Pareto Improvements in the Contest for College Admissions*

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Abstract

Many countries base college admissions on a centrally-administered test. Students invest a great deal of resources to improve their performance on the test, and there is growing concern about the high costs associated with these activities. We consider modifying the test by introducing performance-disclosure policies that pool intervals of performance rankings. Pooling affects the equilibrium allocation of students to colleges,

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which hurts some students and benefits others, but also affects students' effort. We investigate how such policies can improve students' welfare in a Pareto sense, study the Pareto frontier of pooling policies, and identify improvements that are robust to the distribution of college seats.

We illustrate the potential applicability of our results with an empirical estimation that uses data on college admissions in Turkey. We find that a policy that pools a large fraction of the lowest performing students leads to a Pareto improvement in a contest based on the estimated parameters. A laboratory experiment based on the estimated parameters generally supports our theoretical predictions.

1 Introduction

College and university admissions are often determined by students' performance on a centrally administered test. This is commonly the case in Brazil, China, Russia, South Korea, and Turkey. The students with the highest performance are admitted to the best colleges, those ranked below them are admitted to the next best colleges, etc. Other countries consider additional factors, but even then centralized tests typically play an important role.

Consequently, students invest a great deal of effort preparing for these tests. In China, Japan, South Korea, and Taiwan, students attend specialized "cram schools," which focus on improving students' performance on the tests. This consists of rote learning, solving many practice problems, and practicing test-taking strategies tailored to the specific test. Students also hire tutors, buy books, and take specialized courses to improve their test scores. But these costly activities are far less likely to generate substantial gains in students' productive human capital.

Reducing such activities is more difficult than it might initially appear. Passing laws to prohibit or limit them may be both difficult and ineffective.[1] Changing the admissions process may also be impractical. First, it is not clear what a better system would look like. For example, accurate tests lead to better students being admitted to better colleges, and other systems may lead to different outcomes, which may or may not be preferred. Second, implementing a new system may be expensive and technically difficult. Third, a new system that helped some students but hurt others would likely face significant resistance.

This paper investigates simple modifications to centralized tests that make all students better off. We model college admissions as a contest with many players (students) and many prizes (college seats). Students exert costly effort and are admitted to colleges based on the rank order of their performance.[2] We consider performance-disclosure policies, which pool together intervals of performance and assign the same score to all performances in an

[1] In a 2014 New York Times article, (https://www.nytimes.com/2014/08/02/opinion/sunday/ south-koreas-education-system-hurts-students.html), Se-Woong Koo reports that many South Korean presidents tried to limit cram schools' activities, including passing a 10 p.m. mandatory closure time. But even this restriction was circumvented "by operating out of residential buildings or blacking out windows so that light could not be seen from outside."

[2] Our analysis may also be applied to other large contest settings such as large corporate promotion contests (we thank a referee for suggesting this example) and large grant competitions.

interval. Students with the same score are randomly admitted to the corresponding fraction of colleges.[3] For example, a "bottom pooling" policy that pools some fraction of the lowest performing students assigns these students randomly to the same fraction of the lowest-ranked college seats. Performance-disclosure policies do not require changing the tests or introducing new components to the admissions process. They also respect the property that a higher score leads to a better expected college assignment than a lower score. This may help make such policies appealing to policy makers.

We study Pareto improving performance-disclosure policies, which benefit all students. Our notion of Pareto improvements is an interim one, once a student knows her score but before she learns her college assignment. Pareto improving policies often exist, because test preparation is costly. Relative to the baseline contest, introducing a performance-disclosure policy leads to some students being admitted to higher-ranked colleges with positive probability; this makes them better off even if they incur higher costs, provided the cost increase is not too large. Other students are admitted to lower-ranked colleges with positive probability; if they also incur lower costs they are made better off, provided the reduction in the costs is large enough.

We first characterize the Pareto improving policies that pool a single interval of performance. The characterization shows that such pooling is Pareto improving if and only if the student with the highest performance in the interval benefits from the pooling. This in turn happens if the population distribution of student ability conditional on the same interval (in percentile terms) first-order stochastically dominates the uniform distribution. We then generalize this condition to policies with multiple pooling intervals and characterize the distributions of players' types and the distributions of college seats for which Pareto improving policies exist. We also consider mean-preserving contractions (MPCs) of the distribution of college seats, which corresponds to pooling groups of college seats.[4] We characterize the distributions of players' types and the distributions of college seats for which Pareto improving MPCs exist and the Pareto frontier of such MPCs.

[3] This can be viewed as making performance on the test noisier. Morgan et. al. (2022) suggest that other forms of noise can also be socially beneficial.

[4] We thank the referees for encouraging us to consider MPCs.

We then consider robust Pareto improving performance-disclosure policies, which are Pareto improving for any distribution of college seats. We characterize the robust Pareto improving policies and show that the Pareto optimal policy among them is unique. The characterization may be particularly useful for empirical work because it only requires obtaining an estimate of the students' ability distribution.

We illustrate the potential applicability of our results with an empirical estimation that uses data on college admissions in Turkey. We use the framework of Krishna et al. (2018) along with novel techniques to calibrate the model and estimate applicants' ability distribution and the distribution of college seats. We then simulate a college admissions contest with these distributions. Among the many performance-disclosure policies on the Pareto frontier of Pareto improving policies, we focus on the one that maximizes the utility of the applicants with the lowest ability. This is a bottom pooling policy, which pools approximately 63 percent of the lowest test scores and increases applicants' estimated utility by approximately 32 percent. It also maximizes applicants' aggregate welfare among all bottom pooling policies.[5]

Finally, we conduct a laboratory experiment based on the calibrated distributions and the Pareto improving bottom pooling policy. We evaluate subjects' behavior in the baseline contest and with bottom pooling, and find that the behavior is in broad agreement with the theory. A small set of subjects, those with the lowest ability among the subjects who should not be affected by the bottom pooling policy, behave in a way that slightly decreases their monetary payoffs. We aregue that a possible explanation for this behavior may be a preference for randomization, similarly to the findings of Dwenger, Kübler, and Weizsäcker (2016) in the context of school applications. Taken together, our theory, empirical estimation, and experiment suggest that the simple performance-disclosure policies we investigate have the potential to improve the welfare of millions of college admissions applicants.

The rest of the paper is organized as follows. Section 1.1 reviews the related literature. Section 2 introduces the model, presents the equilibrium, and defines the notion of Pareto improvements. Section 3 investigates policies with a single pooling interval. Section 4 investigates policies with multiple pooling intervals. Section 5 derives the conditions for

[5]We additionally show that the robust Pareto improving policy is also a bottom pooling policy, which pools approximately 52 percent of the lowest test scores.

robust Pareto improvements. Section 6 discusses some limitations and extensions of the model. Section 7 describes the empirical exercise. Section 8 presents the experimental results. Section 9 concludes. The appendix contains proofs, considers the Pareto frontier of Pareto-improving policies that pool on multiple intervals, and examines peer effects. The online appendix contains additional details about the empirical exercise, estimation strategy, and counterfactuals, and provides additional details, results, and screenshots from our experiment.

1.1 Contribution to the literature

The work by Che et al. (2018) is the most closely related to the theoretical part of our paper. They study auction formats for a single object that are immune to collusion by bidders, and identify optimal cartels. Given a multi-bidder auction and an equilibrium of the auction, they model a cartel as a mechanism to which the bidders report their types and which bids on their behalf in the auction. The auction is immune to collusion if no such mechanism exists whose outcome is weakly preferred by all types of every bidder to the equilibrium of the auction, with a strict preference for some type of some bidder. Their Theorem 1 provides necessary and sufficient conditions for an auction to be immune to collusion. We study contests with many players and prizes. We use Olszewski and Siegel's (2016) large contest framework to show that any equilibrium is approximated by a single-agent mechanism that implements the assortative allocation of the prizes to agent types. Thus, the motivating questions are different, the settings are different (for example, they have multiple bidders whereas we have a single limiting agent), and the set of manipulations are different (all possible mechanisms in their setting versus pooling intervals of performance in our setting). Nevertheless, our characterization of contests for which no Pareto improvement exists (Theorem 1 and, for mean-preserving contractions of the prize distribution, Theorem 2) is similar to their Theorem 1, and relies on a condition very similar to their condition (PS). The intuition on their pages 411-412 for why the condition is necessary is similar in both results.[6] Our Theorem 3 also characterize the Pareto frontier of Pareto-improving mean-

[6]But the analysis, especially the proof of sufficiency, is different because of the different sets of manipulations. Perhaps most intuitive is the comparison between our setting and the "single-bidder version" of

preserving contractions, and this result is similar to Theorem 2 of Che et al. (2018) although its proof is different. However, the corresponding Pareto frontier of category rankings, which are the main objects of our analysis, differs from the Pareto frontier of mean-preserving contractions (see Example 2 in Appendix C), and our results on the Pareto frontier of category rankings have no counterparts in Che et al. (2018).

Moldovanu et al. (2007) show that the designer of a contest for status may prefer to pool contestants into status categories in order to increase the aggregate performance.[7] In a two-sided matching model with ex-ante symmetric agents and costly signals, Hoppe et al. (2009) provide conditions under which random matching leads to ex-ante higher welfare than assortative matching and show that random matching is Pareto improving for agents on one side if the distribution of types of that side first-order stochastically dominates the uniform distribution. Other papers that compare contests and lotteries from the perspective of contestants' welfare include Taylor et al. (2003), Koh et al. (2006), Hoppe et al. (2011), and Chakravarty and Kaplan (2013). Condorelli (2012) characterizes the ex-ante efficient allocations of heterogeneous objects to heterogeneous agents with private valuations.[8] We are interested in interim Pareto improvements, and the set of ex-ante efficient allocations can be completely different from those that are interim Pareto improving. Olszewski and Siegel (2016) introduced the equilibrium approximation approach to large contests, which we use here.[9] Bodoh-Creed and Hickman (2018) use a similar approach to study quotas and affirmative action in college admissions. One of their findings is that a college assignment lottery would generate higher total student welfare than a college admissions contest. We show that a lottery may be improved upon for all students by partitioning the set of students

their setting in which the strategies of the other bidders are given. Then, if a subset of types in their setting deviates to bidding as some other type, the deviators obtain the same allocation as the other type. In our setting, because the set of prizes is fixed, the deviation allocation depends on the subset of deviating types, and this allocation usually differs from that of the other type.

[7]This happens when the ability distribution is sufficiently concave, in which case our results show that pooling is not Pareto improving. Dubey and Geanakoplos (2010) consider a game of status between students and show that coarse grading policies maximize effort.

[8]His main insights apply when all players' type distributions have monotone hazard rates. We do not require such a condition.

[9]Olszewski and Siegel (2020) use this approach to study performance-maximizing contests. Fang, Noe, and Strack (2020) study the effect of different prize structures on aggregate effort in symmetric all-pay auctions with complete information.

into several categories based on their performance and using a lottery in each category.[10]

Our empirical application is closely related to three strands of literature in empirical industrial organization. First, as we recover the distribution of placement values in the Turkish college admission system, we heavily rely on the method of estimating single-agent dynamic problems introduced in Hotz and Miller (1993); Krishna et al. (2018) adapt this method to the context of Turkey. Second, our estimates of the costs of effort are based on Berry (1994), who shows that mean values of choices can be backed out from the observed shares of agents making these choices. Finally, our approach to estimating the distribution of student ability is novel and thus does not have direct antecedents in the literature. To some degree, we draw our inspiration from Guerre et al. (2000), who infer the distribution of bidder private values from the distribution of observed bids in first-price auctions, which has parallels with the way we back out the distribution of ability from student test scores.

2 The baseline contest

A large number of players (students) compete for prizes (college seats) by taking a test. Each prize is characterized by its known value y in $[0, 1]$, and all the players agree that a prize with a higher value is better. Each player is characterized by her ability (type) x in $[0, 1]$, which affects her cost of performance on the test and/or her prize valuation. After privately observing her type, each player exerts costly effort to achieve her desired performance $t \geq 0$ on the test. The player with the highest performance obtains the highest prize, the player with the second-highest performance obtains the second-highest prize, and so on. Some prizes may be identical, which allows for multiple seats in a given college (or tier of colleges). Thus, each player is admitted to the best college among those with available seats after all the players with a higher performance have been admitted. Ties are resolved by a fair

[10] The optimality of coarse partitions with random lotteries within elements of the partitions has been studied in less closely related papers, including Chao and Wilson (1987), Wilson (1989), and McAfee (2002) in the context of priority classes and Damiano and Li (2007) and Rayo (2013) in monopolistic settings. The effects of different grading policies has been studied by Ostrovsky and Schwarz (2010), Gottlieb and Smetters (2014), Boleslavsky and Cotton (2015), and Harbaugh and Rasmusen (2018). Frankel and Kartik (2019) show that test preparation not available to all students can diminish the signals contained in standardized tests.

lottery. The utility of a type x player who chooses performance t and obtains prize y is

$$g_1(x)\, y - \frac{c(t)}{g_2(x)}, \tag{1}$$

where c is strictly increasing and twice continuously differentiable and $\lim_{t\to\infty} c(t) = \infty$.[11] Function c captures the cost of performance, function $g_1 \geq 0$ captures the effect of the player's type on her prize valuation, and function $g_2 \geq 0$ captures the effect of the player's type on her cost of performance. We let $g_1(x)\, g_2(x) = x$. Two special cases (which are assumed in most of the contest literature) are

$$xy - c(t), \tag{2}$$

in which the player's type only affects her prize valuation, and

$$y - \frac{c(t)}{x}, \tag{3}$$

in which the player's type only affects her performance cost. Utilities (1) for different functions g_1 and g_2 are strategically equivalent because for each type x multiplying (1) by $g_2(x)$ gives (2).[12] For convenience, throughout our theoretical analysis we will use utility (2), and in the empirical analysis of Section 7 we will use utility (3). All of our theoretical results hold without change for any utility (1). Section 6 discusses limitations of the model.

To solve for players' equilibrium behavior, we assume that players' types are drawn independently (but not necessarily identically) across players, and we apply the large contests results of Olszewski and Siegel (2016). These results show that players' equilibrium behavior is approximated by a particular single-agent mechanism, which assortatively allocates prizes to agent types and gives the lowest type a utility of 0. To describe this mechanism, we

[11] The linearity of y is a normalization; we can replace y in players' utility with $h(y)$, where h is strictly increasing and twice continuously differentiable and $h(y) = 0$, without affecting any of the results. We can also replace the assumption that $\lim_{t\to\infty} c(t) = \infty$ with the assumption that $\lim_{t\to\bar{t}} c(t) = \infty$ for some positive $\bar{t}$ that represents a cap on students' maximal effort.

[12] Different functions g_1 have different implications for aggregate welfare, but this makes no difference for our theoretical analysis because we focus on policies that make all students better off (we provide a precise definition in Section 2.1).

denote by F the CDF of the average of players' type distributions and assume that F has a continuous, strictly positive density f. We denote by G the CDF that represents the empirical distribution of prizes, so the size of each atom of G corresponds to the number of seats in a particular college (or tier of colleges), with $G(0)$ representing the fraction of students in excess of the total number of college seats.[13] The *assortative allocation* assigns to each type x prize

$$y^A(x) = G^{-1}(F(x)),$$

where

$$G^{-1}(z) = \inf\{y : G(y) \geq z\} \text{ for } 0 \leq z \leq 1.$$

That is, the quantile in the prize distribution of the prize assigned to type x is the same as the quantile of type x in the (average) type distribution. The unique incentive-compatible mechanism that implements the assortative allocation and gives type $x = 0$ utility 0 specifies for every type x performance

$$t^A(x) = c^{-1}\left(xy^A(x) - \int_0^x y^A(\tilde{x})\, d\tilde{x}\right). \tag{4}$$

This implies that type x obtains utility

$$U(x) = xy^A(x) - c(t^A(x)) = \int_0^x y^A(\tilde{x})\, d\tilde{x}. \tag{5}$$

Olszewski and Siegel (2016) show that in any equilibrium of a contest with many players and prizes, for every player (except a small fraction of the players) the event that the player's type is some x, she chooses a performance close to $t^A(x)$, and obtains a prize close to $y^A(x)$, which gives her a utility close to $U(x)$, has probability close to 1.

The rest of the paper uses the approximating single-agent mechanism to investigate how different performance-disclosure policies affect students' welfare in a Pareto sense, which we define in the next subsection.

[13] If there are n players and k prizes have value y or lower (including "null prizes" with value 0), then $G(y) = k/n$.

2.1 The notion of Pareto improvements

We use the term "Pareto-improving" in reference to types' expected utility in an approximating mechanism. This utility corresponds to players' interim utility - after players' scores are realized but before they learn to which college they are admitted. A performance-disclosure policy is Pareto improving if in the corresonding approximating mechanism all types are better off, and a positive measure of types is strictly better off. Such an improvement implies that in a sufficiently large contest some players are strictly better of (in an interim sense) and no player is worse off by more than an arbitrarily small amount; moreover, the sum of these small amounts is arbitrarily small compared to the gains of the players who are strictly better off. We underscore that because pooling in our performance-disclosure policies leads to lotteries over prizes, by "gains" for a player we mean that the player prefers the lottery to the original disclosure policy, but she may or may not prefer the outcome once the lottery is realized.

3 Pooling on a single interval

We begin by considering pooling a single interval of performance. We denote the interval by $(q^*, q^{**}]$, where q^* and q^{**} are quantiles in the ranking of students' realized performance, with $0 \leq q^* < q^{**} \leq 1$.[14] Students whose performance ranking quantile lies in the interval are treated identically. As a group, these students still obtain the prizes they would in the baseline contest, that is, the prizes whose quantile ranking lies in the quantile interval $(q^*, q^{**}]$ of the prize distribution. But with pooling, these prizes are allocated uniformly at random to the students in the group. The resulting game is therefore different from the baseline contest. Nevertheless, we can describe the contest with pooling in an alternative, equivalent way that allows us to use the same contest framework to study the effect of pooling. We do this by considering an equivalent contest with no performance pooling in which the prizes that correspond to the pooled interval are replaced with the same mass $q^{**} - q^*$ of identical

[14] We use left-open intervals because the assortative allocation $y^A = G^{-1} \circ F$ is left continuous due to the type distribution F being continuous (by assumption) and the prize distribution G being right continuous (as a CDF). The same is true for other assignments of prizes to types.

prizes whose value is equal to the average value of the original prizes in the pooled interval.

Formally, let types x^* and x^{**} be the types at quantiles q^* and q^{**} in the type distribution, that is, $q^* = F(x^*)$ and $q^{**} = F(x^{**})$. We consider the prize distribution that results from replacing the prizes in quantile interval $(q^*, q^{**}]$ of the original prize distribution G with a mass $q^{**} - q^*$ of prize

$$y(q^*, q^{**}) = \frac{\int_{q^*}^{q^{**}} G^{-1}(z)\, dz}{q^{**} - q^*} = \frac{\int_{x^*}^{x^{**}} y^A(x) dF(x)}{F(x^{**}) - F(x^*)}. \tag{6}$$

The corresponding assortative allocation coincides with the original assortative allocation y^A for types lower than x^* and higher than x^{**}, and assigns prize $y(q^*, q^{**})$ to every type in $(x^*, x^{**}]$.

3.1 Welfare comparisons

Pooling affects both the equilibrium prize allocation and players' performance choices. To understand the overall welfare effects, our first result compares each type's utility in the approximating mechanisms with and without pooling.

Proposition 1. *Consider pooling on a quantile interval $(q^*, q^{**}]$. If all the prizes in the interval are identical, then pooling has no effect. If not all prizes in the interval are identical, then the following statements hold, where $q^* = F(x^*)$ and $q^{**} = F(x^{**})$.*

(a) Types lower than x^* are not affected.

(b) Type x in $(x^*, x^{**}]$ weakly benefits from pooling if and only if

$$\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x}) dF(\tilde{x})}{F(x^{**}) - F(x^*)} \geq \frac{\int_{x^*}^{x} y^A(\tilde{x})\, d\tilde{x}}{x - x^*}, \tag{7}$$

and strictly benefits if the inequality is strict.

(c) There is a type $\hat{x}$ in $(x^*, x^{**}]$ such that types in $(x^*, \hat{x})$ benefit from pooling and types in $(\hat{x}, x^{**})$ do not benefit from pooling.

(d) Pooling is Pareto improving if and only if it weakly benefits type x^{**}, that is,

$$\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})dF(\tilde{x})}{F(x^{**}) - F(x^*)} \geq \frac{\int_{x^*}^{x^{**}} y^A\left(\tilde{x}\right)d\tilde{x}}{x^{**} - x^*},$$

and this holds if and only if pooling weakly benefits the highest type $x = 1$.

To understand the idea underlying Proposition 1, whose proof is in Appendix A, notice that players with types lower than x^* are unaffected because their performance and the prize they obtain do not change with pooling. Players with types in $(x^*, \hat{x})$ benefit, but the reason for this may vary across the players. Players with types higher than but close to x^* obtain a prize lottery that is better than the prize they obtain without pooling, which benefits them even though they choose a higher performance than without pooling. Players with types lower than but close to x^{**} obtain a prize lottery that is worse than the prize they obtain without pooling, so if pooling is Pareto improving, these players must choose a sufficiently lower performance with pooling that offsets the loss from the prize lottery.[15] To understand (7), which is the key condition in Proposition 1, multiply each side of (7) by $x - x^*$. The left-hand side of (7) is the prize lottery obtained by every type in $[x^*, x^{**}]$ in the contest with pooling, so the left-hand side of (7) multiplied by $x - x^*$ is the difference between the utilities of type $x > x^*$ and type x^* in the contest with pooling. The right-hand side of (7) multiplied by $x - x^*$ is

$$\int_{x^*}^{x} y^A\left(\tilde{x}\right)d\tilde{x} = \int_{0}^{x} y^A\left(\tilde{x}\right)d\tilde{x} - \int_{0}^{x^*} y^A\left(\tilde{x}\right)d\tilde{x},$$

which is the difference between the utilities of type $x > x^*$ and type x^* in the contest without pooling.[16] This difference in utilities is a linear function with pooling (since the prize $y(q^*, q^{**})$ is constant), and a convex function without pooling (because $y^A\left(\tilde{x}\right)$ increases in $\tilde{x}$), so type x^{**} is the most demanding: if type x^{**} benefits from pooling, all types in $[x^*, x^{**}]$

[15] If pooling is Pareto improving, then it reduces the aggregate cost of performance with utility (3) because the set of prizes is unchanged and all players are made (at least weakly) better off.

[16] Intuitively, type $\tilde{x} + d\tilde{x}$ can pretend to be type $\tilde{x}$, obtain prize $y^A\left(\tilde{x}\right)$, and enjoy a utility increase of $y^A\left(\tilde{x}\right)d\tilde{x}$ relative to type $\tilde{x}$.

benefit. This is illustrated in Figure 1.[17] The utility difference between types $x > x^{**}$ and type x^{**} is unaffected by pooling since their prize allocation does not change, which explains part (d) in the proposition.

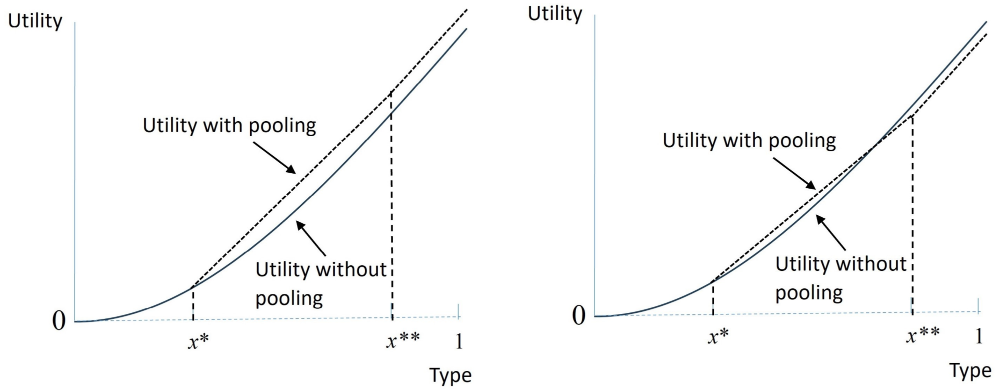


Figure 1: Pooling is Pareto improving (left) and pooling is not Pareto improving (right).

A particular class of pooling intervals are those with lower bound $q^* = 0$. We refer to such intervals as "bottom pooling" intervals, and identify each interval with its upper bound, q^{**}.[18] Such intervals will play an important role in the empirical and experimental parts of the paper.

4 Pooling on multiple intervals

We now consider more general performance disclosure policies, which pool on each of several intervals of performance ranking. We will use the term "category rankings" to describe such policies. One example is pooling above and below the median performance. Another example is pooling performances below the 10-th percentile, between the 10-th percentile and the 20-th percentile, etc. A category ranking induces a partition of the set of prizes, and the prizes within each element of the partition are randomly assigned to the players in the corresponding element of the category ranking.

Formally, a category ranking is a monotone partition $\mathcal{J}$ of the set $[0, 1]$ of quantiles into singletons and K left-open intervals. The intervals are $J_k = (q_k^*, q_k^{**}]$ for $1 \leq k \leq K \leq n$,

[17] The same phenomenon is illustrated in Figure 4 of Che et al. (2018).

[18] Bottom pooling with $q^{**} = 1$ is a lottery over the entire set of prizes. This lottery is also a special case of "top pooling," which is the class of pooling intervals with upper bound $q^{**} = 1$.

where $0 \leq q_1^* < q_1^{**} \leq \cdots \leq q_K^* < q_K^{**} \leq 1$. The interpretation is that the fraction $q_k^{**} - q_k^*$ of players whose performance quantile rankings lie in J_k are grouped together (any rule can be used to break ties in the ranking of two or more players who choose the same performance). Prizes are assigned in decreasing value to the partition elements and distributed according to a fair lottery among the players in each partition element. Similarly to the case of a single interval, this contest is equivalent to a contest with no performance pooling in which the prizes that correspond to each pooled interval J_k are replaced with a mass $q_k^{**} - q_k^*$ of prize

$$y(J_k) = \frac{\int_{q_k^*}^{q_k^{**}} G^{-1}(z)\, dz}{q_k^{**} - q_k^*} = \frac{\int_{x_k^*}^{x_k^{**}} y^A(x) dF(x)}{F(x_k^{**}) - F(x_k^*)}, \tag{8}$$

where $q_k^* = F(x_k^*)$ and $q_k^{**} = F(x_k^{**})$. Thus, the category ranking $\mathcal{J}$ induces a partition $\mathcal{I}$ of the set of types $X = [0,1]$ into singletons and the K intervals $I_k = (x_k^*, x_k^{**}]$, such that in the approximating mechanism of the category ranking all types in interval I_k choose the same performance and obtain the same prize $y(J_k)$, and singleton types obtain the prize they did in the original approximating mechanism. In what follows, it will be convenient to consider such partitions of the set of types into singletons and left-open intervals and the corresponding approximating mechanisms. We will abuse terminology slightly by also referring to such partitions $\mathcal{I}$ of the type interval $[0,1]$ as category rankings.

4.1 Welfare comparisons

For category rankings that include more than one interval, a generalization of the conditions in Proposition 1 provides sufficient conditions for a category ranking to increase the utility of a type and to be Pareto improving, but these conditions are no longer necessary. This is because pooling on an interval, I_1 say, may increase the utility of types in a higher interval, I_2 say, to such a degree that the net effect on all types is positive even if pooling on I_2 in isolation lowers the utility of some types in I_2 relative to the baselines contest. Relatedly, a category ranking that benefits the highest type $x = 1$ is no longer necessarily Pareto improving.[19]

[19] Both of these phenomena arise because with more than one pooling interval the equivalent of part (a) in Proposition 1 no longer holds: for an interval I_k that is not the lowest one in the category ranking (so

To obtain the sufficient conditions, consider a category ranking $\mathcal{I}$ that consists of the $K \geq 2$ intervals $I_1, \ldots, I_K$, where $I_k = (x_k^*, x_k^{**}]$ and $x_k^{**} \leq x_{k+1}^*$ for $k < K$. The effect of the category ranking can be described as follows. For each $k < K$ let G^k be the distribution of prizes when the prizes corresponding to intervals $I_1, \ldots, I_k$ are replaced by their averages. Then, the contest with the category ranking that pools only intervals $I_1, \ldots, I_{k+1}$ is the same as the contest with the single-interval category ranking that pools only interval I_{k+1} but starts with prize distribution G^k. Proposition 1 describes the effect of this single-interval category ranking on a baseline contest with prize distribution G^k. By induction on k we immediately obtain the following result as a corollary of Proposition 1.

Proposition 2. *(a) Type x in $I_k = (x_k^*, x_k^{**}]$ weakly benefits from the category ranking $\mathcal{I} = \{I_1, ..., I_K\}$ if*

$$\frac{\int_{x_k^*}^{x_k^{**}} y^A(\tilde{x}) dF(\tilde{x})}{F(x_k^{**}) - F(x_k^*)} \geq \frac{\int_{x_k^*}^{x} y^A(\tilde{x})\, d\tilde{x}}{x - x_k^*} \text{ and } \frac{\int_{x_j^*}^{x_j^{**}} y^A(\tilde{x}) dF(\tilde{x})}{F(x_j^{**}) - F(x_j^*)} \geq \frac{\int_{x_j^*}^{x_j^{**}} y^A(\tilde{x})\, d\tilde{x}}{x_j^{**} - x_j^*} \text{ for all } j < k,$$

and strictly benefits if any of the inequalities is strict.

(b) The category ranking is Pareto improving if

$$\frac{\int_{x_j^*}^{x_j^{**}} y^A(\tilde{x}) dF(\tilde{x})}{F(x_j^{**}) - F(x_j^*)} \geq \frac{\int_{x_j^*}^{x_j^{**}} y^A(\tilde{x})\, d\tilde{x}}{x_j^{**} - x_j^*} \text{ for all } j \leq K.$$

The next subsection characterizes when Pareto improving category rankings exist and discusses the Pareto frontier of category rankings.

4.2 Existence of Pareto improving category rankings and their Pareto frontier

We now provide a condition that characterizes contests for which Pareto improvements exist. For the condition, denote by $\widehat{F}$ the concavification (concave closure) of F, that is, the lowest-

$k > 1$), the utility of types lower than x_k^* may be lower or higher than in the baseline contest, depending on the effect of pooling on lower intervals.

valued concave function that is (pointwise) weakly higher than F.

Theorem 1. *There does not exist a Pareto improving category ranking if and only if for any interval of types on which $\widehat{F}$ is linear, in the assortative allocation all the types in the interval obtain the same prize, that is, $y^A(\cdot)$ is constant on the interval.*

The proof of Theorem 1 is in Appendix A. For some intuition, recall that by part (d) of Proposition 1, pooling on an interval is Pareto improving if and only if it benefits the highest type in the interval. When not all the types in an interval obtain the same prize, without pooling the highest (and best-performing) type obtains a prize that is (interim) better than the prize lottery induced by pooling. But pooling also eliminates the competition between the types in the interval, since they all obtain the same prize lottery. This competition is more intense, leading to more costly performance, if there are relatively more high types in the interval, since they are willing to pay relatively more for the higher prizes in the interval. The relative frequency of the types in the interval, which determines this trade-off between a worse (interim) prize and a less costly performance, is precisely captured by the comparison between F and $\widehat{F}$.

The condition in Theorem 1 is essentially condition (PS) of Che et al. (2018). The statement of Theorem 1 is very similar to what would be a single-bidder version of their Theorem 1.[20] The proof of the necessity of the condition is similar in both results, and the proof of the sufficiency of the condition in our result is much simpler because the settings are completely different.

For the distribution F that we estimate in Section 7 there is a single (maximal) interval of types on which the concavification $\widehat{F}$ is linear and not all the types in the interval obtain the same prizes. The lower bound of this interval is 0, and we investigate Pareto improving bottom pooling policies that include this interval. The distribution and its concavification are illustrated in Figure 6.

Turning to the Pareto frontier of Pareto improving category rankings, in Appendix C we provide a method for checking whether a Pareto-improving category ranking can be

[20]But their setting stipulates multiple bidders. In a setting like theirs with a single bidder, or with multiple bidders when fixing the equilibrium strategies of all but one of the bidders, the remaining bidder has no profitable manipulation (no "Pareto improvement"), regardless of his type distribution.

further Pareto improved. Using this method, we provide the following sufficient condition for a Pareto-improving category ranking to belong to the Pareto frontier. The proof is in Appendix C.

Proposition 3. *A Pareto-improving category ranking belongs to the Pareto frontier of Pareto-improving category rankings if on any interval of types on which $\widehat{F}$ is linear, all types in the interval obtain the same prize (or prize lottery) in the allocation induced by the category ranking.*

The condition in Proposition 3 is only sufficient. Example 2 in Appendix C shows that the Pareto frontier may also contain category rankings that do not satisfy the condition. As we discuss in the next subsection, however, the condition becomes sufficient and necessary if one considers mean-preserving contractions of the prize distribution instead of category rankings.

4.3 Mean-preserving contractions

Category rankings pool intervals of performance. We investigated the effect of these policies by using the fact that they are equivalent to pooling intervals of prizes. We now consider mean-preserving contractions of the prize distribution G, which generalize category rankings when the latter are viewed as pooling intervals of prizes instead of intervals of performance. Formally, distribution H is a *mean-preserving contraction* (henceforth: MPC) of G (defined on the same domain as G) if H second-order stochastically dominates G. In particular, category rankings can be viewed as MPCs of the original prize distribution. An MPC is an inverse of a mean-preserving spread, the latter of which is obtained by adding to each outcome of the original distribution a random variable with mean zero.[21] Adding a random variable can be interpreted as distributing the "mass" assigned to each outcome over possibly other outcomes. Therefore, an MPC of the prize distribution can be interpreted as dividing the prizes into into groups, and replacing the mass of each group with its expected value (or, equivalently, with a lottery over the prizes in the group). Moreover, any grouping of this kind results in an MPC of the original prizes. Thus, all MPCs of the the prize distribution,

[21] See Mas-Colell et al. (1995), Example 6.D.2 and Proposition 6.D.2.

and not only category rankings, can be implemented with an appropriate grouping policy. But implementing some MPCs involves policies that seem less realistic, or at least more controversial, than category rankings. The following example demonstrates this.

Example 1. *Suppose there are equal masses of colleges of quality 1, 3, 4, and 6. Consider an MPC that groups colleges 1 and 4 together and colleges 3 and 6 together. The average college quality is 2.5 in the first group and 4.5 in the second group. Applicants would be classified as "high performance" or "low performance" depending on whether their performance exceeds the median. Low performance applicants would be assigned to a college in the first group, and high performance applicants would be assigned to a college in the second group. Thus, some high performance applicants would be assigned to college 3 while some low performance applicants would be assigned to college 4. This is likely to be controversial.*

Even though some MPCs may be unrealistic, MPCs may be of theoretical interest. We therefore provide two results, whose proof is in Appendix B. The first result characterizes when Pareto-improving MPCs exist, generalizing Theorem 1.

Theorem 2. *There does not exist a Pareto-improving MPC if and only if for every interval of types on which $\widehat{F}$ is linear, in the assortative allocation all the types in the interval obtain the same prize, that is, $y^A(\cdot)$ is constant on the interval.*

The second result characterizes the Pareto frontier of Pareto-improving MPCs.[22]

Theorem 3. *An MPC belongs to the Pareto frontier of Pareto-improving MPCs if and only if on any interval of types on which $\widehat{F}$ is linear, all types in the interval obtain the same prize (or prize lottery) in the allocation induced by the MPC.*

5 Robust Pareto improvements

The results in the previous sections involve both the type distribution F and the prize distribution G (via the assortative allocation $y^A = G^{-1} \circ F$). We now present more robust

[22]Recall that Proposition 3 only provided a sufficient condition for a Pareto-improving category ranking to belong to the Pareto frontier.

results that involve only the type distribution F. These results may be useful for empirical work because their lack of reliance on G frees the analyst from making assumptions about how various college attributes that students may value are aggregated into a unidimensional prize value.[23] We point out, however, that the results still assume that students agree on the ranking of colleges. We will use the term "robust Pareto improvement" as shorthand for "weakly better for every type for any functions c and G, and a Pareto improvement for some functions c and G."[24] Our main robustness results characterize robust Pareto improving single-interval policies and category rankings, as well as their corresponding Pareto frontiers.

We begin with a characterization of robust Pareto improving single-interval policies.

Proposition 4. *Pooling on a quantile interval $(q^*, q^{**}]$ is robust Pareto improving if and only if*

$$\frac{F(x) - F(x^*)}{F(x^{**}) - F(x^*)} \leq \frac{x - x^*}{x^{**} - x^*} \tag{9}$$

for every x in $(x^*, x^{**}]$, where $q^* = F(x^*)$ and $q^{**} = F(x^{**})$.

Proposition 4 follows from part (d) of Proposition 1 because $y^A(\tilde{x}) = G^{-1}(F(x))$ can be an arbitrary increasing function with values in $[0, 1]$ for an appropriate G, and (9) states that distribution F restricted to the interval $[x^*, x^{**}]$ first-order stochastically dominates (FOSD) the uniform distribution on $[x^*, x^{**}]$.

The following result characterizes robust Pareto improving category rankings.

Proposition 5. *Category ranking $\mathcal{I} = \{I_1, ..., I_K\}$ is robust Pareto improving if and only if for every interval $I_k = (x_k^*, x_k^{**}]$ in $\mathcal{I}$ we have that*

$$\frac{F(x) - F(x_k^*)}{F(x_k^{**}) - F(x_k^*)} \leq \frac{x - x_k^*}{x_k^{**} - x_k^*} \tag{10}$$

for every x in $(x_k^*, x_k^{**}]$.

Sufficiency of the condition in Proposition 5 follows from part (b) of Proposition 2 because (10) states that distribution F restricted to each interval $[x_k^*, x_k^{**}]$ first-order stochastically

[23] We are grateful to a referee for suggesting this comment.

[24] This is different from robustness to the underlying information structure studied in the mechanism design literature.

dominates the uniform distribution on $[x_k^*, x_k^{**}]$. Necessity follows from Proposition 4 by observing that for every interval $I_k = (x_k^*, x_k^{**}]$ there are prize distributions G such that $y^A(\tilde{x}) = G^{-1}(F(x))$ is an arbitrary increasing function on $(x_k^*, x_k^{**}]$ with values in $[0,1]$ and is constant below x_k^* and above x_k^{**}.

Proposition 4, Proposition 5, and Theorem 1 imply the following characterizations of the Pareto frontiers of robust Pareto improving single-interval pooling policies and category rankings. The proofs are in Appendix A.

Theorem 4. *A single-interval pooling policy is on the Pareto frontier of robust Pareto improving single-interval pooling policies if and only if the interval is a maximal interval on which $\widehat{F}$ is linear.*

Theorem 4 shows that to obtain an unimprovable Pareto improving single-interval pooling policy that is robust to the prize distribution, one should pool on a maximal interval on which $\widehat{F}$ is linear. If, however, one is able to estimate the prize distribution, then other single-interval pooling policies may be on the Pareto frontier. Section 7 shows this in our empirical exercise.

Theorem 5. *If the number of maximal intervals on which $\widehat{F}$ is linear is finite, then the Pareto frontier of robust Pareto improving category rankings consists of the single category ranking that pools on every maximal interval on which $\widehat{F}$ is linear. Otherwise, the Pareto frontier is empty.*

The distinction in Theorem 5 between a finite and infinite number of maximal intervals on which $\widehat{F}$ is linear arises because, by definition, a category ranking consists of a finite number of pooling intervals.[25]

[25] This definition suffices for empirical applications and leads to technically simple proofs. Modifying the definition to allow for a countably infinite number of intervals would lead to the Pareto frontier consisting of the single category ranking that pools on every maximal interval on which $\widehat{F}$ is linear even when the number of such intervals is infinite.

6 Discussion: limitations and extensions

Our analysis relies on several assumptions. First, we stipulate a common ordinal ranking of college quality across students.[26] In reality, students often vary in how they rank colleges. Second, we assume that students can choose their performance precisely, without any "noise." This is obviously not the case in practice. In reality, students' test performance is often noisy, and the noisier the relationship between students' preparation efforts and their performance on the test, the less applicable our analysis of Pareto improvements. We make these restrictive assumptions because they are required for our use of Olszewski and Siegel's (2016) large contest framework.[27] Third, we assume that test preparation is costly, as in Spence (1973). This should be interpreted as net of any direct benefit from the preparation activities. This is most appropriate for activities geared specifically toward improving students' performance on the test, as discussed in the introduction. The model is less suitable for countries in which high-school performance or other activities that have significant direct benefits at moderate levels of investment play an important role in college admissions.[28]

In our model, a student's valuation for being admitted to a college does not depend on which other students are admitted to the same college. In Appendix D we consider a setting in which each type x exerts a peer effect $p(x)$, and each student in a college experiences the average effects of the other students in the college. In a large contest, each student is fairly certain about the equilibrium distribution of student types admitted to the various colleges. We can therefore replace the value y of being admitted to a specific college with another value that includes the peer effects generated by the set of students admitted to that college. This generates a new prize distribution, and the rest of the analysis is unchanged.

Our focus on pooling intervals of performance on the test has some practical advantages.

[26] Homogeneous ordinal preferences are also assumed in some matching papers on school choice (for example Lien, Zheng, and Zhong (2017)).

[27] With heterogeneous ordinal rankings, a higher type would not necessarily choose a higher performance. With noisy performance, every effort choice would map to an endogenous distribution over prizes. In both cases, the limiting allocation of prizes to types would no longer be assortative, which is required for our techniques.

[28] But even there the costs may exceed the benefits past a certain point, as Bodoh-Creed and Hickman (2024) demonstrate in the context of college admissions in the United States. They study a rich data set and a contest model in which effort can be productive, but show that for most students most of the effort is in fact wasteful.

First, pooling intervals of performance does not require changing any of the prizes (college seats); it only requires that the prizes corresponding to a set of students with pooled performances be allocated uniformly at random to these students. This makes such a policy potentially easy to implement. Second, pooling intervals of performance maintains the property that a higher performance is better: a higher performance leads to a lottery over prizes that are all better than the prizes in any different lottery associated with a lower performance. As we discuss in Section 4.3, MPCs of the prize distribution, which generalize pooling intervals of prizes, may violate this property.

7 Empirical illustration with Turkish data

We provide an empirical "proof of concept" by applying our theory to obtain Pareto improvements in a college admissions setting. We first estimate the primitives of our theory: a type distribution F, a prize distribution G, and a cost function c. Our estimation uses data on college applications of Turkish high school students.[29] These students invest in tutoring and obtaining admission to selective schools, and take the college entrance exam at the end of high school. They can retake the exam every year, and only the last attempt is considered. We extend the structural model of Krishna et al. (2018) to leverage the richness of the data, including students' choice to retake the exam, and estimate the primitives of the theoretical model.[30] We then use the estimated primitives to simulate a contest based on our theoretical model and derive Pareto improvements for this contest. We first apply our results on robust Pareto improvements and show that the Pareto optimal robust Pareto improving category ranking is a bottom pooling policy. We then consider the Pareto frontier of (non-robust) Pareto improving category rankings and show that the category ranking on the frontier that maximizes the utility of the students with the lowest ability is also a bottom pooling policy, which pools an even larger fraction of the students.

In the structural model, students participate in a large contest in a stationary overlapping

[29] Details regarding the data and the university entrance exam system in Turkey can be found in Krishna et al. (2018).

[30] We distinguish between the *theoretical model*, laid out in Section 2, and the *structural model*, which we estimate below.

generations environment. Students are heterogeneous in ability, and each student solves a single-agent infinite-horizon dynamic problem. In each period, the student faces uncertainty over future exam scores. The student makes an initial effort choice, which corresponds to schooling and tuition and determines her expected competitive standing when she takes the college entrance exam for the first time. Then the student learns her score, which is equal to her expected score plus noise, and decides whether to accept the corresponding placement or retake the exam. If the student retakes the exam, she incurs a retaking cost, which can depend on the number of times she has retaken the exam but is otherwise homogeneous across students. Students who retake the exam draw a new score without new effort choices, and so on.[31] In the steady state of this problem, the mass of students taking the exam at any point in time (fresh high school graduates and retakers) is constant.

We use the structural model and students' test retaking decisions to estimate, for every realized score, the value of obtaining that score in a particular retaking attempt.[32] This value includes the value of placement at that realized score and the option value of retaking the exam. From these estimates, we compute function $W(t)$, which is the value of reaching the *expected* score t in the first exam attempt. We also use the estimates to recover the prize distribution G because these estimates contain information on the value of placement (the prize). This part of the estimation process borrows from Krishna et al. (2018).

We then use the structural model to infer students' mean cost of effort, $C(t)$, at each score t. This is distinct from the cost $c(t)$ in the theoretical model, as we clarify below. To recover $C(t)$, we use students' private tutoring and high school choices (before the exam).[33] These choices are costly, but generate benefits captured by $W(t)$ (estimated above), which depend only on the expected score in the first exam attempt. The costs are the unknown parameters to be estimated. The investment choices are discrete, so we employ a discrete choice setting (mixed logit). We estimate the logit model, and from the observed choice shares, we back out the net values of investments, which are the values of placement minus the investment costs. We remove these net values from the gross benefits of investments reflected in $W(t)$

[31] Students who retake also obtain a learning shock to their score that is estimated to have a positive mean and decreases with the number of retaking attempts.

[32] This value may change across the first several attempts.

[33] There is no data on the cost of effort, so we have to estimate this cost indirectly from students' behavior.

and obtain the cost $C(t)$ of score t by averaging them across all the students with score t.

After estimating functions $W(t)$ and $C(t)$, we use the theoretical model to obtain its remaining primitives: the type distribution $F(x)$ and cost function $c(t)$. This part of the estimation has no counterpart in Krishna et al. (2018) and is a contribution in itself. It can be used independently of the procedure used to estimate $W(t)$ and $C(t)$ in the previous step. To obtain the type distribution $F(x)$ and cost function $c(t)$, we suppose that the students compete in a large contest that corresponds to the theoretical model with prizes given by the estimated function $W(t)$, and derive the type distribution $F(x)$ such that when each type chooses an optimal score, the score distribution matches the one in the data, and the cost of each score t is given by the estimated cost $C(t)$. The assumed optimality of students' choices gives us the function $x(t)$, which maps each score t to the type $x(t)$ that chooses it. This, together with $C(t)$, allows us to back out the cost function $c(t)$.

Having estimated prize function $G(t)$, cost function $c(t)$, and type distribution $F(x)$, we simulate a contest that corresponds to the theoretical model with these primitives and derive the effort, prize allocation, and utility of each type. We then use our theoretical results to investigate Pareto improvements for this contest. In what follows, we elaborate on these steps, focusing on the intuition and leaving the details to Online Appendices 1 and 2.

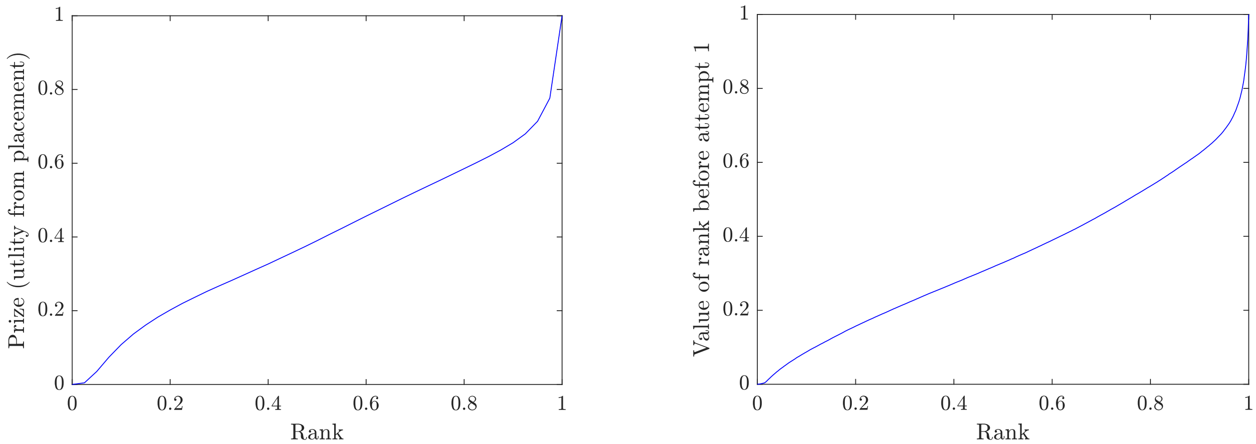
7.1 Estimation

7.1.1 Estimating G and W using the structural model

Each prize y corresponds to the value of a seat in the college system.[34] To back out the distribution G of y and function $W(t)$, which describes the value of obtaining an expected score of t in the first exam attempt, we first estimate a value function that maps every realized rank (realized score quantile) in the exam in a particular retaking attempt to a value that includes the value of placing with this rank, the option value of retaking the exam, and the cost of retaking following that attempt. Given a student's realized score rank on the exam, the student is more likely to retake the exam if the value function increases more sharply

[34] All values are in utility terms relative to the value of the best available seat, which is normalized to 1. The value of the worst available seat is normalized to zero, and all the costs and value functions are measured in these units.

at this rank. The placement value of a student with a rank r is $G^{-1}(r)$. Once we know the value function, we obtain $G^{-1}(r)$ and the retaking costs that rationalize this function by using Bellman's equation that describes the student's decision whether to retake the exam. Thus, the value function, retaking costs, and $G(y)$ are pinned down by the observed exam retaking rates. Since the cost of retaking is assumed homogeneous, it is pinned down by the average retaking rates. The variation in retaking rates by student rank, in turn, pins down the curvature of $G(y)$.



(a) Inverse CDF of prize distribution, $G^{-1}(r)$, mapping one's rank to prize.

(b) Value function before the first exam attempt, $W(t(r))$.

Figure 2: The value of placement and the value of rank in the first attempt

Figure 2a depicts the estimate of $G^{-1}(r)$, the utility of placement, obtained after normalizing the range of prizes to $[0, 1]$ and approximating $G^{-1}(r)$ in a flexible non-parametric way. This function increases sharply at the very top. Students with close-to-perfect scores retake, which can only be rationalized by a sharply increasing utility of placement.

Taking an expectation of the value function over score rank realizations, we obtain the value of a given expected rank in the first attempt, $W(t(r))$, where $t(r)$ is the expected score needed to obtain rank r ($W(t(r))$ is formally defined in equation (28) in Online Appendix 1). This value includes the utility of placement and the option value of retaking and its cost. It is depicted in Figure 2b.[35]

[35]The rank in Figure 2a is restricted to the set of students who accept placement, while that in Figure 2b includes all students taking the exam.

7.1.2 Estimating C using the structural model

To back out the remaining two primitives of the theoretical model, the cost function and the distribution of types, we rely on the observable data on pre-test effort. There are two ways in which students can increase their test scores: by going to private or selective public schools, and by taking extra preparatory courses after school. Both ways are costly: selective schools require entrance exams of their own and involve much effort, and private schools and preparatory courses charge tuition. We refer to selective public schools as 'exam schools.'

There is a clear relationship between pre-test investment and test outcomes. Figure 3 plots the shares of students taking preparatory courses and/or being enrolled in selective and private schools conditional on the exam score. Students at the bottom of the score distribution are predominantly educated in public schools; only about a third of them take preparatory courses before the exam. Students at the top of the score distribution are predominantly educated in exam schools, and nearly all of them take extra preparatory courses. Private schools are the middle ground between public schools and exam schools.

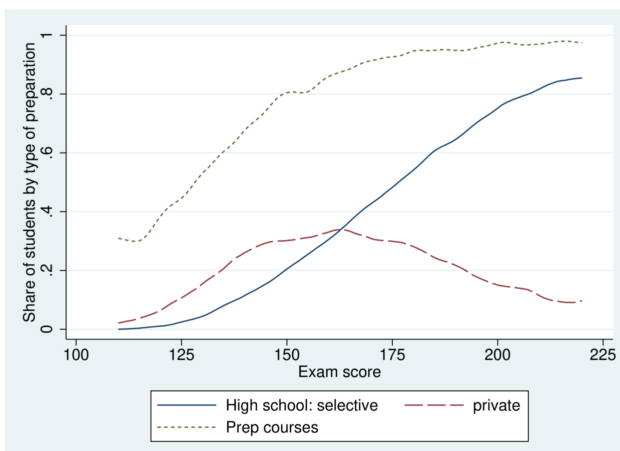


Figure 3: Schooling investments before the first exam attempt

To determine the costs of the three types of pre-test investment, we use a mixed logit discrete choice model with data on individual choices of students and the value of expected rank derived from the utility of placement estimated above. Each middle-school student chooses between public, private, and exam high schools, with or without extra preparatory courses, which results in six effort options in total. Each effort option is associated with an expected gain in score estimated from a regression with the score on the left-hand side, controlling for demographics, middle-school GPA, and other relevant observables. The utility gain from effort is the increase in the value of the expected rank, which comes from Figure 2b. The student maximizes the net gain, which is the difference between the above utility gain and the associated cost of investment.[36]

The net utilities in the logit model are directly related to the shares in the data of the options chosen by the students, that is, the net utilities are obtained by inverting the shares. Having net utilities and gross gains allows us to infer the costs incurred as their difference. These inferred costs vary across students both because of variations in student background and the random shocks to scores and costs of effort in the structural model. Having obtained the cost of each of the six effort levels and the parameters of the cost shocks, we use these parameters and the data on each student's effort level and realized score to compute, for each score, the average effort cost incurred by the students who obtained this score. These inferred costs, $C(t)$, are depicted in Figure 4.

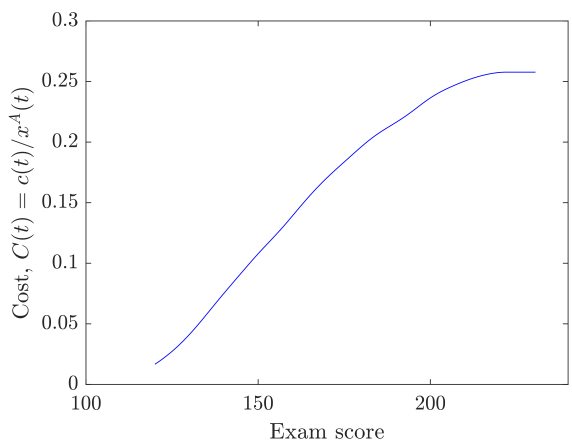


Figure 4: The estimated cost of effort, by score, in the first attempt

[36]Enrolling into an exam school may not be feasible for students who cannot pass the selection exams, especially if they choose the type of high school shortly before these exams. However, since these choices are often made well in advance, students can (and do) prepare for the selection exams. Thus, in principle, exam schools are available to everyone given sufficient effort. The cost of effort and the returns can vary across students, and our structural model allows for this. In particular, the mean cost of effort $\gamma_{g_{i0}}(hs, pt)$ in our model depends on students' middle-school GPA, g_{i0}, which captures the fact that to pass the school selection tests may be harder for students with lower levels of achievement.

7.1.3 Estimating F and c using the theoretical model

We have estimated $G(y)$, $W(t)$, and $C(t)$ using the structural model. To fully calibrate the theoretical model, it remains to estimate $F(x)$ and $c(t)$. To do so, we now think of the Turkish students as competing in a one-shot contest given by our theoretical model with prizes given by $W(t)$, which includes in reduced form the value of any future test retaking.[37] We suppose that the data corresponds to an equilibrium of this model, so that, in particular, $c(t)/x^A(t) = C(t)$.[38] We use this equation to obtain $F(x)$ and $c(t)$ as explained below.

From equation (3) with $W(t)$ instead of y, we obtain that the utility of type x is $W(t) - c(t)/x$. Differentiating this with respect to t, we obtain the optimality condition for type $x = x^A(t)$,

$$\frac{c'(t)}{x^A(t)} = W'(t). \tag{11}$$

Differentiating $C(t) = c(t)/x^A(t)$ and substituting for $\frac{c'(t)}{x^A(t)}$ using (11) gives

$$C'(t) = \frac{c'(t)}{x^A(t)} - \frac{c(t)}{x^A(t)^2}\frac{dx^A(t)}{dt} = W'(t) - \frac{C(t)}{x^A(t)}\frac{dx^A(t)}{dt}.$$

After re-arranging terms, we obtain a differential equation with $x^A(t)$, the equilibrium mapping from score to ability, as the unknown function:

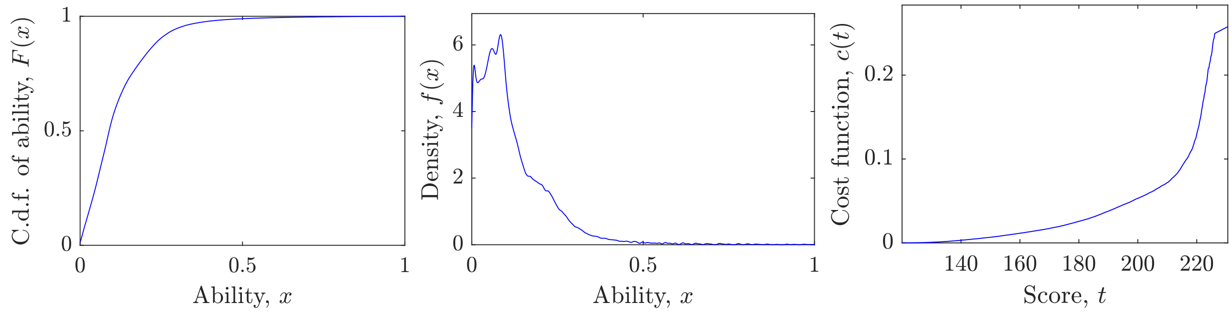
$$\frac{dx^A(t)}{dt} = x^A(t)\frac{W'(t) - C'(t)}{C(t)}.$$

At this point, we know the ratio $\frac{W'(t)-C'(t)}{C(t)}$. We also know that the highest-ability student chooses the highest score. This allows us to integrate the equation numerically to obtain $x^A(t)$. Once we have $x^A(t)$, we invert this function to obtain the distribution of ability since we have the distribution of scores in the data.[39] We also obtain $c(t)$ as $c(t) = C(t)x^A(t)$. These remaining primitives of the model are presented in Figure 5. Panels 5a and 5b plot the

[37] In particular, we suppose that the students can choose a deterministic non-negative score instead of choosing one of the six levels of effort that translate into a noisy score.

[38] Recall that the theoretical model predicts that students with higher ability choose higher scores; $x^A(t)$ associates each score t with the type $x^A(t)$ that chooses it in equilibrium.

[39] The distribution of ability F can be expressed via the observed CDF of scores, $H(t)$, and the inverse function $t^A = \left(x^A\right)^{-1}$: $F(x) = \Pr\{X < x\} = \Pr\{t^A(X) < t^A(x)\} = \Pr\{t < t^A(x)\} = H(t^A(x))$.



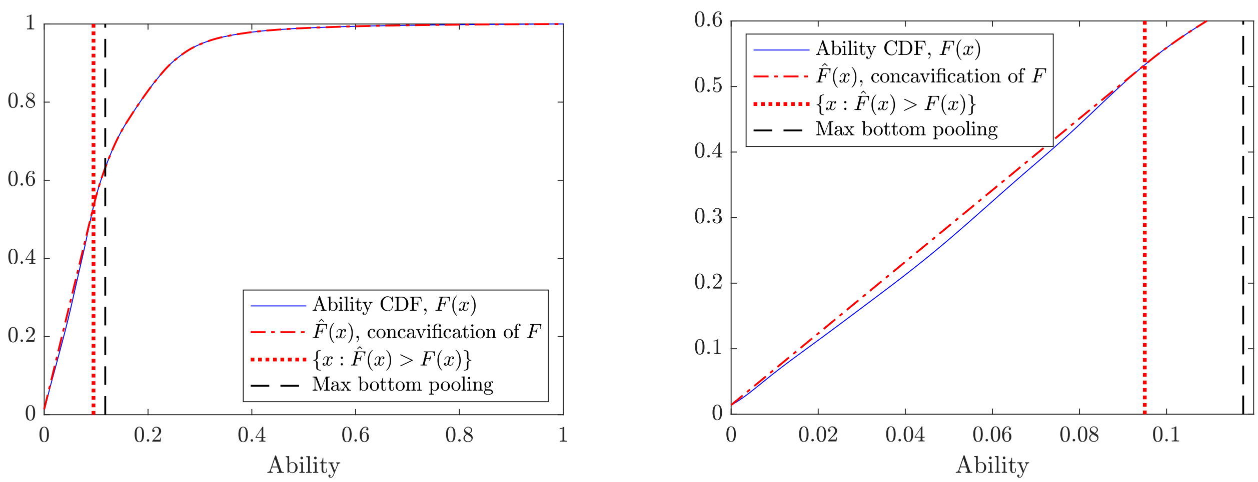
(a) Distribution function, $F(x)$. *(b) Density, $f(x)$.* *(c) Cost function $c(t)$*

Figure 5: Estimated elements of the model

ability distribution $F(x)$ and its density $f(x)$. The estimate for $c(t)$ is shown in Figure 5c.

7.2 Simulating Pareto improvements in the theoretical model

Having estimated $F(x)$, $G(y)$, and $c(t)$, we simulate a contest based on our theoretical model with these primitives and utility $y - c(t)/x$ (that is, (3)).[40] We then investigate Pareto improvements for this contest.



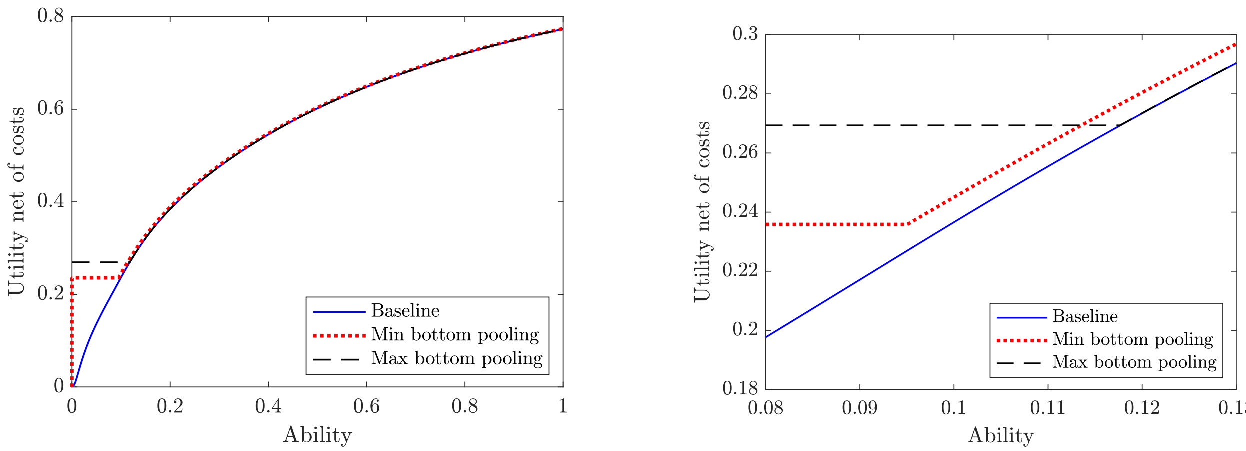
(a) C.d.f. of ability and its concavification *(b) C.d.f. of ability and its concavification, zoomed in*

Figure 6: Concavification of the ability distribution

Our theoretical results on Pareto-improving pooling intervals and category rankings refer to the concavification $\widehat{F}(x)$ of $F(x)$. Figure 6a plots $F(x)$ in solid blue and its concavification

[40] This contest has no exam retaking and uses prize distribution $G(y)$, which does not include the value of retaking, so that players face the rank-to-prize mapping in Figure 2a. One may consider using $W(t)$ instead of $G(t)$ as the prize distribution, but although $W(t)$ correctly reflects the value of placement and retaking in the baseline contest equilibrium, it is not a primitive of the theoretical model. A counterfactual pooling policy would affect retaking incentives, thereby altering the prize distribution relative to the baseline contest.

$\widehat{F}(x)$ in dash-dotted red. The two functions look very similar on $[0, 1]$, but the concavification is in fact significantly above F approximately on the interval $[0, 0.09]$.[41] This interval, which pools approximately 52 percent of the students, is highlighted in Figure 6b and its upper bound is depicted by the dotted red line, which clearly shows that $\widehat{F}(x)$ is linear on this interval and is not linear elsewhere. We refer to this interval as the minimal bottom pooling interval.



(a) Welfare gains for all types *(b) Welfare gains at the top of the pooling interval*

Figure 7: Welfare gains under minimal and maximal bottom pooling

We begin with robust Pareto improvements, which depend only on the type distribution. By Theorem 5, the Pareto frontier of robust Pareto improving category rankings (which may involve pooling on multiple intervals), consists of the single policy that pools every interval on which $\widehat{F}(x)$ is linear. Thus, because $\widehat{F}(x)$ is linear only the minimal bottom pooling interval, the bottom pooling policy that pools on this interval is the unique Pareto optimal robust Pareto improving policy.

We now turn to Pareto-improving policies that depend on the prize distribution and are not necessarily robust. Because the assortative allocation for the estimated type and prize distributions does not assign all types in the minimal bottom pooling interval the same prizes (see Figure 2a), by Theorem 1 Pareto improving policies exist. Moreover, by Proposition 3, the minimal bottom pooling policy (which is the Pareto optimal robust Pareto improving policy) is on the Pareto frontier of (non-robust) Pareto-improving category rankings. This

[41]Everywhere else the distance between $F(x)$ and its concavification, if it is positive, is more than a hundred times smaller. Such regions arise mostly due to rounding and smoothing errors for extreme values of x as the data get sparse in these regions.

Pareto frontier also includes other policies, which benefit different types to different degrees. Among them, we focus on the policy that benefits the lowest types the most.

To identify this policy, notice that the lowest types only benefit from policies that include a bottom-pooling interval.[42] Moreover, the longer the bottom-pooling interval, the greater the benefit of the low types because any bottom pooling interval is associated with a score of zero, and the higher the upper bound of the interval, the better the distribution of prizes associated with the interval. Thus, to benefit the lowest types the most, we first identify the largest Pareto-improving bottom-pooling interval. As we clarify below, this interval strictly contains the minimal bottom pooling interval, and the utility of the type at the top of the interval is the same as in the baseline contest (otherwise the interval can be further increased), and therefore so are the utilities of all higher types. Since $\widehat{F}(x)$ is not linear on any interval above the minimal bottom pooling one, Theorem 1 applied to the prize distribution corresponding to the largest Pareto-improving bottom pooling policy shows that no additional pooling interval can be added while remaining Pareto improving. That is, the longest Pareto-improving bottom pooling interval is the category ranking on the Pareto frontier of Pareto-improving category rankings that benefits the lowest types the most.

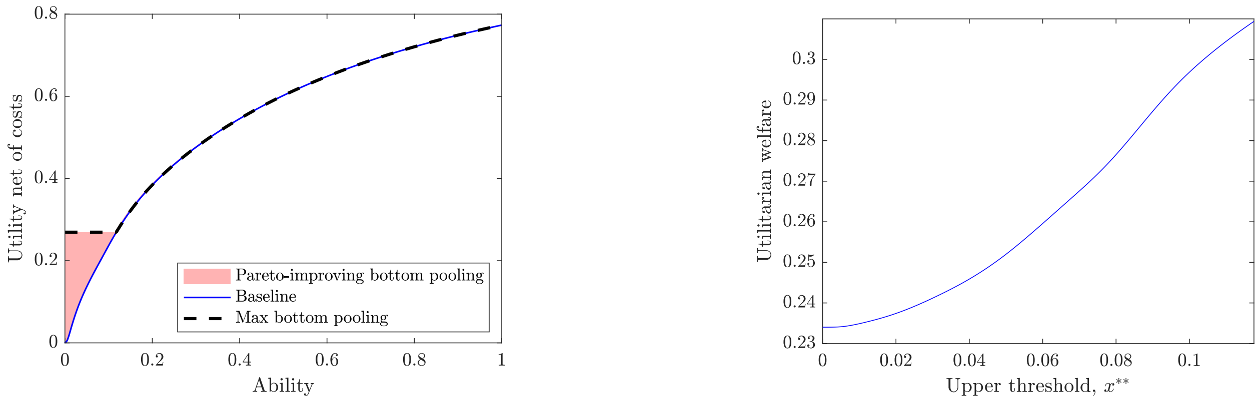
To identify the longest bottom pooling interval, we first pool on the minimal bottom pooling interval.[43] We simulate the baseline equilibrium payoffs, as well as those under minimal bottom pooling. Figure 7a shows these payoffs as a function of student type. The former and the latter payoffs are shown in solid blue and dotted red respectively. Figure 7b highlights these differences by zooming in on lower abilities.

Note that minimal bottom pooling makes everyone better off. It creates slack for those not pooled in terms of their utility relative to the baseline contest. This slack lets us increase the upper bound of the pooled region while keeping the non-pooled above their baseline payoffs. As raising the upper bound raises the expected prize for the pooled types, these types gain,

[42] Any category ranking without bottom pooling leaves unchanged the chosen score, prize allocation, and utility of the types in $[0, x]$ for some $x > 0$.

[43] Recall that there are other intervals on which $\widehat{F}(x)$ is slightly above $F(x)$, which we ignore. To verify that we do not lose anything by restricting ourselves to the lowest interval only, we compare the minimal bottom pooling to the policy that, in addition, pools on all these additional intervals. The payoffs under these two policies are depicted in Figure 3.1 in Online Appendix 3, which shows that they are essentially identical.

while the non-pooled ones lose. At some point, as we continue to raise the upper bound, the slack is exhausted, so that the type at the top of the pooling interval has the same payoff as in the status quo. At this point, we reach the maximal bottom pooling policy, under which the lowest types obtain their maximal payoff across all Pareto-improving policies on the Pareto frontier. The payoffs under this policy are depicted in Figure 7 in dashed black. This policy pools together approximately 63 percent of the students.[44]



(a) Range of net utilities under Pareto-improving bottom pooling vs. maximum bottom pooling

(b) Aggregate utility for all Pareto-improving bottom pooling policies as a function of x^{**}

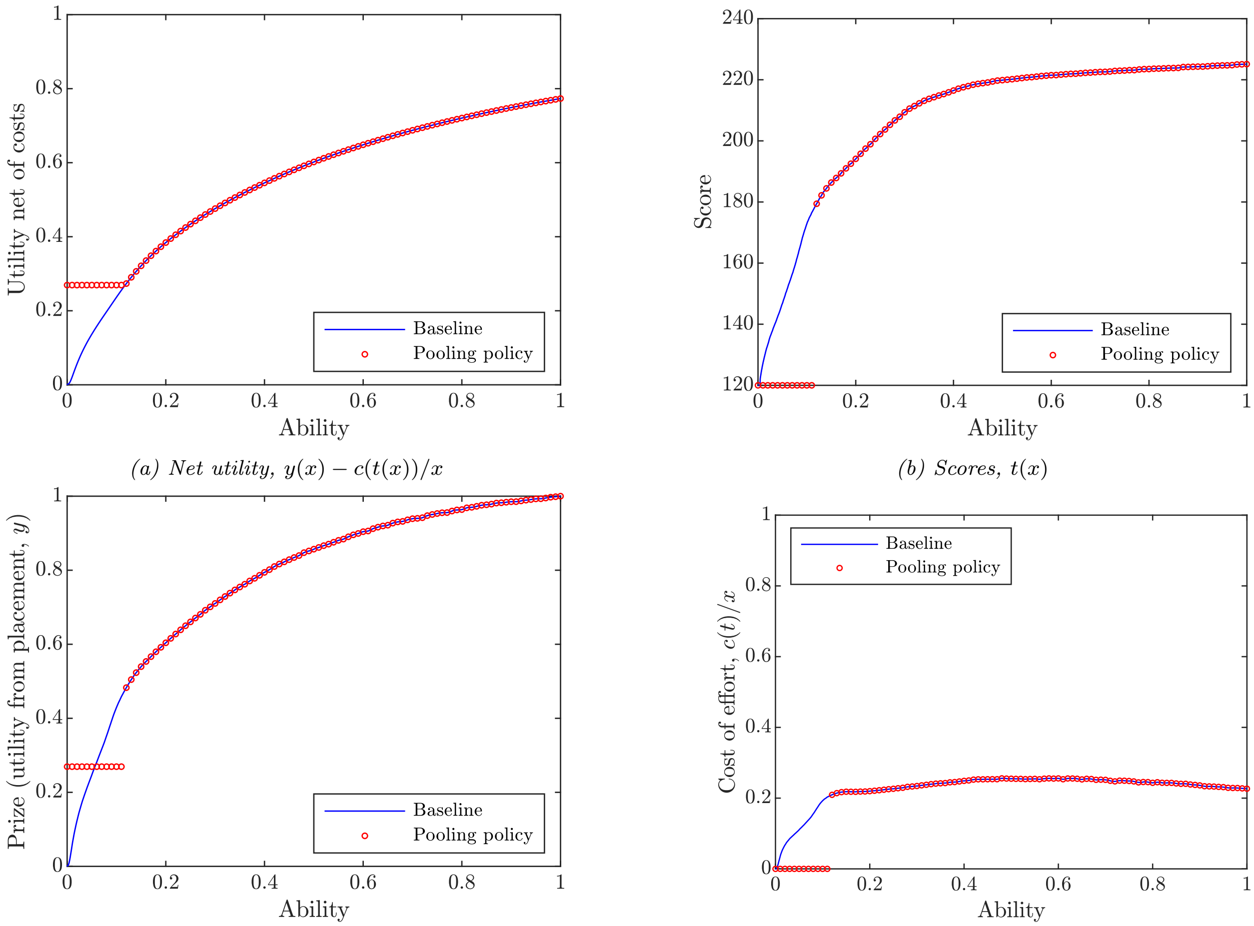
Figure 8: All Pareto-improving bottom pooling policies

In addition to maximizing the utility of the lowest types among all the policies on the Pareto frontier of Pareto-improving category rankings, in our setting the maximal bottom pooling policy also maximizes the aggregate utility across all bottom pooling policies. In Figure 8a, the pink area gives the payoffs obtained across all Pareto-improving bottom pooling policies, while the dashed black line depicts payoffs under maximal bottom pooling (as in Figure 7). Figure 8b depicts the aggregate utility for each upper bound of the Pareto-improving bottom-pooling policies. The figure shows that the aggregate utility increases in the upper bound, and is therefore maximized by maximal bottom-pooling.

To better understand the source of gains under maximal bottom pooling, consider Figure 9. Figure 9a shows types' utilities, $y(x) - c(t(x))/x$, under maximal bottom pooling

[44] As we mention in Section 6, our theoretical model does not allow for noise in scores or test retaking. To check if our main results still hold in a more complex environment, we simulate the maximal bottom pooling policy with the *structural* model from Section 7 instead of the theoretical model from Section 2 and present the results in Online Appendix 3. The structural model explicitly allows for shocks to scores (noise), multiple dimensions in student ability, and the option to retake the university entrance exam. We show that bottom pooling still achieves a Pareto improvement: irrespective of student's initial standing before high school, maximal bottom pooling raises student utility.

and in the baseline contest. Figure 9b shows the equilibrium scores under the two policies. Figures 9c and 9d decompose the payoffs into what is explained by placement, y, and what is explained by the cost of effort, $c(t)/x$.



(a) Net utility, $y(x) - c(t(x))/x$

(b) Scores, $t(x)$

(c) Prize in equilibrium, $y(x)$

(d) Cost of effort in equilibrium, $c(t(x))/x$

Figure 9: Equilibrium payoffs and effort in the baseline contest and under bottom pooling.

Bottom pooling strictly increases the payoff of the types in the pooled interval because pooling induces these types to reduce their investment while still obtaining one of the pooled college seats. Pooling the prizes at the bottom raises the placement payoff for the lower-end types within the pooled interval and reduces the placement payoff for the higher-end types (Figure 9c). However, since the costs of effort are zero for all the pooled types (Figure 9d), everyone in the pooled interval gains, with the lower-end types gaining more than the higher-end types (as evident from Figure 9a). Higher types, those above the pooled interval, are not affected because neither their placements nor their effort is affected by the change in policy from the baseline contest to the maximal bottom pooling one. Overall, the mean utility increases by 32 percent, while the pooled types gain 83 percent on average compared to the baseline contest.

8 Experimental exercise

We conducted a laboratory experiment with 602 subjects based on a discretized version of the calibration exercise and the maximal Pareto improving (non-robust) bottom pooling policy identified in Section 7. While there are substantial differences in stakes between the experiment and real-world decisions that influence college admissions, the experiment may help us identify unanticipated behaviors policy-makers may observe when implementing a pooling policy, and what the resulting welfare implications would be.

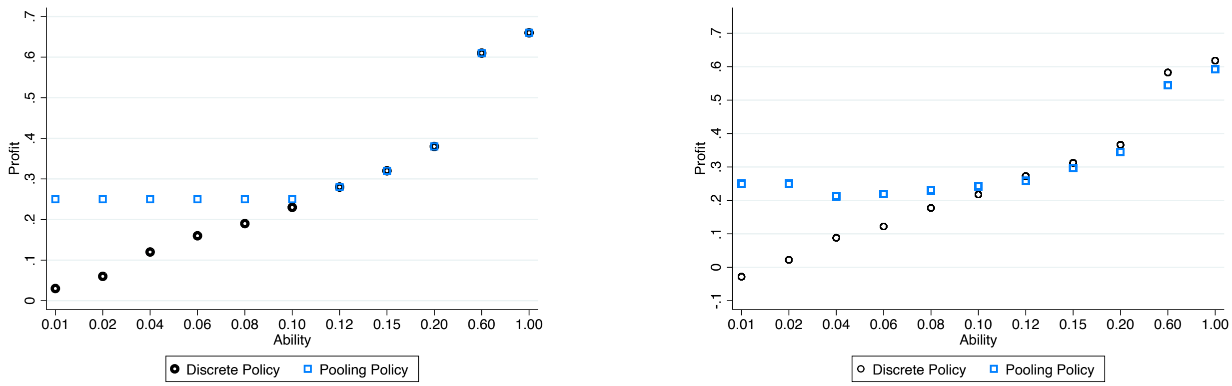
To conduct the experiment, we transformed the game into an individual decision-making problem in a discretized setting without strategic uncertainly. The costs and benefits, exogenous from the subjects' point of view, correspond to a situation in which the admission criteria are known in advance. This is often the case in college admissions settings that involve a large number of applicants and therefore entail little uncertainty.

In the experiment, each college, or tier of pooled colleges, had an admission threshold. Subjects in the experiment chose how much to invest in costly "virtual study materials" to reach their desired threshold under a discrete policy and under a pooling policy. Additional details are in Online Appendices 4 and 5. Online Appendix 6 contains all the experimental materials subjects faced.

8.1 Main experimental results

Figure 10 shows subjects' average utility by ability level under the discretized theoretical pooling policy (panel a) and in the experiment (panel b). Overall, adopting the pooling policy in the experiment increased aggregate welfare by 18.9 percent, closely matching the theoretical prediction of 20 percent[45].

[45] This theoretical prediction is computed for the discretized economy, which has a mix of the eleven ability types shown in Figure 10. Since this economy is not identical to the continuous-type economy in Section 7, the welfare gains reported here are not identical to those in Section 7.



(a) Predictions

(b) Laboratory data

Figure 10: Average profits under the discrete and pooling policies: predictions and experimental data.

Considering the impact of the pooling policy on each ability level separately, we find strong agreement with the theoretical predictions for low-ability subjects: their welfare increases by over 70 percent, close to the theoretical prediction of about 65 percent in the discretized model. The theoretical prediction for high-ability subjects is that their utility should be the same across the two policies. Instead, their welfare in the experiment decreased by 1.7 percent under the pooling policy. This was due to the behavior of subjects with ability levels of 0.12, the lowest level among subjects with high ability (whose utility was predicted to not change). These subjects opted to invest nothing and be assigned to the pooled set of colleges, so they faced a lottery, instead of investing and obtaining a better and deterministic college seat (further details are in Online Appendix 4).

Mapped to the Turkish student population, the experimental results imply that at least 85 percent of applicants should see their utility strictly or weakly increase, and at most 15 percent of the population may see their utility slightly decrease.

We explore possible reasons for the apparently suboptimal behavior of subjects with ability 0.12. We argue in Online Appendix 4 that this behavior is unlikely due to mistakes, experimental procedures, or risk-seeking behavior, and instead is consistent with preferences for randomization. Dwenger, Kübler and Weizsäcker (2016) explored such preferences in the context of school applications and showed that up to 50 percent of individuals choose lotteries between available allocations, indicating an explicit preference for randomization.[46]

[46]We also highlight that in our experiment the optimal choice for subjects with ability 0.12 without pooling was still available in the round with pooling. Thus, these subjects could obtain the same utility in both

9 Conclusion

This paper investigates how to improve college admissions based on centralized tests. Our main message is that coarse performance disclosure policies can benefit all students, regardless of their ability, when test preparation is costly. These policies take a simple form and are easy to implement. As a "proof of concept," we empirically estimated the key theoretical constructs using Turkish college admissions data. We used our theoretical results to simulate the equilibrium outcome of a college admissions contest based on these estimates, and demonstrated how to identify Pareto improving policies. We showed that a policy that pools together the majority of the lowest-performing students would benefit the lower-ability students the most, raising the welfare of the pooled students without impacting the welfare of the other students. We also conducted a laboratory experiment based on these empirical findings, which largely confirmed our theoretical predictions. Overall, our work suggests that Pareto improving performance disclosure policies of the kind we investigated often exist and have the potential to improve college admissions systems.

The data and code underlying this research is available on Zenodo at:
https://doi.org/10.5281/zenodo.14618581

rounds by maintaining the same behavior. By revealed preference, those subjects with ability 0.12 who switched to the lottery in the round with pooling were likely made better off, even though their monetary payoff decreased slightly, which is consistent with a wide range of preferences.

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Appendices

A Proofs of Proposition 1, Theorem 1, Theorem 4, and Theorem 5

Proof of Proposition 1. Part (a) follows because with pooling types $x \leq x^*$ choose effort $t^A(x)$ and obtain prize $y^A(x)$. For part (b), note that the utility of type x^* is the same in the approximating mechanisms with and without pooling. By (5), in the approximating mechanism of the original contest, the utility of a type x in $(x^*, x^{**}]$ exceeds that of type x^* by $\int_{x^*}^{x} y^A(\tilde{x})\, d\tilde{x}$. In the approximating mechanism with pooling, the utility of type x exceeds that of type x^* by

$$(x - x^*)\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})\, dF(\tilde{x})}{F(x^{**}) - F(x^*)},$$

Thus, pooling increases the utility of type x if and only if (7) holds.

For part (c), note that the derivative with respect to x of the utility gain of type x is

$$\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})\, dF(\tilde{x})}{F(x^{**}) - F(x^*)} - y^A(x)\,. \tag{12}$$

The fraction in (12) is a weighted average of $y^A(\tilde{x})$ over types in $(x^*, x^{**}]$, so (12) is positive for types x close to x^*, monotonically decreases as x increases, and becomes negative for types x close to x^{**}. Thus, the utility gain from pooling for types x in $(x^*, x^{**}]$ first increases and then decreases in the type. In particular, if not all prizes in the pooled interval are identical, the utility of all types x in (x^*, x^{**}) strictly increases if the utility of type x^{**} weakly increases. And the difference between the utilities of type $x > x^{**}$ and type x^{**} is $\int_{x^{**}}^{x} y^A(\tilde{x})\, d\tilde{x}$ in both approximating mechanisms, so pooling weakly benefits type x^{**} if and only if it weakly benefits type $x = 1$ if and only if it is Pareto improving, which gives part (d).

Proof of Theorem 1. Suppose first that there is a type interval $(x^*, x^{**}]$ on which $\widehat{F}$ is linear but not all types in the interval obtain the same prize, and without loss of generality suppose that $[x^*, x^{**}]$ is maximal, that is, $\widehat{F}$ is not linear on any interval that strictly contains $[x^*, x^{**}]$. By definition of $\widehat{F}$, we have that $\widehat{F}(x^*) = F(x^*)$, $\widehat{F}(x^{**}) = F(x^{**})$, and $\widehat{F}(x) \geq F(x)$ for every x in $[x^*, x^{**}]$. Thus, restricted to $[x^*, x^{**}]$, F first-order stochastically dominates $\widehat{F}$. Therefore,

$$\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x}) dF(\tilde{x})}{F(x^{**}) - F(x^*)} \geq \frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})\, d\widehat{F}(\tilde{x})}{\widehat{F}(x^{**}) - \widehat{F}(x^*)} = \frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})\, \frac{\widehat{F}(x^{**}) - \widehat{F}(x^*)}{x^{**} - x^*} d\tilde{x}}{\widehat{F}(x^{**}) - \widehat{F}(x^*)} = \frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})\, d\tilde{x}}{x^{**} - x^*},$$

where the first equality follows because $\widehat{F}$ is linear on $[x^*, x^{**}]$, so part (d) of Proposition 1 holds and pooling on $(x^*, x^{**}]$ is Pareto improving.

For the other direction, suppose that on every interval on which $\widehat{F}$ is linear all types obtain the same prize, and assume for contradiction that there exists a Pareto improving category ranking $\mathcal{I}$. Denote by $(x^*, x^{**}]$ the lowest type interval in the category ranking $\mathcal{I}$ that includes some types that strictly benefit from the category ranking.[47] Since $\mathcal{I}$ is Pareto improving, all types not higher than x^* are indifferent between the baseline contest and the category ranking. This implies that pooling on the single interval $(x^*, x^{**}]$ is Pareto improving. In particular, not all types in the interval obtain the same prize, otherwise pooling on $(x^*, x^{**}]$ has no effect. Let quantiles q^* and q^{**} and types x' and x'' be such that $q^* = \widehat{F}(x') = F(x^*)$ and $q^{**} = \widehat{F}(x'') = F(x^{**})$. Because $F \leq \widehat{F}$, we have that $x' \leq x^*$ and $x'' \leq x^{**}$. In addition, $G^{-1} \circ F = G^{-1} \circ \widehat{F}$. Indeed, $G^{-1}(F(x)) = G^{-1}(\widehat{F}(x))$ whenever $F(x) = \widehat{F}(x)$. And for a type x with $F(x) < \widehat{F}(x)$, by definition $\widehat{F}$ is linear on an interval that includes x. Consider the maximal interval on which $\widehat{F}$ is linear that includes x. All the types in the interval obtain the same prize, and at the endpoints of the interval F and $\widehat{F}$ coincide (by definition of $\widehat{F}$), so on this interval $G^{-1} \circ F$ and $G^{-1} \circ \widehat{F}$ coincide. These observations imply that

$$\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})dF(\tilde{x})}{F(x^{**}) - F(x^*)} = \frac{\int_{x'}^{x''} G^{-1}(\widehat{F}(x))d\widehat{F}(\tilde{x})}{\widehat{F}(x'') - \widehat{F}(x')} \leq \frac{\int_{x'}^{x''} G^{-1}(\widehat{F}(x))d\tilde{x}}{x'' - x'} \leq \frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})d\tilde{x}}{x^{**} - x^*}, \tag{13}$$

where the equality follows because both expressions are equal to the expected prize $\int_{q^*}^{q^{**}} G^{-1}(z)\,dz/(q^{**} - q^*)$ in quantile interval $[q^*, q^{**}]$, the first inequality follows because $\widehat{F}$ is concave so is first-order stochastically dominated by the uniform distribution when both are restricted to the interval $[x', x'']$, and the second inequality follows because $x' \leq x^*$, $x'' \leq x^{**}$, and $G^{-1} \circ F = G^{-1} \circ \widehat{F}$. Moreover, the second inequality is strict if $x' < x^*$ or $x'' < x^{**}$ because not all types in the interval $(x^*, x^{**}]$ obtain the same prize. And the first inequality is strict if $x' = x^*$ and $x'' = x^{**}$ because then not all types in $(x', x'']$ obtain the same prize and $\widehat{F}$ is strictly concave on $[x', x'']$.

[47]Such an interval exists otherwise all types weakly prefer the baseline contest to the category ranking, so the category ranking is not Pareto improving.

We conclude that

$$\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})dF(\tilde{x})}{F(x^{**}) - F(x^*)} < \frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})d\tilde{x}}{x^{**} - x^*},$$

so type x^{**} is strictly harmed by pooling on type interval $(x^*, x^{**}]$ (as are types slightly lower than x^{**}), contradicting that pooling on $(x^*, x^{**}]$ is Pareto improving.

Proof of Theorem 4. We first observe that any robust Pareto improving pooling interval is contained in an interval on which $\widehat{F}$ is linear. To see this, take an interval $(x^*, x^{**}]$ that is not so contained. Consider a distribution G of prizes that gives the same prizes to types in any interval on which $\widehat{F}$ is linear, and gives different prizes to all other types.[48] Then the proof of the "if" direction of Theorem 1 shows that pooling on $(x^*, x^{**}]$ hurts type x^{**} (and nearby types) because not all types in $(x^*, x^{**}]$ obtain the same prize (since $(x^*, x^{**}]$ is not contained in an interval on which $\widehat{F}$ is linear). Thus, pooling on $(x^*, x^{**}]$ is not robust Pareto improving. Now, pooling on a maximal interval on which $\widehat{F}$ is linear is robust Pareto improving by definition of $\widehat{F}$ and Proposition 4.

For the Pareto frontier, we show that if $(x', x'']$ and $(x^*, x^{**}]$ are robust Pareto improving pooling intervals and $(x', x''] \subseteq (x^*, x^{**}]$, then $(x^*, x^{**}]$ is robust Pareto preferred to $(x', x'']$. This is because for any prize distribution G, pooling on interval $(x', x'']$ is equivalent to using a baseline contest in which types $(x', x'']$ are allocated identical prizes that are the average of the prizes they are allocated under G. Starting from this modified prize distribution and pooling on the interval $(x^*, x^{**}]$ is equivalent to pooling on the interval $(x^*, x^{**}]$ directly, which is robust Pareto improving. Finally, consider any two distinct maximal intervals on which $\widehat{F}$ is linear. For each interval consider a prize distribution that gives all types below the interval the same prize, all types above the interval the same prize, and different prizes to the types in the interval. With this prize distribution pooling on the interval is Pareto improving, but pooling on the other interval has no effect. Thus, neither of the two robust Pareto improving intervals is robust Pareto preferred to the other.

Proof of Theorem 5. Similarly to the proof of Theorem 4, any interval that is part of a robust Pareto improving category ranking is contained in an interval on which $\widehat{F}$ is linear, pooling on maximal intervals on which $\widehat{F}$ is linear is robust Pareto improving by definition of

[48] For example, start with assigning prize $y = x$ for every type x, and then for any maximal interval $(x', x'']$ on which $\widehat{F}$ is linear set the prize of every type in the interval to be the average of $y = x'$, and $y = x''$.

$\widehat{F}$ and Proposition 5, if every interval in one category ranking is contained in some interval of another category ranking then the second is Pareto preferred to the first, and if one category ranking contains a maximal interval on which $\widehat{F}$ is linear that is not contained in some other category ranking, then the other category ranking is not robust Pareto preferred to the first. Finally, taking a category ranking that consists of one or more maximal intervals on which $\widehat{F}$ is linear and adding to it another maximal interval on which $\widehat{F}$ is linear generates a category ranking that is robust Pareto preferred to the original category ranking. This is because the effect of the original category ranking is identical to the effect of using a baseline contest in which the prizes allocated to each pooled interval of types are replaced with the same mass of the average of these prizes, and then pooling on the additional maximal interval on which $\widehat{F}$ is linear. This proves the result.

B Proofs of Theorems 2 and 3 (mean-preserving contractions)

To prove Theorem 2, we will need the following two lemmas. The first lemma seems to belong to "statistics folklore". We give its proof for completeness; for simplicity, we will restrict attention to continuous, strictly increasing G and H.

Lemma 1. *Suppose that H and G are two CDFs of distributions with the same domain, say $[0,1]$. Then, H second-order stochastically dominates G if and only if*

$$\int_0^z [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})]d\widetilde{z} \leq 0 \tag{14}$$

for all z, with equality for $z = 1$.

Proof. We will first prove necessity. Observe first that (14) holds whenever $G^{-1}(z) = H^{-1}(z)$. Indeed, in the system of coordinates with z on the vertical axis and with $G^{-1}(z)$ and $H^{-1}(z)$ on the horizontal axis, $\int_0^z G^{-1}(\widetilde{z})d\widetilde{z}$ is the area between the graph of G^{-1}, the vertical axis, and the horizontal line at the level of z. This area is equal to the area of the rectangle $[0, G^{-1}(z)] \times [0, z]$ minus the area between the graph of G, the horizontal axis, and the vertical line at the level of $G^{-1}(z)$. Similarly, $\int_0^z H^{-1}(\widetilde{z})d\widetilde{z}$ is equal to the area of the rectangle $[0, H^{-1}(z)]\times[0,z]$ minus the area between the graph of H, the horizontal axis, and the vertical line at the level of $H^{-1}(z)$. Thus, (14) follows from the fact that

$$\int_0^x G(\widetilde{x})d\widetilde{x} \geq \int_0^x H(\widetilde{x})d\widetilde{x}$$

for all x (in particular $x = G^{-1}(z) = H^{-1}(z)$) when H second-order stochastically dominates G.

Note here that the last inequality holds with equality for $x = 1$, by second-order dominance, which yields (14) with equality for $z = 1$.

Suppose now that $G^{-1}(z) > H^{-1}(z)$. Let $\overline{z} = \min\{\widetilde{z} \geq z : G^{-1}(\widetilde{z}) = H^{-1}(\widetilde{z})\}$. Then

$$\int_0^{\overline{z}} [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})]d\widetilde{z} \geq \int_0^z [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})]d\widetilde{z},$$

because $G^{-1}(\widetilde{z}) \geq H^{-1}(\widetilde{z})$ for $\widetilde{z} \in [z, \overline{z}]$. Thus, since (14) holds for $\overline{z}$, it holds for z.

Suppose finally that $G^{-1}(z) < H^{-1}(z)$. Let $\underline{z} = \max\{\widetilde{z} \leq z : G^{-1}(\widetilde{z}) = H^{-1}(\widetilde{z})\}$. Then

$$\int_0^z [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})]d\widetilde{z} \leq \int_0^{\underline{z}} [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})]d\widetilde{z},$$

because $G^{-1}(\widetilde{z}) \leq H^{-1}(\widetilde{z})$ for $\widetilde{z} \in [\underline{z}, z]$. Thus, since (14) holds for $\underline{z}$, it holds for z. This completes the proof of necessity.

The proof of sufficiency obtains by applying the proof of necessity to the functions $\widetilde{G} = H^{-1}$ and $\widetilde{H} = G^{-1}$. □

Lemma 2. *Let $f, g : [0,1] \to \mathbb{R}$ be bounded Lebesgue measurable functions. Suppose that f is weakly increasing and g has the property that $\int_x^1 g(\widetilde{x})d\widetilde{x} \geq 0$ for every $x \in (0,1]$ and $\int_0^1 g(\widetilde{x})d\widetilde{x} = 0$. Then $\int_0^1 f(\widetilde{x})g(\widetilde{x})d\widetilde{x} \geq 0$. Moreover, $\int_0^1 f(\widetilde{x})g(\widetilde{x})d\widetilde{x} > 0$ if one of the following conditions is satisfied: (a) $\int_{\overline{x}}^1 g(\widetilde{x})d\widetilde{x} > 0$ for some $\overline{x}$, and f is not constant on any interval $(\underline{x}, \overline{x}]$; (b) $\int_{\underline{x}}^1 g(\widetilde{x})d\widetilde{x} > 0$ for some $\underline{x}$, and f is not constant on any interval $[\underline{x}, \overline{x})$.*

Proof. Assume w.l.o.g. that f takes values in $(0,1)$. Otherwise, consider an affine transformation $cf + d$ of f with a positive slope $c > 0$, which takes values in $(0,1)$. The lemma for $cf + d$ implies the lemma for f. Represent f as the pointwise limit of functions $f_n = \sum_{i=1}^{n}(1/n)\chi_{A_i^n}$, where $A_i^n = \{x \in [0,1] : f(x) > i/n\}$ and $\chi_{A_i^n}$ takes value 1 on A_i^n and value 0 on $[0,1]\backslash A_i^n$. Since f is weakly increasing, $A_i^n = (x_i^n, 1]$ or $A_i^n = [x_i^n, 1]$ for some x_i^n.

Since f, f_n, and g are bounded,

$$\begin{aligned}\int_0^1 f(\widetilde{x})g(\widetilde{x})d\widetilde{x} &= \lim_n \int_0^1 g(\widetilde{x})\left[\sum_{i=1}^{n}(1/n)\chi_{A_i^n}(\widetilde{x})\right]d\widetilde{x} \\ &= \lim_n \sum_{i=1}^{n}(1/n)\int_0^1 g(\widetilde{x})\chi_{A_i^n}(\widetilde{x})d\widetilde{x} \\ &= \lim_n \sum_{i=1}^{n}(1/n)\int_{x_i^n}^1 g(\widetilde{x})d\widetilde{x},\end{aligned} \tag{15}$$

This yields the first part of the lemma, that is, $\int_0^1 f(\widetilde{x})g(\widetilde{x})d\widetilde{x} \geq 0$, because$\int_{x_i^n}^1 g(\widetilde{x})d\widetilde{x} \geq 0$ for all i.

For part (a) of the second part of the lemma, notice that if $\int_{\overline{x}}^1 g(\widetilde{x})d\widetilde{x} > 0$ for some $\overline{x}$, then there is a constant $c > 0$ such that $\int_x^1 g(\widetilde{x})d\widetilde{x} > c$ for all x from an interval $(\underline{x}, \overline{x}]$. If f is not constant on this interval, then for any large enough n there is a fraction of i's that is bounded away from zero such that $x_i^n \in (\underline{x}, \overline{x}]$. This implies that the last sum in (15) is bounded away from zero, uniformly across all large enough n's. The argument for part (b) is analogous. □

We can now prove Theorem 2.

Proof of Theorem 2. The "only if" follows from the "only if" of Theorem 1. For the other direction, suppose that for every interval of types on which $\widehat{F}$ is linear, in the assortative allocation all the types in the interval obtain the same prize but there exists an MPC H of G that is Pareto-improving. We can assume w.l.o.g. that the allocation under H is also constant on every interval $[x', x'']$ on which $\widehat{F}$ is linear; otherwise, it can be replaced by its Pareto-improving contraction that pools each maximal such interval $[x', x'']$. This pooling composed with H is a Pareto-improving MPC of G as a composition of two mappings with these two properties.

The utility of type $x = 1$ when the prizes are distributed according to G and according to H are

$$\int_0^1 G^{-1} \circ F(x)\, dx \text{ and } \int_0^1 H^{-1} \circ F(x)\, dx,$$

respectively. Since in the assortative allocation all the types of any interval on which $\widehat{F}$ is linear obtain the same prize, none of the two values will be affected when we replace F with $\widehat{F}$.

By substituting $z = \widehat{F}(x)$, we obtain that the utility of type $x = 1$ is

$$\int_0^1 G^{-1}(z) \frac{1}{\widehat{f}(\widehat{F}^{-1}(z))} dz \text{ and } \int_0^1 H^{-1}(z) \frac{1}{\widehat{f}(\widehat{F}^{-1}(z))} dz,$$

respectively. The difference between the two values is

$$\int_0^1 [G^{-1}(z) - H^{-1}(z)] \frac{1}{\widehat{f}(\widehat{F}^{-1}(z))} dz. \tag{16}$$

Since H second-order stochastically dominates G, we have that $\int_0^z [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})] d\widetilde{z} \le 0$ for all $z < 1$, and $\int_0^1 [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})] d\widetilde{z} = 0$. So, $\int_z^1 [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})] d\widetilde{z} \ge 0$ for all $z < 1$ and $\int_0^1 [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})] d\widetilde{z} = 0$. Since $\widehat{F}$ is concave, $1/\widehat{f}(\widehat{F}^{-1}(z))$ is a weakly increasing function of z. Thus, (16) is nonnegative by Lemma 2.

However, to show that the contraction H strictly decreases the utility of type $x = 1$ and obtain a contradiction, we must show that (16) is strictly positive. To show this, we apply the second part of Lemma 2. To be Pareto improving, H must be a nontrivial contraction, that is, it must be that $\int_z^1 [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})] d\widetilde{z} > 0$ for some $z \in (0, 1)$. Moreover, we can assume w.l.o.g. that $z = F(x')$ or $z = F(x'')$ for a maximal interval $[x', x'']$ on which $\widehat{F}$ is linear. Indeed, since both G^{-1} and H^{-1} are constant on $[F(x'), F(x'')]$ for each each such interval $[x', x'']$, if $\int_z^1 [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})] d\widetilde{z} > 0$ for some $z \in [F(x'), F(x'')]$, then $\int_z^1 [G^{-1}(\widetilde{z}) - H^{-1}(\widetilde{z})] d\widetilde{z} > 0$ for $z = F(x')$ or $F(x'')$. If $\overline{z} := z = F(x')$, then $\widehat{f}(\widehat{F}^{-1}(z))$ cannot be constant on any interval $(\underline{z}, \overline{z}]$ because $[x', x'']$ is maximal. If $\underline{z} := z = F(x')$, then $\widehat{f}(\widehat{F}^{-1}(z))$ cannot be constant on any interval $[\underline{z}, \overline{z})$ because $[x', x'']$ is maximal.

To prove Theorem 3 we need the following lemma.

Lemma 3. *Let $f : [0, x] \to \mathbb{R}_+$ and $h : [0, x] \to \mathbb{R}$ be bounded Lebesgue measurable functions. Suppose that f is weakly decreasing and h has the property that $\int_0^y h(\widetilde{y}) d\widetilde{y} \ge 0$ for every*

$y \in (0, x]$. *Then* $\int_0^x f(\widetilde{x})g(\widetilde{x})d\widetilde{x} \geq 0$.

Proof. Assume w.l.o.g. that f takes values in $[0, 1)$. Otherwise, consider a linear transformation cf of f with a positive slope $c > 0$, which takes values in $(0, 1)$. The lemma for cf implies the lemma for f. Represent f as the pointwise limit of functions $f_n = \sum_{i=1}^{n}(1/n)\chi_{A_i^n}$, where $A_i^n = \{y \in [0, x] : f(y) > i/n\}$ and $\chi_{A_i^n}$ takes value 1 on A_i^n and value 0 on $[0, x]\backslash A_i^n$. Since f is weakly decreasing, $A_i^n = [0, x_i^n]$ or $A_i^n = [0, x_i^n)$ for some x_i^n.

Since f, f_n, and g are bounded,

$$
\begin{aligned}
\int_0^x f(\widetilde{x})h(\widetilde{x})d\widetilde{x} &= \lim_n \int_0^x h(\widetilde{x}) \left[\sum_{i=1}^{n}(1/n)\chi_{A_i^n}(\widetilde{x})\right] d\widetilde{x} \\
&= \lim_n \sum_{i=1}^{n}(1/n) \int_0^x g(\widetilde{x})\chi_{A_i^n}(\widetilde{x})d\widetilde{x} \qquad (17)\\
&= \lim_n \sum_{i=1}^{n}(1/n) \int_0^{x_i^n} g(\widetilde{x})d\widetilde{x},
\end{aligned}
$$

This completes the proof, because $\int_0^{x_i^n} g(\widetilde{x})d\widetilde{x} \geq 0$ for all n and i. □

We can now prove Theorem 3.

Proof of Theorem 3. The "only if" direction follows from part (d) of Proposition 1, similarly to the proof of the "only if" direction of Theorem 1. For the other direction, consider an MPC H of G, and suppose that for every interval of types on which $\widehat{F}$ is linear, in the allocation induced by H all the types in the interval obtain the same prize, but there exists an MPC $\widetilde{H}$ of G that Pareto improves on H. We can assume w.l.o.g. that the allocation induced by $\widetilde{H}$ is also constant on every interval $[x', x'']$ on which $\widehat{F}$ is linear; otherwise, it can be replaced by a Pareto-improving MPC that pools each maximal such interval $[x', x'']$.[49]

We will show that $\widetilde{H}$ must be an MPC of H. Since both H and $\widetilde{H}$ are contractions of G,

$$\int_0^1 (\widetilde{H})^{-1}(\widetilde{x})d\widetilde{x} = \int_0^1 G^{-1}(\widetilde{x})\, d\widetilde{x} = \int_0^1 H^{-1}(\widetilde{x})\, d\widetilde{x}.$$

[49]This pooling is a Pareto-improving MPC of G as a Pareto-improving MPC of the Pareto-improving MPC $\widetilde{H}$ of G.

So, by Lemma 1, it remains to show that

$$\int_0^x [(\widetilde{H})^{-1}(\widetilde{x}) - H^{-1}(\widetilde{x})]d\widetilde{x} \geq 0 \tag{18}$$

for all $x \in (0,1)$.

Since $\widetilde{H}$ Pareto improves H, by (5) it must be that

$$\int_0^x (\widetilde{H})^{-1} \circ F(\widetilde{x})\, d\widetilde{x} \geq \int_0^x H^{-1} \circ F(\widetilde{x})\, d\widetilde{x} \tag{19}$$

for all $x \in [0,1]$. And since both H and $\widetilde{H}$ are constant on every interval $[x', x'']$ on which $\widehat{F}$ is linear, we can replace F with $\widehat{F}$ in (19), that is,

$$\int_0^x (\widetilde{H})^{-1} \circ \widehat{F}(\widetilde{x})\, d\widetilde{x} \geq \int_0^x H^{-1} \circ \widehat{F}(\widetilde{x})\, d\widetilde{x}, \tag{20}$$

for all $x \in [0,1]$. By substituting $\widetilde{y} = \widehat{F}(\widetilde{x})$, (20) is equivalent to

$$\int_0^y [(\widetilde{H})^{-1}(\widetilde{y}) - H^{-1}(\widetilde{y})]\frac{1}{\widehat{f}(\widehat{F}^{-1}(\widetilde{y}))}d\widetilde{y} \geq 0 \tag{21}$$

for all $y \in [0,1]$.

Now, apply Lemma 3 to $h(y) = [(\widetilde{H})^{-1}(y) - H^{-1}(y)]/\widehat{f}(\widehat{F}^{-1}(y))$ and $f(y) = \widehat{f}(\widehat{F}^{-1}(y))$ (which is weakly decreasing in y since $\widehat{F}$ is concave) to obtain (18).

Thus, $\widetilde{H}$ is an MPC of H. This is a contradiction to Theorem 2 applied to prize distribution H, because for every interval $[x', x'']$ on which $\widehat{F}$ is linear all types in $[x', x'']$ obtain the same prize in the allocation induced by H.

C Pareto frontier of category rankings

First, using Proposition 2, we will provide a method for checking whether a category ranking belongs to the Pareto frontier of category rankings. Next, we use this method to show that there is no Pareto-improving, category ranking of a category ranking that is constant on each interval $(x', x'']$ on which $\widehat{F}$ is linear. Finally, we give an example showing that there may exist Pareto-frontier category rankings that are not constant on some intervals $(x', x'']$ on which $\widehat{F}$ is linear.

Let $\mathcal{I}$ be a category ranking, and let $x^* < x^{**}$ be a pair of types such that $x^* = a$ for an interval $I = (a, b] \in \mathcal{I}$ or $x^* = d$ for $\{d\} \in \mathcal{I}$, and $x^{**} \in I' \in \mathcal{I}$ with $I' \neq I$. We define a new category ranking $\mathcal{I}(x^*, x^{**})$ that groups all types between x^* and x^{**} into one category as follows: (i) if $x^* = a$ for an interval $I = (a, b]$, and $I' = (a', b']$, then replace I, I', and all the elements of $\mathcal{I}$ between I and I' with $(x^*, x^{**}]$ and $(x^{**}, b']$; (ii) if $x^* = d$ for $\{d\} \in \mathcal{I}$, and $I' = (a', b']$, then replace I' and all the elements of $\mathcal{I}$ between $\{d\}$ and I' with $(x^*, x^{**}]$ and $(x^{**}, b']$; if $x^* = a$ for an interval $I = (a, b]$, and $I' = \{x^{**}\}$, then replace I, I', and all the elements of $\mathcal{I}$ between I and I' with $(x^*, x^{**}]$; if $x^* = d$ for $\{d\} \in \mathcal{I}$, and $I' = \{x^{**}\}$, then replace I' and all the elements of $\mathcal{I}$ between $\{d\}$ and I' with $(x^*, x^{**}]$.

Proposition 6. *A category ranking $\mathcal{I}$ belongs to the Pareto frontier of category rankings if and only if there is no pair of types $x^* < x^{**}$ such that*

$$x^* = a \text{ for some } I = (a, b] \in \mathcal{I} \text{ or } x^* = d \text{ for some } I = \{d\} \in \mathcal{I} \text{ and } x^{**} \in I' \neq I \in \mathcal{I},$$
$$\text{and type } x^{**} \text{ weakly prefers ranking } \mathcal{I}(x^*, x^{**}) \text{ to ranking } \mathcal{I}.$$

Proof of Proposition 6. It will be helpful to provide first a general formula for the utility of type $x \in [0, 1]$ under category ranking $\mathcal{I}$. This utility exceeds $U(x)$ given by (5) by the expression

$$\sum_{(\tilde{a},\tilde{b}]\in\mathcal{I},\tilde{a}<\tilde{b}<x} \left[(\tilde{b} - \tilde{a}) \frac{\int_{\tilde{a}}^{\tilde{b}} y^A(\tilde{x})\, dF(\tilde{x})}{F(\tilde{b}) - F(\tilde{a})} - \int_{\tilde{a}}^{\tilde{b}} y^A(\tilde{x})\, d\tilde{x} \right] + \tag{22}$$

$$(x - a) \frac{\int_a^b y^A(\tilde{x})\, dF(\tilde{x})}{F(b) - F(a)} - \int_a^x y^A(\tilde{x})\, d\tilde{x} \text{ for } x \in (a, b] \in \mathcal{I}.$$

This formula follows directly from the fact that types $\tilde{x} \in (a, b] \in \mathcal{I}$ obtain a fair lottery over prizes $y^A(\tilde{x}')$ for $\tilde{x}' \in (a, b]$.

We will first show that when a pair $x^* < x^{**}$ satisfies the condition in Proposition 6, the category ranking $\mathcal{J} = \mathcal{I}(x^*, x^{**})$ Pareto improves over $\mathcal{I}$. Types $x \in [0, x^*]$ are indifferent between the two category rankings, because their allocation and performance are the same in both cases. By assumption, the utility of type x^{**} is no lower under $\mathcal{J}$ than under $\mathcal{I}$. We will now show that the utility of types $x \in (x^*, x^{**})$ is strictly higher under $\mathcal{J}$ than under $\mathcal{I}$. Indeed, the derivative on $(x^*, x^{**}]$ of type x's utility under $\mathcal{J}$, $U^{\mathcal{J}}(x)$, is constant and equal to

$$\frac{\int_{x^*}^{x^{**}} y^A(\tilde{x})\, dF(\tilde{x})}{F(x^{**}) - F(x^*)}.$$

In turn, the derivative on $(x^*, x^{**}]$ of type x's utility under $\mathcal{I}$, $U^{\mathcal{I}}(x)$, is equal to $y^A(x)$ if x does not belong to any non-degenerate interval $(a, b] \in \mathcal{I}$, and is equal to

$$\frac{\int_a^b y^A(\tilde{x})\, dF(\tilde{x})}{F(b) - F(a)}$$

if $x \in (a, b] \in \mathcal{I}$. This means that the derivative increases in x, and increases strictly except on intervals $(a, b] \in \mathcal{I}$. So, $U^{\mathcal{I}}(x)$ is a convex non-linear function. Since $U^{\mathcal{J}}(x)$ is linear on $(x^*, x^{**}]$, $U^{\mathcal{I}}(x^*) = U^{\mathcal{J}}(x^*)$, and $U^{\mathcal{I}}(x^{**}) \leq U^{\mathcal{J}}(x^{**})$, we obtain that $U^{\mathcal{I}}(x) \leq U^{\mathcal{J}}(x)$ for all $x \in (x^*, x^{**}]$, and the inequality is strict for all types $x \in (x^*, x^{**}]$. Similarly, the derivative of $U^{\mathcal{J}}(x)$ on $(x^{**}, b']$ exceeds that of $U^{\mathcal{I}}(x)$ if $a' < x^{**} < b'$ for some $(a', b'] \in \mathcal{I}$, and the two derivatives are equal for $x > b'$, which completes the proof that $\mathcal{J}$ Pareto improves over $\mathcal{I}$.

Suppose now that another category ranking $\mathcal{I}'$ Pareto improves over $\mathcal{I}$. Recall that $\mathcal{I}$ consists of singletons and a finite number of intervals $(x_1, x_1'], (x_2, x_2'], ..., (x_k, x_k']$, with $x_i' < x_{i+1}$. Denote by x' the highest type such that $\mathcal{I}$ and $\mathcal{I}'$ coincide up to x', and suppose that x' is the lower endpoint of an interval $(x_l, x_l']$ in $\mathcal{I}$. (A similar argument to the one that follows applies if x' is a singleton.)

Then x' must be the lower endpoint of a non-trivial interval in $\mathcal{I}'$. Denote this interval by $(x^*, x^{**}]$, where $x' = x^* < x^{**}$. Otherwise, for types x slightly higher than x_l the utility of these types under $\mathcal{I}$ would exceed their utility under $\mathcal{I}'$ by (22). It also cannot be that $x^{**} < x_l'$, since it would then follow from (22) that x^{**} strictly prefers $\mathcal{I}$ to $\mathcal{I}'$.

Thus $x_l' < x^{**}$, and since $\mathcal{I}'$ Pareto improves over $\mathcal{I}$, type x^{**} weakly prefers $\mathcal{I}'$ to $\mathcal{I}$. And since (by (22)) the payoff of type x^{**} under any ranking depends only on the intervals up to the one that contains x^{**}, type x^{**} is indifferent between ranking $\mathcal{I}'$ and ranking $\mathcal{J} = \mathcal{I}(x^*, x^{**})$, and therefore prefers ranking $\mathcal{J}$ to ranking $\mathcal{I}$.

Proposition 6 provides a method for checking whether a category ranking belongs to the Pareto frontier of category rankings. Proposition 6 and the arguments developed for the proof of Theorem 2 enable us to derive the simpler to pursue sufficient condition in Proposition 3 for a category ranking to be an element of the Pareto frontier of category rankings.

Proof of Proposition 3. Theorem 2 establishes that there is no Pareto-improving, mean-preserving contraction of the category ranking $\mathcal{I}$ that is constant on each interval $(x', x'']$ on which $\widehat{F}$ is linear. In particular, there is no such category ranking. However, this does not yet mean that $\mathcal{I}$ is on the Pareto frontier. It could be dominated by a category ranking that is not a contraction of $\mathcal{I}$. The rest of our proof is devoted to showing that this is impossible.

Suppose that the allocation induced by $\mathcal{I}$ is constant on all intervals on which $\widehat{F}$ is linear, but $\mathcal{I}$ is not on the Pareto frontier. Then there exist a pair of types $x^* < x^{**}$ described in Proposition 6. In particular, type x^{**} weakly prefers ranking $\mathcal{I}(x^*, x^{**})$ to ranking $\mathcal{I}$. Define another ranking $\mathcal{I}'$ that is obtained from ranking $\mathcal{I}$ by changing the interval $I' = (a', b']$ (from Proposition 6). If $x^{**} \in (a', b')$, then $I' = (a', b']$ is replaced with two intervals: $(a', x^{**}]$ and $(x^{**}, b']$. Otherwise, that is, if $I' = \{x^{**}\}$ or $x^{**} = b'$, then $\mathcal{I}' = \mathcal{I}$.

We will now show that types $x \leq x^{**}$ weakly prefer ranking $\mathcal{I}(x^*, x^{**})$ to ranking $\mathcal{I}'$. Note first that type x^{**} weakly prefers $\mathcal{I}$ to $\mathcal{I}'$. This is so, because type x^* is indifferent between the two rankings, and the difference between the payoff of $x \in [x^*, x^{**}]$ and the payoff of x^* increases faster under $\mathcal{I}$ than under $\mathcal{I}'$. Compare now ranking $\mathcal{I}(x^*, x^{**})$ to ranking $\mathcal{I}'$. Types $x \leq a$ (where $I = (a, b]$ in Proposition 6) are indifferent. Further, as observed in the proof of Proposition 6, the payoff of type x is a convex function of x. Since this payoff function is linear on $[a, x^{**}]$ for $\mathcal{I}(x^*, x^{**})$, and types a and x^{**} weakly prefer $\mathcal{I}(x^*, x^{**})$, so must do all types $x \in [a, x^{**}]$. (Type x^{**} prefers $\mathcal{I}(x^*, x^{**})$ to $\mathcal{I}$ by Proposition 6, and we have noticed earlier that x^{**} prefers $\mathcal{I}$ to $\mathcal{I}'$.)

Suppose first that x^{**} does not belong to the interior of any interval on which $\widehat{F}$ is linear. Then, ranking $\mathcal{I}(x^*, x^{**})$ restricted to interval $[0, x^{**}]$ Pareto dominates ranking $\mathcal{I}'$ restricted to this interval. However, this contradicts Theorem 4 from Appendix D, because $\mathcal{I}(x^*, x^{**})$ is a mean-preserving contraction of $\mathcal{I}'$ on $[0, x^{**}]$, and the allocation induced by $\mathcal{I}'$ is constant on any interval on which $\widehat{F}$ is linear.

So, suppose that x^{**} belongs to the interior of an interval on which $\widehat{F}$ is linear. Take the longest interval $(x', x'']$ that contains x^{**} on which $\widehat{F}$ is linear. It exists because the union of intervals that contain x^{**} on which $\widehat{F}$ is linear also has the two properties. This longest interval must be contained in an interval of ranking $\mathcal{I}$, because the allocation induced by $\mathcal{I}$ is constant on any interval on which $\widehat{F}$ is linear. So, $(x', x^{**}]$ is contained in an interval of ranking $\mathcal{I}'$ restricted to $[0, x^{**}]$. (More precisely, it is contained in $I' \cap [0, x^{**}]$.) This implies that the allocation induced by $\mathcal{I}'$ on $[0, x^{**}]$ is constant on any interval on which $\widehat{F}$ is linear. Since ranking $\mathcal{I}(x^*, x^{**})$ restricted to $[0, x^{**}]$ is a mean-preserving contraction of ranking $\mathcal{I}'$ restricted to $[0, x^{**}]$, $\mathcal{I}(x^*, x^{**})$ cannot Pareto dominate $\mathcal{I}'$ on $[0, x^{**}]$.

The following example shows that the Pareto frontier of Pareto-improving category rankings may include category rankings that do not satisfy the condition in Proposition 3.

Example 2. *Let G be the CDF of two prizes: 0 and 1, with mass $1/2$ on each. Let F be a strictly increasing CDF that satisfies the following conditions: (a) $F(1/2) = 1/2$; (b) there is an $x^* > 3/4$ such that $1 = (1/2)/F(1/2) > x^*/F(x^*)$ and $x/F(x) > x^*/F(x^*)$ for all $x > x^*$. We will not describe any parametric CDF with these properties, but it is easy to show that such CDFs exist just by drawing them.*

Consider a category ranking $\mathcal{I}$ that consists of two intervals: $[0, x^*]$ and $(x^*, 1]$. We claim that this category ranking is on the Pareto frontier of category rankings. To see why, notice first that any category ranking $\mathcal{J}$ that Pareto dominated $\mathcal{I}$ would have to have an interval $[0, x]$ for some $x \geq x^*$. Indeed, types sufficiently close to 0 would be worse off otherwise. (By condition (a), they would obtain a prize of 1 with a lower probability.) Notice next that pooling the types from $(x, 1]$ would have no welfare effects. Therefore, if there is a category ranking that Pareto dominates $\mathcal{I}$, then a category ranking $\mathcal{J}$ that consists of two intervals: $[0, x]$ and $(x, 1]$, where $x > x^*$, also Pareto dominates $\mathcal{I}$.

We will show that type 1 is worse off under $\mathcal{J}$ than under $\mathcal{I}$, and in this manner we will obtain a contradiction. Under both category rankings, type 1 obtains prize 1. The *performance required to obtain this prize in $\mathcal{J}$ and in $\mathcal{I}$ is determined by the indifference condition of types x and x^*, respectively. The first condition is*

$$x\frac{F(x)-F(1/2)}{F(x)} = x - t,$$

and the second condition obtains by replacing x with x^*. Thus,

$$t = x\frac{F(1/2)}{F(x)} \text{ and } t^* = x^*\frac{F(1/2)}{F(x^*)}.$$

This completes the argument because $t > t^*$ by the second part of condition (b).

It remains to show that $\mathcal{I}$ Pareto improves on the assortative allocation. Indeed, types from $[1/2, 1]$ are better off because of the first part of condition (b). And types from $[0, 1/2]$ are better off because $\mathcal{I}$ gives them a chance to obtain a prize of 1 at zero effort.

D Peer effects

We can model peer effects in a way that does not change any of our results and requires only a transformation of the prize distribution. The idea is that each student exerts a type-dependant effect on all her peers (those attending the same college), and the effects are additive. We will show that such peer effects fit into our framework, as does the change in the endogenous set of peers brought about by pooling. Of course, other ways of modeling peer effects would lead to different impacts of pooling because of the change in peers that pooling induces.

We will consider a limit prize distribution that consists of a finite number of atoms, where each atom represents a mass of seats in a particular college. Students who attend a particular college experience peer effects from other students attending the same college. To model this, denote by $I(y)$ the set of players admitted to university y (for a particular realization of types and bids). The utility of a player of type x admitted to university y by bidding t is

$$xy + x\frac{\sum_{i\in I(y)} p(x_i)}{|I(y)|} - c(t) = x\underbrace{\left(y + \frac{\sum_{i\in I(y)} p(x_i)}{|I(y)|}\right)}_{\tilde{y}} - c(t),$$

where $p(x_i)$ captures the peer effect exerted by a player of type x_i. We refer to $\tilde{y}$ as the effective prize for player i, which is the sum of the value of the college and the average peers effects of the other students attending the college. Note that the effective prize depends on the allocation of prizes to students.

For the approximation, consider a mechanism that implements the assortative allocation of prizes to types. Then, for each prize y in the support of the limit prize distribution G we have that the effective prize is

$$\tilde{y} = y + \frac{\int_{x_L^y}^{x_H^y} p(\tilde{x})\, dF(\tilde{x})}{F(x_H^y) - F(x_L^y)} = \frac{\int_{x_L^y}^{x_H^y} (y + p(\tilde{x}))\, dF(\tilde{x})}{F(x_H^y) - F(x_L^y)}, \tag{23}$$

where (x_L^y, x_H^y) is the interval of types that are allocated prize y in the assortative allocation (so $x_L^y = F^{-1}(\lim_{y'\uparrow y} G(y'))$ and $x_H^y = F^{-1}(G(y))$). Now, replace the limit prize distribution G with distribution $\tilde{G}$ in which every prize y is replaced with the effective prize $\tilde{y}$. The assortative allocation y^A is replaced with $\tilde{y}^A$, so $\tilde{y}^A(x)$ is the effective prize for type x under the assortative allocation. While Olszewski and Siegel's (2016) large contest framework does not formally accommodate prizes whose values depend on their allocation, it is easy to show that the behavior specified by the mechanism forms an ε-equilibrium for sufficiently large contests. And all our results on the characterization of Pareto improvements continue to hold for this mechanism, as we now show.

To see this, it is enough to consider two consecutive prizes and determine the effect of pooling all the types that are allocated these prizes. Denote by $y < y'$ two consecutive prizes in the support of the limit prize distribution G, so $y = y^A(x)$ for x in $(x_L^y, x_H^y]$ and $y' = y^A(x)$ for x in $(x_L^{y'}, x_H^{y'}]$ (with $x_H^y = x_L^{y'}$). By pooling types on interval $[x_L^y, x_H^{y'}]$, the two prizes y and y' are combined to create an average prize y''. The corresponding effective prize is

$$
\begin{aligned}
\tilde{y}'' =& \frac{\int_{x_L^y}^{x_H^y} y dF(\tilde{x}) + \int_{x_L^{y'}}^{x_H^{y'}} y' dF(\tilde{x}) + \int_{x_L^y}^{x_H^{y'}} p(\tilde{x}) dF(\tilde{x})}{F(x_H^{y'}) - F(x_L^y)} \\
=& \frac{\left(F(x_H^y) - F(x_L^y)\right)\tilde{y} + \left(F\left(x_H^{y'}\right) - F\left(x_L^{y'}\right)\right)\tilde{y}'}{F(x_H^{y'}) - F(x_L^y)} \\
=& \frac{\int_{x_L^y}^{x_H^y} \tilde{y}^A(\tilde{x}) dF(\tilde{x}) + \int_{x_L^{y'}}^{x_H^{y'}} \tilde{y}^A(\tilde{x}) dF(\tilde{x})}{F(x_H^{y'}) - F(x_L^y)} \\
=& \frac{\int_{x_L^y}^{x_H^{y'}} \tilde{y}^A(\tilde{x}) dF(\tilde{x})}{F(x_H^{y'}) - F(x_L^y)},
\end{aligned}
$$

where the first equality follows from (23). As in the proof of Proposition **1**, pooling is Pareto improving if and only if

$$
\frac{\int_{x_L^y}^{x_H^{y'}} \tilde{y}^A(x) dF(x)}{F(x_H^{y'}) - F(x_L^y)} \geq \frac{\int_{x_L^y}^{x_H^{y'}} \tilde{y}^A(x) dx}{x_H^{y'} - x_L^y}.
$$