \left[ \frac{\mathcal{U}_T(b^*, s')}{\mathbb{E}_{s'|s} \mathcal{U}_T(b^*, s')} \frac{P^{T*}}{P^{T*'}} \frac{e}{\mathcal{E}(b^*, s')} \right], \quad (22c)$$

$$R \geq 1, \quad (22d)$$

$$u_T\left(c^T, c^N, F^{-1}\left(\frac{c^N}{A}\right)\right) = \beta\delta R^* (1 + \tau) \mathbb{E}_{s'|s} \mathcal{U}_T(b^*, s') \quad (22e)$$

where  $\mathcal{U}_T(b^*, s') \equiv u_T(\mathcal{C}^T(b^*, s'), \mathcal{C}^N(b^*, s'), \mathcal{H}(b^*, s'))$ . The key difference compared with problem (19) is that now the exchange rate is a choice for the central bank. Moreover, as mentioned above, the optimal policy at any given state depends on the expectations of future policies because these affect the extent to which the zero lower bound binds. Notice that by setting  $\tau$  optimally, the central bank can control borrowing decisions, and therefore *the last constraint in (22) is not a binding implementability constraint*.

In a Markov perfect equilibrium, as defined below, the conjectured policies for future central banks have to be consistent with the actual policies chosen.

**Definition 2.** A Markov perfect equilibrium is defined by policies  $\{\mathcal{E}, \mathcal{C}^T, \mathcal{C}^N, \mathcal{H}\}$  and a value function  $V$  such that (i) the value function  $V$  solves (22), given future policies  $\{\mathcal{E}, \mathcal{C}^T, \mathcal{C}^N, \mathcal{H}\}$ , and (ii)  $\{\mathcal{E}, \mathcal{C}^T, \mathcal{C}^N, \mathcal{H}\}$  are the optimal policy functions in problem (22).

The following proposition characterizes the optimal policy of the central bank.<sup>10</sup>

**Proposition 2** (Optimal monetary and macroprudential policy). *Consider the optimal monetary and macroprudential policy. We have that the labor wedge satisfies  $\psi_t \geq 0$  for all  $t$  and  $\psi_t = 0$*

<sup>10</sup>To avoid clutter, variables indexed by  $t$  in the proposition represent functions of the state  $(b_t^*, s_t)$ .

if the zero lower bound (ZLB) does not bind at date  $t$ . Formally, we have

$$\psi_t = \frac{\tilde{\omega}_t}{1 - \tilde{\omega}_t} \frac{\tilde{\zeta}_t}{\gamma_t c_t^T}, \quad (23)$$

where  $\tilde{\zeta}_t$  is the non-negative Lagrange multiplier on the ZLB constraint in problem (22).

In addition, the optimal tax on debt is given by

$$\tau_t = \frac{1}{\beta \delta_t R_t^* \mathbb{E}_t u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1})} \left\{ -(1 + \Theta_t) \frac{\tilde{\zeta}_t}{\gamma_t c_t^T} + \beta \delta_t R_t^* \mathbb{E}_t \left[ \frac{\tilde{\zeta}_{t+1}}{\gamma_{t+1} c_{t+1}^T} \right] \right\}, \quad (24)$$

where  $\Theta_t \equiv \gamma_t P_t^T c_t^T \mathbb{E}_t \frac{\partial}{\partial b_{t+1}^*} \left[ \frac{u_T(t+1)}{P^{T(t+1)} \mathbb{E}_t u_T(t+1)} \right]$ .

*Proof.* In Appendix A.2. □

There are several important lessons in Proposition 2. First, the central bank implements an allocation with a zero labor wedge whenever the zero lower bound constraint is not binding. To see this, notice that if the ZLB constraint is slack, we can eliminate all constraints from Problem (22) except for the resource constraint (22a). Thus, the first-order condition with respect to  $c^N$  yields (16), which implies a zero labor wedge. Given the optimal allocations, we can back out  $e$  and  $R$  using (10) and (22b).

A second lesson is that the economy never experiences overheating (i.e., a negative labor wedge). Intuitively, the zero lower bound imposes a constraint on the ability to depreciate the exchange rate, but the central bank can always appreciate the exchange rate and reduce the demand for non-tradables by raising the nominal interest rate. On the other hand, if the zero lower bound binds, the central bank is unable to depreciate the exchange rate by lowering the nominal interest rate and faces a positive labor wedge.

Regarding macroprudential policy, equation (24) shows that the tax on debt crucially depends on the current and next-period (non-negative) Lagrange multipliers on the zero lower bound constraints. This implies that when the ZLB is not binding, i.e.,  $\tilde{\zeta}_t = 0$ , the tax on debt is always weakly positive, and strictly positive if there is a positive probability that the ZLB will bind in the future. Conversely, when the ZLB is currently binding but is not expected to bind in the next period with positive probability, the tax is strictly negative.

To shed further light on these results, we use the relationship between the labor wedge and  $\tilde{\zeta}$  in (23) and replace it in (24) to obtain

$$\tau_t = \frac{1}{\beta \delta_t R_t^* \mathbb{E}_t u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1})} \left\{ -(1 + \Theta_t) \frac{1 - \tilde{\omega}_t}{\tilde{\omega}_t} \psi_t + \beta \delta_t R_t^* \mathbb{E}_t \left[ \frac{1 - \tilde{\omega}_{t+1}}{\tilde{\omega}_{t+1}} \psi_{t+1} \right] \right\},$$

which is an equation analogous to the one characterizing the optimal tax under an arbitrary exchange rate policy (21). That is, it is possible to write the tax as a function of how savings affect the next-period labor wedge or as a function of how savings affect the tightness of the zero lower bound. The two expressions are linked by the central bank optimization and are, in fact, equivalent. Notice, however, that one difference between the two tax expressions (21) and (24) is that the latter carries an additional term,  $\Theta$ , related to the restriction that the policy must be consistent with a Markov equilibrium where governments choose the optimal policy sequentially. The additional term captures that an increase in savings alters both next-period consumption and the exchange rate followed by the next central bank.

Expression (24) in Proposition 2 links the use of macroprudential policy to the incidence of a binding zero lower bound (ZLB) and the emergence of a labor wedge. But under what conditions does this actually arise in equilibrium? The key mechanism is the presence of shocks that cause fluctuations in the natural rate of interest, potentially driving it below one. In particular, a liquidity trap arises in the model whenever the interest rate consistent with the flexible price allocation falls below one. Formally, the gross natural interest rate (in units of non-tradables) is given by:

$$\bar{R}_t = \frac{1}{\beta\delta_t} \frac{u_N(\bar{c}_t^T, \bar{c}_t^N, \bar{h}_t)}{\mathbb{E}_t u_N(\bar{c}_{t+1}^T, \bar{c}_{t+1}^N, \bar{h}_{t+1})}$$

where variables with a bar denote allocations under flexible prices. Shocks to the discount factor or other preference parameters, shocks to the natural level of output (such as TFP or labor supply shocks), and shocks affecting the demand for tradables (such as changes in  $R^*$  or  $y^T$ ) can all lower the gross natural rate below one, thereby triggering the ZLB constraint. See Appendix C.1 for further discussion.<sup>11</sup>

In Figure 1, we illustrate numerically the tax on debt.<sup>12</sup> The figure shows that the tax on debt is non-monotonic in the current level of bond holdings (the figure also shows the current labor wedge on a different axis). There are three distinct regions. For low bond holdings, the economy is in a liquidity trap region in which  $R = 1$  and  $\psi > 0$ . In this region, the tax is increasing in bond holdings. It is initially negative, as the current labor wedge exceeds the expected future ones and eventually becomes positive once bonds increase sufficiently, at which point the planner finds optimal to tax rather than subsidize

<sup>11</sup>A knife-edge case in which the ZLB never binds requires simultaneously: (i) no shocks to the discount factor; (ii) no shocks to the natural level of non-tradable output; and (iii) separable preferences between tradable and non-tradable consumption and labor.

<sup>12</sup>The figure sets  $\delta_t = 1$ ,  $R_t^* = R^*$ , and considers a negative one-standard-deviation shock to  $y_t^T$ . The calibration will be described below. The overall pattern, however, is general and does not hinge on specific parameters.

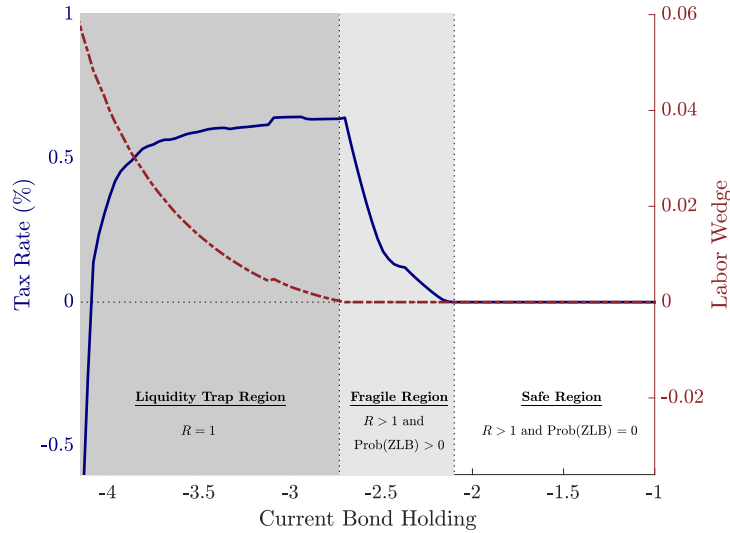


Figure 1: Optimal macroprudential policy

Note: Figure presents the tax schedule as a function of  $b_t^*$  for  $\delta_t = 1$ ,  $R_t^* = R^*$  and a negative one-standard-deviation shock to  $y_t^T$ .

inflows. For intermediate levels of bond holdings, the economy is in a fragile region in which  $R > 1$  but the ZLB constraint may become binding in the next period. In this region, the tax is strictly positive and decreasing in bond holdings. Intuitively, in this region, the planner wants to shift resources to the future when the economy may face a recession and a binding ZLB. Moreover, as current bond holdings increase, this leads to higher bond holdings tomorrow and thus a lower expected labor wedge tomorrow and a lower tax on debt today. For sufficiently high bond holdings, the economy is in a safe region in which the tax becomes zero because there is a zero probability of a binding ZLB in the next period.

### 3.3 Optimal Monetary Policy without Macroprudential Policy

In the previous section, we analyzed the joint use of monetary and macroprudential policy. We saw the optimal policy implies a zero labor wedge whenever the zero lower bound is not binding. We now study optimal monetary policy when the central bank *does not* have access to macroprudential policy. The key question that emerges is whether the central bank should use monetary policy prudentially as a substitute for macroprudential policy and, if so, what this implies for the choice of the interest rate. In particular, *does a prudential monetary policy call for higher or lower interest rates?*

As we did before, we consider the optimal problem under lack of commitment. Relative

to problem (22), the central bank now carries the Euler equation (22e) as a binding implementability constraint, given that  $\tau$  is restricted to be zero. This distinction will generate notable differences in the optimal policy. As characterized in the following proposition, in the absence of macroprudential policy, monetary policy should potentially be used as a prudential tool to stimulate savings and reduce the likelihood of future liquidity traps.

**Proposition 3** (Optimal monetary policy without macroprudential policy). *When the central bank does not have access to macroprudential policy, the optimal monetary policy satisfies*

$$\psi_t = \frac{\left(\frac{\sigma_t}{\bar{\gamma}_t} - 1\right)\tilde{\omega}_t}{(1 - \tilde{\omega}_t)\frac{\sigma_t}{\bar{\gamma}_t} + \tilde{\omega}_t} \mathbb{E}_t \sum_{k=1}^{\infty} \beta^k \left( \prod_{j=0}^{k-1} \frac{\delta_{t+j} R_{t+j}^*}{1 + \bar{\Theta}_{t+j}} \right) \frac{\zeta_{t+k}}{\gamma_{t+k} c_{t+k}^T}, \quad (25)$$

where  $\bar{\Theta}_t \equiv \beta \delta_t R_t^* \frac{1}{u_{TT}(t)} \mathbb{E}_t \frac{\partial \mathcal{U}_T(b_{t+1}^*, s_{t+1})}{\partial b_{t+1}^*}$ ,  $\bar{\gamma}_t^{-1} \equiv \gamma_t^{-1} + \frac{c_t U_{ch}(t)}{A_t F'(h_t) u_N(t)}$ , and  $\zeta_t$  is the non-negative Lagrange multiplier on the ZLB constraint.

*Proof.* In Appendix A.3. □

Proposition 3 shows that to the extent that the central bank faces a binding zero lower bound in the future (i.e.,  $\zeta_{t+k} > 0$  for any  $k > 0$ ), it will deviate from a zero labor wedge today. This contrasts with the joint optimal macroprudential policy, where the labor wedge is kept at zero if the zero lower bound constraint is not binding, as shown above. The reason why the central bank departs from a zero labor wedge away from the ZLB is that the central bank seeks to reduce borrowing in order to make the economy less vulnerable to a ZLB in the future. That is, without taxes on capital inflows, the central bank resorts to a prudential monetary policy intervention that distorts output today to lower borrowing and relax the ZLB in the future.

To better understand the nature of the prudential monetary intervention, consider first the case where preferences are separable between consumption and leisure,  $U_{ch} = 0$ . Equation (25) states that whether monetary policy leans with or against the wind depends on the relative values of the elasticities of substitution. When the intertemporal elasticity of substitution is higher than the intratemporal one,  $\sigma_t > \gamma_t$ , the central bank slows down the economy away from a liquidity trap (i.e.,  $\psi_t > 0$ ).<sup>13</sup> In contrast, when the intertemporal elasticity of substitution is lower than the intratemporal one,  $\sigma_t < \gamma_t$ , the central bank stimulates the economy away from a liquidity trap (i.e.,  $\psi_t < 0$ ). Finally, if  $\sigma_t = \gamma_t$ , the central bank targets a zero labor wedge.

<sup>13</sup>Notice that  $\Theta_t > 0$  under the assumption that consumption policy functions,  $\mathcal{C}^T(b_t^*, s_t)$  and  $\mathcal{C}^N(b_t^*, s_t)$ , are increasing in initial wealth  $b_t^*$ .

The logic for these results is that whether an increase in the interest rate mitigates excessive borrowing ahead of a liquidity trap episode depends on two opposing forces. First, there is an intertemporal substitution effect by which given prices and income, households save more externally. Second, there is an income general equilibrium effect by which the resulting contraction in current aggregate demand reduces output and leads to higher external borrowing.<sup>14</sup> When the intertemporal elasticity of substitution,  $\sigma_t$ , exceeds the intratemporal elasticity,  $\gamma_t$ , the central bank internalizes that tightening monetary policy would reduce household borrowing. Therefore, in the absence of macroprudential policy, the central bank tightens monetary policy when it faces the risk of a liquidity trap in the future. When the ranking of elasticities is reversed, the central bank lowers the interest rate and overheats the economy to make it less vulnerable to a liquidity trap in the future.

With non-separable preferences between consumption and labor, condition (25) in the proposition now takes into account that labor also affects the marginal utility from tradables. Specifically, if higher labor raises the marginal utility for consumption, then stimulating output would lead to more capital inflows, given everything else constant. Therefore, an even higher intertemporal elasticity of substitution is needed to render a contractionary monetary policy optimal ahead of a liquidity trap (relative to the separable case).

Finally, it is worth contrasting an aspect of taxes on capital inflows and prudential monetary interventions. While taxes on capital inflows are imposed only if the zero lower bound binds in the next period, a prudential monetary intervention is engineered as long as the ZLB is binding in a future distant date. To put it differently, monetary policy needs to act even more preemptively than macroprudential policy. The reason for this result is that monetary policy is a blunter instrument than macroprudential policy. A binding zero lower bound in some future state  $k$  implies that the central bank needs to reduce overborrowing at  $k - 1$ . With macroprudential policy, the central bank introduces a tax on borrowing at  $k - 1$  while preserving a zero labor wedge. But, without macroprudential policy, the central bank must introduce a labor wedge at  $k - 1$ . Doing so implies that from the perspective of  $k - 2$ , the central bank also needs to deviate from a zero labor wedge. Proceeding backward, this implies a strong history-dependent result: as long as there is a binding ZLB in some future state, the central bank will deviate from full employment at any period before.

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<sup>14</sup>We provide a formal decomposition of these channels in the working paper version (Bianchi and Coulibaly, 2021).

### 3.4 Discussion and Extensions

Let us make a few remarks about the results and highlight certain extensions.

**Policy interactions.** The preceding analysis suggests important interactions between monetary and macroprudential policies. Specifically, we have shown that in the absence of macroprudential policy, monetary policy must play a dual role. It seeks to stabilize output around the efficient level today and, at the same time, capital inflows so as to increase aggregate demand in the future and help stabilize output at the zero lower bound.

We note that while the optimal policy effectively uses two instruments (the exchange rate and the tax on inflows) to achieve two goals (efficient output and level of borrowing), there is a strong interdependence between the two instruments. For example, if a shock makes the possibility of a zero bound more likely, a higher tax on inflows becomes desirable to mitigate the overborrowing problem. At the same time, the higher tax implies a reduction in aggregate demand, which requires a depreciation of the nominal exchange rate to keep the output at the efficient level. Conversely, when a zero lower bound becomes less likely, the optimal policy would prescribe a lower tax on capital inflows and a more appreciated exchange rate.

**Commitment to monetary policy.** We know from the forward guidance literature (e.g., Eggertsson and Woodford, 2003) that a commitment to lower interest rates in the future can help relax the zero lower bound today. We argue now that even if the central bank has the ability to commit to future monetary policy, it is still strictly optimal for the central bank to use capital controls. To do so, we consider the problem of a central bank that chooses the sequence of exchange rates and taxes on capital flows at time  $t = 0$ . As shown in Appendix B.1, the optimality conditions yield

$$\psi_t = \frac{\tilde{\omega}_t}{\gamma_t c_t^T (1 - \tilde{\omega}_t)} \left[ \tilde{\xi}_t - \frac{\tilde{\xi}_{t-1}}{\beta \delta_{t-1}} \right]. \quad (26)$$

This targeting rule is analogous to the one under discretion, (23). Condition (26) highlights how a depreciation at time  $t$  tightens the zero lower bound constraint at time  $t$ , whereas it relaxes it at  $t - 1$ . Under commitment, the central bank has incentives to promise a depreciation in the future when it hits a zero lower bound. Ex-post, however, this is suboptimal, as the government will try to stabilize output at the efficient level.

Our key result on optimal capital control is preserved when the government can

commit. In fact, as shown in Appendix B.1, the optimal tax on debt has the same form as (21). That is, the optimal taxes on capital flows again target the current and future labor wedges. Nonetheless, the ability to commit to monetary policy does affect the value of the labor wedge and therefore the magnitude of the tax under commitment differs from the one under discretion.<sup>15</sup>

**Tradable production.** We have considered a setup where the tradable output follows an endowment. While this is partly for simplicity, it also captures a situation where exports are sticky in foreign currency so that exports do not respond to a nominal exchange rate depreciation.<sup>16</sup> More generally, however, to the extent that variations in the exchange rate affect tradable production, this will influence capital flows. In particular, a temporary increase in tradable production—through consumption smoothing—leads to an increase in the trade surplus and the net foreign asset position.

Our main result on the role of prudential monetary intervention can be extended to a setup with tradable production. Specifically, assume that there are firms that produce tradable output and solve  $\max_n P_t^T A_t F(n_t) - W_t h_t$ , where  $F$  is a strictly increasing and strictly concave production function. In addition, assume that the disutility from labor depends on total hours worked in both sectors. In this environment, a depreciation of the currency will push up tradable production and the trade surplus *given a level of wages*. However, a nominal depreciation shifts domestic demand toward non-tradables and stimulates labor demand in the non-tradable sector, leading to upward pressure on wages. Whether a depreciation leads to an increase in tradable production depends on the elasticity of labor supply and the extent to which there are rigidities in wages. That is, the consideration of an endogenous tradable production does not necessarily imply a more expansionary monetary policy. Moreover, like in our baseline model, a prudential monetary intervention continues to target a higher net foreign asset position to make the economy less vulnerable to a liquidity trap in the future.

## 4 Quantitative Analysis

In this section, we evaluate the quantitative implications of the theory. We first describe the calibration of the model and the ability of the model to replicate important features from the data. Finally, we assess the effectiveness of prudential monetary interventions

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<sup>15</sup>A quantitative comparison of the taxes under commitment and discretion is left for future research.

<sup>16</sup>See, for example, Auclert, Rognlie, Souchier and Straub (2021).

and macroprudential policy.

## 4.1 Calibration

The time period is a quarter. We calibrate the model using United Kingdom data between 1990 and 2018 as an example of an advanced small open economy.<sup>17</sup>

**Partial price adjustments.** Our baseline model assumes that prices are perfectly sticky. In this section, we consider partial price adjustments, as in Rotemberg (1982). We assume the non-tradable good is a composite of a continuum of intermediate goods produced by monopolistically competitive firms

$$y_t^N = \left( \int_0^1 (y_{jt}^N)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where  $y_{jt}^N$  is a certain variety of intermediates and  $\varepsilon > 1$  denotes the elasticity of substitution between two varieties.

Intermediates are produced with labor with a linear production function,  $y_{jt} = An_{jt}$  with constant productivity ( $A = 1$ ) and face costly price adjustments. In particular, we follow Bilbiie (2024) in assuming that intermediate producers pay a cost relative to the previous period's average price index:

$$\frac{\varphi}{2} \left( \frac{P_{jt}^N}{P_{t-1}^N} - 1 \right)^2 y_{jt}^N,$$

where  $P_{jt}^N$  is the price of the variety  $j$ . We have the following Phillips curve:

$$(1 + \pi_t^N)\pi_t^N = \frac{\varepsilon}{\varphi} \left[ \frac{W_t}{P_t^N} - 1 \right], \quad (27)$$

where  $\pi_t^N \equiv P_t^N / P_{t-1}^N - 1$ .<sup>18</sup> The static nature of the Phillips curve (27) allows us to highlight the forward-looking role for the optimal monetary policy that emerges because of the

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<sup>17</sup>We note that the problem of the zero lower bound has indeed been more pervasive for advanced economies, although a side effect of the recent increase in central bank credibility in emerging markets appears to be the increase in vulnerability to liquidity traps, as can be seen from the recent experiences of countries such as Chile and Peru (see Matthew Bristow, "Paul Krugman Says the Liquidity Trap Has Spread to Emerging Markets," Bloomberg, May 12, 2020).

<sup>18</sup>In deriving (27), we use that firms face a demand curve  $y_{jt}^N = \left( \frac{P_{jt}^N}{P_t^N} \right)^{-\varepsilon} y_t^N$  and choose  $\{P_{jt}^N\}$  to maximize

aggregate demand externality in our framework while preserving the tradeoff between output and inflation.

**Functional forms.** Households' preferences over consumption and hours worked and the consumption aggregator takes the following form:

$$U(c_t, h_t) = \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{h_t^{1+\phi}}{1+\phi},$$

$$c_t = \left[ \omega (c_t^T)^{1-\frac{1}{\gamma}} + (1-\omega) (c_t^N)^{1-\frac{1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}.$$

**Stochastic processes.** The stochastic processes for  $y_t^T$ ,  $R_t^*$ , and  $\delta_t$  are assumed to be independent.<sup>19</sup> The processes for tradable output and the interest rate are modeled as a first-order univariate autoregressive process. The tradable output  $y_t^T$  is measured with the cyclical component of value added in agriculture, mining, fishing, and manufacturing from the Office for National Statistics (ONS). The world interest rate  $R_t^*$  is measured with the US federal funds rate deflated with the expected CPI inflation.<sup>20</sup> The estimated processes are respectively  $\ln y_t^T = 0.8258 \ln y_{t-1}^T + \varepsilon_t^y$  with  $\varepsilon_t^y \sim N(0, 0.001229)$  and  $\ln(R_t^*/R^*) = 0.9173 \ln(R_{t-1}^*/R^*) + \varepsilon_t^{R^*}$  with  $\varepsilon_t^{R^*} \sim N(0, 0.000007)$ . The estimated value of the annualized average real interest rate is 1.4% per year.

Following Eggertsson and Woodford (2003), the discount factor shock  $\delta_t$  is assumed to follow a two-state Markov process,  $\delta_t \in \{\delta^L, \delta^H\}$  with  $\delta^L < \delta^H$ . We normalize  $\delta^L$  to 1, which represents the normal regime in which households discount the future at a rate of  $\beta$ . We set  $\delta^H = 1.004$ . Given the calibrated values for the transition probability (to be described below), we obtain a standard deviation of discount factor shock of 0.19% in the range of those used in the literature (see e.g., Nakata, 2017).

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$$\sum_{t=0}^{\infty} \frac{u_T(t)}{P_t^T} \left\{ \left[ (1+\zeta) P_{jt}^N - W_t \right] \left( \frac{P_{jt}^N}{P_t^N} \right)^{-\varepsilon} y_t^N - \frac{\varphi}{2} \left( \frac{P_{jt}^N}{P_{t-1}^N} - 1 \right)^2 P_t^N y_t^N \right\},$$

where  $\zeta = 1/(\varepsilon - 1)$  is a constant production subsidy that offsets monopoly distortion in pricing. Using the first-order condition with respect to  $P_{jt}^N$  and imposing symmetry yields (27). Notice that under  $\varphi \rightarrow \infty$ , we recover the baseline model with complete price rigidity.

<sup>19</sup>We set  $P^{T*}$  constant and equal to one. This is without loss of generality because changes in  $P^{T*}$  affect the exchange rate but not the equilibrium allocations or welfare under optimal policy.

<sup>20</sup>The real rate is defined as  $R_t^* = \tilde{R}_t^* \mathbb{E}_t \frac{1}{1+\pi_{t+1}^*}$  where  $\tilde{R}_t^*$  is the federal funds rate,  $\pi_t^*$  is the US CPI inflation, and  $\mathbb{E}_t \frac{1}{1+\pi_{t+1}^*}$  is obtained as the one-period-ahead forecast of an estimated AR(1) process for inflation.

Table 1: Calibration

Description	Value	Source/Target
Intertemporal elasticity	$\sigma = 1$	Standard value
Elasticity of substitution T-N	$\gamma = 0.44$	Stockman and Tesar (1995)
Inverse Frisch elasticity	$\phi = 3$	Gali and Monacelli (2005)
Elasticity of subs. varieties	$\varepsilon = 6$	Gali and Monacelli (2005)
High discount rate	$\delta^H = 1.004$	Baseline value
Weight on tradables in CES	$\omega = 0.25$	Share of tradable output
Price-adjustment cost	$\varphi = 980$	Slope of Phillips curve
Discount factor	$\beta = 0.992$	Average NFA-GDP ratio
Transition prob. $\delta^L$ to $\delta^L$	$\mathbb{P}(\delta^L \delta^L) = 0.15$	Frequency of hitting the ZLB
Transition prob. $\delta^H$ to $\delta^H$	$\mathbb{P}(\delta^H \delta^H) = 0.55$	Duration of liquidity traps

**Calibration of parameters.** Table 1 presents the values for the parameters. We set the elasticity of substitution between tradable and non-tradable consumption  $\gamma$  to 0.44, in line with the cross-country estimates of Stockman and Tesar (1995). As in Gali and Monacelli (2005), we set the inverse of the labor supply elasticity to  $\phi = 3$  and the elasticity of substitution between differentiated goods to  $\varepsilon = 6$ . The latter implies a price mark-up of 20%. We calibrate  $\varphi$  so that the slope of the linearized Phillips curve equals 0.032, following the estimate from Gagliardone, Gertler, Lenzu and Tielens (2023).<sup>21</sup>

The remaining parameters are calibrated so that the simulated model can match certain data moments. The weight on tradable consumption in the CES function  $\omega$  is calibrated to match an average 24% share of tradable output in the total value of production as observed in the UK. This implies  $\omega = 0.25$ . We set the transition probability matrix of the discount factor shocks to match (i) the empirical observation that the UK spent 24% of the time at the ZLB between 1990Q1 and 2018Q4, and (ii) an average duration of 7 quarters in the range of the empirical findings by Dordal-i Carreras, Coibion, Gorodnichenko and Wieland (2016). This results in transition probabilities  $\mathbb{P}(\delta^L|\delta^L) = 0.15$  and  $\mathbb{P}(\delta^H|\delta^H) = 0.55$ .<sup>22</sup> Finally, we set  $\beta$  to match an average net foreign asset position (NFA) as a share of GDP of  $-17\%$ . This calibration results in a value of  $\beta = 0.992$ . Notice that in this baseline calibration, the average quarterly output expressed in units of tradables is about 4.4.

<sup>21</sup>Gagliardone et al. (2023) estimate the slope of the Phillips curve using Belgian data. They find, using aggregate instruments (aggregate money and oil shocks) that the slope is between 0.032 and 0.058.

<sup>22</sup>Note that preference shocks  $\delta_t$  are not necessary to trigger a binding ZLB constraint. In Appendix C.2.2, we also study a Cole-Obstfeld parametrization where  $\gamma = \sigma = 1$  that abstracts from discount factor shocks and includes productivity shocks. We find similar normative quantitative implications to our baseline model.

## 4.2 Data and Model Statistics

Before presenting the quantitative normative implications, we examine the ability of the model to replicate empirical regularities in the data. To do so, we simulate the model—using the optimal prudential monetary policy as a reference—and compare untargeted moments with the data counterparts.

Table 2: Statistics: Data vs. Model

	Severity of liquidity trap episodes				Std. deviations relative to GDP		Correlations with GDP	
	$y$	$c$	$h$	$\pi$	$c$	TB	$c$	TB
Data	-4.6%	-2.9%	-2.8%	-0.4%	0.89	1.94	0.81	0.10
Model	-4.5%	-3.8%	-3.3%	-0.3%	0.79	2.59	0.81	0.64

*Note:* For the severity of ZLB episodes, we take the difference between the peak before the recession (average over a year) and the trough of the recession (average over a year). In the data, the peak corresponds to 2008Q1 for output and consumption (2005Q1 for employment) and the trough of the ZLB episode corresponds to 2009Q2 for output and consumption (2011Q3 for employment). Output in the model is computed as  $(P_t^T y_t^T + P_t^N y_t^N) / P_t$  where  $P_t$  is composite price index. For computing the unconditional moments of output, consumption, and employment in the model we use percentage deviations from the ergodic mean. For their counterparts in the data, we use the cyclical component of the HP-detrended series. Inflation corresponds to the average change in non-tradable inflation and GDP deflator in the model and data respectively.

Table 2 reports untargeted moments in the model both conditional on a ZLB episode and unconditional. In line with our calibration, the data counterpart for the ZLB episode corresponds to the Great Recession in the UK. As the table shows, the model predicts significant declines in output and consumption during ZLB episodes, as well as deflation. Despite being untargeted moments, we can also see that the magnitudes are quite close to those observed in the UK.

In addition, Table 2 also shows that the model matches well unconditional business cycle moments in the data. In particular, the model reproduces the observed cyclicalities of consumption and the trade balance, and it roughly matches the volatility of consumption and the trade balance relative to output.

## 4.3 The Prudential Role of Monetary Policy

We examine here the stabilization gains from prudential monetary interventions. To do so, we compare the regime with optimal monetary policy (and no macroprudential policy)

with a regime in which the central bank closes the labor wedge and replicates the flexible price allocation whenever the ZLB does not bind. We refer to the latter as the inflation targeting regime.<sup>23</sup>

Figure 2 presents the labor wedge (panel [a]) and the probability of a binding ZLB constraint next period (panel [b]) as a function of current bond holdings for the two regimes. The gray region in each panel corresponds to the debt region for which the ZLB binds under the prudential monetary policy regime. As Panel (a) shows, away from the ZLB, the central bank leans against the wind under the prudential monetary policy regime. Under inflation targeting, the ZLB is binding for a larger range of bonds and once the ZLB is not binding, the labor wedge equals zero. Panel (b) shows that the prudential monetary intervention reduces the probability of a binding ZLB constraint next period for any level of bonds, relative to the inflation targeting regime.

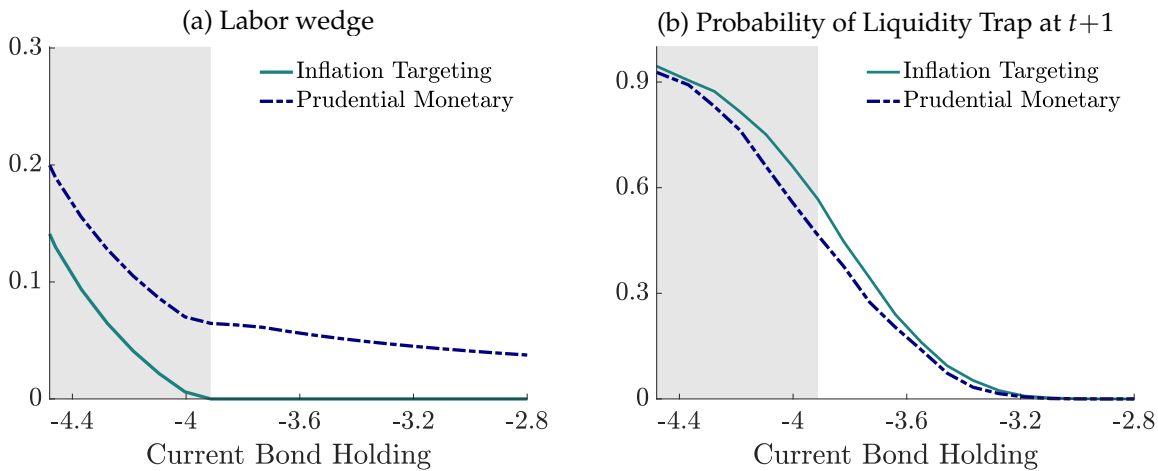


Figure 2: Vulnerability to liquidity traps

*Note:* The figure considers mean values for the shocks for  $y_t^T$  and  $R_t^*$  and  $\delta_t = 1$ .

In line with Proposition 3, when the intratemporal elasticity of substitution  $\gamma$  is lower than the intertemporal elasticity of substitution  $\sigma$ , the prudential intervention requires a positive labor wedge away from the ZLB in order to reduce the likelihood of a liquidity trap. In Figure 3, we show how the prudential policy varies with the intratemporal elasticity of substitution  $\gamma$ , keeping all other parameters constant. Panel (a) presents how the relative duration and frequency of liquidity traps vary with  $\gamma$  and panel (b) presents the welfare gains. Given a baseline value of  $\sigma = 1$ , when  $\gamma = 1$ , there are no gains from prudential monetary policy. When  $\gamma < 1$ , we observe larger benefits from prudential monetary policy

<sup>23</sup>To see that stabilizing output and inflation is feasible away from the ZLB, notice we can rewrite eq. (27) as  $\varphi(1 + \pi_t^N)\pi_t^N = -\frac{\varepsilon\psi_t}{u_T(t)}$ .

relative to the case with  $\gamma > 1$ . The welfare gains in terms of permanent consumption can reach about 0.04 percentage points.

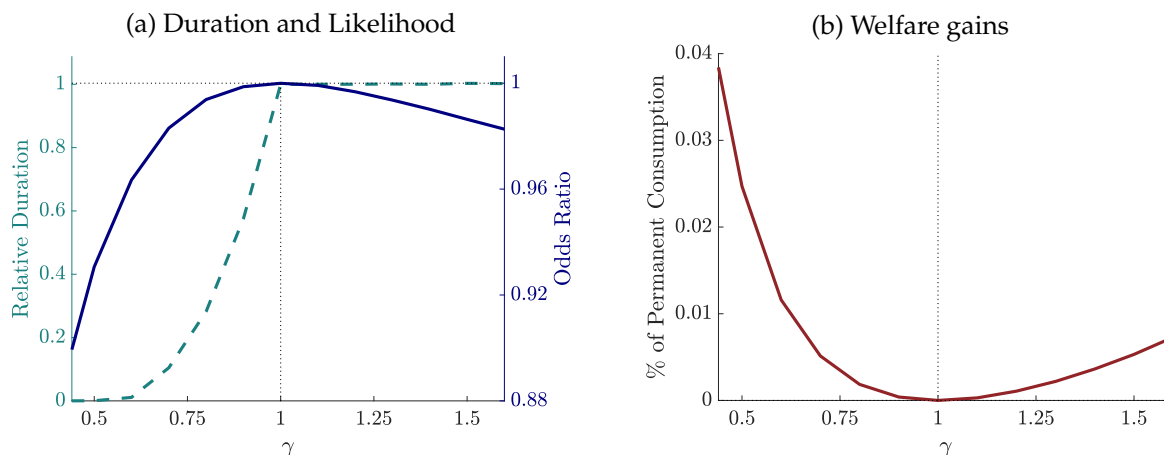


Figure 3: Gains from prudential monetary policy

*Note:* Panel (a) shows the duration of liquidity traps under prudential monetary policy relative to a full-employment policy (left-axis, broken line) and the likelihood of experiencing a liquidity trap episode under prudential monetary policy relative to an inflation-targeting policy (right-axis, solid line).

#### 4.4 The Gains from Macroprudential Policy

In this section, we analyze the gains from macroprudential policy. Specifically, we compare the economy with optimal monetary policy alone with the economy with both optimal monetary and capital control policy.

**Counterfactual event analysis.** Figure 4 compares the dynamics of the economies with and without macroprudential policy in the run-up to a liquidity trap. We construct comparable 30-quarter event windows for the two economies as follows. First, we simulate the economy without macroprudential policy for 1,000,000 periods. Second, we identify episodes of binding ZLB and plot the average of all macro variables within the window. This is represented by the blue broken line in panels (d)-(h). Third, for each episode of a binding ZLB, we take the initial bond position at  $t - 10$  and the sequences of shocks that the economy went through in the 30-quarter window and pass them through the policy functions of the economy under the optimal macroprudential policy. This counterfactual exercise is represented in the solid red line, which plots the average of all these simulations.

Panels (a), (b), and (c) present the average of the shocks that drive the liquidity trap events. As the figure shows, these events are preceded by a decline in tradable output,

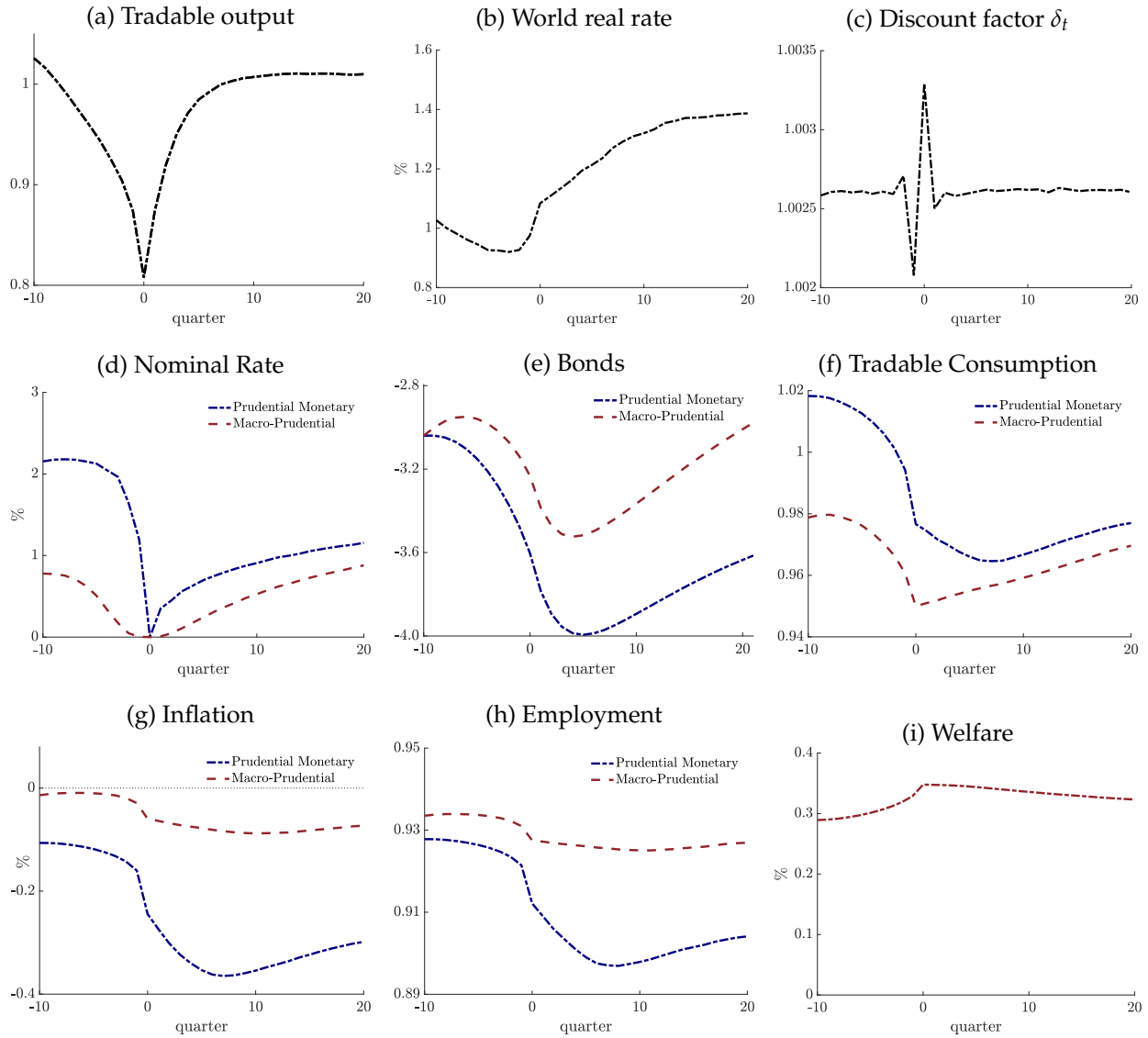


Figure 4: Event analysis

*Note:* Period 0 represents the first period where the ZLB becomes binding. Panels (b) and (d) report the annualized net domestic nominal rate and net world interest rate. Panel (g) reports the annualized inflation rate in non-tradables. Panel (i) corresponds to the welfare gains of using macroprudential policy.

low real rates, and a low discount factor, which lead to an increase in indebtedness in the economy without macroprudential policy, as shown in panel (e).<sup>24</sup> Moreover, we also see that the liquidity trap is preceded by relatively high nominal rates (panel [d]) as the central bank leans against the wind, anticipating an increased likelihood of a liquidity trap in

<sup>24</sup>To further clarify what drives the economy into a liquidity trap, we solve the model when considering one shock at a time. We find that, keeping all parameter values as in our baseline, shocks to tradable output *alone* can drive the economy into a liquidity trap. See Appendix C.2 for full details on these results.

the future. When the ZLB becomes binding, we see on average a larger drop in tradable output, a reversal in the discount factor shock, and a modest increase in the world real rate. Moreover, a persistent fall in employment and deflation follows. The fact that the economy is highly indebted implies that consumption recovers only gradually and expectations of deflation feed back into lower output in those simulations where the ZLB remains binding.

In terms of the counterfactual analysis, panel (e) also shows that the use of macroprudential policies results in a significant increase in bond holdings ahead of liquidity traps. Recall that both economies start from the same debt level at  $t = -10$  (i.e., 10 quarters before the ZLB becomes binding). As a result of the increase in bond holdings, we can see that the recession faced by the economy is significantly milder. Moreover, it is interesting to note that the central bank has higher savings in the run-up to the ZLB. This is because the central bank can use macroprudential policy to reduce the level of indebtedness (rather than tightening monetary policy to reduce capital inflows).

The use of macroprudential policies delivers significant welfare gains both in the run-up to liquidity traps and during liquidity traps. In panel (i), we plot the compensating consumption variations that would make a household living in an economy without macroprudential policies as well-off as a household living in an economy with macroprudential policies. The panel shows that households living in an economy under macroprudential policy experience welfare gains of about 0.35 percentage points (in terms of permanent consumption) during liquidity trap episodes.<sup>25</sup>

**Duration and frequency of liquidity traps.** Table 3 compares the likelihood and duration of liquidity trap episodes with and without macroprudential policy. By taxing borrowing when the economy is vulnerable, the central bank reduces the share of time spent at the ZLB from 24.1% to 23.6% and the frequency of liquidity traps from 3.6% to 2.8%.<sup>26</sup> Perhaps surprisingly, although there are fewer liquidity traps with macroprudential policy, these episodes tend to last longer. The increase in duration reflects two effects. One is that given the reduction in indebtedness, ZLB episodes with macroprudential policy occur when there is a combination of more severe adverse shocks. The second effect is that the central bank often subsidizes inflows during a ZLB episode to alleviate the recession, which in

<sup>25</sup>Formally, the welfare gain from moving from a regime A to a regime B for a given state  $(b^*, s)$ , corresponds to the value of  $q$  that satisfies

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \prod_{k=0}^{t-1} \delta_k \right) \left[ \log((1+q)c_t^A) - \frac{(h_t^A)^{1+\phi}}{1+\phi} \right] = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \prod_{k=0}^{t-1} \delta_k \right) \left[ \log(c_t^B) - \frac{(h_t^B)^{1+\phi}}{1+\phi} \right]$$

<sup>26</sup>The frequency is the unconditional probability that a liquidity trap episode begins in a given quarter.

turn slows down the deleveraging and the economy remains relatively more exposed.<sup>27</sup>

Table 3: Statistics with and without Macroprudential Policy

	ZLB episodes			NFA to GDP ratio	Average Tax on debt
	Share of time	Frequency	Duration		
Without Macroprudential	24.1%	3.6%	1.7	-17.1%	-
With Macroprudential	23.6%	2.8%	2.1	-10.8%	0.1%

*Notes:* Frequency is the unconditional probability, in percent per quarter, that a new ZLB episode begins. Duration is expressed in years.

Macroprudential policy implies an average tax rate on inflows of 0.1%, with a correlation between the tax rate on debt and the nominal interest rate of -0.4. The negative correlation reflects that during positive income shocks, it is optimal to raise the nominal interest rate to stabilize output and to lower the tax on borrowing.

**Welfare costs of liquidity traps.** The severity of recessions following a liquidity trap suggests a large welfare cost from the ZLB constraint. Figure 5 compares the welfare costs of the ZLB with and without macroprudential policy (in terms of permanent consumption) for a range of bond holdings. On average, the welfare cost of the liquidity traps falls from 0.37 percentage points of permanent consumption to about 0.04 percentage points when monetary policy is supplemented with macroprudential policy.<sup>28</sup>

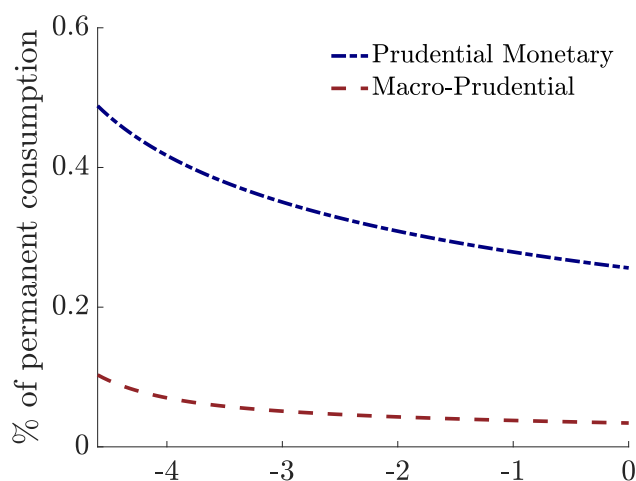


Figure 5: Welfare cost of the ZLB

<sup>27</sup>The average tax during a liquidity trap is -0.02%.

<sup>28</sup>Appendix B.2 analyzes the sensitivity of the results to adding steady-state inflation.

**Takeaway.** The key takeaway is that when monetary policy bears the entire burden of prudential policies, the economy exhibits much more severe liquidity traps. While monetary policy would be able to reduce more sharply the amount of capital inflows, which lead to liquidity traps, the distortions imposed by prudential monetary policy are too severe to justify a more active use. By using macroprudential policy, the central bank can induce a more stable path for output and consumption without sacrificing an efficient level of production.

## 5 International Spillovers

The goal of this section is to study how foreign countries' policies and shocks can affect the domestic economy's welfare. We will show that capital controls can help provide insulation from external disturbances.

A key observation from the central bank optimal policy problem is that the only foreign variable that is relevant for a small open economy's welfare is the world real interest rate. Notice that the world price of tradables,  $P_t^{T*}$ , appears in constraints (22b) and (22c), but in response to changes in  $P_t^{T*}$ , the central bank can adjust  $e_t$  to keep  $P_t^T$  constant and thus offset any effects on welfare.<sup>29</sup> That is, changes in policy abroad affect welfare in the SOE only insofar as they affect the world real rate. Because the world real rate constitutes an intertemporal price, we refer to the spillovers emerging in our framework as a *financial channel of international spillovers*. We summarize this observation in the following remark.

**Remark 1** (Financial Spillovers). *Changes in policy abroad affect welfare in the SOE only to the extent that they affect the world real rate.*

To highlight the key mechanisms underlying the financial spillovers, we will evaluate the effects of exogenous changes in  $R_t^*$ . In particular, we will consider small changes around an equilibrium where the country is neither a net borrower nor a net saver (i.e.,  $b_1^* = b_0^* = 0$ ). In this context, any first-order effects on welfare emerge purely from the general equilibrium considerations within the small open economy (and not from the tradable resource constraint).

Note that by applying the envelope condition on the value function for the central bank, we obtain that the welfare effects of a change in  $R^*$  can be traced to the Lagrange multipliers on the ZLB constraint ( $\zeta$ ) and the household Euler equation for foreign bonds

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<sup>29</sup>This also applies to the problem of optimal monetary policy (A.12).

$(v_0)$ :

$$\frac{dV_0}{dR_0^*} = \frac{1}{R_0^*} \left[ \tilde{\zeta}_0 + u_T(c_0^T, c_0^N, h_0)v_0 \right], \quad (28)$$

As can be seen from Problem (A.12) in the Appendix, a positive (negative) value for  $v_0$  reflects that the planner's value would increase if households borrow more (less). Note that  $v_t = 0$  when the central bank has access to capital controls.

In the following analysis, we will distinguish situations where the central bank operates away or at the ZLB and show how the results depend on whether capital controls are available or not.

## 5.1 Away from the Zero Lower Bound

Consider first the case when the economy is away from the zero lower bound. We have the following proposition.

**Proposition 4** (Insulation with macroprudential policies). *Consider a small change in the world real rate  $dR_0^*$  when the small open economy starts from a zero initial bond position. When the economy is away from the zero lower bound  $R_0 > 1$ , we have the following welfare effects:*

i) *If the central bank sets macroprudential policies optimally, there are no effects on welfare. That is,  $\frac{dV_0}{dR_0^*} = 0$ .*

ii) *In the absence of macroprudential policy, the effect on welfare is given by*

$$\frac{dV_0}{dR_0^*} = \frac{1}{R_0^*} \cdot \frac{\sigma_0}{(1 - \tilde{\omega}_0)\sigma_0 + \tilde{\omega}_0\gamma_0} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \left( \prod_{j=0}^{t-1} \frac{\delta_j R_j^*}{1 + \bar{\Theta}_j} \right) \frac{\gamma_0 c_0^T}{\gamma_t c_t^T} \tilde{\zeta}_t > 0. \quad (29)$$

where recall that  $\tilde{\zeta}$  is the non-negative Lagrange multiplier on the ZLB constraint.

*Proof.* In Appendix A.4. □

Part (i) of the proposition elucidates how macroprudential policy renders an economy insulated from a change in the world real rate. The underlying intuition for this result is that with macroprudential policy, the central bank can implement the socially desired level of borrowing. Thus, to the extent that the economy is neither a net borrower nor net

saver, any changes in borrowing resulting from variations in the world real rate have only second-order effects.<sup>30</sup>

When the central bank lacks macroprudential policy, borrowing decisions are no longer socially optimal. In particular, as discussed above, when there is a positive probability of a liquidity trap in the future, households borrow excessively because they do not internalize that more savings would help stabilize aggregate demand in a liquidity trap. As eq. (29) shows, this implies that welfare decreases with a lower interest rate if  $\xi_t > 0$  for some  $t$ .<sup>31</sup>

Intuitively, a lower world real rate leads households to borrow more and this exacerbates the overborrowing problem.

## 5.2 At the Zero Lower Bound

Consider next a situation when the central bank is already at the zero lower bound. We have the following proposition.

**Proposition 5** (Spillovers at the ZLB). *Consider a small change in the world real rate  $dR_0^*$  when the small open economy starts from a zero initial bond position. When the economy is at the ZLB, we have the following results:*

i) *If the central bank sets macroprudential policies optimally, we have*

$$\frac{dV_0}{dR_0^*} = \frac{\xi_0}{R_0^*} > 0. \quad (30)$$

ii) *In the absence of macroprudential policy, the effect on welfare is given by*

$$\frac{dV_0}{dR_0^*} = \frac{1}{R_0^*} \frac{\sigma_0}{(1 - \tilde{\omega}_0)\sigma_0 + \tilde{\omega}_0\gamma_0} \left[ \frac{\tilde{\omega}_0}{\sigma_0} (\gamma_0 - \sigma_0)\xi_0 + \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \left( \prod_{j=0}^{t-1} \frac{\delta_j R_j^*}{1 + \Theta_j} \right) \frac{\gamma_0 c_0^T}{\gamma_t c_t^T} \xi_t \right]. \quad (31)$$

*Proof.* In Appendix A.5. □

Part (i) shows that if the central bank sets capital controls optimally, welfare increases with a higher world real interest rate. When the economy is at the zero lower bound, a rise in the world real rate helps relax the zero lower bound constraint.<sup>32</sup> On the other hand,

<sup>30</sup>Notice from (28) that if the ZLB is not binding  $\xi_0 = 0$  and if the central bank has access to capital controls  $v_0 = 0$ , rendering  $\frac{dV_0}{dR_0^*} = 0$ .

<sup>31</sup>Notice that (29) can be obtained by replacing (A.21) in (28).

<sup>32</sup>Notice that without uncertainty, combining (10) with the ZLB constraint yields  $R_t = R_t^* e_{t+1}/e_t \geq 1$ .

a higher real interest rate may lead to lower aggregate demand. However, to the extent that the central bank can affect aggregate demand by adjusting taxes on capital inflows, an increase in the world real rate can only benefit the small open economy.

Part (ii) shows that when the central bank does not have access to macroprudential policy, the welfare effect of a change in the world real rate is potentially ambiguous at the ZLB. To the extent that households may be borrowing too little in a liquidity trap (i.e.,  $v_0 < 0$ ), a higher world real rate may exacerbate the lack of aggregate demand and lead to a reduction in welfare. To see this more clearly, suppose that  $\zeta_t = 0$  for  $t \geq 1$  so that there are no liquidity traps in the future. As equation (31) shows, if  $\gamma_0 < \sigma_0$ , an increase in the world real rate reduces domestic welfare. Intuitively, a high intertemporal elasticity of substitution implies that households reduce demand more strongly in response to the higher world real rate, overturning the positive effect that a higher world real rate has on the ZLB constraint through the interest parity condition. However, when  $\zeta_t > 0$  for some  $t \geq 1$ , the overall result is ambiguous. On the other hand, if  $\gamma_0 > \sigma_0$  the welfare effect of an increase in the world real rate is positive (as in the case with macroprudential policy).

### 5.3 Capital Control Wars and Currency Wars

The analysis so far considers an exogenous world real rate. However, we can use the results obtained above to extract lessons for the world general equilibrium effects of central bank policies. As highlighted in Bianchi and Coulibaly (2024), we can evaluate the welfare effect of foreign central bank policies by first inferring their effect on the world real interest rate and then determining whether the change in the world real interest rate has positive or negative effects on the small open economy.

Let us first evaluate the global effects of macroprudential policies. When central banks abroad tax capital inflows to reduce the likelihood of a future liquidity trap, this increases global savings and reduces the world real interest rate. Consider first an economy away from the ZLB. Proposition 4 implies that the increase in foreign taxes on inflows reduces domestic welfare if the domestic central bank does not use macroprudential policy. On the other hand, foreign macroprudential policy has no effects on domestic welfare if the domestic central bank deploys macroprudential policy optimally. Thus, away from the ZLB, macroprudential policy provides full insulation.

When the economy is at the ZLB, Proposition 5 implies that the increase in foreign taxes on capital inflows leads to a reduction in welfare when the central bank deploys macroprudential policy optimally. This result suggests that a world economy where

macroprudential policies are available entails tradeoffs. On one hand, a central bank can raise welfare by using macroprudential policies, but on the other hand, the use of macroprudential policies abroad induces negative welfare effects. The possibility that *laissez-faire* can dominate a world with macroprudential policies was first uncovered in [Fornaro and Romei \(2019\)](#). As they highlight, the logic is that when high-income countries try to protect themselves by forcing households to save more, this depresses the world real interest rate and tightens the ZLB constraint of the low-income countries. They dub this phenomenon “global paradox of thrift.”<sup>33</sup>

One important difference in our analysis with respect to that of [Fornaro and Romei \(2019\)](#) is that the country at the ZLB is not necessarily against its borrowing constraint. This implies that in response to a decrease in the world real rate, the economy could borrow more and potentially offset the adverse effect of the lower world real rate on the ZLB constraint. Thus, the overall welfare effect is ambiguous in our framework. One useful scenario to consider is the case with  $\gamma_0 = \sigma_0$  and where the economy is experiencing a one-time liquidity trap (i.e.,  $\zeta_{t+1} = 0$  for all  $t \geq 0$ ). To understand the implications, consider (31), which characterizes the welfare effects at the ZLB of a change in the world real rate in the absence of domestic macroprudential policy. One can see that under the scenario considered, a marginal decrease in the world real rate has no effect on domestic welfare (in the absence of domestic macroprudential policy). It thus follows that if the central bank is able to use macroprudential policy, it can achieve a welfare improvement. That is, in this scenario, a central bank facing a liquidity trap is unaffected by foreign taxes on capital inflows. Being able to use macroprudential policies can only improve welfare.<sup>34</sup>

Let us conclude by discussing the spillovers from monetary policy. In line with the analysis above, monetary policy abroad affects domestic welfare only insofar as it affects the world real rate. In Section 3.3, we show how in the absence of macroprudential policy, central banks alter their monetary policy to run a trade surplus to reduce their vulnerability to a future liquidity trap. In a world general equilibrium, when many central banks pursue these policies, this leads to a reduction in the world real rate. In the absence of capital controls, this has adverse welfare implications for other countries, as uncovered in Proposition 4. Thus, we have a *currency war* where each central bank tries to manipulate its currency to raise its trade surplus. In general equilibrium, this ends up backfiring

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<sup>33</sup>Their finding resonates with Keynes’ view that international coordination was needed to avert countries running large current account surpluses as that would depress aggregate demand.

<sup>34</sup>Equation (31) suggests that the higher is  $\sigma$ , the higher the benefits from a lower world real rate at the ZLB. As consumption becomes increasingly substitutable over time, it becomes more beneficial to shift consumption towards the present, provided that the borrowing constraint is not limiting. In general equilibrium, this reallocation can stimulate employment and enhance overall welfare.

because central banks deviate from an efficient level of output to try to run a surplus, but this policy proves unsuccessful when all countries are pursuing the same policy.<sup>35</sup>

We highlight again that the incentives to use monetary policy to alter the trade surplus are present when the central bank lacks the use of macroprudential policy. It thus follows that the availability of capital controls can help insulate an economy from monetary spillovers and prevent the outbreak of the aforementioned currency wars.

## 6 Conclusion

We provide an integrated analysis of monetary and macroprudential policies in an open economy subject to an occasionally binding zero lower bound constraint. When macroprudential policy is available, monetary policy stabilizes the output gap away from a liquidity trap. Without macroprudential policy, monetary policy faces a tradeoff between stabilizing output today and curbing capital inflows to reduce the vulnerability to a liquidity trap in the future. However, the optimal monetary policy may call for lower or higher nominal interest rates. Our analysis underscores that international spillovers operate through the world real rate and shows that macroprudential policies can help shield the economy from the adverse effects of foreign policies.

### Data Availability Statement.

The data and code underlying this article are available on Zenodo, at <https://doi.org/10.5281/zenodo.19237842>

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<sup>35</sup>We note that even though currency wars imply that countries seek to raise their trade surplus with a prudential motive, depending on the elasticities of substitution, this may lead countries to run a more expansionary or contractionary monetary policy. Bianchi and Coulibaly (2024) show how the Nash equilibrium may exhibit over- or under-tightening relative to the optimal cooperative monetary policy. See also Fornaro and Romei (2023) for another important study of monetary policy coordination in the presence of international spillovers through the world real rate.

## References

- Acharya, Sushant and Julien Bengui**, “Liquidity Traps, Capital Flows,” *Journal of International Economics*, 2018, 114, 276–298.
- Amador, Manuel, Javier Bianchi, Luigi Bocola, and Fabrizio Perri**, “Exchange Rate Policies at the Zero Lower Bound,” *Review of Economic Studies*, 2020, 87 (4), 1605–1645.
- Aoki, Kosuke, Gianluca Benigno, and Nobuhiro Kiyotaki**, “Monetary and Financial Policies in Emerging Markets,” Technical Report, Working paper 2016.
- Auclert, Adrien, Matt Rognlie, Martin Souchier, and Ludwig Straub**, “Monetary Policy and Exchange Rates with Heterogeneous Agents: Sizing up the Real Income Channel,” 2021. Mimeo, MIT.
- Basu, Suman Sambha, Emine Boz, Gita Gopinath, Francisco Roch, and Filiz Unsal**, “A Conceptual Model for the Integrated Policy Framework,” 2020. Working Paper no. 2021/121, International Monetary Fund.
- Benigno, Pierpaolo and Federica Romei**, “Debt Deleveraging and the Exchange Rate,” *Journal of International Economics*, 2014, 93 (1), 1–16.
- Bianchi, Javier**, “Overborrowing and Systemic Externalities in the Business Cycle,” *American Economic Review*, 2011, 101 (7), 3400–3426.
- **and Louphou Coulibaly**, “Liquidity Traps, Prudential Policies, and International Spillovers,” 2021. Working Paper no. 780, Federal Reserve Bank of Minneapolis.
- **and —**, “Financial Integration and Monetary Policy Cooperation,” 2024. NBER Working Paper 32009.
- Bilbiie, Florin**, “Monetary Policy and Heterogeneity: An Analytical Framework,” *Forthcoming, Review of Economic Studies*, 2024.
- Blanchard, Olivier**, “Currency Wars, Coordination, and Capital Controls,” in “Steven J. Davis, Edward S. Robinson, and Bernard Yeung (eds.), *The Asian Monetary Policy Forum: Insights for Central Banking*” World Scientific 2021, pp. 134–157.
- Caballero, Ricardo, Emmanuel Farhi, and Pierre-Olivier Gourinchas**, “Global Imbalances and Policy Wars at the Zero Lower Bound,” *The Review of Economic Studies*, 2021, 88 (6), 2570–2621.

- Collard, Fabrice, Harris Dellas, Behzad Diba, and Olivier Loisel**, “Optimal Monetary and Prudential Policies,” *American Economic Journal: Macroeconomics*, 2017, 9 (1), 40–87.
- Cook, David and Michael B Devereux**, “Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap,” *American Economic Journal: Macroeconomics*, 2013, 5 (3), 190–228.
- Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles**, “Unconventional Fiscal Policy at the Zero Bound,” *American Economic Review*, 2013, 103 (4), 1172–1211.
- Corsetti, Giancarlo, Eleonora Mavroeidi, Gregory Thwaites, and Martin Wolf**, “Step Away from the Zero Lower Bound: Small Open Economies in a World of Secular Stagnation,” *Journal of International Economics*, 2019, 116, 88–102.
- , **Gernot Mueller, and Keith Kuester**, “The Case for Flexible Exchange Rates after the Great Recession,” 2019. Mimeo, Cambridge.
- Coulibaly, Louphou**, “Monetary Policy in Sudden Stop-Prone Economies,” 2023. *American Economic Journal: Macroeconomics*.
- Devereux, Michael B and James Yetman**, “Capital Controls, Global Liquidity Traps, and the International Policy Trilemma,” *The Scandinavian Journal of Economics*, 2014, 116 (1), 158–189.
- Eggertsson, Gauti B. and Michael Woodford**, “The Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 2003, 34 (1), 139–235.
- , **Neil R. Mehrotra, Sanjay R. Singh, and Lawrence H. Summers**, “A Contagious Malady? Open Economy Dimensions of Secular Stagnation,” *IMF Economic Review*, 2016, 64 (4), 581–634.
- Egorov, Konstantin and Dmitry Mukhin**, “Optimal Policy under Dollar Pricing,” *American Economic Review*, 2023, 113 (7), 1783–1824.
- Fanelli, Sebastian**, “Monetary Policy, Capital Controls, and International Portfolios,” 2023. Mimeo, MIT.
- Farhi, Emmanuel and Iván Werning**, “Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates,” 2012. NBER Working Paper 18199.
- **and Iván Werning**, “A Theory of Macroprudential Policies in the Presence of Nominal Rigidities,” *Econometrica*, 2016, 84 (5), 1645–1704.

- **and Iván Werning**, “Taming a Minsky Cycle,” 2020. Mimeo, MIT.
- Ferrero, Andrea, Richard Harrison, and Benjamin Nelson**, “House Price Dynamics, Optimal LTV Limits and the Liquidity trap,” *Review of Economic Studies*, 2024, 91 (2), 940–971.
- Fornaro, Luca**, “International Debt Deleveraging,” *Journal of the European Economic Association*, 2018, 16 (5), 1394–1432.
- **and Federica Romei**, “The Paradox of Global Thrift,” *American Economic Review*, 2019, 109 (11), 3745–3779.
- **and –**, “Monetary Policy in an Unbalanced Global Economy,” 2023. Working Paper, Centre de Recerca en Economia Internacional.
- Gagliardone, Luca, Mark Gertler, Simone Lenzu, and Joris Tielens**, “Anatomy of the Phillips curve: micro evidence and macro implications,” Technical Report, National Bureau of Economic Research 2023.
- Gali, Jordi and Tommaso Monacelli**, “Monetary Policy and Exchange Rate Volatility in a Small Open Economy,” *Review of Economic Studies*, 2005, 72 (3), 707–734.
- i Carreras, Marc Dordal, Olivier Coibion, Yuriy Gorodnichenko, and Johannes Wieland**, “Infrequent but Long-Lived Zero Lower Bound Episodes and the Optimal Rate of Inflation,” *Annual Review of Economics*, 2016, 8, 497–520.
- Jeanne, Olivier**, “The Global Liquidity Trap,” 2009. Mimeo, Johns Hopkins University.
- Kollmann, Robert**, “Liquidity Traps in a World Economy,” 2021. Working Paper no. 5/2021, Centre for Applied Macroeconomic Analysis, Australian National University.
- Korinek, Anton and Alp Simsek**, “Liquidity Trap and Excessive Leverage,” *American Economic Review*, 2016, 106 (3), 699–738.
- Krugman, Paul**, “It’s Baaack: Japan’s Slump and the Return of the Liquidity Trap,” *Brookings Papers on Economic Activity*, 1998, 29 (2), 137–205.
- Nakata, Taisuke**, “Uncertainty at the Zero Lower Bound,” *American Economic Journal: Macroeconomics*, 2017, 9 (3), 186–221.
- Rotemberg, Julio**, “Sticky Prices in the United States,” *Journal of Political Economy*, 1982, 90 (6), 1187–1211.

- Rubio, Margarita and Fang Yao**, “Macroprudential Policies in a Low Interest Rate Environment,” *Journal of Money, Credit and Banking*, 2020, 52 (6), 1565–1591.
- Schmitt-Grohé, Stephanie and Martin Uribe**, “Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment,” *Journal of Political Economy*, 2016, 124 (5), 1466–1514.
- Stockman, Alan and Linda Tesar**, “Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements,” *American Economic Review*, 1995, 85, 168–185.
- Van der Gote, Alejandro**, “Interactions and Coordination between Monetary and Macroprudential Policies,” *American Economic Journal: Macroeconomics*, 2021, 13 (1), 1–34.
- Werning, Ivan**, “Managing a Liquidity Trap: Monetary and Fiscal Policy,” 2011. NBER Working Paper 17344.

## A Proofs

### A.1 Proof of Proposition 1

The problem of the government consists in choosing  $\{\tau_t\}$  to maximize households' welfare subject to the equilibrium conditions (4), (5), (6), (7) and (14). We solve the relaxed problem of the government (19) and then back out  $\tau_t$  using (6).

Let  $\lambda_t$  and  $\vartheta_t$  be the Lagrangian multipliers associated with the first and second constraint in problem (19) respectively. Taking the first-order conditions, we arrive at

$$[c_t^T] : \lambda_t = u_T(c_t^T, c_t^N, h_t) + \frac{\bar{P}^N}{P_t^T} \frac{\vartheta_t}{\gamma_t c_t^T} \quad (\text{A.1})$$

$$[c_t^N] : 0 = u_N(c_t^T, c_t^N, h_t) + \frac{u_h(c_t^T, c_t^N, h_t)}{A_t F'(h_t)} - \frac{\bar{P}^N}{P_t^T} \frac{\vartheta_t}{\gamma_t c_t^N} \quad (\text{A.2})$$

$$[b_{t+1}^*] : \frac{\lambda_t}{R_t^*} = \beta \delta_t \mathbb{E}_t \lambda_{t+1} \quad (\text{A.3})$$

Combining (A.1) and (A.2) to eliminate  $\vartheta_t$ , and using the second constraint in (19), we get

$$\lambda_t = u_T(c_t^T, c_t^N, h_t) + \frac{\bar{P}^N c_t^N}{P_t^T c_t^T} \psi_t \quad (\text{A.4})$$

We then substitute (A.4) into (A.3), to obtain

$$u_T(c_t^T, c_t^N, h_t) + \frac{\bar{P}^N c_t^N}{P_t^T c_t^T} \psi_t = \beta \delta_t R_t^* \mathbb{E}_t \left[ u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1}) + \frac{\bar{P}^N c_{t+1}^N}{P_{t+1}^T c_{t+1}^T} \psi_{t+1} \right] \quad (\text{A.5})$$

We can now derive the optimal tax rate on debt by combining (6) and (A.5) which leads to

$$\tau_t = \frac{1}{\beta \delta_t R_t^* \mathbb{E}_t u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1})} \left\{ -\frac{1 - \tilde{\omega}_t}{\tilde{\omega}_t} \psi_t + \beta \delta_t R_t^* \mathbb{E}_t \left[ \frac{1 - \tilde{\omega}_{t+1}}{\tilde{\omega}_{t+1}} \psi_{t+1} \right] \right\}. \quad \square$$

### A.2 Proof of Proposition 2

Because  $\tau_t$  only enters (22e), we solve here the relaxed problem of the government where we ignore (22e), then set  $\tau$  to back out the tax consistent with that constraint. We denote by  $\lambda_t$ ,  $\vartheta_t$ ,  $\xi_t$  and  $\chi_t$  the Lagrangian multipliers associated with (22a), (22b), (22c) and (22d) respectively. The optimality condition for  $R_t$  yields  $\xi_t = \chi_t R_t^*$ . Using it to eliminate  $\chi_t$ ,

and applying the envelope condition, we obtain the following optimality conditions

$$[e_t] : \quad \zeta_t = \frac{\bar{P}^N}{P_t^T} \vartheta_t \quad (\text{A.6})$$

$$[c_t^N] : \quad 0 = u_N(c_t^T, c_t^N, h_t) + \frac{u_h(c_t^T, c_t^N, h_t)}{A_t F'(h_t)} - \frac{\bar{P}^N}{P_t^T} \frac{\vartheta_t}{\gamma_t c_t^N} \quad (\text{A.7})$$

$$[c_t^T] : \quad \lambda_t = u_T(c_t^T, c_t^N, h_t) + \frac{\bar{P}^N}{P_t^T} \frac{\vartheta_t}{\gamma_t c_t^T} \quad (\text{A.8})$$

$$[b_{t+1}^*] : \quad \lambda_t = -\zeta_t e_t \mathbb{E}_t \frac{\partial}{\partial b_{t+1}^*} \left[ \frac{\mathcal{U}_T(b_{t+1}^*, s_{t+1})}{\mathcal{E}(b_{t+1}^*, s_{t+1})} \frac{P_t^{T*}}{P_{t+1}^{T*}} \right] + \beta \delta_t R_t^* \mathbb{E}_t \lambda_{t+1} \quad (\text{A.9})$$

We combine (A.6) and (A.7), and use (22b), to arrive at

$$\zeta_t = \gamma_t c_t^T \frac{\bar{P}^N c_t^N}{P_t^T c_t^T} \psi_t \iff \zeta_t = \gamma_t c_t^T \frac{1 - \tilde{\omega}_t}{\tilde{\omega}_t} \psi_t \quad (\text{A.10})$$

Next, we turn to deriving the optimal tax. Substituting (A.8) into (A.9), we obtain

$$u_T(c_t^T, c_t^N, h_t) + (1 + \Theta_t) \frac{\zeta_t}{\gamma_t c_t^T} = \beta \delta_t R_t^* \mathbb{E}_t \left[ u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1}) + \frac{\zeta_{t+1}}{\gamma_{t+1} c_{t+1}^T} \right] \quad (\text{A.11})$$

with  $\Theta_t \equiv \gamma_t P_t^T c_t^T \mathbb{E}_t \frac{\partial}{\partial b_{t+1}^*} \left[ \frac{\mathcal{U}_T(b_{t+1}^*, s_{t+1})}{P^T(b_{t+1}^*, s_{t+1}) \mathbb{E}_t \mathcal{U}_T(b_{t+1}^*, s_{t+1})} \right]$ . We then substitute (6) into (A.11) to get

$$\tau_t = \frac{1}{\beta \delta_t R_t^* \mathbb{E}_t u_T(c_{t+1}^T, c_{t+1}^N, h_{t+1})} \left\{ -(1 + \Theta_t) \frac{\zeta_t}{\gamma_t c_t^T} + \beta \delta_t R_t^* \mathbb{E}_t \left[ \frac{\zeta_{t+1}}{\gamma_{t+1} c_{t+1}^T} \right] \right\}. \quad \square$$

### A.3 Proof of Proposition 3

In the absence of macroprudential policies, the time-consistent problem for optimal monetary policy is given by

$$\begin{aligned}
V(b^*, s) &= \max_{R, e, b^*, c^N, c^T} u\left(c^T, c^N, F^{-1}\left(\frac{c^N}{A}\right)\right) + \beta\delta \mathbb{E}_{s'|s} V(b^*, s'), & (\text{A.12}) \\
&\text{subject to} \\
c^T &= y^T + b^* - \frac{b^*}{R^*}, & (\times\lambda) \\
\frac{\bar{p}^N}{eP^{T*}} &= \frac{u_N(c^T, c^N, F^{-1}(\frac{c^N}{A}))}{u_T(c^T, c^N, F^{-1}(\frac{c^N}{A}))}, & (\times\vartheta) \\
u_T\left(c^T, c^N, F^{-1}\left(\frac{c^N}{A}\right)\right) &= \beta\delta R^* \mathbb{E}_{s'|s} \mathcal{U}_T(b^*, s'), & (\times\nu) \\
R^* &= R \mathbb{E}_{s'|s} \left[ \frac{\mathcal{U}_T(b^*, s')}{\mathbb{E}_{s'|s} \mathcal{U}_T(b^*, s')} \frac{P^{T*}}{P^{T*s'}} \frac{e}{\mathcal{E}(b^*, s')} \right], & (\times\chi) \\
R &\geq 1, & (\times\zeta)
\end{aligned}$$

where the corresponding multipliers are in parentheses.

We define below the Markov perfect equilibrium in this economy.

**Definition A.1** (Markov perfect equilibrium absent capital controls). A Markov perfect equilibrium is defined by government policy functions  $\{\mathcal{R}, \mathcal{E}, \mathcal{B}, \mathcal{C}^T, \mathcal{C}^N, \mathcal{H}\}$  and value function  $V$  such that (i) the value function  $V$  solves (A.12) given future policies  $\{\mathcal{E}, \mathcal{C}^T, \mathcal{C}^N, \mathcal{H}\}$ , and (ii)  $\{\mathcal{E}, \mathcal{C}^T, \mathcal{C}^N, \mathcal{H}\}$  are the optimal policy functions in problem (A.12).

First-order conditions yield

$$[e_t]: \quad \zeta_t = \frac{\bar{p}^N}{P_t^T} \vartheta_t, \quad (\text{A.13})$$

$$[c_t^N]: \quad 0 = u_N(c_t^T, c_t^N, h) + \frac{u_h(c_t^T, c_t^N, h)}{A_t F'(h_t)} - \frac{\bar{p}^N}{P_t^T} \frac{\vartheta_t}{\gamma_t c_t^N} - \frac{\tilde{\omega}_t}{c_t^T} \left( \frac{1}{\tilde{\gamma}_t} - \frac{1}{\sigma_t} \right) u_N(c_t^T, c_t^N, h) v_t, \quad (\text{A.14})$$

where we used  $\frac{du_T(c_t^T, c_t^N, \frac{1}{A_t} F^{-1}(c_t^N))}{dc_t^N} = \frac{\tilde{\omega}_t(\tilde{\gamma}_t^{-1} - \sigma_t^{-1})}{c_t^T} u_N(c_t^T, c_t^N, h_t)$  and  $\tilde{\gamma}_t^{-1} \equiv \gamma_t^{-1} + \frac{c_t U_{ch}(t)}{A_t F'(h_t) u_N(t)}$ . Combining (A.13) and (A.14) and using  $\psi_t$  as defined in (18), we arrive at

$$\frac{\zeta_t}{\gamma_t c_t^N} = \frac{u_N(c_t^T, c_t^N, h_t)}{u_T(c_t^T, c_t^N, h_t)} \left[ \psi_t - \frac{\tilde{\omega}_t(\sigma_t - \tilde{\gamma}_t)}{\sigma_t \tilde{\gamma}_t} u_T(c_t^T, c_t^N, h_t) \frac{v_t}{c_t^T} \right]. \quad (\text{A.15})$$

Therefore, when the ZLB does not bind,  $\zeta_t = 0$ , we get

$$\psi_t = \frac{\tilde{\omega}_t(\sigma_t - \tilde{\gamma}_t)}{\sigma_t \tilde{\gamma}_t} u_T(c_t^T, c_t^N, h_t) \frac{v_t}{c_t^T}. \quad (\text{A.16})$$

The optimality conditions for  $c_t^T$  and  $b_{t+1}^*$  are respectively given by

$$\lambda_t = u_T(c_t^T, c_t^N, h_t) - u_{TT}(c_t^T, c_t^N, h_t) v_t + \frac{\bar{P}^N}{P_t^T} \frac{\vartheta_t}{\gamma_t c_t^T}, \quad (\text{A.17})$$

$$\lambda_t = \beta \delta_t R_t^* \mathbb{E}_t \lambda_{t+1} - \tilde{\zeta}_t P_t^T \mathbb{E}_t \frac{\partial \frac{\mathcal{U}_T(b_{t+1}^*, s_{t+1})}{P^T(b_{t+1}^*, s_{t+1}) \mathbb{E}_t \mathcal{U}_T(b_{t+1}^*, s_{t+1})}}{\partial b_{t+1}^*} + v_t \beta \delta_t R_t^* \mathbb{E}_t \frac{\partial \mathcal{U}_T(b_{t+1}^*, s_{t+1})}{\partial b_{t+1}^*}. \quad (\text{A.18})$$

Plugging (A.17) into (A.18), and using (A.13) along with the implementability constraint associated with the multiplier ( $\times v$ ), we get

$$-(1 + \bar{\Theta}_t) u_{TT}(c_t^T, c_t^N, h_t) v_t = \beta \delta_t R_t^* \mathbb{E}_t \left[ -u_{TT}(c_{t+1}^T, c_{t+1}^N, h_{t+1}) v_{t+1} + \frac{\tilde{\zeta}_{t+1}}{\gamma_{t+1} c_{t+1}^T} \right], \quad (\text{A.19})$$

where  $\bar{\Theta}_t \equiv \beta \delta_t R_t^* \frac{1}{u_{TT}(t)} \mathbb{E}_t \frac{\partial \mathcal{U}_T(b_{t+1}^*, s_{t+1})}{\partial b_{t+1}^*}$ . Iterating forward (A.19) and using the transversality condition,  $\lim_{j \rightarrow \infty} \beta^j \mathcal{U}_T(b_{t+j}^*, s_{t+j}) v_{t+j} = 0$ , we obtain

$$v_t = \frac{1}{-u_{TT}(t)} \mathbb{E}_t \sum_{k=1}^{\infty} \beta^k \prod_{j=0}^{k-1} \left( \frac{\delta_{t+j} R_{t+j}^*}{1 + \bar{\Theta}_{t+j}} \right) \frac{\tilde{\zeta}_{t+k}}{\gamma_{t+k} c_{t+k}^T}, \quad (\text{A.20})$$

$$u_T(c_t^T, c_t^N, h_t) \frac{v_t}{c_t^T} = \frac{\sigma_t \gamma_t}{(1 - \tilde{\omega}_t) \sigma_t + \tilde{\omega}_t \gamma_t} \mathbb{E}_t \sum_{k=1}^{\infty} \beta^k \prod_{j=0}^{k-1} \left( \frac{\delta_{t+j} R_{t+j}^*}{1 + \bar{\Theta}_{t+j}} \right) \frac{\tilde{\zeta}_{t+k}}{\gamma_{t+k} c_{t+k}^T}. \quad (\text{A.21})$$

Finally, we substitute (A.21) into (A.16) to get the optimal monetary policy in its target form:

$$\psi_t = \frac{\gamma_t}{\tilde{\gamma}_t} \frac{\tilde{\omega}_t(\sigma_t - \tilde{\gamma}_t)}{(1 - \tilde{\omega}_t) \sigma_t + \tilde{\omega}_t \gamma_t} \mathbb{E}_t \sum_{k=1}^{\infty} \beta^k \prod_{j=0}^{k-1} \left( \frac{\delta_{t+j} R_{t+j}^*}{1 + \bar{\Theta}_{t+j}} \right) \frac{\tilde{\zeta}_{t+k}}{\gamma_{t+k} c_{t+k}^T}. \quad \square$$

which corresponds to expression (25) in the proposition.

## A.4 Proof of Proposition 4

The value of the central bank is given by (A.12). By applying the envelope theorem and using  $b_1^* = 0$ , we arrive at

$$\frac{\partial V_0}{\partial R_0^*} = \frac{1}{R_0^*} \left[ u_T(c_0^T, c_0^N, h_0) v_0 + \zeta_0 \right]. \quad (\text{A.22})$$

Away from the ZLB, we have  $\zeta_0 = 0$ . Moreover, the Euler equation is not binding if the central bank can choose  $\tau_0$  optimally. We thus have  $v_0 = 0$  and  $\frac{\partial V_0}{\partial R_0^*} = 0$ . This completes part (i).

In the absence of macroprudential policy, we use (A.21) to substitute for  $v_0$  and obtain (29). This completes part (ii).  $\square$

## A.5 Proof of Proposition 5

Recall that the effect of  $dR_0^*$  on welfare is given by (A.22). If the central bank sets macroprudential policy optimally, which implies  $v_0 = 0$ , we obtain (30) in Proposition 5. This completes part (i).

Absent macroprudential policy, we solve for  $v_0$  by first substituting (A.17) into (A.18). We then iterate it forward and using that  $\lim_{j \rightarrow \infty} \beta^j \mathcal{U}_T(b_{t+j}^*, s_{t+j}) v_{t+j} = 0$ , we obtain

$$u_T(c_0^T, c_0^N, h_0) \frac{v_0}{c_0^T} = \frac{\sigma_0 \gamma_0}{(1 - \tilde{\omega}_0) \sigma_0 + \tilde{\omega}_0 \gamma_0} \left[ -\frac{\zeta_0}{\gamma_0 c_0^T} + \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \prod_{j=0}^{t-1} \left( \frac{\delta_j R_j^*}{1 + \bar{\Theta}_j} \right) \frac{\zeta_t}{\gamma_t c_t^T} \right]. \quad (\text{A.23})$$

Substituting (A.23) into (A.22), we obtain (31) in Proposition 5. This completes part (ii).  $\square$