The Architecture of Social Networks and the Diffusion of Innovations

Bryony Reich *

University of Cambridge and Northwestern University

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Abstract

For many technologies and behaviors, an agent's benefit from adopting depends on his contacts adopting, and the benefit to his contacts of adopting depends on their contacts adopting. This paper examines how the architecture of these connections shapes the success or failure of the diffusion of innovations. We start with a standard model of diffusion with the key addition that some agents can coordinate their decisions. This captures the idea that people often talk and make decisions together with friends or family to adopt technologies. We show that insularity of connections, that is, the extent to which agents tend to concentrate their connections to a narrow set of other agents, determines contagion. However, whether insularity helps or hinders depends on the technology being diffused. For technologies that are valuable even without many contacts adopting, we find insular connections hinder adoption, but for technologies that are valuable only when many contacts adopt, insular connections facilitate adoption.

*Bryony Reich: bryony.reich@kellogg.northwestern.edu. I would like to thank Alberto Alesina, Ana Babus, Daniel Barron, Marco Bassetto, Antonio Cabrales, Terri Kneeland, Mike Powell, Lars Nesheim, Jonathan Newton, Sanjeev Goyal, Imran Rasul, Michela Tincani, and Jörgen Weibull for comments and guidance, as well as many seminar participants for helpful comments. Hossein Alidaee provided excellent research assistance. I gratefully acknowledge financial support from the UK Economic and Social Research Council (grant number ES/K001396/1).

1 Introduction

Diffusion is a social phenomenon. For many technologies and behaviors, an individual's utility from adopting depends on his contacts adopting, and the benefit to his contacts of adopting in turn depends on their contacts adopting, and so on. Such complementarities in adoption raise the potential for "snowballing," where adoption spreads from contact to contact throughout the network (Easley and Kleinberg, 2010). However, complementarities also raise the possibility of "coordination failure" (Morris, 2000), where individuals do not adopt because their contacts in the network have not adopted. Such coordination failure can lead to under-adoption of fundamental technologies like latrines (Guiteras et al., 2019), reliance on outdated, inefficient communication technologies (Olson, 2021), and the failure of behaviors, like protest, to spread.[1][2]

Our goal is to understand how the architecture of a network shapes the success or failure of diffusion. We start with a standard model of contagion with strategic complementarities (Morris, 2000), with the key addition that some subsets of agents can coordinate their decision to adopt. We use this framework to provide new understanding of the role of network structure in diffusion. We show that insularity of connections, that is, the extent to which agents tend to concentrate their connections to a narrow set of other agents, determines contagion. However, whether insularity helps or hinders depends on the technology being diffused. For technologies that are valuable even without many contacts adopting, we show insular connections hinder adoption, but for technologies that are valuable only when many contacts adopt, this result is reversed and insular connections facilitate adoption.

Our framework builds on the standard threshold diffusion model (Morris, 2000; Granovetter, 1978). A population of agents are connected in a network. A technology or behavior is associated with a "threshold of adoption" that determines what share of an agent's neighbors in the network must adopt the new technology for him to be better off by adopting. To incorporate the possibility that some subsets of agents can decide jointly whether or not to adopt, our framework introduces a second "social object", a set of decision-making groups. This specifies who in the population may make decisions jointly. Diffusion occurs in two ways. 1) An agent adopts at time t if a large enough share (above the threshold) of his

[1] Numerous examples of strategic complementarities in adoption exist. An individual's benefit from adopting a mobile phone depends on his contacts also adopting (Björkegren, 2019). A farmer is more likely to adopt a new crop or agricultural technology when neighboring farmers do (Bandiera and Rasul, 2006; Beaman et al., 2020). The use of birth control (Munshi and Myaux, 2006), type of contract used (Young and Burke, 2001), adoption of protestantism during the Reformation (Becker et al., 2020), and participation in protest (Bursztyn et al., 2021), are all shown to depend positively on adoption by friends and neighbors.

[2] Guiteras et al. (2019) shows that a household's utility from adopting a latrine depends positively on neighbors adopting and that, as a result, "coordination failure can keep open defecation the prevailing social norm even when everyone in that community would be better off if they simultaneously adopted latrines."

contacts have adopted at time $t-1$. 2) A group of agents that can make a joint decision adopts at time t if, by adopting, each agent in the group then has a large enough share of his contacts who have adopted.

The addition of joint decision-making to the standard contagion model captures a key adoption behavior typically omitted from diffusion models: friends, family, and neighbors often talk about new technologies and behaviors and may make decisions to adopt together. For example, friends or family members discuss new communication technologies and may collectively agree to adopt if it improves their ability to communicate among themselves (see Olson (2021) and Turrell (2017) on joint adoption of messaging applications among friends). Colleagues may together choose to adopt Dropbox to share files conveniently (Warner, 2011). School friends can decide to attend a protest together.[3] Joint decision-making among neighbors to each adopt a latrine is so important that it is now a central feature of safe sanitation campaigns (Sengupta, 2017).[4]

Diffusion with strategic complementarities illustrates a large coordination problem: a society would do better if everyone adopted the improving technology, but it can be difficult to move to this point. The idea that agents might sometimes discuss and make adoption decisions together is a way of saying that some groups of agents can solve their local coordination problem. To flexibly capture a variety of environments, our framework places minimal restrictions on which sets of agents can make decisions together (for example, some sets of agents may be able to make joint decisions while others may not, some decision-making groups may be connected in the network, others not).

Using our model, we first show that more joint decision-making increases adoption. Because a group of individuals who can make a joint decision can solve their local coordination problem, joint decision-making naturally mitigates the coordination problem at the heart of the diffusion of technologies with strategic complementarities and increases adoption.

Our main analysis examines how network structure impacts adoption. We show that the insularity of agents' connections, the extent to which agents' connections are concentrated to a narrow set of other agents, is the feature of network structure that shapes diffusion. However, we show that there are two competing facets of insularity: on the one hand, insular connections hinder the spread of a technology; on the other, insular connections can facilitate joint adoption.

[3]The decision to participate in protest has been made jointly by groups of individuals for centuries. For example, in major labor unrest that spread from village to village throughout England in the 1830s, villagers gathered in local "beer houses" to jointly agree to participate in protests the next day (Holland, 2005).

[4]Adoption of safe sanitation exhibits strategic complementarities among neighbors, (Guiteras et al., 2019). To deal with this, development organizations commonly organize workshops to get neighboring households together to each agree to adopt safe sanitation.

To illustrate, consider a small set of agents that cannot make a joint decision together. An agent is willing to adopt the technology if a sufficient share of his neighbors adopt, and so if each agent in that set has many of his connections concentrated within the set, then no agent in the set is willing to adopt until others in the set adopt. The agents' "over-connectedness" within this set prevents the new technology from entering. Instead, suppose that this small set of agents can make a joint decision together. Exactly because each agent has many of his connections within the set, adopting together is profitable. Notice that this concentration of connections to the set does not mean that agents are more likely to be able to make joint decisions. Rather the concentration of connections makes it beneficial to adopt jointly if agents are able to do so.

To understand which facet of insularity dominates, we examine whether there is more or less adoption as the network architecture becomes more or less insular. Our formal definition of an increase in insularity compares the relative connections of two agents. We say that a "local increase in insularity" occurs when one agent's connections become more concentrated to a narrow set and another agent's connections become less concentrated to a superset of the narrow set, to the extent that the first agent has a greater concentration of his connections to the narrow set than the other agent has to the larger set. To compare two networks overall, we say that one network is less insular than another, Network Y is less insular than Network Z, if there are no local increases in insularity for any agents in Network Y relative to Z. Network Y is strictly less insular than Network Z if, in addition, there is a local increase in insularity for some agents in Network Z relative to Y.

Our main finding is that the role of insularity in diffusion depends on the technology being diffused. For technologies with low thresholds of adoption, less insular networks facilitate greater adoption. In contrast, for technologies with high thresholds of adoption, this result is reversed and more insular networks facilitate greater adoption. The final step of intuition for this result is to notice that technologies with high or low thresholds of adoption diffuse in different ways. Consider a technology with a very high threshold such that an individual needs nearly all of his contacts to adopt to be willing to adopt. Joint adoption is crucial to getting this technology adopted and, since insular connections facilitate joint adoption, insular networks are advantageous to diffusion. In contrast, a low threshold technology can spread from person to person, largely without the need for joint decision-making. Insular connections are therefore not needed to get a low-threshold technology adopted and, if those agents cannot make a joint decision, insularity can hinder diffusion.

2 Related Literature

The contribution of this paper is to incorporate coordinated decision-making among subsets of agents into a threshold model of contagion and to use this framework to compare contagion across different network structures. Specifically, this network comparison 1) provides a classification of network structures as more or less insular, 2) delivers a tractable way to compare contagion across different networks, and 3) contributes new findings on the role of network structure in diffusion.

This paper contributes to a large literature on threshold models of diffusion. Morris (2000) was the first paper to provide a characterization of contagion on a general network. This is extended by Acemoglu et al. (2011) and builds on work using a related framework, including Blume (1993, 1995), Young (1996, 2011), Anderlini and Ianni (1996), Goyal (1996), and Montanari and Saberi (2010). Recent advances on diffusion with strategic complementarities include Sadler (2020) on forward-looking agents, Leister et al. (2022) on uncertainty over technologies, and Campbell et al. (2025) on endogeneity of complementarities. Our key departure is to incorporate joint decision-making and to derive new findings on the role of network structure.

Work that incorporates joint decision-making into networks is rare. In different settings to ours, Kets et al. (2011) and Ambrus et al. (2014) examine coalitional deviations on networks. Within the context of stochastic coordination games, Belloc and Bowles (2013) and Newton and Angus (2015) study the potential for coalitions to speed up or slow down adoption of a norm, but do not examine the role of network structure.

Theoretical work on financial contagion also models threshold processes over networks of financial institutions (e.g. Elliott et al. (2014) and Acemoglu et al. (2015)).[5] These papers do not incorporate joint decision-making (although there is evidence of cooperative decision-making among financial institutions to prevent contagion, see Rogers and Veraart (2013) and Kanik (2024)). Like us, Elliott et al. (2014) and Acemoglu et al. (2015) compare contagion across different network structures. However, their structural features of interest and mechanisms are different to ours and do not focus on insularity.

Our analysis speaks to two interventions: a firm's choice of which market to enter with a product and whether to encourage joint decision-making. These differ from previous interventions studied in models of strategic complementarities, which include seeding (Lim et al., 2016), pricing (Fainmesser and Galeotti, 2020), and a general framework to examine how optimal interventions vary with network structure (Galeotti et al., 2020). We discuss this literature further in Section 8.

[5]See Jackson and Pernoud (2021) for a review.

3 Model

A population consists of a finite set of agents $N = \{1, 2, \ldots, n\}$. Each agent i takes action $a_i \in \{0, 1\}$: he either maintains the status quo technology or behavior (action 0) or adopts a new technology or behavior (action 1).

The Society.

Agents are embedded in a society, described by two different types of interaction: a network and a set of decision-makers.

The network, denoted $\{w_{ij}\}$, captures the strategic complementarities between agents' decisions. For any pair of agents (i, j), the weight $w_{ij} \in [0, 1]$ describes how important j's adoption of the new technology is to i's payoff from adopting the new technology. We say j is a neighbor of i if $w_{ij} > 0$ and assume $w_{ij} = w_{ji}$, $w_{ii} = 0$, and the network is connected.

The set of decision-makers, denoted $\mathcal{C}$, is a subset of the power set of N and captures which sets of agents can make decisions together. An element of $\mathcal{C}$, a "decision-making unit," is a set of agents $C \subseteq N$ that can make a joint decision whether to adopt together (the process of joint decision-making is detailed in the description of the diffusion process below). The set of decision-making units, $\mathcal{C}$, is subject to two assumptions. First, every subset of a decision-making unit is itself a decision-making unit. Second, since an agent can make a decision to adopt or not by himself, each agent is a decision-making unit. This is formalized in Assumption 1.

Assumption 1. *The set of decision-makers, $\mathcal{C}$, is a subset of the power set of N, subject to: a) $\{i\} \in \mathcal{C}$, for all $i \in N$; and b) if $C \in \mathcal{C}$, then for any $C' \subseteq C$ also $C' \in \mathcal{C}$.*

Consider some examples of the set of decision-makers. *Example 1.* Only individuals can make decisions. Then $\mathcal{C} = \{i : i \in N\}$. *Example 2.* Any pair of neighbors can make a joint decision, but larger groups cannot. Then $\mathcal{C} = \{\{i : i \in N\}, \{(i, j) : w_{ij} > 0\}\}$. *Example 3.* Any group of agents where everyone in the group is a neighbor of everyone else in the group can make a joint decision. Then $\mathcal{C} = \{\{i : i \in N\}, \{C \subseteq N : w_{ij} > 0 \text{ for all } i, j \in C\}\}$.[6]

The Technology.

A new technology is summarized by its threshold of adoption $Q \in [0, 1]$, such that an agent is better off adopting "technology Q" if more than share Q of his neighbors adopt.

Our interpretation of this threshold Q is that the technology exhibits strategic complementarities. Suppose an individual's utility from adopting the new technology increases in the weighted share of his neighbors who adopt, $\frac{\sum_{j \in N} w_{ij} a_j}{\sum_{j \in N} w_{ij}}$, while his utility from keeping the

[6]Although it is the case in our examples, we do not require the set of decision-makers and network to overlap: two agents can be in the same decision-making unit but need not have a link in the network.

status quo is normalized to zero. Namely,

$$u(a_i, a_{-i}) = \begin{cases} 0 & \text{if } a_i = 0 \\ \\ v\left(\frac{\sum_{j\in N} w_{ij}a_j}{\sum_{j\in N} w_{ij}}\right) & \text{if } a_i = 1 \end{cases} \tag{1}$$

where $v(\cdot)$ is continuous and strictly increasing. Then Q is the unique value that satisfies $v(Q) = 0$ and $Q < 1$ implies that the new technology strictly improves on the status quo, provided enough people adopt it.

The population, actions, and payoffs $u(\cdot, \cdot)$, together with the network $\{w_{ij}\}$, describe a static coordination game. An agent's best response in this game is action 1 if more than Q of his neighbors take action 1 and action 0 otherwise. We next define our diffusion process which prescribes how agents adopt action 1 and incorporates joint decision-making. This diffusion process selects a Nash equilibrium of the static coordination game.[7]

The Diffusion Process.

At each time $t \in \mathbb{N}$, each agent i takes action $a_i^t \in \{0, 1\}$. The vector $a^t = (a_1^t, ..., a_n^t)$ describes who takes action 0 or 1 at time t. At time $t = 0$, all agents take action 0. The set of agents who take action 1 at time t is $A^t = \{i : a_i^t = 1\}$. The recurring piece of notation in this paper is $P_i(S) = \frac{\sum_{j\in S} w_{ij}}{\sum_{j\in N} w_{ij}}$, the weighted fraction of i's neighbors within a set $S \subseteq N$. We also refer to this as the proportion or share of i's connections to set S.

Diffusion Process *At each* $t \geq 1$, *for each* $i \in N$,

1. $a_i^t = 1$ *if* $i \in C \in \mathcal{C}$ *where* $C \subseteq N \backslash A^{t-1}$ *and, for each* $j \in C$, $P_j(A^{t-1} \cup C) > Q$;
2. $a_i^t = a_i^{t-1}$ *otherwise.*

The diffusion process captures both singleton and group adoption. Each period, any agent i who has not yet adopted will adopt as a singleton if more than fraction Q of his neighbors adopted in the previous period. Each period, a decision-making unit $C \in \mathcal{C}$ with two or more agents will adopt jointly if every agent in the group does better by adopting. That is, if, for every agent in that group, the fraction of his neighbors in C together with the fraction of his neighbors outside of C who adopted in the previous period is greater than Q.

When the set of decision-makers is only singleton agents, $\mathcal{C} = \{i : i \in N\}$, this model is a standard threshold model of contagion (e.g. Morris (2000)). We do not seed the population;

[7] That this is a Nash equilibrium of the static coordination game can be seen from the definition of the adoption process.

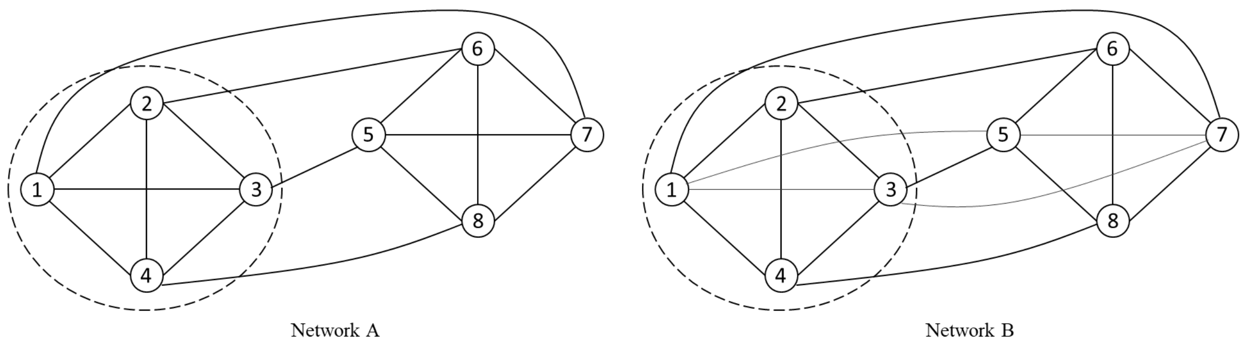

Figure 1: Networks A and B. Thicker links are weight 1. Thinner links are weight 1/2, that is, $w_{13} = w_{57} = w_{15} = w_{37} = 1/2$ in Network B. Decision-making groups are circled (dashed line).

instead, a group of individuals may kick-start the process by deciding to adopt jointly.[8] In our diffusion process, individuals switch from 0 to 1 but cannot switch back. In Supplementary Appendix C we show this is equivalent to a best response updating process where individuals can switch from 0 to 1 and from 1 back to 0.

The Outcome.

Our interest lies in the extent of adoption. We examine the final number of adopters of a technology Q in society $(\{w_{ij}\}, \mathcal{C})$ when the diffusion process comes to a stop. The diffusion process is deterministic and so the final set of adopters of technology Q in society $(\{w_{ij}\}, \mathcal{C})$ is unique. We say an individual i "adopts technology Q" if i is in the final set of adopters. We say individual i "adopts technology Q at time t" to refer to the period at which individual i switches from 0 to 1.

4 A Simple Example

This simple two-network example illustrates the diffusion process and our main result. In Figure 1 in Network A there are eight agents. A line drawn between any two agents is weight 1. Notice that agents $\{1, 2, 3, 4\}$ form a visually well-connected set, where each agent has a link to all others in the set and a single link to an agent outside that set. Similarly for set $\{5, 6, 7, 8\}$. Network B is created by reducing the weight of the horizontal links in these two sets (the link between nodes 1 and 3 and the link between nodes 5 and 7 are reduced from weight 1 to weight 1/2) and by adding two additional links of weight 1/2 between the two

[8]In our model, when the only decision-making units are singleton agents, there is no adoption. In this case, to get contagion started, seeding would be required. Threshold models of diffusion typically focus, like Morris (2000), on the equilibrium of the static coordination game with least adoption, given any seeding. For work studying similarities across all equilibria of a coordination game see Jackson and Storms (2026).

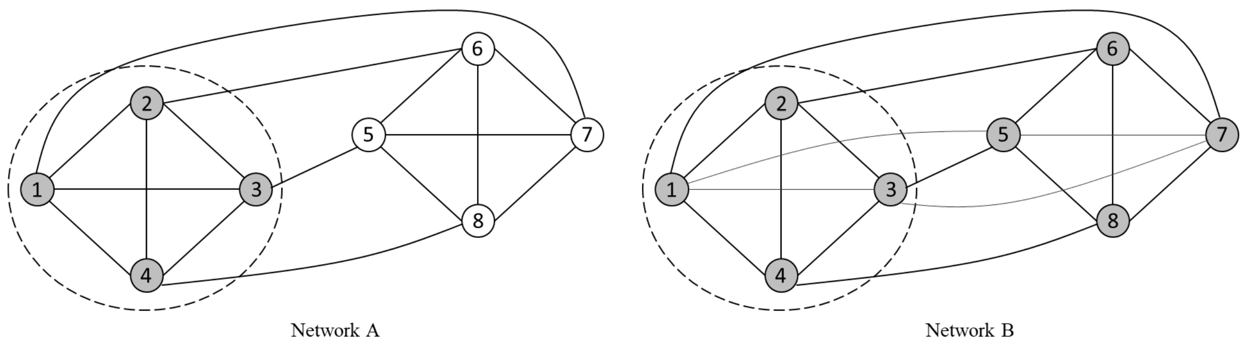

Figure 2: Adoption of a technology $Q = 3/10$. Shaded agents adopt.

sets (between nodes 1 and 5 and between nodes 3 and 7).

In Network A, agent 3 has a share of 3/4 of his connections to agents $\{1, 2, 4\}$ and only 1/4 of his connections to the remaining agents $\{5, 6, 7, 8\}$. In Network B this is equalised somewhat, since the share of agent 3's connections to set $\{1, 2, 4\}$ is reduced and his share of connections to the larger set $\{5, 6, 7, 8\}$ is increased. Symmetrically for agents 1, 5, and 7. Because agents 1, 3, 5 and 7 have a higher concentration of their links within a narrow set in Network A, we say they have more insular connections in Network A than in Network B. The connections of agents 2, 4, 6, and 8 do not change between the two networks. We say that Network A is more insular than Network B. This informal discussion of insularity is consistent with the formal definition that we introduce in Section 6.

The decision-making units are the same in Networks A and B: each agent can make a decision to adopt as a singleton, the circled set $\{1, 2, 3, 4\}$ can make a joint decision to adopt together, and so can any subset of set $\{1, 2, 3, 4\}$.

We examine the diffusion of a technology with threshold $Q = 3/10$ on Networks A and B. This is illustrated in Figure 2. In Network A, set $\{1, 2, 3, 4\}$ can make a joint decision and since each agent in set $\{1, 2, 3, 4\}$ has 3/4 of his neighbors within this set, the set will adopt technology $Q = 3/10$ jointly at $t = 1$. At $t = 2$ there is no further adoption: agents 5, 6, 7, and 8 cannot make a joint decision and each agent has only 1/4 of his neighbors who have already adopted, which is less than the threshold $Q = 3/10$.

In Network B, set $\{1, 2, 3, 4\}$ can make a joint decision and each agent has 5/8 of their neighbors within the set, which is greater than $Q = 3/10$. The set will adopt jointly at $t = 1$. At $t = 2$, agent 5 will adopt as a singleton since he has 3/8 of his neighbors who have adopted at $t = 1$ (which is higher than the threshold $Q = 3/10$). Similarly, for agent 7. At $t = 3$ agents 6 and 8 also adopt.

Compare this to the adoption of a technology with a higher threshold of $Q = 7/10$ illustrated in Figure 3. In Network A, set $\{1, 2, 3, 4\}$ adopts technology $Q = 7/10$ jointly

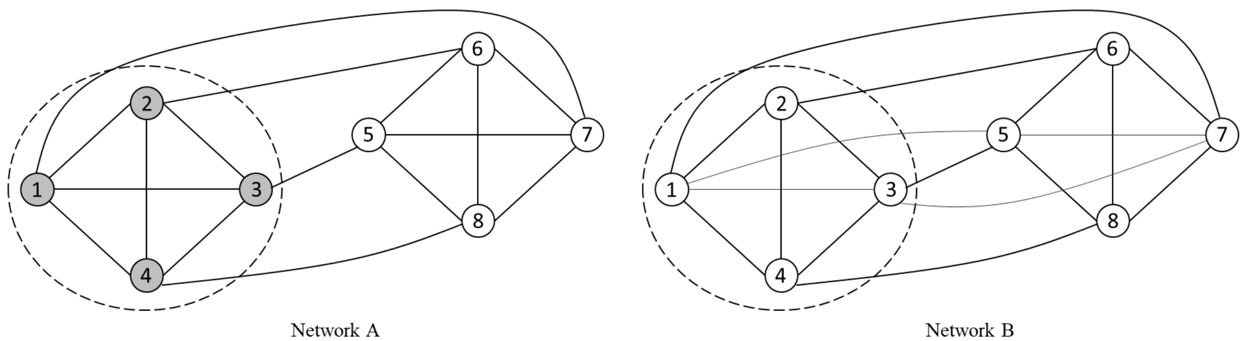

Figure 3: Adoption of a technology $Q = 7/10$. Shaded agents adopt.

since each agent has 3/4 of his neighbors adopting. Technology $Q = 7/10$ does not spread to other agents. In Network B there is no adoption of technology $Q = 7/10$. Individual 3 has only 5/8 of his neighbors in set $\{1, 2, 3, 4\}$ which is less than $Q = 7/10$ and so he is not willing to adopt jointly.

This example highlights the two opposite effects of insular connections on diffusion. When the threshold Q is low, the less insular connections in Network B allow the technology to spread from one part of the network to agents 5 and 7 and then to 6 and 8. When the threshold Q is high, the more insular connections in Network A facilitate joint adoption by set $\{1, 2, 3, 4\}$, while this set does not adopt in Network B.

This example illustrates our main result. There is greater adoption of the technology with the low threshold in Network B, the less insular network, and greater adoption of the technology with the high threshold in Network A, the more insular network. We will show this result holds generally.

5 Characterization

We start by characterizing the set of adopters of a technology Q in a society $(\{w_{ij}\}, \mathcal{C})$.

First, we provide a measure of connectedness of a subset of agents which will be central to the characterization. For a set of agents S, we say that a subset of those agents, R, is "p-connected in set S" if at least one agent in subset R has at least share p of his connections to the remainder of set S.

Definition 1. *A subset of agents $R \subseteq S$ is p-connected in set S if some agent $i \in R$ has at least share p of his connections to set $S \backslash R$, that is, $P_i(S \backslash R) \geq p$.*

A singleton agent is therefore p-connected in set S if that agent has at least share p of his connections to other agents in S. While a group, R, of two or more agents is p-connected in

S if at least one of those agents has fraction p of his connections to agents in set S outside of R. Informally speaking, p-connectedness requires that a group R cannot visually be "an island in S" since a minimal level of connection is required between agents in R and agents in the remainder of set S.

Consider the network in Figure 4. There are 14 agents and links shown have weight 1. Take the set of agents numbered 5 to 14, denote it by $S = \{5, 6, ..., 14\}$. Agent 5 is 3/4-connected in set S, while agent 12 is 2/3-connected in S. The circled subset $\{10, 11, 12, 13\}$ is 1/2-connected in S since agent 10 has half of his connections to the remainder of set S. While the circled subset $\{5, 6, 7, 8\}$ is only 1/3-connected in S.

We say that a set S is p-subgroup cohesive if every decision-making unit in S is p-connected in S.

Definition 2. *A set of agents S is p-subgroup cohesive if each $C \subseteq S$, where $C \in \mathcal{C}$, is p-connected in S.*

Since each singleton agent is a decision-making unit, the definition of subgroup cohesion of a set requires that every agent has at least fraction p of his connections to the set. In addition, it requires that any decision-making unit with two or more agents is p-connected. Putting these two together, informally speaking, a subgroup cohesive set is an island that is sufficiently inter-connected that there are no decision-making islands within.

Consider again the network in Figure 4 and suppose the circled (dashed line) subsets can make a joint decision. The set of decision-making units is thus all singleton agents, each circled set, and all subsets of each circled set. Does the set $S = \{5, 6, ..., 14\}$ satisfy the definition of 1/2-subgroup cohesive? The answer is no. Each singleton agent is 1/2-connected in S and the decision-making unit $\{10, 11, 12, 13\}$ is 1/2-connected in S, but the decision-making unit $\{5, 6, 7, 8\}$ is not connected enough to the rest of set S (it is only 1/3-connected). Set S is 1/3-subgroup cohesive.

The following lemma states that in society $(\{w_{ij}\}, \mathcal{C})$ any set that is sufficiently subgroup cohesive will not adopt technology Q.

Lemma 1. *A set $S \subseteq N$ will not adopt technology Q if S is $(1 - Q)$-subgroup cohesive.*

Even if everyone outside of a set S that is $(1 - Q)$-subgroup cohesive has adopted, each agent in S has at most Q neighbors adopting and so is not willing to adopt as a singleton unless others in S adopt. Similarly, any decision-making group C has at least one agent with more than proportion $1 - Q$ of his connections to set $S \backslash C$ and so that decision-making unit is not willing to adopt technology Q unless other decision-making units in S adopt. Thus, a set that is sufficiently subgroup cohesive faces coordination failure of the type where no one (no agent or decision-making unit) in S adopts because no one else in S has adopted.

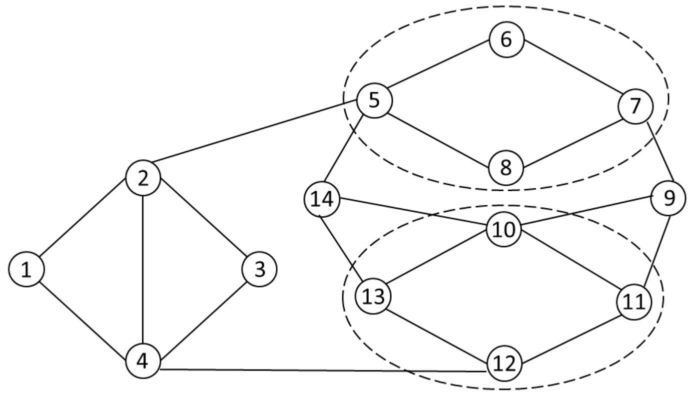

Figure 4: A network with 14 agents. Links shown have weight 1.

Notice that if the only decision-making units in a set S are singleton agents, the definition of p-subgroup cohesive coincides with the definition of p-cohesive from Morris (2000) (a p-cohesive set is a set in which each agent has at least fraction p of his connections to the set) and Lemma 1 tells us that a group that is sufficiently cohesive will not adopt, per results in Morris (2000).

Lemma 1 leads to the characterization of the set of adopters of a technology Q in a society $(\{w_{ij}\}, \mathcal{C})$. Define S_Q as the union of all $(1-Q)$-subgroup cohesive sets. Proposition 1 states that no agent in S_Q adopts, while all other agents adopt technology Q.

Lemma 2. *Define S_Q as the union of all $(1-Q)$-subgroup cohesive sets in society $(\{w_{ij}\}, \mathcal{C})$. The set S_Q is $(1-Q)$-subgroup cohesive and is the unique inclusion-wise maximal element of the set of $(1-Q)$-subgroup cohesive sets in $(\{w_{ij}\}, \mathcal{C})$.*

Proposition 1. *Agents in S_Q do not adopt technology Q. All other agents adopt technology Q.*

What indication does this characterization provide for how network structure shapes diffusion? Each individual agent in a highly subgroup cohesive set has a high share of his connections to that set. Since an agent with a high share of his connections to the set does not want to adopt unless others in the set also adopt, this "over-connectedness" to a set can prevent a new technology from entering via the individual agent. However, a highly subgroup cohesive set also requires that there is no "decision-making island" within. A subset of agents who can make a joint decision and form a decision-making island, such that each agent has a low share of his connections to the wider set, will adopt the technology jointly. In this case, an agent's high share of connections to the subset make it beneficial for the agent to adopt jointly with that subset.

The characterization captures both a negative and a positive impact of insularity on diffusion. On the negative side, an agent with a high share of his connections to a set will not adopt until others in the set adopt and in this way insular connections can hinder the spread of a technology. On the positive side, exactly because this agent is so highly connected to the set, these insular connections mean the agent is willing to adopt the new technology jointly with the other agents in that set.

The characterization indicates that insularity of individuals' connections matters to diffusion, but that its impact could go in either direction. This leads us to the next section and main analysis which explores changes in the insularity of connections within a network to determine which effect dominates.

6 The Architecture of Society

We now turn to our main analysis. We consider how differences in the network and the set of decision-makers impact diffusion.

First, consider how adoption changes when we increase the amount of joint decision-making $\mathcal{C}$, keeping the network $\{w_{ij}\}$ the same. This captures the idea that in some societies joint decision-making may be more prevalent, as well as interventions that increase joint decision-making.

Proposition 2. *Adoption of technology Q is weakly higher in society $(\{w_{ij}\}, \mathcal{C}')$ than in society $(\{w_{ij}\}, \mathcal{C})$, where $\mathcal{C} \subseteq \mathcal{C}'$.*

A group that can make a joint decision can solve their local coordination problem. For example, while I may not find it beneficial to adopt a new messaging application alone, if my group of friends propose that we all adopt the new messaging application then it may be beneficial. Joint decision-making thus mitigates the coordination problem at the heart of the diffusion of a technology with strategic complementarities. Quite naturally then, more joint decision-making in a society increases adoption.

Our main question asks how adoption changes as the network architecture becomes more or less insular, keeping the set of decision-makers the same. The first challenge in answering this question is to find a way to formally compare network structures in terms of their insularity.

Our informal notion of insularity is the extent to which agents concentrate their connections to a narrow set of other agents. Insularity thus implies a disparity in agents' connections such that they are "over-connected" to some agents and "under-connected" to others. Informally, we consider an increase in insularity occurs when agents increase connections to

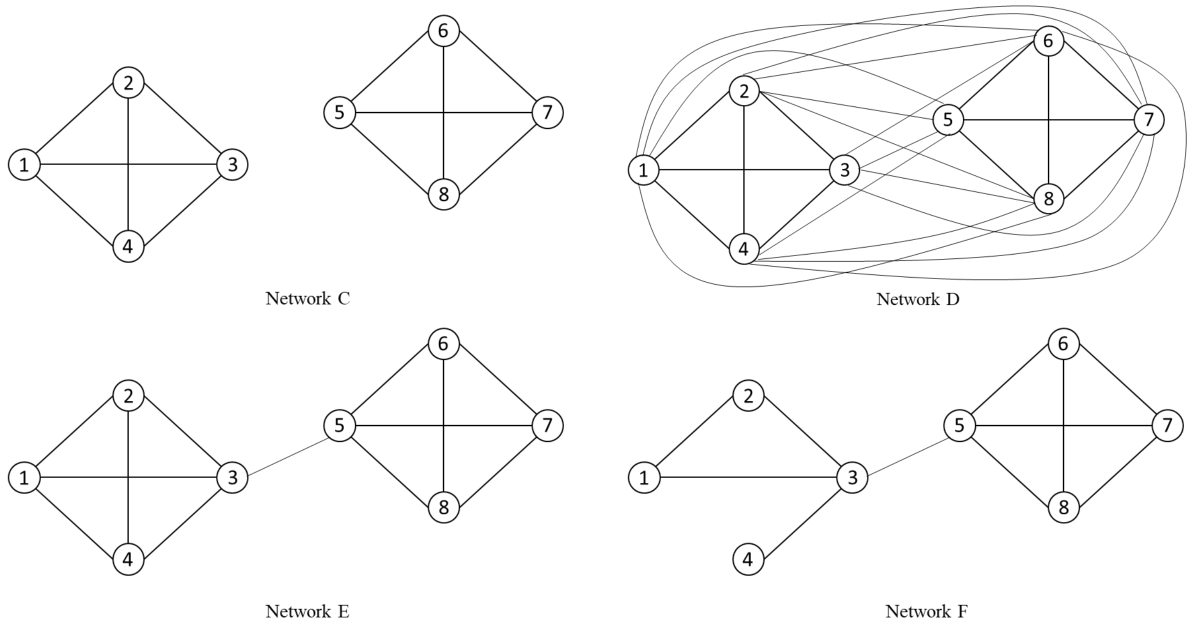

Figure 5: Networks C, D, E and F. Thicker links are weight 1. Thinner links are weight λ, where $0 < \lambda < 1$.

be "over-connected" or further increase connections where already over-connected, and decrease connections to be "under-connected" or where already under-connected. A reduction in insularity occurs with the reverse move towards equalizing links.

How can we formalize this notion to classify networks as more or less insular? We first illustrate some considerations necessary in such a definition using the examples in Figure 5. When we compare two network structures, we fix the agents in the population, $N = \{1, 2, ..., n\}$, and vary only the links between them, w_{ij}.

Compare Network C and Network D in Figure 5. In Network C, agents $1, 2, 3, 4$ each have a link of weight 1 to one another but are not connected to agents $5, 6, 7, 8$, who, similarly, each have a link of weight 1 to one another. Network D consists of the same agents and same links, plus additional links of weight λ, where $0 < \lambda < 1$, from each agent $1, 2, 3, 4$ to each agent $5, 6, 7, 8$. In Network D relative to Network C, agents' links are more equally distributed: for each agent $1, 2, 3, 4$, the proportion of his connections to the set of agents $\{1, 2, 3, 4\}$ reduces from 1 to $\frac{3}{3+4\lambda}$ and the proportion of his connections to agents $\{5, 6, 7, 8\}$ increases from 0 to $\frac{4\lambda}{3+4\lambda}$. Similarly, for agents $5, 6, 7, 8$. In fact, Network D can be formed in a systematic way from Network C: Network D is a convex combination of Network C and the complete network with links of weight 1, following the definition below.

Definition 3. *Let $\{w\}$ denote the complete network with $w_{ij} = w \in (0, 1]$ for all $i \neq j \in N$ and $w_{ii} = 0$ for all $i \in N$. Network $\{\bar{w}_{ij}\}$ is the λ-convex combination of a general network*

$\{w_{ij}\}$ and the complete network $\{w\}$ if $\bar{w}_{ij} = (1-\lambda)w_{ij} + \lambda w$ for $\lambda \in (0,1)$.

In a complete network, each agent's links are equally distributed between all other agents and thus the λ-convex combination of a network $\{w_{ij}\}$ and a complete network moves towards an equalizing of agents' links, "ironing out" the original network $\{w_{ij}\}$ to reduce connections when agents are over-connected and increase them when under-connected.[9] The reverse direction, moving from the λ-convex combination network to the original network $\{w_{ij}\}$, increases agents' connections towards being over-connected and reduces connections towards being under-connected. Networks and their convex combinations with a complete network thus provide a systematic and intuitive comparison of more and less insular networks.

However, we would like a definition that allows for comparison of a wider range of networks, beyond convex combinations with a complete network. A broader comparison requires dealing with the complex multidimensionality of networks. For a population of n agents, there are $\frac{n(n-1)}{2}$ different links of potentially different weights. Therefore, two networks with the same agents can vary in such a multitude of different ways that it can be difficult to meaningfully compare their structures.[10] Even the simple examples in Figure 5 illustrate this complexity of comparisons. Compared to Network C, Network E has an additional link of weight λ connecting nodes 3 and 5, while Network F has a variety of differences, including an additional link of weight λ connecting 3 and 5, and removal of the links connecting 1 and 4 and connecting 2 and 4.

Our approach to compare the potential variety of differences throughout two network structures is to first define a "local" increase in insularity in one network relative to another, which specifies whether there is an increase in insularity among some agents (Definition 4). We then say that one network structure is overall more or less insular than another when all local changes between the two networks move in the same direction, towards more or less insularity (Definition 5).

The definition of a local increase in insularity is based on our informal notion that an increase in insularity occurs when agents increase connections to be (or where already) over-connected and decrease connections to be (or where already) under-connected. Such a definition requires a measure of over-connected and under-connected. Since over- and under-connected is a relative concept, dependent on the network, our measure contrasts the relative connections of two agents. We consider agent i is over-connected to set S and

[9] Specifically, take agent i in a set S comprising $k-1$ other agents. The proportion of those other agents in the population is $\frac{k-1}{n-1}$. If the proportion of i's connections to S is higher than $\frac{k-1}{n-1}$ in network $\{w_{ij}\}$, then in the λ-convex combination with a complete network, the proportion of i's connections to S strictly decreases towards $\frac{k-1}{n-1}$. Analogously, if the proportion of his connections is less than $\frac{k-1}{n-1}$ in network $\{w_{ij}\}$.

[10] A comparison between a network and its convex combination with a complete network provides a way to manage this multidimensionality since the changes between one network and the other are very systematic.

agent j is under-connected to a superset $R \supset S$ when agent i has a higher fraction of his connections to the strictly smaller set S than the fraction of j's connections to superset R, where $i \in S$, $j \in R$. This "crossing," such that one agent's connections to a smaller subset are more concentrated than another agent's connections to a strictly larger superset, captures our notion of the agents being over- and under-connected with respect to those sets. For example, in Network C, agent 3 is over-connected (has all his connections) to set $S = \{1, 2, 3, 4\}$ while agent 5 is under-connected (has a third of his connections) to the superset $R = \{1, 2, 3, 4, 5, 6\}$.

Definition 4 then states that there is a "local increase in insularity" in one network relative to another when two agents' connections become more insular within some sets: that is, one agent's connections become more concentrated in a smaller set, while the other agent's connections become less concentrated among a larger superset, to the extent that the first agent's connections end up more concentrated to the smaller set than the second agent's connections are to the larger set.

Definition 4. *There is a local increase in insularity in network $\{w_{ij}\}$ relative to network $\{\tilde{w}_{ij}\}$ when, for some agents i and j and sets S and R, where $i \in S$, $j \in R$ and $S \subset R \subseteq N$, statements 1. and 2. hold.*

1. *The proportion of agent i's connections to set S is strictly higher and the proportion of agent j's connections to set R is strictly lower in network $\{w_{ij}\}$ than in network $\{\tilde{w}_{ij}\}$.*

2. *In network $\{w_{ij}\}$ the proportion of agent i's connections to S is weakly higher than the proportion of agent j's connections to R.*

There is a local increase in insularity in Network C relative to Network D whereby agent 3 strictly increases the proportion of his links to agents in set $\{1, 2, 3, 4\}$ and agent 5 strictly reduces the proportion of his links to agents $\{1, 2, 3, 4, 5, 6\}$ in Network C, such that agent 3 has a higher proportion of his connections to the smaller set $\{1, 2, 3, 4\}$ than agent 5 has to the superset $\{1, 2, 3, 4, 5, 6\}$. There is a similar local increase in insularity for agents 3 and 5 in Network A relative to Network B in Figure 1.

To compare the overall structure of two networks, any local differences in one network relative to the other should move insularity in the same direction. This is stated in Definition 5.

Definition 5. *A network $\{\tilde{w}_{ij}\}$ is less insular than another network $\{w_{ij}\}$ ($\{w_{ij}\}$ is more insular than $\{\tilde{w}_{ij}\}$) if 1. holds, and strictly so if 1. and 2. hold.*

1. *There is no local increase in insularity in network $\{\tilde{w}_{ij}\}$ relative to network $\{w_{ij}\}$.*

2. There is a local increase in insularity in network $\{w_{ij}\}$ relative to network $\{\tilde{w}_{ij}\}$.

The definition of a network as less insular than another, gives a weak inequality in the sense that it requires that no agent's connections are more insular in the less insular network. The stronger requirement, strictly less insular, additionally requires that some agent's connections are more insular in the more insular network.[11]

We next illustrate Definition 5 with some examples. Network D is less insular than Network C, since there is no local increase in insularity in Network D relative to Network C, and is also strictly less insular, since there is a local increase in insularity for agents 3 and 5 in Network C relative to D. Similarly, Network E is strictly less insular than Network C. In our initial example in Section 4, Network B is less insular than Network A. As discussed in Section 4, in Network A, each agent 1, 3, 5, and 7 has a higher concentration of links to a narrow set. This generates local increases in insularity in Network A relative to Network B: for example, agent 3 increases the share of his connections to set $\{1, 2, 3, 4\}$ and reduces his connections to the larger set $\{3, 5, 6, 7, 8\}$ while agent 5 increases the share of his connections to the smaller set $\{5, 6, 7, 8\}$ and reduces his connections to set $\{1, 2, 3, 4, 5\}$.

In contrast, Network C and Network F in Figure 5 cannot be compared by Definition 5. There is a local increase in insularity in Network C relative to Network F in one part of the network (when we compare agents 3 and 5 as above) but also a local increase in insularity in Network F relative to Network C in another part of the network (the proportion of agent 2's links to agent 1 increases from 1/3 in Network C to 1/2 in Network F, while the proportion of agent 4's links to set $\{1, 2, 4\}$ falls from 2/3 to 0). We therefore do not have a meaningful way to rank these two networks in terms of their insularity.[12]

Lemma 3 confirms that Definition 5 encompasses a comparison of any network $\{w_{ij}\}$ and its convex combinations with a complete network. By Definition 5, the convex combination network is always less insular than the original network $\{w_{ij}\}$.

Lemma 3. *For any network $\{w_{ij}\}$ that is not complete, the λ-convex combination of $\{w_{ij}\}$ and the complete network $\{w\}$ is less insular.*

Lemma 4 shows that Definition 5 also encompasses other intuitive cases of more and less insular networks. Consider an individual with ten colleagues who spends half his time with two of them. A reduction in the amount of time the individual spends with those two

[11]The requirements on the definition of a less insular network are also weaker in the sense that it is possible to find two networks where each network is less insular than the other, which is not possible with the definition of strictly less insular. For two networks that are each less insular than the other, the following Theorem 1 holds but holds trivially, in that there is weakly more adoption in one network than the other for all Q. This can be seen in the proof of Theorem 1.

[12]In that sense, Definition 5 is a partial characterization of networks as more or less insular.

colleagues makes his connections to them more proportionate to their size in the population. Lemma 4 shows that changes that result in agents' connections becoming more proportionate relative to population size result in a less insular network. Definition 6 first specifies a change that leads an agent i to have more proportionate connections to a set S.[13]

Definition 6. *An agent i in a set $S \subseteq N$ that comprises $k-1$ other agents has more proportionate connections to S in network $\{\tilde{w}_{ij}\}$ than in network $\{w_{ij}\}$ if:*

- *when the proportion of i's connections to S is greater than $\frac{k-1}{n-1}$ in network $\{w_{ij}\}$ then the proportion of his connections to S in network $\{\tilde{w}_{ij}\}$ is weakly lower, but not below $\frac{k-1}{n-1}$;*
- *when the proportion of i's connections to S is less than $\frac{k-1}{n-1}$ in network $\{w_{ij}\}$ then the proportion of his connections to S in network $\{\tilde{w}_{ij}\}$ is weakly higher, but not above $\frac{k-1}{n-1}$.*

Lemma 4. *For any networks $\{\tilde{w}_{ij}\}$ and $\{w_{ij}\}$ where, for all $i \in N$ and all sets $S \subseteq N$, with $i \in S$, agent i has more proportionate connections to $S \subseteq N$ in network $\{\tilde{w}_{ij}\}$ than $\{w_{ij}\}$, then network $\{\tilde{w}_{ij}\}$ is less insular.*

Our main result given in Theorem 1 compares diffusion across more and less insular networks, holding fixed the set of decision-making units $\mathcal{C}$. We find that less insular networks facilitate greater adoption of technologies with low thresholds, while more insular networks facilitate greater adoption of technologies with high thresholds.

Theorem 1. *For societies $(\{w_{ij}\}, \mathcal{C})$ and $(\{\tilde{w}_{ij}\}, \mathcal{C})$, where network $\{\tilde{w}_{ij}\}$ is less insular than network $\{w_{ij}\}$, there exists a threshold $\mu \in [0,1]$ such that:*

- *for technologies $Q < \mu$, adoption is weakly higher in the society with the less insular network;*
- *for technologies $Q \geq \mu$, adoption is weakly lower in the society with the less insular network.*

Theorem 1 finds that the advantage of less insular connections in diffusing low threshold technologies becomes a disadvantage in the diffusion of high threshold technologies. At

[13] Notice that Definition 5 allows for broader comparisons beyond simply proportionate changes per Definition 6. This can be seen by comparing Networks C and E. The addition of the link of weight $0 < \lambda < 1$ between nodes 3 and 5 does not result in 3 and 5 having more proportionate connections to each other if $\lambda > 1/2$, since this moves their connections above their proportion in the population. However, Network E is less insular than Network C according to Definition 5.

the heart of this theorem lies the two different roles insular connections play in diffusion that appeared in our example in Section 4 and characterization in Section 5. Consider, for example, an agent with a high share of his connections to a narrow group of other agents. Increasing the share of his connections to the group makes it less beneficial for that agent to adopt as a singleton, unless others in the group adopt. This makes it harder for a technology to spread from outside into the group via singleton adoption. In contrast, if this narrow group can make a joint decision, increasing the agent's share of his connections to this narrow group increases his benefit of jointly adopting the technology with that group. To highlight the subtlety in this intuition, insularity of connections does not mean that agents are better able to make joint decisions. Rather, more concentrated connections render it more profitable for an agent to adopt together with those to whom his connections are concentrated.[14]

The second step of intuition relies on noticing that different technologies and behaviors diffuse differently. Consider the extreme case of a technology with a very high threshold of adoption, meaning that an agent needs most of his contacts to adopt to be willing to adopt himself. In turn, his contacts need most of their contacts to be willing to adopt. Thus, joint decision-making among agents is crucial to the adoption and spread of this technology. Because insular connections facilitate joint adoption, more insular networks are beneficial for spreading a high-threshold technology. Contrast this with a technology with a very low threshold, which means that an agent will adopt even without many of his contacts adopting. This technology will spread from contact to contact largely without the need for joint decision-making and so insular connections are not needed to get this technology adopted. Since insular connections are not needed and can inhibit spread when agents cannot make joint decisions, less insular networks are more effective at diffusing low threshold technologies.

This intuition can be seen in our initial example in Section 4. When the threshold of adoption is low, $Q = 3/10$, once diffusion has started, it can spread from one part of the network to another without the need for joint decision-making. It spreads only via the less insular connections in Network B. When the threshold of adoption is high, $Q = 7/10$, joint decision-making is important in getting this technology adopted and the more insular connections in Network A render the decision-making group willing to adopt.

[14]If it is true empirically that insularity of connections means those agents are better able to make joint decisions, then, in the context of this model, a more insular society would also be one where there is more joint decision-making. Here, when we vary the network, we hold fixed the set of decision-making units.

7 Heterogeneous Thresholds

Our model assumes a homogeneous threshold of adoption Q for each agent. In this section, we show that our main result, Theorem 1, continues to hold when agent thresholds vary somewhat. We then show the conditions required for the specification of Theorem 1 to break.

The population N and society $(\{w_{ij}\}, \mathcal{C})$ are the same as the main model. A technology is summarized by its threshold of adoption $Q \in [0,1]$, but now each agent $i \in N$ also has an idiosyncratic component $\theta_i \in \mathbb{R}$, which is common across technologies Q. An agent is better off adopting technology Q if more than fraction $Q + \theta_i$ of his neighbors adopt. The diffusion process adjusts accordingly.

Diffusion Process *At each $t \geq 1$, for each $i \in N$,*

1. $a_i^t = 1$ *if $i \in C \in \mathcal{C}$ where $C \subseteq N \backslash A^{t-1}$ and, for each $j \in C$, $P_j(A^{t-1} \cup C) > Q + \theta_j$;*
2. $a_i^t = a_i^{t-1}$ *otherwise.*

The complete analysis of heterogeneous thresholds and all proofs are contained in Supplementary Appendix B. Supplementary Appendix B extends the characterization in Section 5 to the case of heterogeneous thresholds. The intuition behind subgroup cohesive sets in preventing diffusion persists in the case of heterogeneous thresholds, however, the concentration of links required within a group is moderated by each agent's idiosyncratic component. For example, agents with high idiosyncratic components do not require such a high share of their neighbors within a set to prevent singleton adoption.

Proposition 3 shows that our main result, Theorem 1, holds when agents have idiosyncratic components but the values are bounded. For example, this applies to settings like the adoption of a communication technology which is valuable only if many of an individual's contacts adopt, so that individual thresholds vary somewhat but remain high for all agents. In comparing two networks, we now fix not only the population N but also the idiosyncratic components $(\theta_1, ..., \theta_n)$.

Proposition 3. *There exists $\gamma > 0$ such that if $\theta_i \in [-\frac{\gamma}{2}, \frac{\gamma}{2}]$ for all $i \in N$, then for societies $(\{w_{ij}\}, \mathcal{C})$ and $(\{\tilde{w}_{ij}\}, \mathcal{C})$, where network $\{\tilde{w}_{ij}\}$ is less insular than network $\{w_{ij}\}$, there exists a threshold $\mu \in [0,1]$ such that:*

- *for technologies $Q < \mu$, adoption is weakly higher in the society with the less insular network;*

- *for technologies $Q \geq \mu$, adoption is weakly lower in the society with the less insular network.*

If there is no bound on the values of the idiosyncratic components, then one agent can have a very low personal threshold of adoption $Q+\theta_i$ and another agent a very high personal threshold of adoption $Q+\theta_j$, for the same technology Q. In this case, the specification of the main result may not hold. The intuition for why it may not hold is as follows. A high idiosyncratic component results in a high threshold of adoption for that agent while a low idiosyncratic component results in a low threshold of adoption. Intuition then comes directly from Theorem 1: more insular connections tend to facilitate diffusion among agents with high idiosyncratic components while less insular connections tend to facilitate diffusion among agents with low idiosyncratic components. The intuition driving Theorem 1 continues to hold with idiosyncratic thresholds, however, the specification of Theorem 1 no longer necessarily holds. When some agents have very high idiosyncratic components and some agents have very low idiosyncratic components, the same technology diffuses in different ways across the same network and we lose the clean comparison between two networks in Theorem 1 that is possible with a homogeneous threshold.

Proposition 4 shows this formally, providing necessary conditions on the idiosyncratic components for our main result (Theorem 1) not to hold.

Proposition 4. *Consider societies $(\{w_{ij}\}, \mathcal{C})$ and $(\{\tilde{w}_{ij}\}, \mathcal{C})$, where network $\{\tilde{w}_{ij}\}$ is less insular than network $\{w_{ij}\}$. For some $Q_2 > Q_1$,*

- *adoption of technology Q_1 is strictly lower in the society with the less insular network, and*

- *adoption of technology Q_2 is strictly higher in the society with the less insular network,*

only if there exists an agent i who adopts technology Q_1 in $(\{w_{ij}\}, \mathcal{C})$ but not in $(\{\tilde{w}_{ij}\}, \mathcal{C})$, another agent j who adopts technology Q_2 in $(\{\tilde{w}_{ij}\}, \mathcal{C})$ but not in $(\{w_{ij}\}, \mathcal{C})$, and $\theta_i > \theta_j + Q_2 - Q_1$.

Proposition 4 states that a necessary condition for the main finding to break is that there are two agents, i and j, one with a significantly higher idiosyncratic component than the other. Agent i, with the high idiosyncratic component θ_i, adopts the low threshold technology Q_1 in the more insular network but not in the less insular network. Agent j, with the low idiosyncratic component, adopts the high threshold technology Q_2 in the less insular network but not in the more insular network. Only when these conditions hold, the specification of Theorem 1 may not hold. However, even under these conditions, the general intuition behind Theorem 1 persists.

8 Discussion and Conclusion

When new technologies or behaviors spread, some subsets of agents can coordinate their decisions to adopt or not. In this context, we explored how network structure shapes diffusion and showed how insularity of connections determines the extent to which it spreads or fails. We conclude with some policy implications and suggestions of future avenues for research.

An immediate implication of our results is to inform firms and policymakers which markets to enter. For a firm with a low (high) threshold technology, targeting less (more) insular societies will maximize uptake.[15] For example, when Facebook launched, the value of adopting Facebook relied heavily on an individual's contacts also adopting, suggesting a high threshold of adoption and making it optimal for Facebook to target insular societies. This is consistent with Facebook's choice to initially launch for students only within the closed settings of certain universities. The paper does not analyze seeding and so the implication above should be taken in the context of no seeding. One natural avenue for future work is to examine optimal seeding in the context of network structure, joint decision-making, and the threshold Q.

Our framework finds that joint decision-making unambiguously increases adoption. This raises a novel intervention in the diffusion literature: encouraging joint decision-making. In practice, this is not novel. There are a proliferation of programs that are designed to encourage joint decision-making between neighbors in the adoption of safe sanitation (Pickering et al., 2015).[16] Firms design features of their technology or pricing that could encourage joint decision-making. For example, giving a discount when multiple family members adopt a technology (e.g. mobile phone payment plans) may encourage family members to discuss and make adoption decisions together. During periods of protest, governments can try to discourage joint decision-making by prohibiting in person gatherings or online communication.[17]

This raises a second avenue for future research. If encouraging or discouraging joint decision-making is costly, for which network structures and technologies is it most profitable to do so? The answer is not obvious: more insular groups find it more profitable to jointly adopt, but adoption by a more insular group does not facilitate as much follow-on diffusion.

[15]Given that the extent of insularity of groups is a major dimension along which social networks vary and is well-documented (Gorodnichenko and Roland, 2017), this is a relevant choice.

[16] In a field experiment, Bakhtiar et al, (2021) show that encouraging joint agreement among neighbors to adopt and maintain safe sanitation is effective at increasing adoption.

[17]During protests that spread throughout England in the 1830s, officials complained that local "beer houses" had increased participation by facilitating joint decision-making among villagers (Holland, 2005). Analysis of Russian protests in 2011 suggests social media facilitated joint decision-making by getting individuals together online to jointly agree to protest together offline, and this increased participation in the protests (Enikolopov et al., 2020).

This also relates to work on targeted pricing interventions (Fainmesser and Galeotti, 2020) or other subsidies (Galeotti et al., 2020). To what extent is it useful to target subsidies at joint decision-making groups versus others? A next step to answer this is to bring pricing into a joint decision-making framework.

Finally, a direct implication of our findings is for technological specialization. All else equal, less insular networks have higher adoption of low threshold technologies while more insular networks have higher adoption of high threshold technologies. To speak further to this specialization, future work should consider which technological features result in high or low thresholds of adoption. We end with a broad avenue for future research, to understand the role of the network and this specialization when technology adoption, as well as innovation, build upon one another.

References

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2015) "Systemic Risk and Stability in Financial Networks," *American Economic Review*, 105 (2), 564–608.

Acemoglu, Daron, Asuman Ozdaglar, and Ercan Yildiz (2011) "Diffusion of Innovations in Social Networks," *Proc. of IEEE Conference on Decision and Control*.

Ambrus, Attila, Markus Mobius, and Adam Szeidl (2014) "Consumption Risk-Sharing in Social Networks," *American Economic Review*, 104 (1), 149–82.

Anderlini, Luca and Antonella Ianni (1996) "Path Dependence and Learning from Neighbors," *Games and Economic Behavior*, 13 (2), 141–177.

Bakhtiar, M. Mehrab, Raymond Guiteras, James Levinsohn, and Ahmed Mushfiq Mobarak (2021) "Social and Financial Incentives for Overcoming a Collective Action Problem," *mimeo*.

Bandiera, Oriana and Imran Rasul (2006) "Social Networks and Technology Adoption in Northern Mozambique," *Economic Journal*, 116 (514), 869–902.

Beaman, Lori, Ariel BenYishay, Jeremy Magruder, and Ahmed Mushfiq Mobarak (2021) "Can Network Theory-Based Targeting Increase Technology Adoption?" *American Economic Review*, 111 (6), 1918–43.

Becker, Sascha O., Yuan Hsiao, Steven Pfaff, and Jared Rubin (2020) "Multiplex Network Ties and the Spatial Diffusion of Radical Innovations: Martin Luther's Leadership in the Early Reformation," *American Sociological Review*, 85 (5), 857–894.

Belloc, Marianna and Samuel Bowles (2013) "The Persistence of Inferior Cultural-Institutional Conventions," *American Economic Review*, 103 (3), 93–98.

Björkegren, Daniel (2019) "The Adoption of Network Goods: Evidence from the Spread of Mobile Phones in Rwanda," *The Review of Economic Studies*, 86 (3), 1033–1060.

Blume, Lawrence E. (1993) "The Statistical Mechanics of Strategic Interaction," *Games and Economic Behavior*, 5 (3), 387–424.

——— (1995) "The Statistical Mechanics of Best-Response Strategy Revision," *Games and Economic Behavior*, 11 (2), 111–145.

Bursztyn, Leonardo, Davide Cantoni, David Y. Yang, Noam Yuchtman, and Y. Jane Zhang (2021) "Persistent Political Engagement: Social Interactions and the Dynamics of Protest Movements," *American Economic Review: Insights*, 3 (2), 233–50.

Campbell, A, D Thornton, and Y Zenou (2025) "Strategic Influence: The Diffusion of Prosocial and Antisocial Behaviors," *CEPR Discussion Paper No. 20425*.

Easley, D and J Kleinberg (2010) *Networks, Crowds, and Markets: Reasoning About a Highly Connected World.*, New York, NY, USA: Cambridge University Press.

Elliott, Matthew, Benjamin Golub, and Matthew O. Jackson (2014) "Financial Networks and Contagion," *American Economic Review*, 104 (10), 3115–53.

Enikolopov, Ruben, Alexey Makarin, and Maria Petrova (2020) "Social Media and Protest Participation: Evidence From Russia," *Econometrica*, 88 (4), 1479–1514.

Fainmesser, Itay P. and Andrea Galeotti (2020) "Pricing Network Effects: Competition," *American Economic Journal: Microeconomics*, 12 (3), 1–32.

Galeotti, Andrea, Benjamin Golub, and Sanjeev Goyal (2020) "Targeting Interventions in Networks," *Econometrica*, 88 (6), 2445–2471.

Gorodnichenko, Yuriy and Gerard Roland (2017) "Culture, Institutions, and the Wealth of Nations," *The Review of Economics and Statistics*, 99 (3), 402–416.

Goyal, Sanjeev (1996) "Interaction Structure and Social Change," *Journal of Institutional and Theoretical Economics*, 152 (3), 472–494.

Granovetter, Mark (1978) "Threshold Models of Collective Behavior," *American Journal of Sociology*, 83 (6), 1420–1443.

Guiteras, Raymond, James Levinsohn, and Ahmed Mushfiq Mobarak (2019) "Demand Estimation with Strategic Complementarities: Sanitation in Bangladesh," *working paper.*

Holland, Michael (2005) "The Captain Swing Project," in Holland, Michael ed. *Swing Unmasked: The Agricultural Riots of 1830 to 1832 and their Wider Implications*: FACHRS Publications, UK.

Jackson, Matthew O. and Evan C. Storms (2026) "Behavioral Communities and the Atomic Structure of Networks," *American Economic Journal: Microeconomics*, 18 (1).

Jackson, Matthew and Agathe Pernoud (2021) "Systemic Risk in Financial Networks: A Survey," *Annual Review of Economics*, 13, 171–202.

Kanik, Zafer (2024) "From Lombard Street to Wall Street: Systemic Risk, Rescues, and Stability in Financial Networks," *mimeo.*

Kets, Willemien, Garud Iyengar, Rajiv Sethi, and Samuel Bowles (2011) "Inequality and Network Structure," *Games and Economic Behavior*, 73 (1), 215–226.

Leister, C Matthew, Yves Zenou, and Junjie Zhou (2022) "Social Connectedness and Local Contagion," *Review of Economic Studies*, 89 (1), 372–410.

Lim, Y., A. Ozdaglar, and A. Teytelboym (2016) "A Simple Model of Cascades in Networks," *Working Paper.*

Montanari, A. and A. Saberi (2010) "The Spread of Innovations in Social Networks," *Proceedings of the National Academy of Sciences*, 107 (47), 20196–20201.

Morris, Stephen (2000) "Contagion," *Review of Economic Studies*, 67 (1), 57–78.

Munshi, Kaivan and Jacques Myaux (2006) "Social norms and the fertility transition," *Journal of Development Economics*, 80 (1), 1–38.

Newton, Jonathan and Simon D Angus (2015) "Coalitions, Tipping points and the Speed of Evolution," *Journal of Economic Theory*, 157, 172–187.

Olson, Parmy (2021) "Telegram and Signal Will Never Match the Ubiquity of WhatsApp," *The Washington Post, https://www.washingtonpost.com/business/telegram-and-signal-will-never-match-the-ubiquity-of-whatsapp/ 2022-05-26.*

Pickering, Amy, Habiba Djebbari, Carolina Lopez, Massa Coulibaly, and Maria Alzua (2015) "Effect of a Community-led Sanitation Intervention on Child Diarrhoea and Child Growth in Rural Mali: A Cluster-randomised Controlled Trial," *The Lancet Global Health*, 3, e701–e711.

Rogers, L. C. G. and L. A. M. Veraart (2013) "Failure and Rescue in an Interbank Network," *Management Science*, 59 (4), 882–898.

Sadler, Evan (2020) "Diffusion Games," *American Economic Review*, 110 (1), 225–270.

Sengupta, Amit (2017) "What is Community Led Total Sanitation (CLTS) Top Things to Know," *CLTS Foundation*, http://www.cltsfoundation.org.

Turrell, Mark (2017) "Bye Whatsapp, Hello Telegram (And Later... Bye TG, Hello WhatsNext)," *The Huffington Post, https://www.huffpost.com/entry/bye-whatsapp-hello-telegram/ 2022-10-05.*

Warner, Andrew (2011) "How Dropbox Became The Startup Steve Jobs Wished To Own," *Mixergy, https://mixergy.com/interviews/drew-houston-dropbox-interview/ 2022-10-24.*

Young, H. Peyton (1996) "The Economics of Convention," *Journal of Economic Perspectives*, 10 (2), 105–122.

——— (2011) "The Dynamics of Social Innovation," *Proceedings of the National Academy of Sciences*, 108 (4), 21285–21291.

Young, H. Peyton and Mary A. Burke (2001) "Competition and Custom in Economic Contracts: A Case Study of Illinois Agriculture," *American Economic Review*, 91 (3), 559–573.

Appendix A

Lemma 1 *A set $S \subseteq N$ will not adopt technology Q if S is $(1-Q)$-subgroup cohesive.*

Proof of Lemma 1 By contradiction, suppose some agent in S adopts. Then there exists a time $t-1$ at which no agent in S adopts and a time $t \geq 1$ at which an agent $i \in S$ adopts. Therefore, by the diffusion process, there exists $C \subseteq N \setminus A^{t-1}$, where $i \in C$ and $C \in \mathcal{C}$, such that, for all $j \in C$, $P_j(A^{t-1} \cup C) > Q$. For all $j \in C \cap S$, we can rewrite this as $1-Q > P_j(N) - P_j(A^{t-1} \cup (C \setminus S)) - P_j(C \cap S) \geq P_j(S \setminus (C \cap S))$, where the final inequality follows since $S \subseteq N \setminus (A^{t-1} \cup (C \setminus S))$. Then we have $P_j(S \setminus (C \cap S)) < 1-Q$ for all $j \in C \cap S$.

A contradiction since $C \cap S \in \mathcal{C}$ and S is $(1-Q)$-subgroup cohesive. $\square$

Lemma 2 *Define S_Q as the union of all $(1-Q)$-subgroup cohesive sets in society $(\{w_{ij}\}, \mathcal{C})$. The set S_Q is $(1-Q)$-subgroup cohesive and is the unique inclusion-wise maximal element of the set of $(1-Q)$-subgroup cohesive sets in $(\{w_{ij}\}, \mathcal{C})$.*

Proof of Lemma 2 Take two sets S' and S'' that are $(1-Q)$-subgroup cohesive. We show the union is $(1-Q)$-subgroup cohesive. Then, by iteration, S_Q is $(1-Q)$-subgroup cohesive. Take any $C \subseteq S' \cup S''$ where $C \in \mathcal{C}$. Without loss suppose $C \cap S' \neq \emptyset$. By assumption, $C \cap S' \in \mathcal{C}$ and so, by the definition of subgroup cohesion, there exists $i \in C \cap S'$ such that $P_i(S' \backslash (C \cap S')) \geq 1-Q$. Since $S' \backslash (C \cap S') = S' \backslash C \subseteq (S' \cup S'') \setminus C$ then there exists $i \in C \cap S'$ such that $P_i((S' \cup S'') \backslash C) \geq P_i(S' \backslash (C \cap S')) \geq 1-Q$. Thus $S' \cup S''$ is $(1-Q)$-subgroup cohesive. $\square$

Proposition 1 *Agents in S_Q do not adopt technology Q. All other agents adopt technology Q.*

Proof of Proposition 1 By Lemmas 1 and 2 no agent in S_Q adopts. We show by contradiction that all other agents adopt technology Q. Suppose some non-empty set $S_0 \in N \backslash S_Q$ does not adopt. By Lemma 2, $S_Q \cup S_0$ is not $(1-Q)$-subgroup cohesive. Therefore, by the definition of subgroup cohesion, there exists $C \subseteq S_0 \cup S_Q$, where $C \in \mathcal{C}$ such that, for all $i \in C$, $P_i((S_0 \cup S_Q) \backslash C) < 1-Q$. This is rewritten as, for all $i \in C$, $P_i(N \backslash (S_0 \cup S_Q)) + P_i(C) > Q$, and so, by the diffusion process, agents in $C \subseteq S_0 \cup S_Q$ adopt. $\square$

Proposition 2 *Adoption of technology Q is weakly higher in society $(\{w_{ij}\}, \mathcal{C}')$ than in society $(\{w_{ij}\}, \mathcal{C})$, where $\mathcal{C} \subseteq \mathcal{C}'$.*

Proof of Proposition 2 In society $(\{w_{ij}\}, \mathcal{C}')$, let S_Q be the set of agents that do not adopt technology Q. By Lemma 2 and Proposition 1, S_Q is $(1-Q)$-subgroup cohesive: for each $C \subseteq S_Q$ where $C \in \mathcal{C}'$ there exists $i \in C$ such that $P_i(S_Q \backslash C) \geq 1-Q$. Since each $C \in \mathcal{C}$ is also an element of $\mathcal{C}'$, then the above also holds for each $C \subseteq S_Q$ where $C \in \mathcal{C}$. Therefore, by Lemma 1, S_Q is part of the set of non-adopters of technology Q in $(\{w_{ij}\}, \mathcal{C})$. $\square$

Lemma 3 *For any network $\{w_{ij}\}$ that is not complete, the λ-convex combination of $\{w_{ij}\}$ and the complete network $\{w\}$ is less insular.*

Proof of Lemma 3

The proof proceeds in two steps. Step 2 shows that there are no local increases in insularity in the λ-convex combination network relative to network $\{w_{ij}\}$ and so, by Definition 5, the λ-convex combination network is less insular.

Step 1: Let a, b, c, A, B, C be positive real numbers and $\gamma \in (0,1)$, then $\frac{A}{B+C} > \frac{a}{b+c}$ if and only if $\frac{A}{B+C} > \frac{(1-\gamma)A+\gamma a}{(1-\gamma)B+(1-\gamma)C+\gamma b+\gamma c} > \frac{a}{b+c}$.

Proof of Step 1: Let $\frac{A}{B+C} > \frac{a}{b+c}$. Suppose $\frac{(1-\gamma)A+\gamma a}{(1-\gamma)B+(1-\gamma)C+\gamma b+\gamma c} \leq \frac{a}{b+c}$. Rearranging gives $\frac{A}{B+C} \leq \frac{a}{b+c}$. A contradiction. Similarly, suppose $\frac{A}{B+C} \leq \frac{(1-\gamma)A+\gamma a}{(1-\gamma)B+(1-\gamma)C+\gamma b+\gamma c}$. Rearranging gives $\frac{A}{B+C} \leq \frac{a}{b+c}$. A contradiction. These inequalities can also be used to show that if $\frac{A}{B+C} > \frac{(1-\gamma)A+\gamma a}{(1-\gamma)B+(1-\gamma)C+\gamma b+\gamma c} > \frac{a}{b+c}$ then $\frac{A}{B+C} > \frac{a}{b+c}$.

Step 2: For any pair of agents i and j where $i \in S$ and $S \subset R \subseteq N$, such that agent i has $\frac{(1-\lambda)\sum_{k\in S} w_{ik}+\lambda w(n_S-1)}{(1-\lambda)\sum_{k\in N} w_{ik}+\lambda w(n-1)} > \frac{\sum_{k\in S} w_{ik}}{\sum_{k\in N} w_{ik}}$ and agent j has $\frac{(1-\lambda)\sum_{k\in R} w_{jk}+\lambda w(n_R-1)}{(1-\lambda)\sum_{k\in N} w_{jk}+\lambda w(n-1)} < \frac{\sum_{k\in R} w_{jk}}{\sum_{k\in N} w_{jk}}$, where n_T denotes the number of nodes in a set $T \subseteq N$, then $\frac{(1-\lambda)\sum_{k\in R} w_{jk}+\lambda w(n_R-1)}{(1-\lambda)\sum_{k\in N} w_{jk}+\lambda w(n-1)} > \frac{(1-\lambda)\sum_{k\in S} w_{ik}+\lambda w(n_S-1)}{(1-\lambda)\sum_{k\in N} w_{ik}+\lambda w(n-1)}$.

Proof of Step 2: Using Step 1 we have $\frac{n_S-1}{n-1} > \frac{(1-\lambda)\sum_{k\in S} w_{ik}+\lambda w(n_S-1)}{(1-\lambda)\sum_{k\in N} w_{ik}+\lambda w(n-1)} > \frac{\sum_{k\in S} w_{ik}}{\sum_{k\in N} w_{ik}}$ and $\frac{\sum_{k\in R} w_{jk}}{\sum_{k\in N} w_{jk}} > \frac{(1-\lambda)\sum_{k\in R} w_{jk}+\lambda w(n_R-1)}{(1-\lambda)\sum_{k\in N} w_{jk}+\lambda w(n-1)} > \frac{n_R-1}{n-1}$. Thus, since $n_R > n_S$, we have $\frac{(1-\lambda)\sum_{k\in R} w_{jk}+\lambda w(n_R-1)}{(1-\lambda)\sum_{k\in N} w_{jk}+\lambda w(n-1)} > \frac{(1-\lambda)\sum_{k\in S} w_{ik}+\lambda w(n_S-1)}{(1-\lambda)\sum_{k\in N} w_{ik}+\lambda w(n-1)}$. □

Lemma 4 *For any networks $\{\tilde{w}_{ij}\}$ and $\{w_{ij}\}$ where, for all $i \in N$ and all sets $S \subseteq N$, with $i \in S$, agent i has more proportionate connections to $S \subseteq N$ in network $\{\tilde{w}_{ij}\}$ than $\{w_{ij}\}$, then network $\{\tilde{w}_{ij}\}$ is less insular.*

Proof of Lemma 4 Consider any agents i and j where $i \in S$ and $S \subset R \subseteq N$, such that agent i has $\frac{\sum_{k\in S} \tilde{w}_{ik}}{\sum_{k\in N} \tilde{w}_{ik}} > \frac{\sum_{k\in S} w_{ik}}{\sum_{k\in N} w_{ik}}$ and agent j has $\frac{\sum_{k\in R} \tilde{w}_{jk}}{\sum_{k\in N} \tilde{w}_{jk}} < \frac{\sum_{k\in R} w_{jk}}{\sum_{k\in N} w_{jk}}$, where n_T denotes the number of nodes in a set $T \subseteq N$. By assumption, agent i has more proportionate connections to $S \subseteq N$ in network $\{\tilde{w}_{ij}\}$ and therefore the following inequality holds, $\frac{n_S-1}{n-1} \geq \frac{\sum_{k\in S} \tilde{w}_{ik}}{\sum_{i\in N} \tilde{w}_{ik}} > \frac{\sum_{k\in S} w_{ik}}{\sum_{k\in N} w_{ik}}$. By the same argument, $\frac{n_R-1}{n-1} \leq \frac{\sum_{k\in R} \tilde{w}_{jk}}{\sum_{k\in N} \tilde{w}_{jk}} < \frac{\sum_{k\in R} w_{jk}}{\sum_{k\in N} w_{jk}}$. Since $n_R > n_S$ we have $\frac{\sum_{k\in R} \tilde{w}_{jk}}{\sum_{k\in N} \tilde{w}_{jk}} > \frac{\sum_{k\in S} \tilde{w}_{ik}}{\sum_{k\in N} \tilde{w}_{ik}}$ and therefore there is no local increase in insularity in $\{\tilde{w}_{ij}\}$ relative to $\{w_{ij}\}$. □

Preliminary Lemmas

We next provide some preliminary lemmas that are used in the proof of Theorem 1. Let A_Q (respectively $A_{Q'}$) denote the set of agents that adopt technology Q (respectively Q') in society $(\{w_{ij}\}, \mathcal{C})$.

Lemma 5. *For $Q' \geq Q$, $A_{Q'} \subseteq A_Q$ and $N \backslash A_Q \subseteq N \backslash A_{Q'}$.*

Proof of Lemma 5 By Lemma 2 and Proposition 1, $N \backslash A_Q$ is $(1-Q)$-subgroup cohesive. By definition of subgroup cohesion, the set $N \backslash A_Q$ is also $(1-Q')$-subgroup cohesive for any $Q' \geq Q$. Thus, by Lemma 1, set $N \backslash A_Q$ does not adopt technology Q' and so $N \backslash A_Q \subseteq N \backslash A_{Q'}$. Using this, any agent in the complement of $N \backslash A_{Q'}$ is also in the complement of $N \backslash A_Q$ and so $A_{Q'} \subseteq A_Q$. □

Lemma 6. *For any $S \subset A_Q$ there exists $C \subseteq N \backslash S$ where $C \in \mathcal{C}$ such that, for all $i \in C$, $P_i(S) + P_i(C) > Q$. Then $C \subseteq A_Q$.*

Proof of Lemma 6 Suppose not, and for all $C \subseteq N \backslash S$ where $C \in \mathcal{C}$ there exists an $i \in C$, such that $P_i(S) + P_i(C) \leq Q$. This inequality can be rewritten as $P_i(N \backslash S) - P_i(C) \geq 1 - Q$ and therefore $N \backslash S$ is $(1-Q)$-subgroup cohesive. By Lemma 1 no agent in $N \backslash S$ adopts and we have a contradiction.

To show that all agents in any such C adopt technology Q in society $(\{w_{ij}\}, \mathcal{C})$, suppose instead that, for some $i \in C$, $i \notin A_Q$. Since $S \subset A_Q$, we have for all $i \in C \backslash A_Q$

$$P_i(A_Q) + P_i(C \backslash A_Q) \geq P_i(S) + P_i(C) > Q$$

and by the diffusion process all $i \in C \backslash A_Q$ adopt technology Q in society $(\{w_{ij}\}, \mathcal{C})$. A contradiction. □

Lemma 7. *For all $C \subseteq N \backslash A_Q$ where $C \in \mathcal{C}$, for any $S \subseteq A_Q$, there exists an $i \in C$ such that $P_i(S) + P_i(C) \leq Q$.*

Proof of Lemma 7 By Lemma 2 and Proposition 1, $N \backslash A_Q$ is $(1-Q)$-subgroup cohesive. We can rewrite the definition of $(1-Q)$-subgroup cohesion as, for each $C \in \mathcal{C}$, there exists $i \in C$ such that $P_i(A_Q) + P_i(C) \leq Q$ and, since $S \subseteq A_Q$, $P_i(S) + P_i(C) \leq Q$. □

Theorem 1 *For societies $(\{w_{ij}\}, \mathcal{C})$ and $(\{\tilde{w}_{ij}\}, \mathcal{C})$, where network $\{\tilde{w}_{ij}\}$ is less insular than network $\{w_{ij}\}$, there exists a threshold $\mu \in [0,1]$ such that:*

- *for technologies $Q < \mu$, adoption is weakly higher in the society with the less insular network;*

- *for technologies $Q \geq \mu$, adoption is weakly lower in the society with the less insular network.*

Proof of Theorem 1 Let A_Q denote the set of agents that adopt a technology Q in society $(\{w_{ij}\}, \mathcal{C})$ and $\tilde{A}_Q$ denote the set of agents that adopt a technology Q in society $(\{\tilde{w}_{ij}\}, \mathcal{C})$. As above, $P_i(S) = \frac{\sum_{j \in S} w_{ij}}{\sum_{j \in N} w_{ij}}$ is the weighted fraction of i's neighbors within a set $S \subseteq N$ in network $\{w_{ij}\}$ and, symmetrically, $\tilde{P}_i(S) = \frac{\sum_{j \in S} \tilde{w}_{ij}}{\sum_{j \in N} \tilde{w}_{ij}}$ is the weighted fraction of i's neighbors within a set $S \subseteq N$ in network $\{\tilde{w}_{ij}\}$.

The proof proceeds in four steps.

Step 1: If more agents adopt technology Q in $(\{\tilde{w}_{ij}\}, \mathcal{C})$ than $(\{w_{ij}\}, \mathcal{C})$, then $A_Q \subseteq \tilde{A}_Q$. If more agents adopt technology Q in $(\{w_{ij}\}, \mathcal{C})$ than $(\{\tilde{w}_{ij}\}, \mathcal{C})$ then $\tilde{A}_Q \subseteq A_Q$.

First, consider the case where strictly more agents adopt technology Q in $(\{\tilde{w}_{ij}\}, \mathcal{C})$ than $(\{w_{ij}\}, \mathcal{C})$, and show that $A_Q \subset \tilde{A}_Q$. Since strictly more agents adopt technology Q in $(\{\tilde{w}_{ij}\}, \mathcal{C})$, then $A_Q \cap \tilde{A}_Q \subset \tilde{A}_Q$ and, by Lemma 6, there exists $C_1 \subseteq N \backslash (A_Q \cap \tilde{A}_Q)$ where $C_1 \in \mathcal{C}$ such that for all $i \in C_1$

$$\frac{\sum_{j \in A_Q \cap \tilde{A}_Q} \tilde{w}_{ij} + \sum_{j \in C_1} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}} > Q \tag{2}$$

and $C_1 \subseteq \tilde{A}_Q$. Since $C_1 \cap A_Q = \emptyset$, by Lemma 7, there exists $i \in C_1$ such that

$$\frac{\sum_{j \in A_Q \cap \tilde{A}_Q} w_{ij} + \sum_{j \in C_1} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}} \leq Q. \tag{3}$$

Using inequalities (2) and (3), for some $i \in C_1$,

$$\frac{\sum_{j \in A_Q \cap \tilde{A}_Q} \tilde{w}_{ij} + \sum_{j \in C_1} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}} > Q \geq \frac{\sum_{j \in A_Q \cap \tilde{A}_Q} w_{ij} + \sum_{j \in C_1} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}}. \tag{4}$$

Now suppose $A_Q \not\subset \tilde{A}_Q$ and we will show a contradiction. By Lemma 6, there exists

$C_2 \subseteq N \backslash (A_Q \cap \tilde{A}_Q)$, where $C_2 \in \mathcal{C}$, such that for all $i \in C_2$

$$\frac{\sum_{j \in A_Q \cap \tilde{A}_Q} w_{ij} + \sum_{j \in C_2} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}} > Q, \tag{5}$$

and $C_2 \subseteq A_Q$. Since $C_2 \cap \tilde{A}_Q = \emptyset$, by Lemma 7, for some $i \in C_2$

$$\frac{\sum_{j \in \tilde{A}_Q} \tilde{w}_{ij} + \sum_{j \in C_2} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}} \leq Q. \tag{6}$$

Using inequalities (5) and (6), for some $i \in C_2$,

$$\frac{\sum_{j \in \tilde{A}_Q} w_{ij} + \sum_{j \in C_2} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}} > Q \geq \frac{\sum_{j \in \tilde{A}_Q} \tilde{w}_{ij} + \sum_{j \in C_2} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}}, \tag{7}$$

where the first inequality follows from (5) and since $A_Q \cap \tilde{A}_Q \subseteq \tilde{A}_Q$.

Using inequalities (7) and (4), for some $i \in (A_Q \cap \tilde{A}_Q) \cup C_1$ and $j \in \tilde{A}_Q \cup C_2$ we have $\tilde{P}_i((A_Q \cap \tilde{A}_Q) \cup C_1) > P_i((A_Q \cap \tilde{A}_Q) \cup C_1)$, $\tilde{P}_j(\tilde{A}_Q \cup C_2) < P_j(\tilde{A}_Q \cup C_2)$, and $\tilde{P}_i((A_Q \cap \tilde{A}_Q) \cup C_1) > Q \geq \tilde{P}_j(\tilde{A}_Q \cup C_2)$. Since $(A_Q \cap \tilde{A}_Q) \cup C_1 \subseteq \tilde{A}_Q \subset \tilde{A}_Q \cup C_2$, this defines a local increase in insularity in network $\{\tilde{w}_{ij}\}$ relative to network $\{w_{ij}\}$ which contradicts the definition of $\{\tilde{w}_{ij}\}$ as less insular than $\{w_{ij}\}$.

Second, consider the case where the same number of agents adopt technology Q in $(\{\tilde{w}_{ij}\}, \mathcal{C})$ and $(\{w_{ij}\}, \mathcal{C})$. We show $A_Q = \tilde{A}_Q$. Suppose $A_Q \neq \tilde{A}_Q$, then $A_Q \cap \tilde{A}_Q \subset \tilde{A}_Q$ and $A_Q \not\subseteq \tilde{A}_Q$, and using the same analysis above, we have a contradiction.

Finally, consider the case where strictly more agents adopt technology Q in $(\{w_{ij}\}, \mathcal{C})$ than $(\{\tilde{w}_{ij}\}, \mathcal{C})$ and show $\tilde{A}_Q \subset A_Q$. By Lemma 6, there exists $C_1 \subseteq N \setminus (A_Q \cap \tilde{A}_Q)$ where $C_1 \in \mathcal{C}$ such that for all $i \in C_1$

$$\frac{\sum_{j \in A_Q \cap \tilde{A}_Q} w_{ij} + \sum_{j \in C_1} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}} > Q, \tag{8}$$

and $C_1 \subseteq A_Q$. Since $C_1 \cap \tilde{A}_Q = \emptyset$, for some $i \in C_1$

$$\frac{\sum_{j \in \tilde{A}_Q} \tilde{w}_{ij} + \sum_{j \in C_1} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}} \leq Q. \tag{9}$$

Using inequalities (8) and (9), for some $i \in C_1$

$$\frac{\sum_{j \in \tilde{A}_Q} w_{ij} + \sum_{j \in C_1} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}} > Q \geq \frac{\sum_{j \in \tilde{A}_Q} \tilde{w}_{ij} + \sum_{j \in C_1} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}} \tag{10}$$

where the first inequality follows from (8) and since $A_Q \cap \tilde{A}_Q \subseteq \tilde{A}_Q$.

Suppose $\tilde{A}_Q \not\subset A_Q$. There exists $C_2 \subseteq N \setminus (A_Q \cap \tilde{A}_Q)$ where $C_2 \in \mathcal{C}$ such that for all $i \in C_2$

$$\frac{\sum_{j \in A_Q \cap \tilde{A}_Q} \tilde{w}_{ij} + \sum_{j \in C_2} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}} > Q$$

but for some $i \in C_2$

$$\frac{\sum_{j \in A_Q \cap \tilde{A}_Q} w_{ij} + \sum_{j \in C_2} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}} \leq Q.$$

Then for some $i \in C_2$

$$\frac{\sum_{j \in A_Q \cap \tilde{A}_Q} \tilde{w}_{ij} + \sum_{j \in C_2} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}} > Q \geq \frac{\sum_{j \in A_Q \cap \tilde{A}_Q} w_{ij} + \sum_{j \in C_2} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}}. \tag{11}$$

Using inequalities (10) and (11), there exists some $i \in (A_Q \cap \tilde{A}_Q) \cup C_2$ and $j \in \tilde{A}_Q \cup C_1$ such that $\tilde{P}_i((A_Q \cap \tilde{A}_Q) \cup C_2) > P_i((A_Q \cap \tilde{A}_Q) \cup C_2)$, $\tilde{P}_j(\tilde{A}_Q \cup C_1) < P_j(\tilde{A}_Q \cup C_1)$, and $\tilde{P}_i((A_Q \cap \tilde{A}_Q) \cup C_2) > Q \geq \tilde{P}_j(\tilde{A}_Q \cup C_1)$. Since $(A_Q \cap \tilde{A}_Q) \cup C_2 \subseteq \tilde{A}_Q \subset \tilde{A}_Q \cup C_1$, this defines a local increase in insularity in network $\{\tilde{w}_{ij}\}$ relative to network $\{w_{ij}\}$ which contradicts the definition of $\{\tilde{w}_{ij}\}$ as less insular than $\{w_{ij}\}$.

Step 2: If there exists a technology Q' such that there is strictly more adoption in $(\{\tilde{w}_{ij}\}, \mathcal{C})$ than $(\{w_{ij}\}, \mathcal{C})$ then there is weakly more adoption in $(\{\tilde{w}_{ij}\}, \mathcal{C})$ than $(\{w_{ij}\}, \mathcal{C})$ for all $Q \leq Q'$.

By contradiction, suppose that for some $Q \leq Q'$ there is strictly more adoption in network $\{w_{ij}\}$. Then for such a Q, by Step 1 $\tilde{A}_Q \subset A_Q$, and by Lemmas 6 and 7 there exists $C_2 \subseteq N \backslash \tilde{A}_Q$ where $C_2 \in \mathcal{C}$ and $i \in C_2$ such that

$$\frac{\sum_{j \in \tilde{A}_Q} w_{ij} + \sum_{j \in C_2} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}} > Q \geq \frac{\sum_{j \in \tilde{A}_Q} \tilde{w}_{ij} + \sum_{j \in C_2} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}}. \tag{12}$$

Similarly, since $A_{Q'} \subset \tilde{A}_{Q'}$, there exists $C_1 \subseteq N \backslash A_{Q'}$ where $C_1 \in \mathcal{C}$ and $i \in C_1$ such that

$$\frac{\sum_{j \in A_{Q'}} \tilde{w}_{ij} + \sum_{j \in C_1} \tilde{w}_{ij}}{\sum_{j \in \mathcal{N}} \tilde{w}_{ij}} > Q' \geq \frac{\sum_{j \in A_{Q'}} w_{ij} + \sum_{j \in C_1} w_{ij}}{\sum_{j \in \mathcal{N}} w_{ij}}. \tag{13}$$

Using inequalities (12) and (13) we have that for some $i \in A_{Q'} \cup C_1$ and $j \in \tilde{A}_Q \cup C_2$, $\tilde{P}_i(A_{Q'} \cup C_1) > P_i(A_{Q'} \cup C_1)$, $\tilde{P}_j(\tilde{A}_Q \cup C_2) < P_j(\tilde{A}_Q \cup C_2)$, and $\tilde{P}_i(A_{Q'} \cup C_1) > Q' \geq Q \geq \tilde{P}_j(\tilde{A}_Q \cup C_2)$. Since $A_{Q'} \cup C_1 \subseteq \tilde{A}_{Q'} \subseteq \tilde{A}_Q \subset \tilde{A}_Q \cup C_2$ this defines a local increase in insularity in network $\{\tilde{w}_{ij}\}$ relative to network $\{w_{ij}\}$ which contradicts the definition of $\{\tilde{w}_{ij}\}$ as less insular than $\{w_{ij}\}$.

Step 3: If there exists a technology Q' such that there is strictly more adoption in $(\{w_{ij}\}, \mathcal{C})$ than $(\{\tilde{w}_{ij}\}, \mathcal{C})$ then there is weakly more adoption in $(\{w_{ij}\}, \mathcal{C})$ for all $Q \geq Q'$.

Suppose instead for some $Q'' > Q'$ there is strictly more adoption in $(\{\tilde{w}_{ij}\}, \mathcal{C})$ than $(\{w_{ij}\}, \mathcal{C})$, then by Step 2 there must be weakly more adoption in $(\{\tilde{w}_{ij}\}, \mathcal{C})$ than $(\{w_{ij}\}, \mathcal{C})$ for all $Q \leq Q''$. A contradiction.

Step 4: there exists a threshold $\mu \in [0, 1]$ that satisfies the conditions of the Theorem.

Let $\bar{Q}$ denote the lowest value of $Q \in [0, 1]$ at which there is strictly more adoption in network $\{w_{ij}\}$ if such a Q exists. Then set $\mu = \bar{Q}$ and by Step 2 and 3 the result follows. If there is no $Q \in [0, 1]$ at which there is strictly more adoption in network $\{w_{ij}\}$ then set $\mu = 1$ since at $\mu = 1$ there is no adoption and so, trivially, adoption is weakly higher in $\{w_{ij}\}$. $\square$