Search Complementarities, Aggregate Fluctuations, and Fiscal Policy

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Abstract

We document five novel facts about the role of search effort in forming trading relationships among firms by combining a variety of micro and macro datasets. These facts strongly suggest the presence of search complementarities. To study the implications of these facts for aggregate fluctuations, we build a dynamic general equilibrium model, disciplined by our new firm-level evidence on search effort. The model matches key aspects of the macro and micro data that have remained unaccounted for by standard models, including the time-varying bimodal distribution of output and the strong, nonlinear propagation of shocks. Also, changes to the volatility of shocks have nonlinear effects on macroeconomic fluctuations that advance a novel interpretation of the Great Moderation. Finally, we provide a new account of the state-dependent effects of fiscal policy.

Keywords: Search complementarities, aggregate fluctuations, macroeconomic volatility, government spending.

JEL classification: C63, C68, E32, E37, E44, G12.

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1 Introduction

The production of goods and services in the value-added chains pervasive in modern economies results from trading relationships among firms. For instance, manufacturing an airplane requires thousands of specialized inputs, from carbon fiber reinforced thermoplastics (CFRPs) to semiconductors. The planes, in turn, are used by airlines, which must link them with many inputs, like ticket reservation servers or onboard food and drinks, to deliver air travel services.

Forming these trading relationships requires search effort by firms. An airplane manufacturer must find a CFRP producer and a CFRP producer must find a buyer for its products. This search effort goes well beyond locating partners. We have in mind, among other things, the effort buyers make in analyzing vendors (e.g., assessing the quality of CFRPs delivered by a new supplier and checking their suitability for proprietary production processes) and in completing contractual arrangements, certifications, and regulatory compliance procedures. For the suppliers, we have in mind the effort related to advertising and branding, participating in trade fairs, tendering offers, adapting production processes to buyer requirements, and setting up supply procedures to process and track orders. The ample space dedicated to these topics in operations management textbooks (e.g., Heizer et al., 2016, or Stevenson, 2018) proves how seriously practitioners take the building of trading relationships. Are practitioners right? What do the data say about the role of search effort in forming trading relationships? And what are the implications for aggregate fluctuations?

To answer these questions, we use the Occupational Employment Survey, the American Productivity and Quality Center and the FactSet Supply Chain Relationships databases, the BEA input-output tables, and the Center for Research in Security Prices and the Compustat Fundamentals Annual data. By being the first to combine these datasets to study the link between search effort and trading relationships, we uncover five novel empirical facts.

Fact 1 is that a higher search effort by a firm forecasts a firm creating more trading relationships. Fact 2 is the positive correlation between a firm's trading relationships and its market value and sales. While our research design does not ascertain the causality behind these correlations, an intuitive interpretation of Facts 1 and 2 is that firms that exert greater search effort build more trading relationships and command greater market value and higher sales. This interpretation is suggestive because these correlations persist even when we employ lagged values of our regressors as well as time and firm fixed effects.

Fact 3 is that the correlation between the increase in a firm's search effort and the increase in trading relationships is stronger when the search effort in the industries connected with the firm is also higher. Fact 4 is the significant positive correlation in the increase of the search efforts of interconnected industries. There are two natural interpretations of Facts 3 and 4. One interpretation is that firms' behavior is correlated because aggregate shocks hit them. But this interpretation is hard to reconcile with the observation that Facts 3 and 4 still appear even when we purge, with a two-stage estimator, the search efforts of prospective suppliers from changes in the search efforts and economic conditions in the firm's industry. The second interpretation is that there are search complementarities in the data. That is, when a firm searches with more intensity for a partner, it is more profitable for potential partners to search with higher intensity (and conversely, when a firm searches with less intensity for a partner, it is best for potential partners to also search with lower intensity).

Finally, Fact 5 is that the micro correlations in Facts 1-4 are also present in the aggregate: output and intermediate inputs comove positively. Fluctuations in intermediate inputs account for 71% of the movements in gross industry output, and this contribution increases during recessions.

To account for these five facts, we build a dynamic general equilibrium model, which we discipline with our new firm-level evidence on search effort. This model casts new light on some classic macroeconomic questions like the shape of aggregate fluctuations or the effects of fiscal policy. In the model, firms that produce intermediate and final goods must build long-lasting trading relationships to produce a final good by exerting costly effort. Motivated by Facts 3 and 4, we assume that the matching function among firms is supermodular even if it has (small) decreasing returns to scale. Under mild conditions compatible with our empirical evidence, supermodularity overcomes the congestion effects of many conventional search environments and generates search complementarities.

Regarding exogenous movements in fundamentals, households are subject to discount factor shocks, while firms experience productivity shocks. Since households own the firms in the economy, the discount factor shocks also affect how firms discount the future. Thus, the return from establishing a trading relationship between firms depends on fundamentals *and* on the search effort of potential trading partners.

The interaction between fundamentals and search effort defines three regions of state variable values: a region where there is a unique *passive* equilibrium (where firms search for partners in the current period with zero effort), a region where there is a unique *active* equilibrium (where firms search for partners in the current period with positive effort), and a region where both equilibria exist. In this case, we will assume that the economy stays in the same equilibrium as in the previous period: if yesterday firms did not search, today firms still do not search; if yesterday firms searched with positive effort, today firms still search. History dependence is a transparent equilibria selection device and a strong predictor of empirical behavior in coordination games similar to ours (see the classic findings in Van Huyck et al., 1990, 1991). Loosely speaking, search complementarities provide a microfoundation for what would appear, at first sight, to be increasing returns to matching à la Diamond (1982).

We close the model with a labor market where firms post job vacancies and fill them with workers from households in an off-the-shelf Diamond-Mortensen-Pissarides (DMP) frictional labor market. The DMP block gives us a framework to analyze unemployment and vacancies but, for simplicity, does not present search complementarities.

Quantitatively, our model has three key elements: the degree of search complementarity in the matching function, the coefficients of the search cost function, and the stochastic properties of the discount factor shock. We discipline all three by calibrating our model to U.S. micro and macro observations. The degree of search complementarity in the matching function replicates the degree of search complementarities in Fact 3. Our parameterization of the search costs function is based on surveys of search effort from the American Productivity and Quality Center administered to 4,000 firms. The discount factor shock, following Hall (2017), matches the stochastic properties of the expected returns of the stock market index.

Our model matches key properties of the U.S. aggregate variables, including the autocorrelations and skewness of their distributions, endogenous movements in labor productivity, and a more realistic unemployment volatility than standard business cycle models. Since firms post more vacancies in the active equilibrium, output is higher and unemployment is lower than in the passive equilibrium. Thus, aggregate shocks can induce large aggregate fluctuations by switching the economy between equilibria. If the model starts from the active equilibrium deterministic steady state, an adverse shock that lowers the household's discount factor from 0.996 to 0.824 (a 17.3% reduction) moves the system to the passive equilibrium, reducing output by roughly 16%. The drop in output is in the ballpark of the one observed for the U.S. in the financial crisis of 2008 measured as a deviation with respect to trend (between 2007.Q4 and 2014.Q4, output per capita fell 12.4% in the U.S. with respect to its post-war trend). Given our calibration, this is a low probability but not a rare event. Smaller shocks fail to move the economy away from the original equilibrium, and the subsequent dynamics are similar to those of conventional business cycle models. Also, we show how shocks to the discount factor –proxied by several indexes– are correlated with unemployment, the creation of trading relationships, and the volume of intermediate inputs and output.[1]

Interestingly, our model links nonlinearly the volatility of exogenous shocks with aggregate outcomes in two distinctive ways that are not present in other business cycle models with complementarities. Since these two sharp and distinctive predictions of our model are also present in the data, they provide strong supportive evidence for our mechanism.

First, when the volatility of shocks is high, the distribution of output is bimodal, as the model switches between low and high search effort with high probability. Thus, we present a mechanism that accounts for the influential results by Adrian et al. (2021), who have documented how the empirical distribution of output has switched between periods of uni- and bimodality.

Second, when the volatility of shocks is low, output is very persistent, as the economy rarely switches between low and high search effort. Hence, search complementarities can transform transitory negative shocks into protracted slumps. This is a key implication because, since the Great Moderation started in 1984, recessions have been more infrequent but also more persistent, particularly for unemployment (Liu et al., 2019). Compared with standard business cycle models (which require an exogenous variation in the persistence of shocks or some form of hysteresis), our economy endogenously delivers this fact, allowing us to reconceive the aftermath of the financial crisis of 2008. According to our model, output remained below trend, and employment-to-population ratios were depressed for a decade because in 2008 the economy moved to an equilibrium with less search and did not abandon it even after the original adverse shocks evaporated.

Given the empirical success of our model, we can also use it to study fiscal policy. In our

[1]All our results come without adding expectational shocks as in Kaplan and Menzio (2016). We do not include them so that we can focus on the interaction between shocks to fundamentals and search complementarities. For the same reason, we postpone for future research the study of non-Markov strategies by firms, alternative equilibrium selection devices, and limit cycles such as those in Beaudry et al. (2016, 2018, 2020).

example above, a CFRP producer can supply an airplane manufacturer or provide materials for constructing a new, seismic-resistant public school in California. If the government raises its expenses (modeled as more government-owned firms such as a new public school), the search incentives for private firms increase, and the economy can switch from a passive equilibrium to an active one. In this case, the fiscal multipliers can be as high as 1.37. On the other hand, if search effort is already high (or the fiscal expansion too small in a passive equilibrium), the fiscal multiplier will be as low as 0.25. Thus, our model explains the strong state dependence of fiscal multipliers in the data documented by Auerbach and Gorodnichenko (2012), Owyang et al. (2013), and Ghassibe and Zanetti (2022).

Search complementarities are an instance of the strategic complementarities defined by Bulow et al. (1985). There is a long tradition in economics of linking strategic complementarities to aggregate fluctuations, going back to Diamond (1982), Weitzman (1982), Howitt (1985), Howitt and McAfee (1987), Howitt and McAfee (1988), Diamond and Fudenberg (1989), and Howitt and McAfee (1992) and explored by Cooper and John (1988), Chatterjee et al. (1993), and Kaplan and Menzio (2016). Recent papers with strategic complementarities, but with mechanisms different from ours, include Schaal and Taschereau-Dumouchel (2018) (with complementarities in production capacity), Sterk (2016) (with complementarities created by the lost skills of unemployed workers), and Eeckhout and Lindenlaub (2019) (with complementarities between on-the-job search and vacancy posting by firms).

How does our paper add to this tradition? First, we address the need for more empirical firm-level evidence that has long afflicted the literature on strategic complementarities. Using new datasets, we document five novel facts about the search efforts of firms and the forming of trading relationships that strongly suggest the existence of strategic complementarities. Second, we study a dynamic equilibrium model of strategic complementarities tightly disciplined by observed data. Furthermore, this model matches key aspects of the macro and micro data, including the time-varying bimodal distribution of output, that have remained unaccounted for. Third, we show how the nonlinear effects of changes to the volatility of shocks on our economy can help us think differently about the Great Moderation. Finally, we provide a novel account of the state-dependent effects of fiscal policy.

2 A simple model of search complementarities

Our point of departure is a simple model of search complementarities. The model will illustrate transparently the main mechanisms we want to highlight. Furthermore, it will help us organize our data analysis in Section 3. Later, in Section 4, we will extend this simple framework to a quantitative dynamic equilibrium model, which we will calibrate and confront with the data.

2.1 Environment

We postulate a discrete-time economy composed of a continuum of islands of unit measure. Two risk-neutral firms populate each island. Firms are owned by a representative household, which consumes aggregate net profits. The two firms are located in separate locations within the island, and they search for each other to establish a trading relationship. If firms do not match, they produce zero output. If they meet, they jointly produce 2 units of output that they evenly split. At the end of each period t, the match is dissolved, and each firm moves to a new, separate location to search for a partner in the next period *ex novo*.

Although realizations of meetings will differ among islands, a law of large numbers will hold in the aggregate, and individual matching probabilities will equal the aggregate share of islands where matches occur. Since no payoff-relevant state variables carry information across periods, and given our focus on Nash equilibria of the stage game (i.e., within any given period), we drop each variable's time index for the moment. Analogously, since we will analyze symmetric equilibria, where all firms exert the same search effort, we drop the island index.

The probability of meeting and forming a trading relationship is given by a matching function that depends on the search effort of each firm within the island. For a search effort $\sigma_1 \in [0,1]$ of firm 1 and a search effort $\sigma_2 \in [0,1]$ of firm 2, the matching probability function is:

$$\pi\left(\sigma_1, \sigma_2\right) = \frac{\sigma_1 + \sigma_2 + \alpha\sigma_1\sigma_2}{2 + |\alpha|}, \tag{1}$$

where the denominator bounds the matching probability between zero and one. This matching function has two key features. First, the matching probability increases linearly in the firms' search efforts, i.e., $\partial\pi\left(\sigma_i, \sigma_{-i}\right)/\partial\sigma_i = \frac{1+\alpha\sigma_{-i}}{2+|\alpha|} > 0$ if $1 + \alpha\sigma_{-i} > 0$. Second, when $\alpha > 0$, search efforts are complementary. That is, the marginal contribution of a firm's search effort to the matching probability increases in the other firm's search effort: $\partial^2\pi\left(\sigma_1, \sigma_2\right)/\partial\sigma_1\partial\sigma_2 =$

$\partial^2\pi(\sigma_1,\sigma_2)/\partial\sigma_2\partial\sigma_1 > 0$. In contrast, search efforts are substitutes when $\alpha < 0$.

Finally, we assume the cost of search effort for firm $i \in \{1,2\}$ is $c(\sigma_i) = \frac{c}{2(2+|\alpha|)}\sigma_i^2$, where $c > 1 + |\alpha|$ to ensure positive profits.

2.2 Nash equilibria

We find the Nash equilibrium of the stage game on each island by solving the problem of firm i that takes as given the search effort of the other firm $-i$, $\overline{\sigma}_{-i}$. The profit function for firm i is:

$$J(\sigma_i, \overline{\sigma}_{-i}) = \frac{\sigma_i + \overline{\sigma}_{-i} + \alpha\sigma_i\overline{\sigma}_{-i}}{2+|\alpha|} - \frac{c}{2(2+|\alpha|)}\sigma_i^2. \tag{2}$$

Maximizing $J(\sigma_i, \overline{\sigma}_{-i})$ with respect to σ_i, we get the best response function $\Pi(\overline{\sigma}_{-i})$ for firm i:

$$\sigma_i^* = \frac{1 + \alpha\overline{\sigma}_{-i}}{c}, \tag{3}$$

which *increases* with the effort of the other firm $\overline{\sigma}_{-i}$ when $\alpha > 0$ (complementarity) and *decreases* with $\overline{\sigma}_{-i}$ when $\alpha < 0$ (substitutability).

A tuple $\{\sigma_1^*, \sigma_2^*\}$ is a pure Nash equilibrium of the stage game if it is a fixed point of the product of the best response functions determined by equation (3). When $c > 1 + |\alpha|$, $\{\sigma_1^*, \sigma_2^*\} = \left\{\frac{1}{c-\alpha}, \frac{1}{c-\alpha}\right\}$ is the unique Nash equilibrium.

2.3 Stochastic shocks

To demonstrate how stochastic shocks impact search efforts, we introduce stochastic shocks to the discount factor and productivity, which will be the source of cyclical fluctuations in the fully fledged model in Section 4.

Discount factor shocks: First, we assume now that production requires a one-time-to-build lag instead of taking place in the immediate aftermath of matching and that the value of production is subject to stochastic discounting. Output is produced at the end of each period, immediately before the match is dissolved. Agents are impatient, and the 2 units of output produced at the end of period t have a value of $2\beta_t$ at time t (when the search cost is incurred), where β_t is the stochastic discount factor and the index t tracks time.

The new expected profit function for firm i is:

$$J\left(\sigma_{i,t},\overline{\sigma}_{-i,t},\beta_t\right)=\beta_t\frac{\sigma_i+\overline{\sigma}_{-i}+\alpha\sigma_i\overline{\sigma}_{-i}}{2+|\alpha|}-\frac{c}{2\left(2+|\alpha|\right)}\sigma_i^2. \tag{4}$$

Similarly to the deterministic case, the best response function $\Pi\left(\overline{\sigma}_{-i,t},\beta_t\right)$ for firm i is:

$$\sigma_{i,t}^*=\frac{\beta_t}{c}\left(1+\alpha\overline{\sigma}_{-i,t}\right). \tag{5}$$

When $\beta_t=1$, equation (5) collapses to equation (3).

For $\beta_t<\frac{c}{1+|\alpha|}$, the new pure Nash equilibrium of the stage game is $\left\{\sigma_{1,t}^*,\sigma_{2,t}^*\right\}=\left\{\frac{1}{c/\beta_t-\alpha},\frac{1}{c/\beta_t-\alpha}\right\}$. The equilibrium search efforts are increasing in β_t as $d\left(\frac{1}{c/\beta_t-\alpha}\right)/d\beta_t>0$. Higher complementarities encapsulated by a larger α imply a stronger amplification effect since $\partial^2\left(\frac{1}{c/\beta_t-\alpha}\right)/\partial\beta_t\partial\alpha>0$.

Productivity shocks: Second, we investigate the case where firms have an idiosyncratic productivity. Let $z_{i,t}$ be the shock for firm i with $cov\left(z_{1,t},z_{2,t}\right)=0$. The new best response function $\Pi\left(\overline{\sigma}_{-i,t},z_{i,t}\right)$ for firm i is:

$$\sigma_{i,t}^*=\frac{z_{i,t}}{c}\left(1+\alpha\overline{\sigma}_{-i,t}\right). \tag{6}$$

The Nash equilibrium is now $\left\{\sigma_{1,t}^*,\sigma_{2,t}^*\right\}=\left\{\frac{c+\alpha z_{2,t}}{c^2/z_{1,t}-\alpha z_{2,t}},\frac{c+\alpha z_{1,t}}{c^2/z_{2,t}-\alpha z_{1,t}}\right\}$ (we impose the condition that $z_{1,t}$ and $z_{2,t}$ be in a range such that $\sigma_{i,t}^*\in[0,1]$). The equilibrium search effort of firm i is strictly increasing in $z_{i,t}$ as $d\left(\frac{c+\alpha z_{-i,t}}{c^2/z_{i,t}-\alpha z_{-i,t}}\right)/dz_{i,t}>0$ for all parameter values that ensure the inner solution, while increasing in $z_{-i,t}$ if and only if $\alpha>0$. Consequently, search efforts are positively correlated between the two firms, i.e., $cov\left(\sigma_{1,t}^*,\sigma_{2,t}^*\right)>0$ if and only if $\alpha>0$.

Taking stock: Our model has several key empirical implications. First, it assumes that the matching probabilities –and hence the number of trading relationships– increase in the search efforts of firms and that those trading relationships raise profits and output. Second, if $\alpha>0$, the marginal benefit of searching is positively correlated with the effort of the connected industries and the search efforts in connected industries comove positively. In the next section, we will document that the two assumptions hold in the data (we will call them Facts 1 and 2) and that the two predictions are empirically present (Facts 3 and 4). We will also add a Fact 5 that shows that these predictions also hold in the aggregate, as we would occur in our model if we aggregate over islands.

3 Five facts about search effort and trading relationships

As mentioned above, this section will document five facts supporting the assumptions and corroborate the key predictions of our simple model in Section 2. These new facts will motivate our quantitative model in Section 4 and discipline its calibration in Section 6.

Fact 1: Search effort forecasts trading relationships

We start by constructing two proxies for search efforts using alternative firm-level datasets. These proxies show that searching activities absorb a substantial amount of firms' resources and that changes in search efforts forecast an increase in the number of trading relationships.

Search effort proxy 1: We derive our first proxy for search efforts from the Occupational Employment Survey (OES) database constructed by the BLS, which reports yearly employment and wages at the 3-digit NAICS industry level, including detailed occupation levels between 2003 and 2021. The database covers 1.1 million establishments and comprises 57% of jobs in the U.S. The BLS collects employment and wage information from establishments in six semiannual panels in three consecutive years. Every six months, a new panel of data is added, and the oldest panel is dropped. That is, if a firm is surveyed and added to the database in year t, the information on the firm's employment and wages will remain in the database until the firm is replaced with another firm in year $t+3$. This updating method results in repeated cross-sectional data every three years.

Following Michaillat and Saez (2015), we approximate a firm k's search effort in matching with suppliers by the number of workers whose occupation is ordering, buying, purchasing, and procurement. Analogously, we approximate a firm k's effort in matching with customers by the number of workers whose occupation is advertising, marketing, sales, demonstration, and promotion. On average, firms allocate 1.4% and 1.9% of employment to these two types of search efforts, respectively.[2]

Denote the employment involved in "buying" in industry i in period t by $emp_{i,t}^{buy}$ and the employment involved in "selling" by $emp_{i,t}^{sell}$ (we do not have information on these variables at the firm level, only at the industry level). Then, define $\Delta\sigma_{i,t}^{buy} = \frac{ln\left(emp_{i,t}^{buy}\right)-ln\left(emp_{i,t-3}^{buy}\right)}{3}$ and $\Delta\sigma_{i,t}^{sell} = \frac{ln\left(emp_{i,t}^{sell}\right)-ln\left(emp_{i,t-3}^{sell}\right)}{3}$ as our measures of search efforts for buying and selling firms in

[2] We exclude the retail and wholesale industries since their trading relationships might not reflect the inter-firm cooperation in production that we aim to study.

each industry i, respectively.[3]

At the firm level, the change in employment in search activities is equal to the change at the industry level plus a firm-specific idiosyncratic component:

$$\Delta\sigma_{i,k,t}^{buy} = \Delta\sigma_{i,t}^{buy} + \Delta\widehat{\sigma}_{i,k,t}^{buy}, \tag{7}$$

and

$$\Delta\sigma_{i,k,t}^{sell} = \Delta\sigma_{i,t}^{sell} + \Delta\widehat{\sigma}_{i,k,t}^{sell}. \tag{8}$$

Since search efforts are measured at the industry level, and the changes in efforts at the firm level ($\Delta\widehat{\sigma}_{i,k,t}^{buy}$ and $\Delta\widehat{\sigma}_{i,k,t}^{buy}$) are unobserved, we assume that firm-level changes in efforts are orthogonal to the observed industry-level changes, i.e., $\left(\Delta\widehat{\sigma}_{i,k,t}^{buy}, \Delta\widehat{\sigma}_{i,k,t}^{sell}\right) \perp \left(\Delta\sigma_{i,t}^{buy}, \Delta\sigma_{i,t}^{sell}\right)$.

Search effort proxy 2: We derive a second proxy for search efforts using the American Productivity and Quality Center (APQC) database for 2018-2021. APQC surveys over 4,000 firms about their practice in sales, marketing, contracting, and procurement and publicly discloses the most recent cross-section moments. The median spending on searching for suppliers at the firm level is about 1.4% of total revenue (coincidently, in the OES, 1.4% of employment is involved in searching for suppliers). The median spending on searching for customers is about 7.5% of total revenue, which is higher than the measurement from the OES data derived from employment within the industry (1.9%). This result is unsurprising as many marketing and sales efforts are outsourced.

The advantage of the APQC dataset is that it contains the detailed data for a wide range of search costs, which we will use to calibrate the cost function in the theoretical model. The drawback of the APQC dataset is its limited time dimension (2018-2021). Thus, while our proxy based on the APQC database provides a detailed cross-sectional measure of efforts in different categories, we will use our proxy based on the OES database to study the changes in search efforts over time. Appendix A.4 documents that our results below are robust to measuring search efforts using advertising expenses as proposed by Hall (2014).

Trading relationships: We obtain the number of trading relationships using FactSet Supply Chain Relationships data. FactSet collects firms' relationship information from public sources

[3] We differentiate the series to remove the trend in employment. We use third log differences due to the BLS's data-updating method. First log differences underestimate the change in employment, as 2/3 of the firms do not update their information in two consecutive years.

such as SEC 10-K annual filings, investor presentations, and press releases since 2003. Using the sample period 2003-2021, we obtain 289,239 distinct customer-supplier trading relationships. Most trading relationships are continuative, with only 15,522 (5%) of them experiencing a reinstatement from previously dissolved relationships.

The observed average duration of relationships is 3.5 years (Figure 1 plots the histogram for the duration of customer-supplier trading relationships). But since our sample ends in 2021, with many relationships still ongoing, 3.5 years is a downward-biased estimate of the persistence of relationships. The number of supplier firms that sell intermediate goods to the customer firm k that operates in industry i in year t is $n^{sup}_{i,k,t}$, while $n^{cus}_{i,k,t}$ is the number of customer firms that purchase intermediate goods from the supplier firm k that operates in industry i in year t.

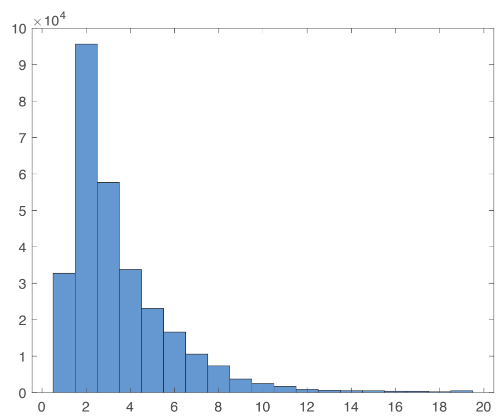


Figure 1: Histogram of duration (year) of trading relationships

Estimation: We now show that changes in our proxies for search efforts predict growth in the number of trading relationships. Since our proxies for search efforts are on firms operating in the U.S., we focus on the subset of U.S. firms within FactSet.

Recall that our model in Section 2 implies that increases in search effort translate into linear increases in matching probabilities. Thus, we estimate:

$$\Delta n^{sup}_{i,k,t} = \beta \Delta \sigma^{buy}_{i,k,t-p} + \chi_t + \gamma_{i,k} + \epsilon_{i,k,t}, \tag{9}$$

where $\Delta n^{sup}_{i,k,t}$ is the first difference in the number of supplier firms, $\Delta \sigma^{buy}_{i,k,t-p}$ is the change in buying effort at the firm level with p-years' lag, χ_t and $\gamma_{i,k}$ are year and firm fixed effects, and $\epsilon_{i,k,t}$ is the component of $\Delta n^{sup}_{i,k,t}$ orthogonal to changes in search efforts and fixed effects plus any error.

Using equation (7) to substitute out $\Delta\sigma^{buy}_{i,k,t-p}$ in equation (9) yields:

$$\Delta n^{sup}_{i,k,t} = \beta\Delta\sigma^{buy}_{i,t-p} + \chi_t + \gamma_{i,k} + \eta_{i,k,t}, \text{ with } \eta_{i,k,t} = \beta\Delta\widehat{\sigma}^{buy}_{i,k,t-p} + \epsilon_{i,k,t}. \tag{10}$$

Equation (10) is a regression where $\Delta\sigma^{buy}_{i,k,t-p}$ is observed with measurement error, i.e., we observe $\Delta\sigma^{buy}_{i,t-p}$ instead of $\Delta\sigma^{buy}_{i,k,t-p}$. But since the firm-specific idiosyncratic component is uncorrelated with $\Delta\sigma^{buy}_{i,t-p}$, our point estimates remain unbiased and consistent.[4] An analogous interpretation holds for the related equations in the next subsections.

Table 1: Search effort forecasts the number of relationships

	(1)	(2)	(3)	(4)
Dependent variable	Change of suppliers ($\Delta n^{sup}_{i,k,t}$)		Change of customers ($\Delta n^{cus}_{i,k,t}$)	
Year lag (p)	0	1	0	1
$\Delta\sigma^{buy}_{i,t-p}$	0.32**	0.62***		
	(0.14)	(0.15)		
$\Delta\sigma^{sell}_{i,t-p}$			0.37*	0.57**
			(0.21)	(0.23)
Time FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.22	0.25	0.11	0.12
Observations	25,515	24,148	29,009	26,495

Note: Yearly data 2003-2021. The dependent variables are the change in the number of suppliers ($\Delta n^{sup}_{i,k,t}$) and customers ($\Delta n^{cus}_{i,k,t}$) matched by each firm for Columns (1)-(2) and (3)-(4), respectively. Standard errors are in parentheses. ** and *** denote significance at the 5% and 1% level, respectively.

Several examples of demand and supply shocks that could enrich our simple model in Section 2 support the formulation of our equation. An example of a demand shock is when a firm (or the firm's industry) has the opportunity to sell its products in a new export market after a trade liberalization or a lowering of tariffs. To achieve sales in the new market, the firm needs to find new suppliers to produce products that may fit the new customers' preferences, and in addition, the firm needs to establish new contracts with shipping companies. An example of a supply shock is when a firm (or its industry) develops a new product to offer in the market, the firm needs to find suitable suppliers to manufacture the new parts that would enable the creation of the new product. In both cases, we expect the firm (or its industry) to exert more search efforts

[4] In the classic errors-in-variables situation, the bias problem appears because the measurement error is uncorrelated with the unobserved variable. Our estimates of the variance, though, are not efficient because of the presence of an extra term $\beta^2 var(\Delta\widehat{\sigma}^{buy}_{i,k,t})$. However, given our point estimates of β, this term is small, and our significance levels have plenty of room to accommodate it.

due to the shock and, thus, increase the number of relationships.

Columns (1) and (2) in Table 1 show the estimation results for equation (10) for 0 and 1−year lags. The coefficient β for $\Delta\sigma^{buy}_{i,k,t-p}$ is positive and statistically significant for both lags (the estimated coefficient remains significant for lags up to $p = 7$). Thus, stronger search efforts predict faster matching with suppliers in the subsequent years. This persistent effect is consistent with the time-consuming formation of trading relationships.

Similarly, to check whether more intensive search efforts for selling forecast acquiring more customers, we estimate:

$$\Delta n^{cus}_{i,k,t} = \beta\Delta\sigma^{sell}_{i,k,t-p} + \chi_t + \gamma_{i,k} + \epsilon_{i,k,t}, \tag{11}$$

where $\Delta n^{cus}_{i,k,t}$ is the first difference in the number of customer firms and $\Delta\sigma^{sell}_{i,k,t-p}$ is the change in selling effort at the firm level with p-years' lag. As before, we use equation (8) to substitute out $\Delta\sigma^{sell}_{i,k,t-p}$ in equation (11), which yields:

$$\Delta n^{cus}_{i,k,t} = \beta\Delta\sigma^{sell}_{i,t-p} + \chi_t + \gamma_{i,k} + \eta_{i,k,t}, \text{ with } \eta_{i,k,t} = \beta\Delta\widehat{\sigma}^{sell}_{i,k,t-p} + \epsilon_{i,k,t}. \tag{12}$$

Columns (3) and (4) in Table 1 show the results for 0 and 1-year lags. The coefficient β for $\Delta\sigma^{sell}_{i,k,t-1}$ is positive and statistically significant.

As a robustness test, we re-ran the regressions in Table 1 (as well as those in Tables 2 and 3 below) using employment shares instead of employment levels as a measure of search effort. The results are reported in Appendix A.2. The findings are similar to those above, but less statistically significant in a few cases. We prefer our baseline specification because employment shares may vary due to shocks unrelated to search. For example, an increase in workers involved in equipment maintenance (for instance, due to a new government safety regulation) mechanically lowers the share of workers involved in search, even if the firm's total search effort is the same.[5]

We also re-ran all our regressions by focusing on a less granular list of search-related occupations and changing the sample by dropping the periods before 2011 and after 2019. The results are robust to these alternative measures and samples as shown in Appendix A, although we lose statistical significance in a few cases.

[5]The fact that employment shares could not robustly predict growth in the number of relationships indicates that employment shares are a poorer measure of search effort since search effort should have predictive power for relationship formation by definition.

Fact 2: Trading relationships are correlated with firm value and sales

Our model in Section 2 assumes that trading relationships increase the firm's output and profits. Table 2 provides evidence for the positive correlation between trading relationships and a firm's economic fundamentals by regressing market value and sales over the number of trading relationships. The dependent variables are market value and sales, obtained from the Center for Research in Security Prices (CRSP) and Compustat Fundamentals Annual data, respectively, and the regressions $ln\left(n^{sup}_{i,k,t}\right)$ and $ln\left(n^{cus}_{i,k,t}\right)$ are the log of the number of suppliers and customers constructed with FactSet data.

Columns (1) and (2) document that a 1% increase in suppliers is associated with a 0.09% and 0.1% rise in market value and sales, respectively. Columns (3) and (4) in Table 2 show that a 1% increase in the number of customers is associated with a 0.1% and 0.08% rise in market value and sales, respectively.

Table 2: Match creation forecasts firm growth

	(1)	(2)	(3)	(4)
Dependent variable	Market value	Sales	Market value	Sales
$ln\left(n^{sup}_{i,k,t}\right)$	0.09***	0.10***		
	(0.01)	(0.01)		
$ln\left(n^{cus}_{i,k,t}\right)$			0.10***	0.08***
			(0.01)	(0.01)
Time FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
R^2	0.92	0.92	0.92	0.96
Observations	22,282	22,034	23,565	23,379

Note: Yearly data 2003-2021. Standard errors are in parentheses. *** denotes significance at the 1% level.

Fact 3: Search efforts are correlated

Is the correlation between the firm's search effort and the increase in trading relationships stronger when potential trading partners in connected industries search more actively, as postulated by our matching function (1)? To answer this question, we identify each industry's supplier and customer industries using the BEA input-output tables, which report the use of intermediate inputs for 66 private industries at the 3-digit NAICS level. For each industry i, let $sup\,(i)$ be the set of supplier industries that sell intermediate goods to industry i. Adapting our previous notation, we denote with $\sigma^{sell}_{sup(i),t}$ the selling efforts in searching for customers in the suppliers' industry to industry i. Since each industry has multiple supplier industries, we measure the average search effort for industry i's supplier industries as the mean of their search efforts weighted by the value of intermediate goods that industry i purchases from them. Then, $\Delta\sigma^{sell}_{sup(i),t} = \sum_{j \in sup(i)} \omega_{i,j,t} \Delta\sigma^{sell}_{j,t}$, where $\omega_{i,j,t}$ is the fraction of the value of intermediate goods that industry i purchases from industry j, and $\Delta\sigma^{sell}_{j,t}$ is the first difference in the selling effort of industry j in searching for customers.

Analogously, we denote the buying effort of industry i's customer industries in searching for suppliers as $\sigma^{buy}_{cus(i),t}$, and compute $\Delta\sigma^{buy}_{cus(i),t} = \sum_{j \in cus(i)} \widehat{\omega}_{i,j,t} \Delta\sigma^{buy}_{j,t}$, where $\widehat{\omega}_{i,j,t}$ is the fraction of the value of intermediate goods that industry i sells to industry j and $\Delta\sigma^{buy}_{j,t}$ is the first difference in the buying effort of industry j in searching for suppliers.

Then, we estimate:

$$\Delta n^{sup}_{i,k,t} = \beta_1 \Delta\sigma^{buy}_{i,k,t-1} + \beta_2 \Delta\sigma^{buy}_{i,k,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1} + \chi_t + \gamma_{i,k} + \epsilon_{i,k,t}, \tag{13}$$

where $\Delta\sigma^{buy}_{i,k,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1}$ is the interaction term between the changes in the firm's search effort and the changes in the search effort of its connected firms. According to regression (13), the marginal contribution of firm k's change in search effort to the formation of relationships is equal to $\beta_1 + \beta_2 \times \Delta\sigma^{sell}_{sup(i),t-1}$. A positive value for β_2 indicates that firm k's change in search effort forecasts a stronger formation of new trading relationships conditional on a higher search effort in the supplier industries.

Using equation (7) to substitute out $\Delta\sigma^{buy}_{i,k,t-1}$ in equation (13) yields:

$$\Delta n^{sup}_{i,k,t} = \beta_1 \Delta\sigma^{buy}_{i,t-1} + \beta_2 \Delta\sigma^{buy}_{i,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1} + \chi_t + \gamma_{i,k} + \eta_{i,k,t}, \tag{14}$$

with $\eta_{i,k,t} = \beta_1 \widehat{\Delta\sigma}^{buy}_{i,k,t-1} + \beta_2 \widehat{\Delta\sigma}^{buy}_{i,k,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1} + \epsilon_{i,k,t}$.

Column (1) of Table (3) shows that β_2 is positive and statistically significant. This estimate is consistent with the hypothesis of supermodularity in the matching function: stronger search by the prospective suppliers makes a firm's search effort more productive.

Below, Fact 4 will document that $\Delta\sigma^{buy}_{i,t}$ and $\Delta\sigma^{sell}_{sup(i),t}$ are positively correlated. This is a natural manifestation of search complementarities. But the correlation can also be a byproduct of common shocks. If this is the case, $\Delta\sigma^{buy}_{i,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1}$ will be positively correlated with $\left(\Delta\sigma^{buy}_{i,t-1}\right)^2$. Then, a $\beta_2 > 0$ in regression (14) may come from a missing quadratic term on $\Delta\sigma^{buy}_{i,t-1}$ rather than from search complementarities.

Table 3: Search efforts are complements

	(1)	(2)	(3)	(4)
Dependent variable	Change of suppliers ($\Delta n^{sup}_{i,k,t}$)		Change of customers ($\Delta n^{cus}_{i,k,t}$)	
	One-stage	Two-stage	One-stage	Two-stage
$\Delta\sigma^{buy}_{i,t-1}$	0.72***	0.89***		
	(0.19)	(0.20)		
$\Delta\sigma^{buy}_{i,t-1} \times \Delta\sigma^{sell}_{sup(i),t-1}$	4.45***	5.87***		
	(1.68)	(1.88)		
$\Delta\sigma^{sell}_{i,t-1}$			0.94***	1.03***
			(0.25)	(0.31)
$\Delta\sigma^{sell}_{i,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1}$			2.72**	9.00***
			(1.07)	(2.36)
Time FE	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes
Adj R^2	0.25	0.25	0.12	0.11
Observations	21,671	21,616	24,222	24,073

Note: Yearly data 2003-2021. The dependent variables are the change in the number of suppliers ($\Delta n^{sup}_{i,k,t}$) for columns (1) and (2) and of customers ($\Delta n^{cus}_{i,k,t}$) for columns (3) and (4). Standard errors are in parentheses. ** and *** denote significance at the 5% and 1% level, respectively.

To address this concern, we conduct a two-stage exercise. In the first stage, we purge $\Delta\sigma^{sell}_{sup(i),t}$ from changes in effort in industry i ($\Delta\sigma^{buy}_{i,t}$), the influence of aggregate conditions in industry i ($y_{i,t}$), and an industry-specific fixed effect (α_i) by running:

$$\Delta\sigma^{sell}_{sup(i),t} = \alpha_i + \beta_i \Delta\sigma^{buy}_{i,t} + \kappa_i y_{i,t} + \Delta\varsigma^{sell}_{sup(i),t}. \tag{15}$$

Thus, the residual $\Delta\varsigma^{sell}_{sup(i),t}$ is the change in search efforts exerted by the suppliers of industry i that is orthogonal to industry j's changes in search efforts and the industry's economic conditions. In the second stage, we replace $\Delta\sigma^{sell}_{sup(i),t-1}$ with $\Delta\varsigma^{sell}_{sup(i),t-1}$ in equation (14).

Table 3 reports the results of our two-stage procedure in column (2). Since β_2 is positive and statistically significant, the heightened search efforts by prospective suppliers are correlated with an increase in a firm's search efforts, even when the search efforts of prospective suppliers are orthogonal to changes in the search efforts and economic conditions in the firm's industry.

Next, we examine whether the change in the effort of the supplier firm forecasts a stronger formation of new trading relationships conditional on higher search effort in the customer industries by estimating the following:

$$\Delta n^{cus}_{i,k,t} = \beta_1 \Delta\sigma^{sell}_{i,k,t-1} + \beta_2 \Delta\sigma^{sell}_{i,k,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1} + \chi_t + \gamma_{i,k} + \epsilon_{i,k,t}, \tag{16}$$

where $\Delta\sigma^{sell}_{i,k,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1}$ is the interaction term between the change in search effort of firm k and the change in the search effort of the customer industries.

Using equation (7) to substitute out $\Delta\sigma^{sell}_{i,k,t-1}$ in equation (16), we get:

$$\Delta n^{cus}_{i,k,t} = \beta_1 \Delta\sigma^{sell}_{i,k,t-1} + \beta_2 \Delta\sigma^{sell}_{i,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1} + \chi_t + \gamma_{i,k} + \eta_{i,k,t}, \tag{17}$$

with $\eta_{i,k,t} = \beta_1 \widehat{\Delta\sigma}^{sell}_{i,k,t-1} + \beta_2 \widehat{\Delta\sigma}^{sell}_{i,k,t-1} \times \Delta\sigma^{buy}_{cus(i),t-1} + \epsilon_{i,k,t}$.

Column 3 in Table 3 shows the estimation results for regression (17). The coefficient β_2 is positive and statistically significant, which documents that heightened search by a prospective customer is correlated with an increase in the firm's search efforts. Column 4 in the table shows the results when we replace $\Delta\sigma^{buy}_{cus(i),t-1}$ with $\Delta\varsigma^{buy}_{cus(i),t-1}$ to purge the estimation from the effect of common shocks. Again, the results are consistent with the hypothesis in Section 2 that the matching process among firms is supermodular in their search efforts.

Fact 4: Positive comovement of search efforts in connected industries

Our simple model in Section 2 implies the positive comovement of the search efforts between connected industries due to strategic complementarities. To check this empirical implication, we estimate for the customer industry:

$$\Delta\sigma^{buy}_{i,t} = \omega \Delta\sigma^{sell}_{sup(i),t} + \upsilon_i + \gamma_t + \epsilon_{i,t},$$

where $\Delta\sigma^{buy}_{i,t}$ is the change in search effort in industry i as a customer industry at period t, ω is our coefficient of interest, and $\Delta\sigma^{sell}_{sup(i),t}$ is the change in search effort of industry i's supplier industries.[6] Controlling for time fixed effects rules out the possibility that the comovements in search efforts are not a consequence of correlated shocks across firms.

The estimate for the coefficient ω is equal to 0.29 (column (1) of Table 4). Its significance at the 1% level is strong evidence that changes in the search efforts of supplier industries are positively correlated with the changes in search efforts of customer industries beyond the presence of common shocks.

Analogously, we estimate the equation for the supplier industry:

$$\Delta\sigma^{sell}_{i,t} = \omega \Delta\sigma^{buy}_{cus(i),t} + \upsilon_i + \gamma_t + \epsilon_{i,t}, \tag{18}$$

[6] Linear trends are removed from both variables to avoid possible common time trends in the change in search efforts between connected industries.

where $\Delta\sigma^{sell}_{i,t}$ is the change in the search effort of industry i as a supplier industry at period t, and $\Delta\sigma^{buy}_{cus(i),t}$ is the change in the search effort of industry i's customer industries. Table 4 shows (in column 3) that ω is positive and statistically significant, which confirms our previous result that changes in search effort are correlated between connected industries.

As with Fact 3, a possible complication with our findings could be the presence of shocks specific to each pair of connected industries that cannot be removed by adding time fixed effects. To address this concern, we use a two-stage procedure to purge the observed search efforts from the influence of common shocks. The first stage is characterized by equation (15) without $\Delta\sigma^{buy}_{i,t}$ as an independent variable. In the second stage, we replace the changes in search efforts with the residual changes in search efforts obtained from the first stage, and study the comovement in residual changes in search efforts that exclude the influence of common shocks. Column (2) in Table 4 shows a positive correlation between changes in search effort in connected industries even after excluding common shocks.

Table 4: Search efforts are positively correlated between connected industries

	(1)	(2)	(3)	(4)
Dependent variable	Buying effort ($\Delta\sigma^{buy}_{i,t}$)		Selling effort ($\Delta\sigma^{sell}_{i,t}$)	
	One-stage	Two-stage	One-stage	Two-stage
$\Delta\sigma^{sell}_{sup(i),t}$	0.29***	0.34***		
	(0.10)	(0.09)		
$\Delta\sigma^{buy}_{cus(i),t}$			0.31***	0.15**
			(0.08)	(0.07)
Time FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes
R^2	0.59	0.58	0.10	0.11
Observations	909	906	790	752

Note: Yearly data 2003-2021. The dependent variables are buying effort ($\Delta\sigma^{buy}_{i,t}$) for columns (1) and (2) and selling effort ($\Delta\sigma^{sell}_{i,t}$) for columns (3) and (4). Standard errors are in parentheses. ** and *** denote significance at the 5% and 1% level, respectively.

We can apply the same two-stage exercise to equation (18). The estimation results are reported in Column (4) in Table 4, which verifies the positive correlation between changes in search effort in connected industries.

Fact 5: Positive comovement of output with intermediate inputs

Fact 5 is that output and intermediate inputs comove in the fashion predicted by search complementarities when we aggregate across all the firms in the economy (i.e., all the islands in Section 2). The BEA compiles a measure of gross output (O) equal to the sum of an industry's value added (VA) and intermediate inputs (II), i.e., $O = VA + II$. BEA data are annual over the period 1997-2021. Figure 2 plots the cyclical component of gross output (blue line), intermediate inputs (red line), and industry value added (yellow line) together with NBER-dated recession periods (grey bands). We extract the cyclical component of the variable using an HP filter. The figure reveals that fluctuations in intermediate inputs are more procyclical than output fluctuations. The Great Recession witnessed a sharp fall in the production of intermediate inputs and in gross production across industries, while the value added remained more stable.

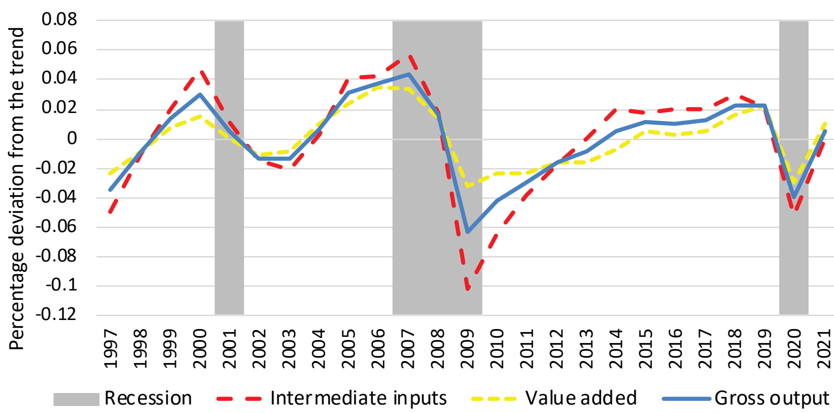


Figure 2: Intermediate inputs, value added, and gross output

To determine the relative contribution of value added and industry input to the overall volatility of gross output, we decompose the variance of the gross industrial output in terms of its covariance terms: $\text{Var}(O) = \text{Cov}(VA, O) + \text{Cov}(II, O)$. Using this identity, together with the definition $O = VA + II$, and plugging in observed data, we find that the contribution of industry inputs to movements in industrial gross output is $\frac{\text{Cov}(II, VA+II)}{\text{Var}(VA+II)} = 0.67$.

Thus, fluctuations in intermediate inputs account for 67% of the movements in gross industry output. This average contribution increases during recessions. For example, in 2008, industry intermediate inputs decreased by 1.9 trillion, making up 84% of the decline in gross industrial output (2.3 trillion).

4 A quantitative model

In this section, we generalize our simple model in Section 2 with a quantitative business cycle model that we can compare with the data. More concretely, we postulate a discrete-time model where firms in the intermediate-goods sector (I) and the final-goods sector (F) are connected through trading relationships. Firms will invest search effort in building these relationships because more effort leads to more relationships (Fact 1 in Section 3) and higher sales and profits (Fact 2). The matching function will be supermodular in the search effort of firms (even if it presents decreasing returns to scale). The supermodularity will create search complementarities that replicate Facts 3 and 4. Since we will deal with general equilibrium, including the presence of households, the model also captures Fact 5.

Households: There is a continuum of households of size 1. Households can either work one unit of time per period for a wage w or be unemployed and receive h utils of home production and leisure. Households do not have preferences for working –or searching for a job– in either sector $i \in \{I, F\}$ and receive the firms' profits.

Households are risk-neutral and discount the future by $\beta\xi_t$ per period, where $\beta < 1$ is a constant and ξ_t is a discount factor shock that follows $\log \xi_t = \rho_\xi \log \xi_{t-1} + \sigma_\xi \epsilon_{\xi,t}$, with $\rho_\xi \leq 1$ and $\epsilon_{\xi,t} \sim \mathcal{N}(0, 1)$. When $\xi_t > 1$, households are *more* patient than average and when $\xi_t < 1$ households are *less* patient than average. Innovations to ξ_t encapsulate demographic shifts, movements in financial frictions, or fluctuations in risk tolerance. Smets and Wouters (2007), Justiniano and Primiceri (2008), Cochrane (2011), Hall (2016, 2017), and Ikeda et al. (2020) provide evidence that those shocks are a central source of aggregate fluctuations. Since households own the firms, firms also employ $\beta\xi_t$ to discount future profits.

Labor matching: At the beginning of each period t, any willing new firm can post a vacancy in either sector $i \in \{I, F\}$ at the cost of χ per period to hire job-seeking households. Each firm posts a vacancy for one worker. Vacancies and job seekers meet in a DMP frictional labor market. Since this DMP block is standard, its only role is to provide a natural framework to discuss unemployment and vacancies without undue complexity.

Given $u_{i,t}$ unemployed households and $v_{i,t}$ posted vacancies in sector i, a constant-returns-to-scale matching technology $m(u_{i,t}, v_{i,t})$ determines the number of hires and vacancies filled in t. The new hires start working in $t + 1$. The job-finding rate, $\mu_{i,t} = m(u_{i,t}, v_{i,t})/u_{i,t} = \mu(\theta_{i,t})$, and the probability of filling a vacancy, $q_{i,t} = m(u_{i,t}, v_{i,t})/v_{i,t} = q(\theta_{i,t})$, are functions of each sector's labor market tightness ratio $\theta_{i,t} = v_{i,t}/u_{i,t}$. Then, $\mu'(\theta_{i,t}) > 0$ and $q'(\theta_{i,t}) < 0$.

At the end of each period t, existing jobs terminate at a rate δ and unfilled vacancies expire. The newly unemployed households are split evenly to search in each sector. Once an unemployed household is assigned to search in one sector, it is not allowed to search in another sector (given the symmetry across sectors and our calibration below, workers do not mind this restriction). Appealing to a law of large numbers, unemployment, $u_t = u_{I,t} + u_{F,t}$, evolves as:

$$u_{t+1} = u_t - \underbrace{\left[\mu_I\left(\theta_{I,t}\right)u_{I,t} + \mu_F\left(\theta_{F,t}\right)u_{F,t}\right]}_{\text{Job creation}} + \underbrace{\delta\left(1-u_t\right)}_{\text{Job destruction}}. \tag{19}$$

Trading relationships: Once job vacancies are filled, a final-goods firm must form a trading relationship with an intermediate-goods firm to manufacture together, starting in $t+1$, the final good sold to households, which is also our numeraire. A technology with variable search effort governs inter-firm matching. Search effort is costly, but it increases the probability of forming a trading relationship. Variable search effort generates search complementarities since the optimal search effort by one firm will be (weakly) increasing in the number of firms searching in the opposite sector and their search effort (see below for details). This stylized matching summarizes more sophisticated inter-firm networks such as those in Jones (2013) and Acemoglu et al. (2012).

In a trading relationship, the intermediate-goods firm uses its worker to produce $y_{I,t} = z_t$, where z_t is the stochastic productivity that follows $\log z_t = \rho_z \log z_{t-1} + \sigma_z \epsilon_{z,t}$, with $\rho_z \leq 1$ and $\epsilon_{z,t} \sim \mathcal{N}(0,1)$. The final-goods firm takes $y_{I,t}$ and, employing its worker, transforms it one-to-one into the final good, $y_{F,t} = y_{I,t} = z_t$. If a firm fails to form a trading relationship in t, it produces no output and continues searching for a partner in $t+1$. At the end of each period, a constant fraction of existing trading relationships is destroyed because either the job is destroyed with probability δ, or the trading relationship fails at a rate $\widetilde{\delta}$.[7] In the former case, the firms dissolve. In the latter case, the firms become single firms, but the jobs survive.

Search effort: Building on Burdett and Mortensen (1980), the number of inter-firm matches is $M\left(\widetilde{n}_{F,t}, \widetilde{n}_{I,t}, \widetilde{\sigma}_{F,t}, \widetilde{\sigma}_{I,t}\right) = \left(\phi + (\psi + \widetilde{\sigma}_{F,t}^{0.5})(\psi + \widetilde{\sigma}_{I,t}^{0.5})\right) H\left(\widetilde{n}_{F,t}, \widetilde{n}_{I,t}\right)$, where $\widetilde{n}_{F,t}$ is the number of single firms in sector F with search effort, $\widetilde{\sigma}_{F,t}$; $\widetilde{n}_{I,t}$ and $\widetilde{\sigma}_{I,t}$ are the analogous variables for the

[7] To simplify, in a trading relationship, the jobs in the intermediate-goods firm and the final-goods firm terminate simultaneously with probability δ or survive simultaneously with probability $1-\delta$. In single firms, the job destruction rate is also δ. We assume that $\delta + \widetilde{\delta} < 1$, and that the separation of job matches and trading relationships is a mutually exclusive event.

I sector (because of random search, all firms within a sector search with the same intensity). The function $H(\cdot)$ has constant returns to scale and is strictly increasing in both terms. We set up its units by choosing $H(1,1)=1$. We will explain momentarily why we specify two different parameters, $\{\phi,\psi\}>0$, and why we set the power of $\widetilde{\sigma}_{i,t}$ to 0.5.

Each firm optimally chooses $\widetilde{\sigma}_{i,t}\geq 0$ to trade off search cost and the profits from matching success. The cost of $\widetilde{\sigma}_{i,t}$ is:

$$c\left(\widetilde{\sigma}_{i,t}\right)=c_0\widetilde{\sigma}_{i,t}^{0.5}+c_1\frac{\widetilde{\sigma}_{i,t}^{(1+\nu)/2}}{1+\nu}, \tag{20}$$

where $c_0>0$ creates a concave cost tranche and $c_1>0$, with $\nu>1$, a convex cost tranche.

Given the inter-firm market tightness ratio $\widetilde{n}_F/\widetilde{n}_I$, the probability that a sector I firm will form a trading relationship with a sector F firm is:

$$\pi_I=\frac{M\left(\widetilde{n}_{F,t},\widetilde{n}_{I,t},\widetilde{\sigma}_{F,t},\widetilde{\sigma}_{I,t}\right)}{\widetilde{n}_I}=\left(\phi+(\psi+\widetilde{\sigma}_{F,t}^{0.5})(\psi+\widetilde{\sigma}_{I,t}^{0.5})\right)H\left(\frac{\widetilde{n}_{F,t}}{\widetilde{n}_{I,t}},1\right), \tag{21}$$

and the probability that a sector F firm will form a trading relationship with a sector I firm is:

$$\pi_F=\frac{M\left(\widetilde{n}_{F,t},\widetilde{n}_{I,t},\widetilde{\sigma}_{F,t},\widetilde{\sigma}_{I,t}\right)}{\widetilde{n}_F}=\left(\phi+(\psi+\widetilde{\sigma}_{F,t}^{0.5})(\psi+\widetilde{\sigma}_{I,t}^{0.5})\right)H\left(1,\frac{\widetilde{n}_{I,t}}{\widetilde{n}_{F,t}}\right). \tag{22}$$

In the symmetric equilibria where $\widetilde{n}_{F,t}=\widetilde{n}_{I,t}$, we have:

$$\pi_{F,t}=\pi_{I,t}=\phi+(\psi+\widetilde{\sigma}_{F,t}^{0.5})(\psi+\widetilde{\sigma}_{I,t}^{0.5})=\phi+\psi^2+\psi\widetilde{\sigma}_{F,t}^{0.5}+\psi\widetilde{\sigma}_{I,t}^{0.5}+\widetilde{\sigma}_{F,t}^{0.5}\widetilde{\sigma}_{I,t}^{0.5}. \tag{23}$$

The parameter ψ determines the impact of $\widetilde{\sigma}_{i,t}$ on the matching probability (23) without considering any interaction with $\widetilde{\sigma}_{-i,t}$. Thus, ψ bounds the marginal return to searching from below when prospective partners search with zero effort, a mechanism that will govern the degree of supermodularity in the model. In comparison, ϕ is a scaling parameter, unrelated to $\widetilde{\sigma}_{i,t}$, that will allow us to match the average inter-firm matching probabilities in the data. This difference separately identifies ϕ and ψ.

Equation (23) has decreasing returns to scale on $\widetilde{\sigma}_{F,t}$ and $\widetilde{\sigma}_{I,t}$. Nonetheless, $\widetilde{\sigma}_{F,t}^{0.5}\widetilde{\sigma}_{I,t}^{0.5}$, the most relevant term for the quantitative analysis, is homogeneous of degree 1. Since we are looking for a microfoundation for the increasing returns to scale assumption in Diamond (1982) through the endogeneity of search effort, homogeneity of degree 1 is an intuitive baseline.[8]

[8]Given our calibration in Section 6, equation (23) also has decreasing returns to scale if we express it in terms of the costs $c(\widetilde{\sigma}_{i,t})$. However, the function is nearly homogeneous of degree 1 for all but minimal cost levels (and hence search effort). Again, this is a natural benchmark.

To simplify notation, we define $\sigma_{i,t} = \widetilde{\sigma}_{i,t}^{0.5}$. Then, equation (23) becomes:

$$\pi_{F,t} = \pi_{I,t} = \phi + (\psi + \sigma_{F,t})(\psi + \sigma_{I,t}). \tag{24}$$

Together with equation (20), this result implies that the net gain from searching can be negative, in which case the firm chooses $\sigma_i = 0$, or positive and the firm picks $\sigma_i > 0$.

These two alternatives imply the existence of multiple equilibria of the stage game (i.e., within period t). One equilibrium is *passive*, with $\sigma_{I,t} = \sigma_{F,t} = 0$, low production, and high unemployment. The other equilibrium is *active*, with $\{\sigma_{I,t}, \sigma_{F,t}\} > 0$, high production, and low unemployment. The expression "equilibrium of the stage game," which we borrow from the literature on repeated games, highlights that we look at possible outcomes within one given period. The rational expectations equilibria for our economy are a sequence of these equilibria of the stage game. Households and firms have rational expectations about this sequence of equilibria and act accordingly.[9]

We select among equilibria through history dependence following Cooper (1994). If the economy was in a passive equilibrium in $t-1$, firms stay in the passive equilibrium in t. Conversely, if the economy was in an active equilibrium in $t-1$, firms continue searching with positive effort in t. Sufficiently large shocks to productivity or the discount factor induce firms to adjust search effort, and the economy shifts from one equilibrium to the other. Otherwise, the economy stays in the same equilibrium as in the previous period.[10] An indicator function, ι_t, with a value of zero if the equilibrium is passive and one if active, keeps track of the equilibrium. More specifically, ι_t is an endogenous stochastic state variable whose probability distribution is jointly determined by ι_{t-1} (due to history-dependence) and the exogenous shocks to the model (which might force a switch of equilibrium). The evolution of ι_t is taken as given by all agents in forming rational expectations about the future.[11]

[9] To avoid repetition, we will eliminate the qualifier "of the stage game" and refer simply to "equilibrium" when no ambiguity occurs.

[10] The selection based on history dependence differs from Schaal and Taschereau-Dumouchel (2018), who use a global game to select the equilibrium. With a global game, the equilibrium is uniquely and monotonically determined by the economic fundamentals, while, in our model, it is jointly determined by economic fundamentals and history. As we will see later, our framework allows spells with strong fundamentals while the economy stays in the low-activity equilibrium despite the coexistence of a high-activity equilibrium.

[11] There might exist mixed-strategy Nash equilibria of the stage game in which firms search with positive variable effort with some probability. We ignore those equilibria because the mixed strategy is not robust: when one sector changes the probability slightly due to a trembling-hand perturbation, the opposite sector would immediately set the probability to either zero or one (Echenique and Edlin, 2004). We leave non-Markov strategies, limit cycles, and alternative equilibria selection devices for future investigation.

The number of trading relationships in $t+1$ comprises firms that survive job separation and trading relationship destruction plus newly formed trading relationships:

$$n_{t+1} = (1-\delta-\widetilde{\delta})n_t + (\phi + (\psi + \sigma_{F,t})\,(\psi + \sigma_{I,t}))\widetilde{n}_{I,t}. \tag{25}$$

The number of single firms in sector i in $t+1$ includes firms that survive job separation ($(1-\delta)\,\widetilde{n}_{i,t}$), newly created single firms whose vacancies are filled by job-seekers ($\mu_i\,(\theta_{i,t})\,u_{i,t}$), and firms whose trading relationships exogenously terminate ($\widetilde{\delta}n_{i,t}$), net of the number of single firms that form trading relationships ($\pi_{i,t}\widetilde{n}_{i,t}$):

$$\widetilde{n}_{i,t+1} = (1-\delta)\,\widetilde{n}_{i,t} + \mu_i\,(\theta_{i,t})\,u_{i,t} + \widetilde{\delta}n_{i,t} - \pi_{i,t}\widetilde{n}_{i,t}. \tag{26}$$

Value functions: We can now define the Bellman equations that determine the value, for each sector i, of an unemployed household ($U_{i,t}$), of an employed household in a single firm ($\widetilde{W}_{i,t}$) and in a trading relationship ($W_{i,t}$), of a filled job in a single firm ($\widetilde{J}_{i,t}$) and in a trading relationship ($J_{i,t}$), and of a vacant job ($V_{i,t}$). We index all these value functions by ι_t since they depend on the type of equilibrium at t.

The value of an unemployed household in sector i and equilibrium ι is:

$$U_{i,t|\iota_t} = h + \beta\xi_t\mathbb{E}_t\left[\mu_{i,t}\widetilde{W}_{i,t+1} + (1-\mu_{i,t})\,U_{i,t+1} \mid \iota_t\right]. \tag{27}$$

In the current period, the unemployed household receives a payment h. The household finds a job with probability $\mu_{i,t}$ and circulates into employment during the next period, or it fails to find employment with probability $1-\mu_{i,t}$ and remains unemployed. To save space, we ignore the state variables when presenting the equations, but they are described in Appendix I.

The value of a household with a job in a single firm in sector i is:

$$\widetilde{W}_{i,t|\iota_t} = \widetilde{w}_{i,t} + \beta\xi_t\mathbb{E}_t\left\{(1-\delta)\left[\pi_{i,t}W_{i,t+1} + (1-\pi_{i,t})\,\widetilde{W}_{i,t+1}\right] + \delta U_{i,t+1} \mid \iota_t\right\}. \tag{28}$$

The first term on the right-hand side (RHS) is the wage $\widetilde{w}_{i,t}$ (to be determined by Nash bargaining). In $t+1$, the match that survives job destruction may either form a trading relationship with a firm in the opposite sector with probability $\pi_{i,t}$, gaining the value $W_{i,t+1}$, or otherwise remain a single firm with probability $1-\pi_{i,t}$, with value $\widetilde{W}_{i,t+1}$. The job is destroyed with probability δ, and the household transitions into unemployment.

The value of a household with a job in a trading relationship in each sector i is:

$$W_{i,t|\iota_t} = w_{i,t} + \beta\xi_t\mathbb{E}_t\left[(1-\delta-\widetilde{\delta})W_{i,t+1} + \widetilde{\delta}\widetilde{W}_{i,t+1} + \delta U_{i,t+1} \mid \iota_t\right]. \tag{29}$$

A worker in this situation receives the wage $w_{i,t}$. In $t+1$, the worker becomes unemployed with probability δ, gaining the value $U_{i,t+1}$. With probability $\widetilde{\delta}$, the trading relationship is terminated, and the value becomes $\widetilde{W}_{i,t+1}$. Otherwise, the match continues, gaining the value $W_{i,t+1}$.

The value of a single firm in sector i is:

$$\widetilde{J}_{i,t|\iota_t} = \max_{\sigma_{i,t}\geq 0}\left\{-\widetilde{w}_{i,t} - c\left(\sigma_{i,t}\right) + \beta\xi_t\left(1-\delta\right)\mathbb{E}_t\left[\pi_{i,t}J_{i,t+1} + \left(1-\pi_{i,t}\right)\widetilde{J}_{i,t+1} \mid \iota_t\right]\right\}. \tag{30}$$

Single firms have zero revenues, but they still need to pay the wage $(\widetilde{w}_{i,t})$ and incur the search costs $c\left(\sigma_{i,t}\right)$. In $t+1$, conditional on surviving job destruction with probability $1-\delta$, the firm forms a trading relationship with probability $\pi_{i,t}$, gaining $J_{i,t+1}$. Otherwise, the firm remains single with value $\widetilde{J}_{i,t+1}$. If the job is destroyed, the firm exits with zero value.

The value of a trading relationship for a sector I firm is:

$$J_{I,t|\iota_t} = z_t p_t - w_{I,t} + \beta\xi_t\mathbb{E}_t\left[(1-\delta-\widetilde{\delta})J_{I,t+1} + \widetilde{\delta}\widetilde{J}_{I,t+1} \mid \iota_t\right]. \tag{31}$$

The firm's earnings are equal to the revenue from the intermediate good, $z_t p_t$, less the wage $w_{I,t}$. Both p_t and $w_{I,t}$ are determined by Nash bargaining. In $t+1$, with probability $\widetilde{\delta}$, the firm is separated from its partner and becomes a single firm, gaining a value of $\widetilde{J}_{I,t+1}$; with probability δ, the job match is destroyed, and the firm exits the market with zero value. Otherwise, the trading relationship continues with value $J_{i,t+1}$.

The value of a trading relationship for a sector F firm is:

$$J_{F,t|\iota_t} = z_t(1-p_t) - w_{F,t} + \beta\xi_t\mathbb{E}_t\left[(1-\delta-\widetilde{\delta})J_{F,t+1} + \widetilde{\delta}\widetilde{J}_{F,t+1} \mid \iota_t\right]. \tag{32}$$

The profit in the final-goods sector comprises revenues from selling z_t units of final goods at a price 1, net of the costs of purchasing intermediate goods $(z_t p_t)$ and paying the wage $(w_{F,t})$. The rest of the equation follows the same interpretation as equation (31).

The value of a vacant job in sector i is:

$$V_{i,t|\iota_t} = -\chi + \beta\xi_t\mathbb{E}_t\left[q\left(\theta_{i,t}\right)\widetilde{J}_{i,t+1} + \left(1-q\left(\theta_{i,t}\right)\right)\max\left(0, V_{I,t+1}, V_{F,t+1}\right) \mid \iota_t\right]. \tag{33}$$

Equation (33) shows that the value of a vacant job comprises the fixed cost of posting a vacancy χ in t. With probability $q\left(\theta_{i,t|\iota_t}\right)$, the vacancy is filled, and a single firm with value $\widetilde{J}_{i,t+1}$ is created. The last term shows that firms that fail to recruit a worker may choose to be inactive or post a vacancy in either sector in $t+1$.

By free-entry, we have $V_{i,t}=0$ and the condition that pins down labor market tightness: $\chi=\beta\xi_t\mathbb{E}_t\left[q\left(\theta_{i,t}\right)\widetilde{J}_{i,t+1}\mid\iota_t\right]$. Appendix B describes the Nash bargaining over wages between firms in trading relationships and workers and prices between the final-goods producer and the intermediate-goods producer within a trading relationship.

Aggregate resource constraint: Finally, the total resources of the economy, equal to $z_t n_t$ (i.e., production per trading relationship times the number of existing trading relationships), are used for aggregate consumption by households, c_t, and to pay for vacancies and inter-firm search:

$$c_t+\sum_{i=I,F}\chi v_{i,t}+\sum_{i=I,F}\widetilde{n}_{i,t}\left(c_0\widetilde{\sigma}_{i,t}^{0.5}+c_1\frac{\widetilde{\sigma}_{i,t}^{(1+\nu)/2}}{1+\nu}\right)=z_t n_t. \tag{34}$$

5 Characterizing the equilibrium

The equilibrium definition for our model is standard, and we include it in Appendix H. Here, we characterize the optimal search strategy of firms and show how multiple equilibria make the effect of shocks persist over time.

Optimal search effort: The value for a firm i of searching with effort $\sigma_{i,t}>0$ when the search effort in the other sector $\sigma_{j,t}$ given an equilibrium ι_t is:

$$\Pi_i\left(\sigma_{i,t}\mid\sigma_{j,t},\iota_t\right)=-\widetilde{w}_{i,t}-c\left(\sigma_{i,t}\right)+\beta\xi_t\left(1-\delta\right)\mathbb{E}_t\left[\pi_{i,t}(J_{i,t+1}-\widetilde{J}_{i,t+1})+\widetilde{J}_{i,t+1}\mid\iota_t\right]. \tag{35}$$

The interior solution $\sigma_{i,t}>0$ (i.e., the best response) satisfies:

$$c_0+c_1\sigma_{i,t}^{\nu}=\widetilde{\beta}\underbrace{\left(\psi+\sigma_{j,t}\right)}_{\text{Search effort in sector } j}\underbrace{\xi_t}_{\text{discount factor shock}}\underbrace{\mathbb{E}_t\left(J_{i,t+1}-\widetilde{J}_{i,t+1}\mid\iota_t\right)}_{\text{Expected capital gain}} \tag{36}$$

where $\widetilde{\beta}=\beta\left(1-\delta\right)/\tau$ (the wage Nash bargaining implies that the firm bears τ fraction of the search cost). The left-hand side (LHS) of equation (36) is the marginal cost of exerting $\sigma_{i,t}$ to build a trading relationship, while the RHS is the expected benefit of searching for a partner. Since $\mathbb{E}_t(J_{i,t+1}-\widetilde{J}_{i,t+1}\mid\iota_t)$ depends positively on z_t, condition (36) shows how higher ξ_t or z_t (fundamentals) and higher $\sigma_{j,t}$ (search complementarities) lead to higher $\sigma_{i,t}$. Because the optimization problem is non-convex, we also have a corner solution $\sigma_{i,t}=0$, either because the firms in the other sector search too little or the discounted expected gains from matching are too small. The next proposition summarizes this argument.

Proposition 1. *The optimal $\sigma_{i,t}$ is equal to:*

$$\sigma_{i,t} = \begin{cases} \left[\frac{\widetilde{\beta}(\psi+\sigma_{j,t})\xi_t \mathbb{E}_t\left(J_{i,t+1}-\widetilde{J}_{i,t+1}|\iota_t\right)-c_0}{c_1}\right]^{\frac{1}{\nu}} & \text{if } \widetilde{\beta}\left(\psi+\sigma_{j,t}\right)\xi_t \mathbb{E}_t\left(J_{i,t+1}-\widetilde{J}_{i,t+1} \mid \iota_t\right) > c_0 \\ 0 & \text{otherwise.} \end{cases} \tag{37}$$

This proposition follows from equation (36) (see Appendices E and F for additional results and the proof of the existence of two equilibria of the stage game). Sufficiently large shocks to either ξ_t or z_t move the system between interior and corner solutions, generating alternate business cycle phases with robust search effort, a large number of trading relationships, and low unemployment with phases marked by no search effort, few trading relationships, and high unemployment.

Transitional dynamics: Figure 3 illustrates the deterministic transitional dynamics of the model embedded in the previous results and given the calibration in Section 6.

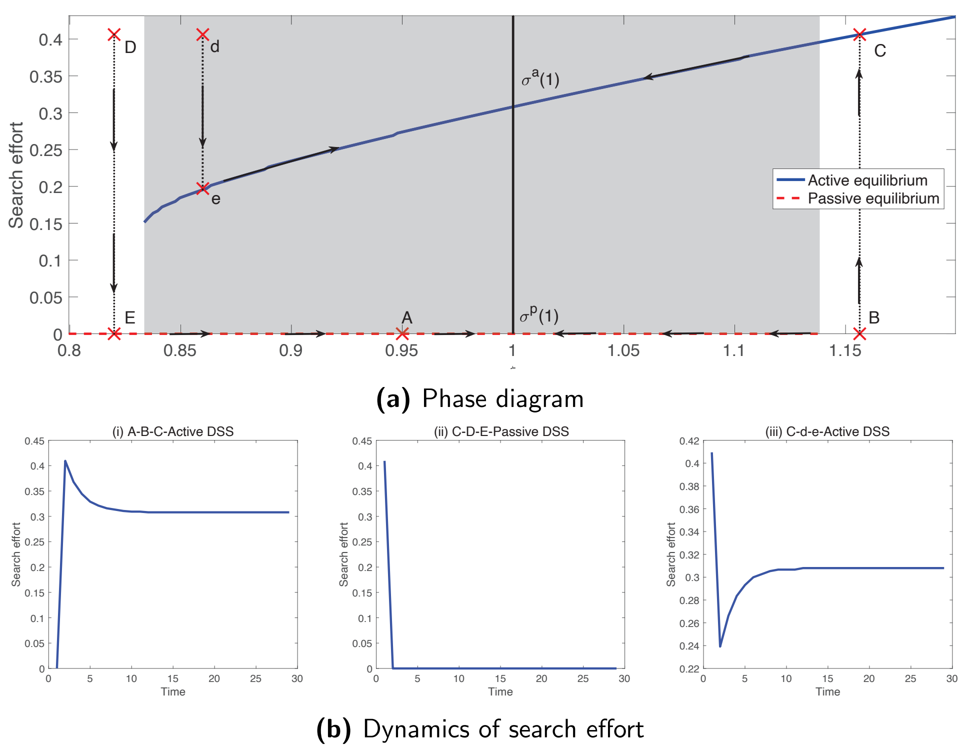


Figure 3: Transitional dynamics for search effort

The upper panel plots the movements in the search effort as a function of ξ_t (a similar figure could be drawn for z_t). The dashed line traces the passive equilibrium path with low search effort, and the solid line the active equilibrium path with high search effort. The arrows indicate the dynamics that the search effort follows to reach the passive deterministic steady state (DSS), $\{1, \sigma^p(1)\}$, and the active DSS, $\{1, \sigma^a(1)\}$, as ξ_t exogenously returns to 1.[12] The system converges to the passive or the active DSS depending on the starting equilibrium.

The shaded area indicates the range of ξ_t that supports multiple equilibria. The passive equilibrium does not exist when ξ_t is sufficiently large. Conversely, the active equilibrium fails to exist when ξ_t is sufficiently small. Thus, temporary shifts to ξ_t that are sufficiently strong to change search effort move the system to a new equilibrium.

Suppose the economy starts at point A and a large positive innovation to ξ_t moves the system to point B. In that case, the passive equilibrium disappears, and the equilibrium of the system becomes active. The economy moves to the active equilibrium at point C, converging to $\{1, \sigma^a(1)\}$ in the long run. Panel (b, i) shows the generalized impulse response functions (GIRF) of the search effort to this innovation: search increases at impact and then decreases, but staying at a higher level (where it will remain until a sufficiently negative innovation to ξ_t returns the system to the passive equilibrium).[13] In comparison, a large negative innovation to ξ_t that moves the system from point C to point D in the upper panel triggers the new passive equilibrium at point E, converging to $\{1, \sigma^p(1)\}$. Panel (b, ii) plots the GIRF of the search effort when the economy starts at C. Innovations to ξ_t that move the system from point C to point d within the shaded area where both equilibria coexist fail to shift the equilibrium because of history dependence, as shown by the GIRF in Panel (b, iii) (for the active equilibrium case). The asymmetric shape of the three GIRFs demonstrates the strong nonlinearity of our model.

6 Calibration

We calibrate the model at a monthly frequency for U.S. data over the post-WWII period. Table 5 summarizes the value and the source or target for each parameter.

The constant β is set to 0.996 to replicate an average annual interest rate of 5%. In keeping with the DMP block of the model standard, we assume a Cobb-Douglas labor market matching function $m(u, v) = u^{1-\alpha}v^{\alpha}$, with $\alpha = 0.3$, the average value in the literature (Petrongolo

[12] The DSS is the steady state to which the economy converges in the absence of shocks. This model has two different DSSs: one with active search and one with passive search. Appendix G shows the solution of the DSSs.

[13] Since the model is not linear, we use the adjective "generalized."

and Pissarides, 2001). To satisfy the Hosios (1990) condition, we set the wage bargaining power to $\tau = \alpha = 0.3$. We follow den Haan et al. (2000) in selecting the inter-firm matching function that ensures that matching probabilities are between 0 and 1: $H(\widetilde{n}_F, \widetilde{n}_I) = (\widetilde{n}_F \cdot \widetilde{n}_I)/[(\widetilde{n}_F^{\kappa}/2 + \widetilde{n}_I^{\kappa}/2)^{1/\kappa}]$. Also after den Haan et al. (2000), we set $\kappa = 1.25$.

Table 5: Parameter calibration

Parameter	Value	Source or Target
β	0.996	5% annual risk-free rate
α	0.3	Petrongolo and Pissarides (2001)
τ	0.3	Hosios condition
χ	0.73	0.42 monthly job-finding rate (Shimer, 2005)
κ	1.25	den Haan et al. (2000)
h	0.32	Thomas and Zanetti (2009)
$\widetilde{\tau}$	0.5	Sectoral symmetry
δ	0.024	5.5% unemployment rate in active DSS
$\widetilde{\delta}$	0.024	3.5 years' duration of trading relationship
ϕ	0.13	22% rate of idleness in recessions
ψ	0.15	Estimation of the matching function in Section 2.2
c_0	0.29	4.45% of revenue spent on search effort
c_1	5	12% rate of idleness in booms
ν	2	Ensure concavity of best response function
ρ_ξ	0.6	Livingston Survey
σ_ξ	0.054	Livingston Survey
ρ_z	$0.88^{1/3}$	BLS
σ_z	0.008	BLS

We pick the cost of posting a vacancy $\chi = 0.73$ to match the monthly job-finding rate in the active DSS, $\mu(\theta) = 0.42$, as in Shimer (2005). Then, we select a job-separation rate $\delta = 0.024$ to match an unemployment rate of 5.5% in the active DSS. We set $h = 0.32$ to include the value of leisure and home production and the unemployment benefit, as in Thomas and Zanetti (2009). Thus, the flow value of unemployment is about 63% of the average wage in the active DSS, which is in the range of replacement rates documented by Hall and Milgrom (2008).

Compared to a standard DMP economy, our model includes seven new parameters: ψ, $\widetilde{\tau}$, $\widetilde{\delta}$, ϕ, c_0, c_1, and ν. We set ψ to 0.15 following our estimated regression values for equation (14), reported in Table 3. We can rewrite the matching function for firm k in sector i as:

$$\pi_{i,k,t} = \underbrace{\phi + \psi^2}_{\text{constant}} + \underbrace{\psi\sigma_{i,k,t}}_{\text{linear term}} + \underbrace{\sigma_{i,k,t}\varsigma_{j,t}}_{\text{interactive term}} + \underbrace{\psi\varsigma_{j,t}}_{\text{error term}},$$

where $\varsigma_{j,t}$ is the component of $\sigma_{j,t}$ that is orthogonal to $\sigma_{i,k,t}$. Imposing $\pi_{i,k,t} \propto \Delta n^{sup}_{i,k,t}$ yields $\psi = \beta_1/\beta_2 = 0.89/5.87$, where β_1 and β_2 are the coefficients in regression (14).

The bargaining share of the intermediate-goods firm $\widetilde{\tau}$ is set to 0.5 to evenly split the total surplus from matching between firms and make the workers indifferent between working in either sector. The inter-firm matches' termination rate $\widetilde{\delta}$ is 0.024 to replicate the 3.5 years' average duration of a match documented in Section 3.

Given the previous parameters, c_1 and ϕ pin down the measure of single firms in the active and passive DSS, respectively. The ratio of single firms to employment is the rate of idleness, i.e., the share of time when employed workers are idle due to a lack of activity (Michaillat and Saez, 2015). According to the Institute for Supply Management, the idleness rate in the U.S. was about 30% for the non-manufacturing sector and 20% for the manufacturing sector during the Great Recession and 12% for both sectors before this event. Thus, we set $\phi = 0.13$ and $c_1 = 5$ to yield a rate of idleness equal to 0.22 and 0.12 in the passive and active DSS, respectively. We calibrate c_0 to 0.29 to generate a search cost of about 4.45% of output. This value is consistent with the fact that suppliers and customers spend 7.5% and 1.4% of revenues searching for trading partners, respectively, as documented above. Finally, $\nu = 2$ ensures the concavity of the best response function of search effort.

Following Hall (2017), we use the Livingston Survey to calibrate the discount factor shock by obtaining the median 12-months-ahead expected return r_t of the stock market index. We compute the discount factor as $\xi_t = 1/(1 + r_t)$. The monthly AR(1) that fits the series of ξ_t has parameters $\rho_\xi = 0.6$ and $\sigma_\xi = 0.054$ (Appendix L compares various measures of the discount factor). Given the rest of the calibration, these values generate a passive equilibrium with 15% probability, consistent with the frequency of recessions in the post-WWII U.S. The quarterly standard deviation (s.d.) of 5% for ξ_t is close to the popular estimate of a quarterly s.d. of 5.7% by Justiniano and Primiceri (2008, Table 1). The persistence of the productivity shock, $\rho_z = 0.88^{1/3}$ matches the observed quarterly autocorrelation of 0.88, and the s.d., $\sigma_z = 0.008$ matches the quarterly s.d. of 0.02, as in Shimer (2005).

Once the model is calibrated, we compute the value functions using value function iteration and exploit the equilibrium conditions of the model to find all variables of interest. See Appendices C and I for details.

7 Quantitative analysis

To study the dynamics of the model, we simulate it for three million months and time-average the resulting variables to generate quarterly data. We start the simulation from the active DSS, focusing on the case when only discount factor shocks are present. Appendix J provides a quantitative analysis of the model with productivity shocks. We relegate that case to the appendix because productivity shocks of plausible magnitude cannot move the system between equilibria, unless those shocks are permanent.

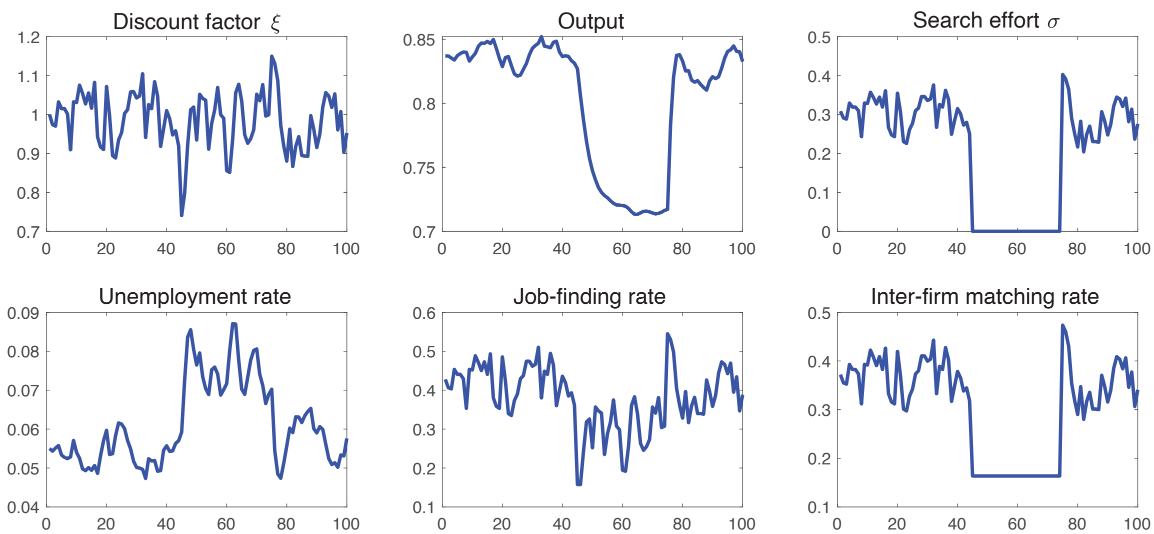


Figure 4: Simulated variables for the first 100 periods with shocks to ξ_t

Figure 4 reports the responses of key variables to shocks to ξ_t (top left panel) for the first 100 periods. The economy begins at a positive search effort with high output, low unemployment, and a high job-finding rate. Then, in period 45, a large negative shock to ξ_t pushes the economy to a prolonged drop in output (top center panel) as trading relationships terminate faster than they are replaced due to low effort (top right panel). Low effort generates a high unemployment rate and low job-finding and inter-firm matching rates (bottom panels). While the mean-reversion of ξ_t increases job-finding and decreases unemployment, the recovery is mild since the economy stays in the passive equilibrium until a large positive discount factor shock that makes households more patient shifts the economy back to the active equilibrium in period 74. In such a way, our model endogenizes, through varying change effort, the idea of a regime-switching process in the evolution of output postulated by Hamilton (1989).

Figure 5 plots the ergodic distribution of selected variables implied by the entire simulation.

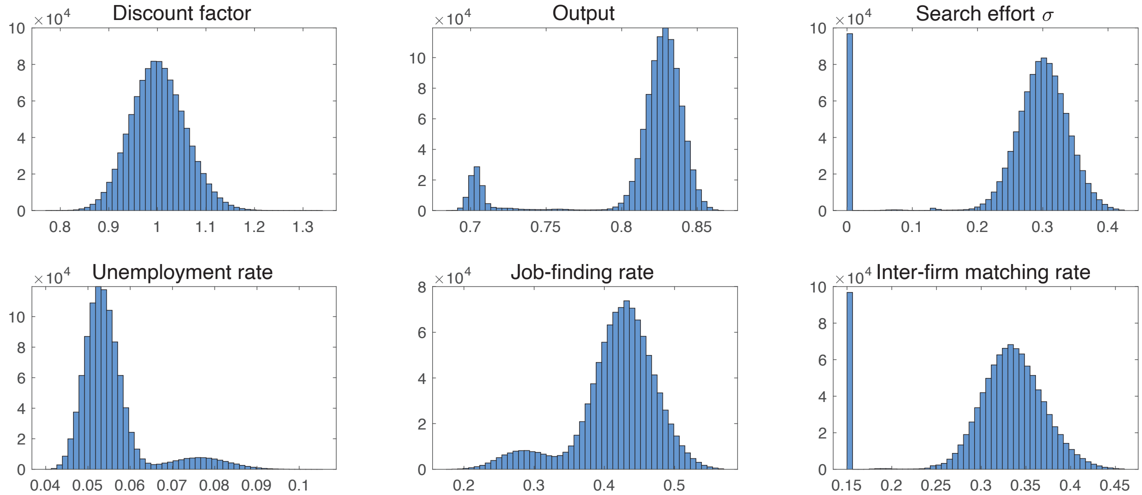


Figure 5: Ergodic distribution with shocks to ξ_t

Endogenous switches between passive and active equilibria generate a distinctive bimodal distribution of aggregate variables. These resemble some of the bimodal distributions documented in Adrian et al. (2021) or the ones from models with increasing returns to scale to search à la Diamond (1982), even if the discount factor shock has a unimodal distribution.[14] Consistent with U.S. data regarding recessions, our model predicts that the economy spends about 85% of the time in the active equilibrium and 15% in the passive equilibrium. In the former, the unemployment rate fluctuates around 5.5%. In the latter, unemployment fluctuates around 7.6%. The job-finding rate moves around 42% in the active equilibrium and 29% in the passive equilibrium.

Figure 6 compares the empirical distribution of real GDP per capita, the unemployment rate, the job-finding rate, and the inter-firm matching rate (continuous line) with the ergodic distribution in the model (discontinuous line).[15] Both the data and the model show skewness and bimodality.[16] This similarity supports the model, particularly if we recall that while we are using shocks *only* to the discount factor in our model, a combination of different shocks drives the dynamics of the actual data. We will revisit this issue in more detail in Section 8.

[14]See Pizzinelli et al. (2020) and Schaal and Taschereau-Dumouchel (2018) for additional evidence on skewness and bimodality in macroeconomic variables.

[15]Real GDP per capita is quarterly from 1960 to 2018 and is linearly detrended in logs. The unemployment and job-finding rates are quarterly from 1960 to 2018. The inter-firm matching rate is measured at the three-digit NAICS industry-year level from 2003 to 2021. We compute the firm-level matching rates as the proportion of new suppliers from each firm's total number of suppliers and average them to the sectoral matching rates.

[16]We cannot reject the bimodality of the distributions for real GDP per capita and the unemployment rate, but we can easily reject unimodality and tri-modality. Further details are available upon request.

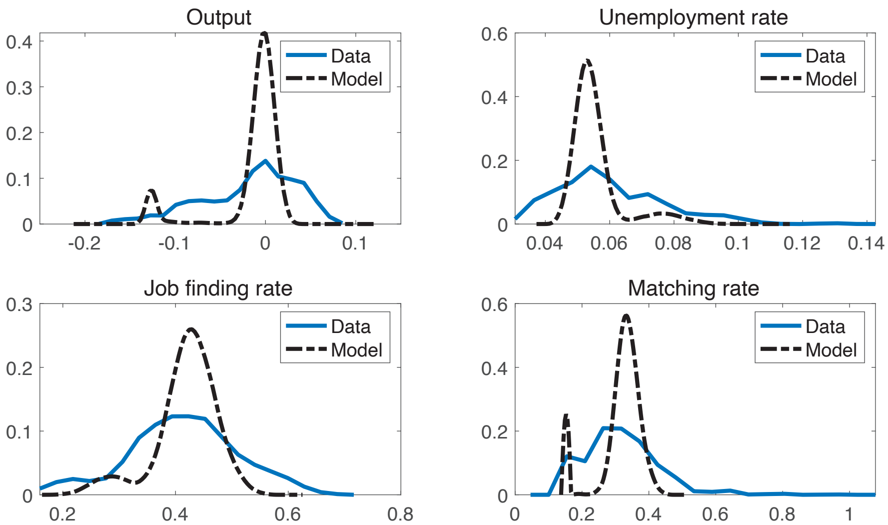


Figure 6: Distribution of unemployment rate and output in the data

Panel (a) of Table 6 reports second moments of observed business cycle statistics following the structure in Shimer (2005, Table 1). Panel (b) reports the second moments of the benchmark model with two DSSs. Appendix K reports the simulation of the model without search complementarities. The results are nearly identical when the filtering is done with a linear trend, as we used in Figure 6.

Table 6: Second moments

		u	v	v/u	lp	ξ
(a) Quarterly U.S. data, 1951-2016						
Autocorrelation coefficient		0.95	0.95	0.95	0.90	–
Standard deviation		0.20	0.21	0.40	0.02	–
	u	1	-0.92	-0.98	-0.25	–
Correlation matrix	v		1	0.98	0.29	–
	v/u			1	0.27	–
	lp				1	–
(b) Benchmark model						
Autocorrelation coefficient		0.84	0.59	0.71	0.96	0.38
Standard deviation		0.10	0.33	0.40	0.03	0.05
	u	1	-0.68	-0.80	-0.82	-0.51
Correlation matrix	v		1	0.98	0.49	0.83
	v/u			1	0.61	0.80
	lp				1	0.13
	ξ					1

Note: Following Shimer (2005), all variables are reported in logs as deviations from an HP trend with $\lambda = 10^5$.

Several lessons arise from Table 6. First, our model generates a robust internal propagation: the autocorrelation coefficients of the aggregate variables are significantly larger than in the model without complementarities and much closer to the observed ones. Complementarities in search effort plus history dependence amplify and prolong the effect of shocks. Second, our model generates empirically plausible s.d.'s for the selected variables that are much larger than those in the model without complementarities. This property of the model comes from the amplification of shocks described above. Third, our model produces endogenous movements in labor productivity ("lp" in the table) because of the time-varying fraction of the trading relationships over the total number of firms, $n_{i,t}/\left(\widetilde{n}_{i,t}+n_{i,t}\right)$. The business cycle statistics for labor productivity generated by our benchmark model are close to those in the data.

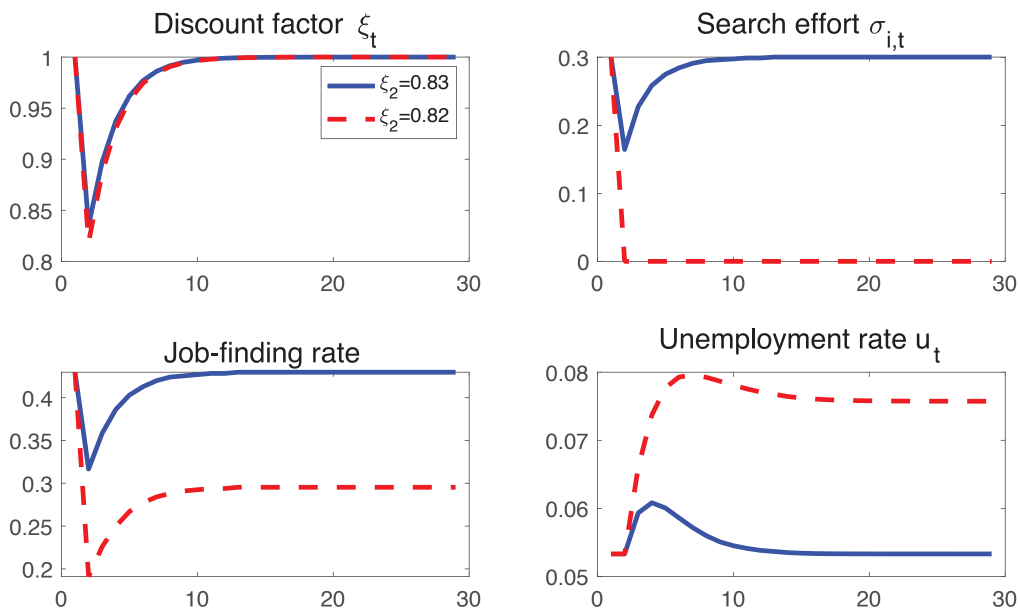


Figure 7: GIRFs to a negative discount factor shock

Figure 7 shows the GIRF of selected variables to a 18% (dashed line) and 17% (solid line) shock to ξ_t, respectively. In $t=1$, the economy starts from the active DSS. In $t=2$, a one-period innovation to the discount factor hits the economy (recall, however, that households and firms have rational expectations that this innovation can arrive with some probability). When the contractionary shock to ξ_t is 17%, the firm's search effort declines in response to the fall in the stream of benefits in forming a trading relationship, generating a mild decline in labor market tightness and a rise in the unemployment rate. This shock is too small to move the system to the passive equilibria, and the variables converge to the original DSS. However, when the fall in ξ_t is sufficiently large, the system moves to the equilibrium with zero search effort, low output, and high unemployment. While the shock is only a bit larger (18% vs. 17%), its effects are quite different: search complementarities induce large nonlinearities in the model.

8 The volatility of shocks and aggregate performance

Our model links the volatility of shocks with aggregate outcomes nonlinearly. This feature has two sharp implications. First, when volatility is high, the distribution of output is bimodal, as the economy often switches between equilibria. However, when volatility is low, the distribution of output is unimodal. This unique prediction distinguishes our model from most other business cycle models, including other models with strategic complementarities. Second, when volatility is low, large shocks have particularly persistent effects. Once a large shock pushes the economy into a new equilibrium, it will remain in it for a very long time because the probability of another large shock that will switch equilibria is low.[17] Again, this is a distinctive property of our model. We show now that both implications of our model hold in the data.

High volatility and bimodality: Our model predicts that the bimodality of the distribution of output will be particularly salient in periods of high volatility. To show that this is also the case in the data, we estimate the one-quarter-ahead conditional distribution of output with the non-parametric approach proposed by Adrian et al. (2021). Denote $y_t = (ln\,(GDP_{t+1})\,, ln\,(pd_{t+1}))$ and $x_t = (ln\,(GDP_t)\,, ln\,(pd_t))$, where $ln\,(GDP_t)$ and $ln\,(pd_t)$ are the quarterly real GDP per capita and price-dividend ratio in logs as deviations from an HP trend, respectively.[18] The log price-dividend ratio is a proxy for the discount factor, the driving shock in our model.

We compute the joint distribution function of y conditional on x as:

$$\widehat{p}\,(y \mid x) = \frac{\frac{1}{T-1}\sum_{t=2}^{T} K_y\,(y - y_t)\,K_x\,(x - x_t)}{\frac{1}{T-1}\sum_{t=2}^{T} K_x\,(x - x_t)},$$

where $K_y\,(\cdot)$ and $K_x\,(\cdot)$ are independent kernels for y and x, respectively, defined as:

$$K_y\,(y - y_t) = \frac{1}{\omega_{1,y}}\varphi\left(\frac{y_1 - y_{1t}}{\omega_{1,y}}\right) + \frac{1}{\omega_{2,y}}\varphi\left(\frac{y_2 - y_{2,t}}{\omega_{2,y}}\right)$$
$$K_x\,(x - x_t) = \frac{1}{\omega_{1,x}}\varphi\left(\frac{x_1 - x_{1t}}{\omega_{1,x}}\right) + \frac{1}{\omega_{2,x}}\varphi\left(\frac{x_2 - x_{2,t}}{\omega_{2,x}}\right),$$

[17] See Appendix N for an illustration of how the duration of each equilibrium is inversely related to the volatility of the shocks. Appendix M reports the histograms of the model's endogenous variables for alternative levels of the volatility of the shocks.

[18] We obtain the monthly p-d ratio from Robert Shiller's website: http://www.econ.yale.edu/ shiller/data.htm, then convert it to quarterly using time-averaging.

where $\omega_{i,y}$ and $\omega_{i,x}$ are the bandwidths for the ith variable of y and x, respectively, and $\varphi(\cdot)$ is the normal pdf. We set the bandwiths to be proportional to the in-sample unconditional standard deviation: $\omega_{1,y} = \omega_{1,x} = c \cdot \sigma(ln(GDP_t))$, $\omega_{2,y} = \omega_{2,x} = c \cdot \sigma(ln(pd_t))$, where c is calibrated to 0.3 as in Adrian et al. (2021).

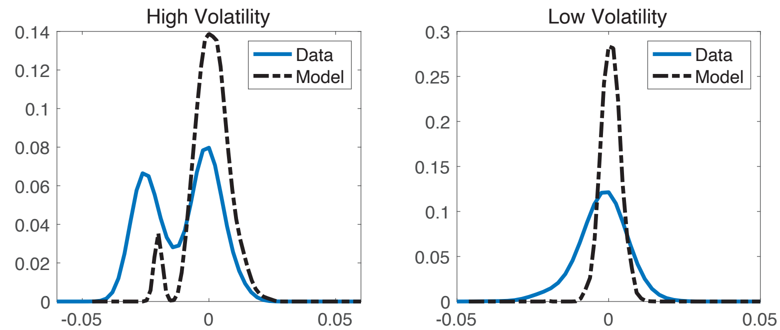


Figure 8: Marginal conditional distribution of output

The solid curves in Figure 8 plot the estimated conditional marginal distribution of output in 2008.Q4 (left panel) and 2014.Q2 (right panel). Bimodality is pronounced in 2008.Q4, when volatility was the highest in the sample 1960.Q1-2021.Q1 ($\sigma(ln(pd_t)) = 0.195$ vs. a sample mean of 0.052).[19] In contrast, there was no bimodality in 2014.Q2, when volatility was the lowest ($\sigma(ln(pd_t)) = 0.013$). This result is general across the sample. The correlation between the Hartigan dip statistic for each quarter's marginal conditional distribution of output and the volatility of the price-dividend ratio is -0.13, which is statistically significant at the 5% level.[20]

To replicate the same exercise using our model, we simulate two sets of 1,000 economies. In the first set, we set $\sigma_\xi = 0.081$, 50% higher than in our benchmark calibration in Table 5. Each economy runs for one quarter (three months) and starts from the active DSS with the initial discount factor two standard deviations below its mean. In the second set, we simulate another 1,000 economies for one quarter when $\sigma_\xi = 0.027$, 50% lower than our benchmark calibration. To make the two sets perfectly comparable, we construct the discount factors in the second simulation as our random draws in the first simulation multiplied by 1/3.

The dashed curves in Figure 8 plot the marginal conditional distribution of output for each of these two simulated samples. The marginal conditional distribution of output implied by the

[19] We compute $\sigma(ln(pd_t))$ as the standard deviation of the monthly p-d ratio in a 12-month window around each quarter.

[20] Since the dip statistic is an increasing function of the probability of unimodality, a negative correlation means unimodality is much more likely to be rejected when volatility is high.

model in the high volatility case (the left panel) is bimodal and fairly close to the solid curve in the same panel. The right panel shows that the conditional distribution is unimodal when volatility is low, similar to the data.

The Great Moderation and the persistence of business cycles: Our model predicts that a lower volatility of fundamentals is associated with more prolonged equilibrium spells. This prediction is consistent with the U.S. data.

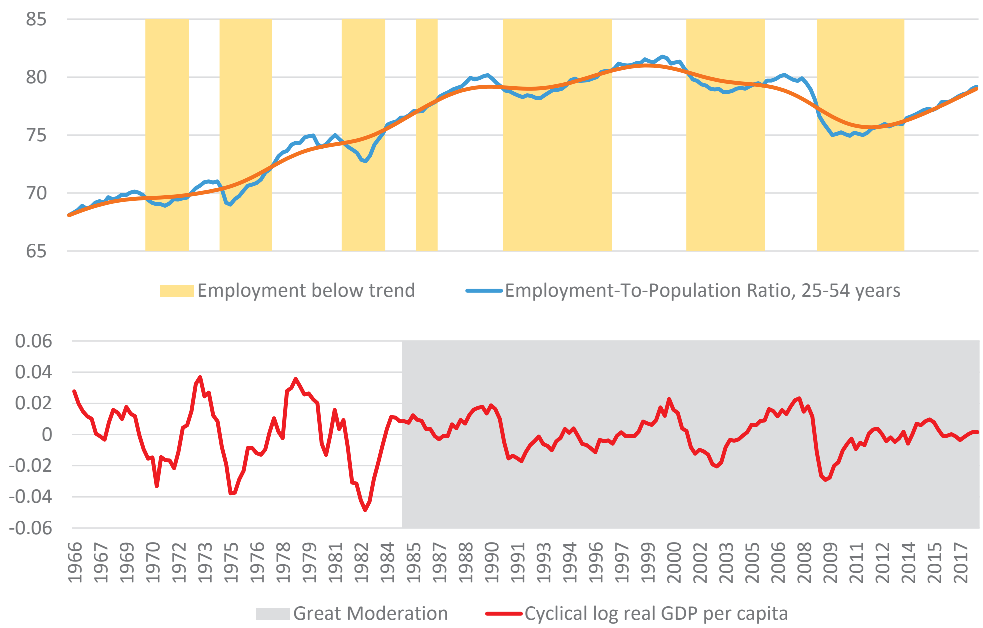


Figure 9: The Great Moderation and labor market downturns

In Figure 9, the upper panel plots the U.S. employment rate (blue curve) and its trend (orange curve) estimated from an HP filter with $\lambda = 1600$ from 1996 to 2017. The light-orange bars indicate labor market downturns. Inspired by the NBER's methodology, we define a labor market downturn as starting when the employment rate falls below the trend for two quarters and ending when the employment rate rises above the trend for two quarters. As noted by many researchers (Jaimovich and Siu, 2012 and references therein), the figure shows how the three labor market downturns after 1984 were longer than the previous ones. Precisely after 1984, the U.S. economy experienced a substantial reduction in aggregate volatility, which Fernández-Villaverde et al. (2015) attribute, in part, to a lower volatility of shocks to fundamentals. To illustrate this point, the bottom panel in Figure 9 plots the cyclical real GDP per capita component, with a grey area indicating the Great Moderation after 1984.

Our model suggests an intrinsic connection between the Great Moderation and the increasing persistence of labor market downturns, like the one that followed the financial crisis of 2008. While the Great Moderation improves macroeconomic stability and reduces the occurrences of recessions, it makes these recessions and the associated labor market downturns more durable.

9 The role of fiscal policy

In our model, government spending that stimulates search effort may permanently move the system from a passive to an active equilibrium, inducing a large fiscal multiplier. To explore this idea, we embed government spending in our economy. We focus on government spending (government consumption expenditures and gross investment) while ignoring transfers because, in our model, output is not demand-determined.

9.1 Government spending as a set of final-goods producers

The government owns single final-goods firms, $\widetilde{n}^G_{F,t}$, that operate together with private firms. The only difference between government-owned and private firms is that the former do not use labor.[21] The formation of private firms remains endogenous, as described by equation (26).

Government spending is equal to the output produced by government-owned firms in trading relationships (i.e., the government "buys" the output of its firms and the government-owned firms use those resources to pay the private intermediate firm and in production) plus the single government-owned firms' search cost. This spending is financed through lump-sum taxes.

We model higher government spending as an exogenous increase in the number of single firms in the final-goods sector. These additional firms can be interpreted as new public projects, such as building a new school. Thus, the law of motion for government single final-goods firms is $\widetilde{n}^G_{F,t} = (1 - \delta - \pi_F)\,\widetilde{n}^G_{F,t-1} + \epsilon^G_t$, where ϵ^G_t are the new government-owned single firms created in t.[22] Like private firms, government-owned firms must form a trading relationship with firms in the intermediate-goods sector to manufacture goods (for example, a public school requires

[21]It would be easy to modify the model to force government-owned firms to hire workers to operate (and, thus, be fully symmetric to private firms). We prefer our assumption of jobless government firms because it allows for a rapid increase in government spending. Job matching requires time, and it would mean that government spending would only phase in slowly. Unfortunately, this slow phase-in would make our quantitative results less comparable with existing findings in the literature.

[22]We assume that government spending shocks hit once per year. With probability $1/12$, ϵ^G_t is drawn from the uniform distribution with the support $[0, \widetilde{n}_{F,t}/2]$. Otherwise, $\epsilon^G_t = 0$. This specification ensures a non-negative measure of government firms and that the inter-firm matching market tightness ratio does not explode.

CFRPs produced by private firms). Trading relationships with government-owned firms follow $n^G_{F,t+1} = \left(1 - \delta - \widetilde{\delta}\right) n^G_{F,t} + \pi_F \widetilde{n}^G_{F,t}$. A government-owned firm exits the market when its trading relationship is terminated (either because the relationship fails with probability $\widetilde{\delta}$ or the job in the private firm disappears with probability δ).

The inflow ϵ^G_t changes the matching probabilities $\pi_{I,t} = \left[\phi + (\psi + \sigma_I)(\psi + \sigma_F)\right] H\left(1, \widetilde{\theta}_t\right)$, in the inter-firm matching market and $\pi_F = \left[\phi + (\psi + \sigma_I)(\psi + \sigma_F)\right] H\left(\frac{1}{\widetilde{\theta}_t}, 1\right)$, where $\widetilde{\theta}_t = (\widetilde{n}_{F,t} + \widetilde{n}^G_{F,t})/\widetilde{n}_{I,t}$ is the new inter-firm matching market tightness ratio.

Since H is increasing in both arguments, $\epsilon^G_t > 0$ increases the matching probability for intermediate-goods firms (more potential partners) and decreases the matching probability for final-goods firms (stiffer competition for partners). These changes in matching probabilities, in turn, move search effort and, potentially, the equilibrium of the economy.

Government spending is equal to $g_t = z_t n^G_{F,t} + \widetilde{n}^G_{F,t}\left(c_0 \widetilde{\sigma}^{0.5}_{F,t} + c_1 \frac{\widetilde{\sigma}^{(1+\nu)/2}_{F,t}}{1+\nu}\right)$. Gross aggregate output comprises government and private production (as per standard national accounting conventions): $y_t = z_t(n^G_{F,t} + n_{F,t})$, and it is used for private consumption, government spending, and search costs. The aggregate resource constraint is $y_t = c_t + g_t + \sum_{i=I,F} \chi v_i + \sum_{i=I,F} \widetilde{n}_{i,t}\left(c_0 \widetilde{\sigma}^{0.5}_{i,t} + c_1 \frac{\widetilde{\sigma}^{(1+\nu)/2}_{i,t}}{1+\nu}\right)$.

9.2 Shocks to government spending and equilibria switches

We assume that the economy is in the passive equilibrium (i.e., $\sigma_I = \sigma_F = 0$) before the arrival of a positive government spending shock, ϵ^G_t. Upon the realization of the shock, the passive equilibrium continues to exist if and only if:

$$\widetilde{\beta}\xi_t \psi H\left(1, \widetilde{\theta}_t\right) \mathbb{E}_t\left(J_{I,t+1} - \widetilde{J}_{I,t+1} \mid \iota = 0\right) < c_0, \tag{38}$$

or

$$\widetilde{\beta}\xi_t \psi H\left(\widetilde{\theta}^{-1}_t, 1\right) \mathbb{E}_t\left(J_{F,t+1} - \widetilde{J}_{F,t+1} \mid \iota = 0\right) < c_0, \tag{39}$$

where $\widetilde{\beta} = \beta(1-\delta)/\tau$. Equation (38) shows that the passive equilibrium disappears if the increase in government-owned single firms sufficiently tightens the inter-firm matching market.

Proposition 2. *Starting from the passive equilibrium, the size of government spending needed* *to move the system to the active equilibrium is:*

$$\frac{\widetilde{n}^{G}_{F,t}}{\widetilde{n}_{I,t}} > \Psi\left[\frac{c_0}{\widetilde{\beta}\xi\psi\mathbb{E}_t\left(J_{I,t+1} - \widetilde{J}_{I,t+1} \mid \iota = 0\right)}\right] - \frac{\widetilde{n}_{F,t}}{\widetilde{n}_{I,t}}, \tag{40}$$

with $\Psi' > 0$.[23]

Equation (40) determines that the magnitude of the policy intervention that moves the economy to an active equilibrium is proportional to the cost-benefit ratio of forming a trading relationship, and it decreases with the measure of private firms in the final-goods sector relative to intermediate-goods firms. A large quantity of private final-goods firms improves the trading relationship prospects for intermediate-goods firms, decreasing the magnitude of government spending needed to move to the active equilibrium.

9.3 The fiscal multiplier

We now measure the economy's response to expansionary fiscal policy shocks and the size of the fiscal multiplier. Once we introduce government spending, we have 12 state variables. Due to this large number of state variables, we implement a dimensionality reduction algorithm inspired by Krusell and Smith (1998) that is of interest in itself and applicable to similar problems. See Appendix I.2 for computational details.

Figure 10 shows the GIRFs to the same 70% (dotted line) and 80% (solid line) shocks to the relative size of the final-goods sector that we just described when the economy starts at the passive DSS (Appendix O shows the responses for the system that starts from the active DSS). Since the 80% fiscal expansion satisfies Proposition 2, it produces a significant and persistent increase in output and a fall in unemployment. Nevertheless, this fiscal expansion crowds out private consumption upon impact. This reaction is due to two mechanisms. First, a rise in government-owned firms reduces, in the short run, the formation of trading relationships that produce goods for private consumption. Second, the equilibrium shift triggers an increase in the cost associated with vacancy posting and forming trading relationships, further reducing private consumption. The first mechanism still exists in the 70% fiscal expansion, inducing a

[23] Denote $h\left(\widetilde{\theta}\right) = H\left(1, \widetilde{\theta}\right)$. Ψ is the inverse function of $h(\cdot)$. As $h(\cdot)$ is strictly increasing in $\widetilde{\theta}$ by assumption, Ψ is also a strictly increasing function. In our calibration: $h(\theta) = 2^{\frac{1}{\kappa}}\left(1 + \widetilde{\theta}_t^{-\kappa}\right)^{-1/\kappa}$, $\Psi(x) = \left(2x^{-\kappa} - 1\right)^{-1/\kappa}$.

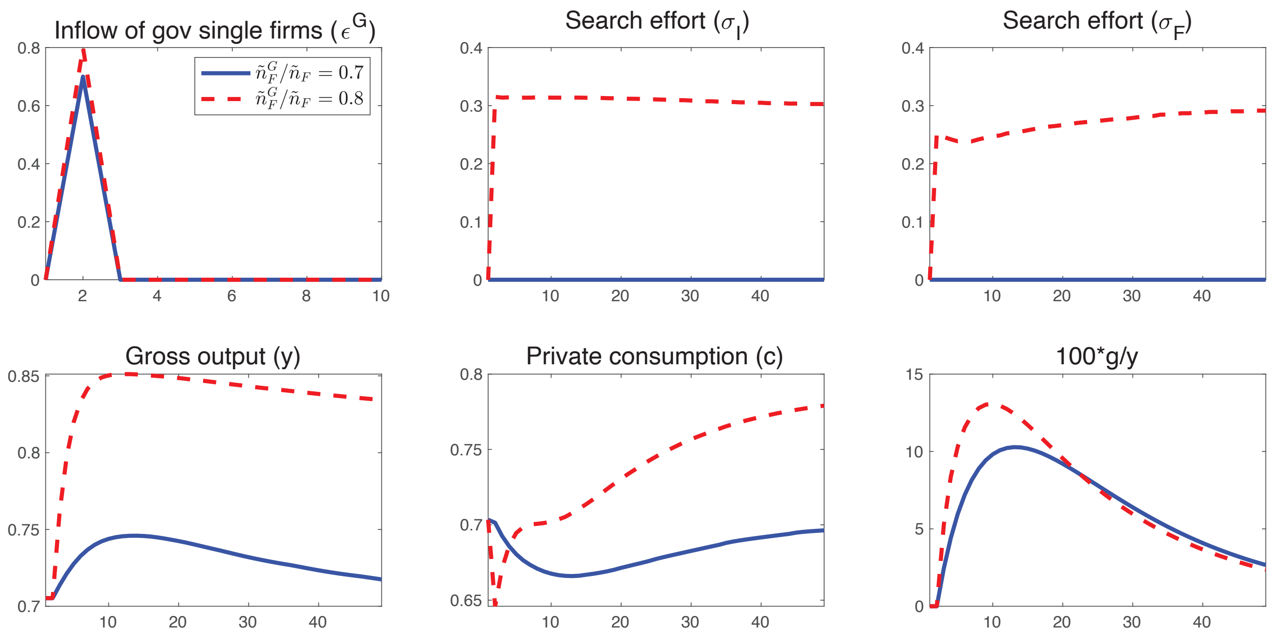
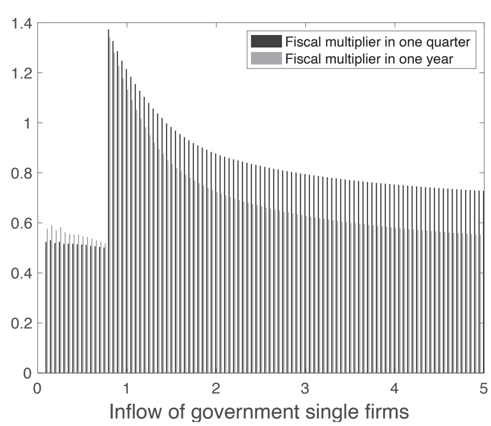


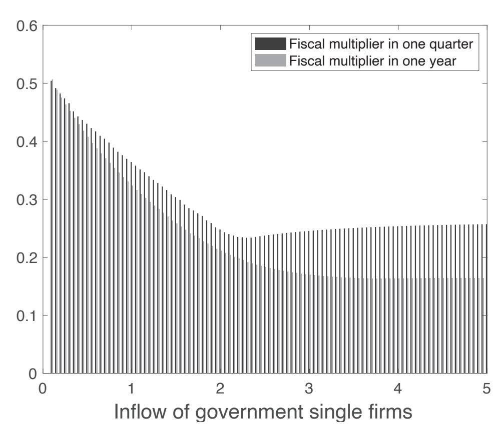
Figure 10: GIRFs to positive government spending shock

slight drop in private consumption.

We calculate the fiscal multiplier for our economy, defined as the ratio of the cumulative change in output over one quarter and one year, generated by the one-period change in government spending triggered by the inflow of government-owned single firms in the final-goods sector (we could compute the fiscal multiplier at other horizons if desired). Panel (a) in Figure 11 shows the fiscal multiplier as a function of the inflow of government-owned single firms when the economy is in the passive equilibrium at the start of the fiscal expansion. Panel (b) replicates the exercise for the case when the economy is in the active equilibrium.



(a) passive equilibrium



(b) active equilibrium

Figure 11: Fiscal multiplier

In the passive equilibrium, a sufficiently large fiscal expansion generates a multiplier larger than one since it triggers a jump in search effort. The fiscal multiplier peaks at the threshold where we shift from the passive to the active equilibrium. In our calibration, the peak quarterly fiscal multiplier, 1.37, is at a 75% increase in the number of government-owned firms, which is equivalent to a 6.4% rise in government spending relative to output in the first quarter (since the increase in government spending is persistent, the overall size of the fiscal intervention is larger than the impact change of 6.4%). Any larger stimulus reduces the fiscal multiplier because the crowding out of private consumption outweighs the increase in output from the fiscal expansion. Similarly, a fiscal expansion below the threshold generates a less-than-unitary fiscal multiplier since it creates a large crowding-out effect and no equilibrium switch.

Panel (b) in Figure 11 shows that the fiscal multiplier is substantially lower in the active equilibrium. The increased costs of forming trading relationships for private firms in the final-goods sector reduce private output, and we have a less than unitary fiscal multiplier for any size of the fiscal stimulus. The multiplier declines with the size of government spending for a crowding-out effect across a wide range of time horizons.

Our results in Figure 11 agree with the recent empirical literature documenting the acute state dependence of fiscal multipliers. See, for example, Auerbach and Gorodnichenko (2012), Owyang et al. (2013), and Ghassibe and Zanetti (2022). Our model accounts for such state dependence of fiscal multipliers.

10 Conclusion

This paper has documented five novel facts about the role of search effort in forming trading relationships by combining a variety of micro and macro datasets. These five facts can be parsimoniously interpreted as suggesting the existence of search complementarities in forming trading relationships. We have built a dynamic general equilibrium model, disciplined with our new firm-level evidence on search effort, to account for those five novel facts and explore its quantitative implications.

The analysis opens exciting avenues for additional research. Empirically, the role that agent and spatial heterogeneity play in search effort and the formation of trading relationship deserves further exploration. Quantitatively, a direct extension would be to embed strategic complementarities in richer business cycle models, including money, nominal rigidities, and financial frictions. We will pursue some of those ideas in future work.

Data Availability Statement

The data and code underlying this research are available in the Zenodo data repository, at https://doi.org/10.5281/zenodo.10655866.

References

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016.

Adrian, T., Boyarchenko, N., and Giannone, D. (2021). Multimodality in macrofinancial dynamics. *International Economic Review*, 62(2):861–886.

Auerbach, A. J. and Gorodnichenko, Y. (2012). Measuring the output responses to fiscal policy. *American Economic Journal: Economic Policy*, 4(2):1–27.

Beaudry, P., Galizia, D., and Portier, F. (2016). Is the macroeconomy locally unstable and why should we care? In *NBER Macroeconomics Annual 2016, Volume 31*, pages 479–530. National Bureau of Economic Research, Inc.

Beaudry, P., Galizia, D., and Portier, F. (2018). Reconciling Hayek's and Keynes' views of recessions. *Review of Economic Studies*, 85(1):119–156.

Beaudry, P., Galizia, D., and Portier, F. (2020). Putting the cycle back into business cycle analysis. *American Economic Review*, 110(1):1–47.

Bulow, J. I., Geanakoplos, J. D., and Klemperer, P. D. (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy*, 93(3):488–511.

Burdett, K. and Mortensen, D. (1980). Search, layoffs, and labor market equilibrium. *Journal of Political Economy*, 88(4):652–72.

Chatterjee, S., Cooper, R., and Ravikumar, B. (1993). Strategic complementarity in business formation: Aggregate fluctuations and sunspot equilibria. *Review of Economic Studies*, 60(4):795–811.

Cochrane, J. H. (2011). Discount rates. *Journal of Finance*, 66(4):1047–1108.

Cooper, R. (1994). Equilibrium selection in imperfectly competitive economies with multiple equilibria. *Economic Journal*, 104(426):1106–1122.

Cooper, R. and John, A. (1988). Coordinating coordination failures in Keynesian models. *Quarterly Journal of Economics*, 103(3):441–463.

den Haan, W. J., Ramey, G., and Watson, J. (2000). Job destruction and propagation of shocks. *American Economic Review*, 90(3):482–498.

Diamond, P. (1982). Aggregate demand management in search equilibrium. *Journal of Political Economy*, 90(5):881–894.

Diamond, P. and Fudenberg, D. (1989). Rational expectations business cycles in search equilibrium. *Journal of Political Economy*, 97(3):606–619.

Echenique, F. and Edlin, A. S. (2004). Mixed equilibria are unstable in games of strategic complements. *Journal of Economic Theory*, 118(1):61–79.

Eeckhout, J. and Lindenlaub, I. (2019). Unemployment cycles. *American Economic Journal: Macroeconomics*, 11(4):175–234.

Fernández-Villaverde, J., Guerrón, P., and Rubio-Ramírez, J. F. (2015). Estimating dynamic equilibrium models with stochastic volatility. *Journal of Econometrics*, 185(1):216–229.

Ghassibe, M. and Zanetti, F. (2022). State dependence of fiscal multipliers: the source of fluctuations matters. *Journal of Monetary Economics*, 132:1–23.

Hall, R. E. (2014). What the cyclical response of advertising reveals about markups and other macroeconomic wedges. Mimeo, Stanford University.

Hall, R. E. (2016). *Macroeconomics of Persistent Slumps*, volume 2 of *Handbook of Macroeconomics*, pages 2131–2181. Elsevier.

Hall, R. E. (2017). High discounts and high unemployment. *American Economic Review*, 107(2):305–330.

Hall, R. E. and Milgrom, P. (2008). The limited influence of unemployment on the wage bargain. *American Economic Review*, 98(4):1653–1674.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 57(2):357–384.

Heizer, J., Render, B., and Munson, C. (2016). *Operations Management: Sustainability and Supply Chain Management*. Pearson Education, 12th edition.

Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies*, 57(2):279–298.

Howitt, P. (1985). Transaction costs in the theory of unemployment. *American Economic Review*, 75(1):88–100.

Howitt, P. and McAfee, R. P. (1987). Costly search and recruiting. *International Economic Review*, pages 89–107.

Howitt, P. and McAfee, R. P. (1988). Stability of equilibria with externalities. *Quarterly Journal of Economics*, 103(2):261–277.

Howitt, P. and McAfee, R. P. (1992). Animal spirits. *American Economic Review*, pages 493–507.

Ikeda, D., Li, S., Mavroeidis, S., and Zanetti, F. (2020). Testing the effectiveness of unconventional monetary policy in Japan and the United States. Papers 2012.15158, arXiv.org.

Jaimovich, N. and Siu, H. E. (2012). The trend is the cycle: Job polarization and jobless recoveries. Working Paper 18334, National Bureau of Economic Research.

Jones, C. I. (2013). Misallocation, economic growth, and input-output economics. In Acemoglu, D., Arellano, M., and Dekel, E., editors, *Advances in Economics and Econometrics: Tenth World Congress*, volume 2, pages 419–456. Cambridge University Press.

Justiniano, A. and Primiceri, G. E. (2008). The time-varying volatility of macroeconomic fluctuations. *American Economic Review*, 98(3):604–641.

Kaplan, G. and Menzio, G. (2016). Shopping externalities and self-fulfilling unemployment fluctuations. *Journal of Political Economy*, 124(3):771–825.

Krusell, P. and Smith, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Liu, P., Theodoridis, K., Mumtaz, H., and Zanetti, F. (2019). Changing macroeconomic dynamics at the zero lower bound. *Journal of Business & Economic Statistics*, 37(3):391–404.

Michaillat, P. and Saez, E. (2015). Aggregate demand, idle time, and unemployment. *Quarterly Journal of Economics*, 130(2):507–569.

Owyang, M. T., Ramey, V. A., and Zubairy, S. (2013). Are government spending multipliers greater during periods of slack? Evidence from twentieth-century historical data. *American Economic Review*, 103(3):129–134.

Petrongolo, B. and Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, 39(2):390–431.

Pizzinelli, C., Theodoridis, K., and Zanetti, F. (2020). State dependence in labor market fluctuations. *International Economic Review*, 16(3):1027–1072.

Schaal, E. and Taschereau-Dumouchel, M. (2018). Coordinating business cycles. Mimeo.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.

Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3):586–606.

Sterk, V. (2016). The dark corners of the labor market. Discussion Papers 1603, Centre for Macroeconomics.

Stevenson, W. J. (2018). *Operations Management*. McGraw-Hill/Irwin, 13th edition.

Thomas, C. and Zanetti, F. (2009). Labor market reform and price stability: An application to the Euro Area. *Journal of Monetary Economics*, 56(6):885–899.

Van Huyck, J. B., Battalio, R. C., and Beil, R. O. (1990). Tacit coordination games, strategic uncertainty, and coordination failure. *American Economic Review*, 80(1):234–248.

Van Huyck, J. B., Battalio, R. C., and Beil, R. O. (1991). Strategic uncertainty, equilibrium selection, and coordination failure in average opinion games. *Quarterly Journal of Economics*, 106(3):885–910.

Weitzman, M. (1982). Increasing returns and the foundations of unemployment theory. *Economic Journal*, 92(368):787–804.