An Elementary Theory of Directed Technical Change and Wage Inequality*

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This paper generalizes central results from the theory of (endogenously) directed technical change to settings where technology does not take a labor-augmenting form and with arbitrarily many levels of skill. Building on simple notions of complementarity, the results remain intuitive despite their generality. The developed theory allows to study the endogenous determination of labor-replacing, that is, automation technology through the lens of directed technical change theory. In an assignment model with a continuum of differentially skilled workers and capital, where capital perfectly substitutes for labor in the production of tasks, any increase in the relative supply of skilled workers stimulates investment into improving the productivity of capital, potentially leading skill premia to increase in relative skill supply. Relatedly, trade with a skill-scarce country discourages improvements in capital productivity, potentially reversing the standard Heckscher-Ohlin effects.

JEL: J24, J31, O33, **Keywords**: Directed Technical Change, Endogenous Technical Change, Wage Inequality, Automation, Assignment Model, Monotone Comparative Statics.

1. Introduction

Since the 1980s, many advanced economies have witnessed substantial increases in wage inequality between groups of workers with different levels of educational attainment. A broad empirical literature attributes parts of this increase to skill-biased technical change.[1] Appealing to skill-biased technical change as an exogenous explanation for the observed changes

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[1]See Bound and Johnson (1992), Katz and Murphy (1992), and Goldin and Katz (2008) on skill-biased technical change in general, and Graetz and Michaels (2018), Acemoglu and Restrepo (2020), and Dauth, Findeisen, Suedekum and Woessner (2019) on the effects of automation technology in particular.

in the wage structure, however, is not entirely satisfactory. After all, the technologies that are used in an economy are eventually chosen by economic agents, about whose decisions economics should have something to say. This is the starting point for the theory of directed technical change (see Acemoglu, 1998; Kiley, 1999). Central results of the theory predict how the skill bias of technical change depends on the supply of skills firms face on the labor market. In particular, they provide conditions under which (i) there is *weak relative equilibrium bias of technology* (weak relative bias, henceforth), meaning that any increase in the relative supply of skill induces skill-biased technical change, and (ii) there is *strong relative equilibrium bias of technology* (strong relative bias, henceforth), meaning that the positive effect of the induced technical change on the skill premium dominates the (typically negative) direct effect, such that the skill premium increases in relative skill supply (e.g. Acemoglu, 2002, 2007). With the notable exception of Acemoglu (2007) (discussed below), these conditions are limited to settings in which aggregate production takes the specific form $G(\theta_1 L_1, \theta_2 L_2) - C(\theta_1, \theta_2)$, where L_1 and L_2 denote the supply of skilled and unskilled labor, and θ_1 and θ_2 represent the endogenous, differentially labor-augmenting technology.

At the same time, the most recent literature on the effects of technical change on wage inequality analyzes labor-replacing (that is, automation) technology, typically in assignment models with labor and capital where capital perfectly substitutes for labor in the production of tasks (e.g. Acemoglu and Autor, 2011; Autor and Dorn, 2013; Acemoglu and Restrepo, 2018a; Feng and Graetz, 2020; Aghion, Jones and Jones, 2019). In these models, the relevant technology variables can in general not be represented as labor-augmenting technology, such that they are outside the scope of the main results on directed technical change described above.

This paper generalizes the central results from directed technical change theory on weak and strong relative bias beyond the special case of purely labor-augmenting technology and thereby makes them applicable to automation technology in Roy-like assignment models.[2] The first part of the paper derives general conditions for weak and strong relative bias that are independent of any functional form restriction, drawing on techniques from the theory of monotone comparative statics (Topkis, 1978, 1998; Milgrom and Shannon, 1994). Besides making directed technical change theory applicable to automation technology, the results clarify the general mechanisms, based on simple notions of complementarity, that underlie weak and strong relative bias. The second part applies these results to obtain novel insights about the endogenous determination of automation technology in a Roy-like assignment model.

The first part starts form a reduced-form characterization of wages and equilibrium technology that is shown to arise from a range of different microfoundations of endogenous technical change, including standard approaches from endogenous growth theory. Building on this reduced-form characterization, conditions are identified under which there is weak relative

[2]At first glance, it may seem that Uzawa's theorem provides a justification for the restriction to labor-augmenting technology. But Uzawa's theorem only applies to the component of technology that grows over time on a balanced growth path, whereas the literature on directed technical change has mainly been concerned with the component of technology that is stationary on a balanced growth path, inducing changes in the stationary long-run distribution of (relative) wages. Moreover, with the labor share and the (risk-free) real interest rate declining over several decades (e.g. Karabarbounis and Neiman, 2014; Caballero, Farhi and Gourinchas, 2017), the general desirability for a model to generate balanced growth is no longer obvious.

bias, that is, any increase in relative skill supply induces skill-biased technical change. The only essential condition is that the skill bias of technology is scale invariant, in the sense that a proportional change in the supply of all skill levels does not induce biased technical change. This condition covers the most general setting from existing results, which features a production function of the form $G(\theta_1 L_1, \theta_2 L_2) - C(\theta_1, \theta_2)$, characterized by purely labor-augmenting technology and an additive cost of technology development (Acemoglu, 2007). Within the class of settings with such an additive cost structure, my condition allows to complement labor-augmenting technology by technologies of other forms – resulting in production functions of the form $G(\theta_1 L_1, \theta_2 L_2, \theta_3) - C(\theta_1, \theta_2, \theta_3)$ with $\theta_3 \in \mathbb{R}^M$ – as long as G and C satisfy certain homotheticity-like properties. Perhaps more importantly, when moving beyond settings with an additive technology cost, the condition allows to dispense with labor-augmenting technology completely. Examples for such settings include those with a multiplicative technology cost, $G(L, \theta)(1 - C(\theta))$, and settings where cost is captured by a restriction of technology to a bounded set Θ; the latter occurs naturally in cases of pure technology adoption or when technology development is constrained by factors that are in fixed supply.[3]

While an increase in the relative supply of skill tends to induce skill-biased technical change, it also has a direct effect on the wage distribution, which typically depresses skill premia. The second set of results provides necessary and sufficient conditions for the occurrence of strong relative bias, meaning that the effect of the induced technical change dominates the direct effect such that skill premia increase with relative skill supply. I show that the induced technical change effect dominates everywhere if and only if the aggregate production function is quasiconvex. Reversely, if and only if aggregate production is quasiconcave, the direct effect dominates everywhere. These conditions provide an interesting analogy to endogenous growth theory, where convexity of aggregate production along rays through the origin (that is, increasing returns to scale) is required to generate persistent growth in a wide class of models (cf. Romer, 1986). As in these models, the aggregate (quasi-)convexity requirement discovered here has implications for the market structures needed in a model to analyze the case where skill premia increase in relative skill supply. In particular, either deviations from perfect competition or external effects between firms' technologies are needed.

While my baseline results apply to local changes (in the sense of differential calculus) in two different labor inputs, I show that they transfer in natural ways to global changes in labor supply and to settings with arbitrarily many different types of labor.

I demonstrate how to apply my directed technical change results by analyzing the endogenous determination of automation technology in the Roy-like assignment model proposed by Teulings (1995) (see Costinot and Vogel, 2010 for decisive progress in comparative statics for this model), augmented to incorporate capital as an additional production factor as in Acemoglu and Autor (2011) or Feng and Graetz (2020). In the model, a continuum of differentially skilled workers and capital, taking the form of machines that perfectly substitute for labor in the production of tasks, are assigned to a continuum of tasks, which in turn are combined to

[3]The results in the first part of the paper imply a LeChatelier Principle for relative demand curves, analogous to the conventional LeChatelier Principle that applies to absolute demand curves (e.g. Milgrom and Roberts, 1996). For an explicit formulation of the implied LeChatelier Principle for relative demand see Loebbing (2016), an earlier version of the present paper.

produce a single final good. In line with recent forecasts on the future automation potential for different tasks (e.g. Frey and Osborne, 2017; Arntz, Gregory and Zierahn, 2016), capital is assumed to have comparative advantage in less complex tasks than labor, such that any increase in the set of tasks performed by capital (automation) displaces low-skilled workers from some of their previous tasks.[4]

I endogenize the productivity of capital in the model by allowing firms to choose capital productivity and the productivity by which they transform tasks into final goods subject to a technology frontier. While this specification of technology does not have the labor-augmenting form required by previous results, it is covered by the generalized directed technical change results from the first part of the paper. Using the results on weak relative bias, I find that any increase in relative skill supply induces an improvement in capital productivity, which in turn raises skill premia.

In the baseline model, where firms' technology choices depend on each other only via (competitive) market prices, aggregate production is quasiconcave. Applying my directed technical change results, this immediately implies that there cannot be strong relative bias, that is, the induced improvement in capital productivity cannot be strong enough to make skill premia increase in relative skill supply. This result is overturned once I allow for external effects between firms' technology choices.[5]

Finally, adopting an insight from Costinot and Vogel (2010), I show that, in the assignment model, the introduction of trade between two countries can be analyzed by the same tools as a change in labor supply. My results can thus be readily applied to study the impact of trade on automation technology. I find that trade with a skill-scarce country reduces capital productivity in a skill-abundant country, leading to a fall in skill premia. This counteracts the usual Heckscher-Ohlin effects that would determine the impact of trade on skill premia absent endogenous technology. Under strong relative bias, the novel directed technical change effect dominates and trade causes skill premia in the skill-abundant country to fall after adjustment of technology.

The remainder of the paper is structured as follows. Section 2 introduces the reduced-form characterization of wages and equilibrium technology that provides the basis for the general results on directed technical change in the following sections. Section 3 presents these results for local labor supply changes and two different levels of skill. Sections 4 and 5 generalize them to global labor supply changes and arbitrarily many levels of skill. Section 6 applies the results to endogenous automation technology in assignment models, and Section 7 concludes.

Related Literature The paper is related to several strands of the existing literature. The first part of the paper extends the literature on directed technical change and wage inequality (e.g. Acemoglu, 1998, 2002 and Kiley, 1999), generalizing the key theoretical results of that literature. Most closely related is Acemoglu (2007), who provides a directed technical change

[4]This assumption is also broadly supported by recent estimates of the impact of industrial robots (Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020) and a wider set of automation technologies in US manufacturing (Lewis, 2011) on the structure of employment and wages.

[5]The same effect can be achieved in a model where technology, embodied in intermediate inputs, is supplied by a monopolistically competitive R&D sector, following the monopolistic competition approach in endogenous growth theory (see Online Appendix G.2).

analysis on a similar level of generality. In contrast to the present paper, Acemoglu (2007) analyzes the effects of technical change induced by changes in the supply of a given skill level on the absolute wage of that skill, rather than on relative wages between different skills. From a purely theoretical perspective, the first part of the present paper can thus be viewed as the completion of a general theory of the effects of skill supply on the direction of technical change, with the first part on absolute wages given by Acemoglu (2007) and the second part on relative wages presented here. The analysis of relative wages is indispensable when the goal is to study implications of endogenous technical change for wage inequality.

The second part of the paper bridges the gap between the literature on directed technical change and the more recent strand of work on technical change and wage inequality in Roy-like assignment models. Costinot and Vogel (2010), Acemoglu and Autor (2011), and Feng and Graetz (2020) analyze the impact of exogenous technical change on wage inequality in such models. More closely related are Acemoglu and Restrepo (2018a), Hémous and Olsen (2020), and Acemoglu and Restrepo (2019). Acemoglu and Restrepo (2018a) and Hémous and Olsen (2020) study the endogenous evolution of automation technology, but focus on its response to exogenous technology shocks rather than to changes in the structure of labor supply and international trade. Acemoglu and Restrepo (2019) analyze the effects of the demographic structure on endogenous automation, with the focus on the effects of automation on productivity and the labor share rather than on wage inequality.

The analysis of trade and automation in Section 6.4 is related to existing work on the effects of trade and technology on wage inequality. Part of this literature employs the assignment framework used here as well, but considers settings with exogenous technology (see Costinot and Vogel, 2015 for a survey of the use of assignment models in international trade).[6] Analyses of trade and wage inequality with endogenous technology are presented by Acemoglu (2003), Yeaple (2005), and Sampson (2014), but none of them uses the assignment model with labor-replacing capital employed here. All three papers find that trade (with a skill-scarce country) induces skill-biased technical change, which is opposite to my result. I discuss the differences between their approaches and my analysis in more detail at the end of Section 6.4.

2. A Simple Framework for Directed Technical Change

I analyze directed technical change in general equilibrium models with endogenous technology, exogenous labor supply, and a single consumption good. Acemoglu (2007) shows that, in a large class of such models, wages and technology are characterized by the same set of equations in equilibrium. Hence, instead of providing a complete, but necessarily specific, microfoundation of technology choices, I directly work with this general set of equilibrium conditions. This allows me to treat a diverse set of models within a simple, unifying framework. For microfounded models fitting into this framework, see Acemoglu (2007) and Online

[6]Closely related is parallel work by Krenz, Prettner and Strulik (2018) who provide a joint analysis of offshoring and automation in an assignment model with capital that perfectly substitutes for labor in task production. Their model, however, does not feature endogenous capital productivity and hence cannot generate strong relative bias and the ensuing reversal of the Heckscher-Ohlin effects.

Appendix F.[7]

Let L denote the exogenous labor supply in the economy. In the main text, there is a finite number N of different skill types, such that $L = (L_1, L_2, ..., L_N) \in \mathbb{R}^N_{++}$, where L_s is the supply level for a given skill type $s \in S = \{1, 2, ..., N\}$. All results go through with a continuum of skills, but some clarifications are needed regarding the meaning of differentiation and partial derivatives in this case. Hence, I treat the continuum case separately in Appendix B.

Throughout the paper, I will call a higher index s a higher level of skill, and a type with a higher s a more skilled type of labor. While this suggests that wages should increase in s, none of my results requires a particular ordering of wages or that the ordering of wages remains constant when labor supply changes.

Production is described by the real-valued aggregate production function $F(L, \theta)$, where $\theta \in \Theta$ represents the (endogenous) technology and Θ denotes the set of feasible technologies. I impose the following assumptions throughout the analysis.

Assumption 1. *The set of feasible technologies Θ is a compact topological space. The aggregate production function F is continuous in technology θ and continuously differentiable in labor supply L. The gradient $\nabla_L F(L, \theta)$ is strictly positive everywhere.*

Compactness of Θ and continuity of F in θ ensure that there always exists a technology that maximizes aggregate production. Existence and strict positiveness of the marginal products of labor ensure that wages and wage ratios are always determined in equilibrium. Finally, the restriction that marginal products of labor are continuous is used exclusively in the proofs of Theorems 2 and 2′, and thus irrelevant for many of my results.[8]

Assumption 1 imposes hardly any restrictions on the set of feasible technologies Θ, supporting a general notion of technology. A technology θ can, for example, represent the allocation of production factors to tasks in an economy (see the example in Online Appendix G); a particular production technique that can be adopted by firms at no cost (see the example in Section 6); or the distribution of costly investment into the quality of a range of intermediate inputs, each embodying a particular technology (see the example in Online Appendix G.2).

Since my results contrast situations where technology adjusts to changes in labor supply with situations where technology is fixed, it is useful to distinguish between an exogenous-technology equilibrium and an endogenous-technology equilibrium. In an exogenous-technology equilibrium, wages are determined by

$$w(L, \theta) = \nabla_L F(L, \theta) \,, \tag{1}$$

while technology θ is exogenous.

[7] All models in Acemoglu (2007) and Online Appendix F are static, but there is an equivalence between the equilibria of the static models and the constant growth paths of corresponding dynamic model versions. In particular, the models in Online Appendix F can be extended to dynamic versions, which generate constant growth paths with stationary relative wages between skill groups. These relative wages are identical to the relative wages that prevail in equilibrium of the static models. My comparative statics results thus also apply on the constant growth paths of appropriately specified dynamic models. See Loebbing (2016, Section 3.2 and Appendix B) for details.

[8] It guarantees that the endogenous-technology production function $F^*(L)$ (see below) is absolutely continuous, which in turn allows me to apply an envelope theorem by Milgrom and Segal (2002).

In an endogenous-technology equilibrium, technology is determined according to

$$\theta^*(L) \in \underset{\theta \in \Theta}{\operatorname{argmax}} F(L, \theta) \,, \tag{2}$$

while wages again satisfy (1), but with the equilibrium technology $\theta^*(L)$ in place of θ. It is useful to introduce the notation

$$w^*(L) := w(L, \theta^*(L))$$

for wages in endogenous-technology equilibrium. Similarly, let

$$F^*(L) := F(L, \theta^*(L))$$

denote aggregate production in endogenous-technology equilibrium. As mentioned above, microfoundations that give rise to these equilibrium conditions are presented in Acemoglu (2007) and Online Appendix F.

The equilibrium technology $\theta^*(L)$ as given by (2) may not be unique. In this case, I assume that $\theta^*(L)$ is some point-valued selection from the set of maximizers in (2). Whenever possible (i.e., whenever an order is defined and the maximizer set has a supremum), the selection should be made from the supremum of the maximizer set. Otherwise, it can be arbitrary. The results could also be derived in terms of sets of relative wages induced by sets of equilibrium technologies, but this would complicate the exposition without much apparent gain.

Before delving into the analysis, it is instructive to briefly review the most general existing results on directed technical change and wage inequality in the present environment. These results are structured along two hypotheses – weak and strong relative equilibrium bias of technology (Acemoglu, 2002, 2007) – and provide conditions for each of the two hypotheses to be true.

The results are restricted to a setting with two skill levels, $N = 2$, and local changes in labor supply. The weak relative bias hypothesis states that any increase in relative skill supply induces skill-biased technical change. Acemoglu (2007) shows that the hypothesis is true if technology takes a purely labor-augmenting form. My conditions for weak relative bias will cover this as a special case and thus imply the following result as a corollary.

Corollary 1 (Local Weak Bias with Two Skills and Labor-Augmenting Technology, Acemoglu 2007, Theorem 1)**.** *Suppose $N = 2$, $\Theta \subset \mathbb{R}^2_+$ and F takes the form*

$$F(L, \theta) = G(\theta_1 L_1, \theta_2 L_2) - C(\theta) \,,$$

with G twice continuously differentiable, concave, and homothetic, and C twice continuously differentiable, strictly convex, and homothetic.

Then, for any initial labor supply L and any change ΔL such that $\Delta L_2 / L_2 \geq \Delta L_1 / L_1$ (an increase in relative skill supply), the induced technical change $\Delta\theta^* = \nabla_L \theta^*(L) \Delta L$ locally raises the skill premium:

$$\nabla_\theta \frac{w_2(L, \theta^*(L))}{w_1(L, \theta^*(L))} \Delta\theta^* \geq 0 \,.$$

The important restriction here is that technology is purely labor-augmenting and constrained by an additive cost $C(\theta)$. In Section 3, I present a condition for weak relative bias that covers this setup as a special case. When maintaining the focus on settings with an additive cost of technology, my condition allows to augment the setting by technologies that are not labor-augmenting, giving rise to production functions of the form $G(\theta_1 L_1, \theta_2 L_2, \theta_3) - C(\theta)$. When dispensing with the additive cost structure – and instead considering settings where technology is constrained, for example, by a multiplicative cost or only by the boundaries of the set Θ – the labor-augmenting technology can be dispensed with completely.

While an increase in the relative supply of skilled labor induces skill-biased technical change, it also has a direct effect on the skill premium. If aggregate production is concave in labor, as is typically the case at a point of equilibrium, this direct effect is negative. It is thus natural to ask whether the negative direct effect or the positive induced technical change effect dominates; that is, whether, after adjustment of technology, the skill premium decreases or increases in relative skill supply.

The strong relative bias hypothesis states that the directed technical change effect dominates and the skill premium increases in relative skill supply. For labor-augmenting technologies, Acemoglu (2007) shows that this is the case whenever the elasticity of substitution between the two skill types is large enough.

Corollary 2 (Local Strong Bias with Two Skills and Labor-Augmenting Technology, Acemoglu 2007, Theorem 1)**.** *Suppose $N = 2$, $\Theta = \mathbb{R}^2_{++}$ and F takes the form*

$$F(L,\theta) = G(\theta_1 L_1, \theta_2 L_2) - C(\theta) ,$$

with G twice continuously differentiable, concave, and homothetic, and C twice continuously differentiable, strictly convex, and homothetic.

Denote the local elasticity of substitution between the two labor inputs by

$$\sigma(L,\theta) := -\frac{\partial \log (L_2/L_1)}{\partial \log (F_{L_2}/F_{L_1})}$$

and the local elasticity of substitution between the two technology variables in the cost function C by

$$\delta(L,\theta) := \frac{\partial \log (\theta_2/\theta_1)}{\partial \log (C_{\theta_2}/C_{\theta_1})} .$$

Then, for any initial labor supply L and any change ΔL such that $\Delta L_2/L_2 \geq \Delta L_1/L_1$, the skill premium locally increases after adjustment of technology if and only if the elasticities of substitution are large enough, that is,

$$\nabla_L \frac{w_2^*(L)}{w_1^*(L)} \Delta L \geq 0$$

if and only if $\sigma(L, \theta^*(L)) - 1/\delta(L, \theta^*(L)) \geq 2$.

In Section 3, I provide a generalized condition for strong relative bias that applies beyond the case of labor-augmenting technologies and additive technology cost. When applied to that special case, of course, the generalized condition is equivalent to the condition in Corollary 2.

3. Directed Technical Change with Local Labor Supply Changes and Two Skill Levels

To facilitate the comparison with existing results, I develop my results at first for two skill levels and local labor supply changes, the case treated by Corollaries 1 and 2. In Section 4, I extend the results to global changes in labor supply and in Section 5 I provide extensions to settings with many skill levels.

3.1. Weak Relative Bias

My results show that the following scale-invariance condition on the skill bias of the equilibrium technology is sufficient for weak relative bias.

Assumption 2. *For any labor supply L and any $\lambda \in \mathbb{R}_{++}$,*

$$\frac{F_{L_2}(L, \theta^*(L))}{F_{L_1}(L, \theta^*(L))} = \frac{F_{L_2}(L, \theta^*(\lambda L))}{F_{L_1}(L, \theta^*(\lambda L))} .$$

In words, Assumption 2 says that a proportional change in labor supply levels does not induce biased technical change (i.e., technical change that alters the skill premium). I discuss in detail below which restrictions Assumption 2 imposes on the form of aggregate production F. At first, however, I show that Assumption 2 indeed guarantees weak relative bias.

Proposition 1 (Local Weak Bias with Two Skills)**.** *Suppose that $N = 2$, $\Theta \subset \mathbb{R}^M$ (for arbitrary $M \in \mathbb{N}$), F is twice continuously differentiable, and the equilibrium technology θ^* is differentiable in labor supply L. Moreover, suppose that Assumption 2 holds.*

Then, for any initial labor supply L and any change in labor supply ΔL with $\Delta L_2/L_2 \geq \Delta L_1/L_1$, the induced technical change $\Delta\theta^* = \nabla_L \theta^*(L)\Delta L$ locally raises the skill premium:

$$\nabla_\theta \frac{w_2(L, \theta^*(L))}{w_1(L, \theta^*(L))} \Delta\theta^* \geq 0 .$$

Proof. The proof (i) shows that Assumption 2 allows to focus on labor supply changes in direction of the isoquant, and (ii) applies an envelope argument along the isoquant to derive the desired result.

For the first step, note that we can decompose the labor supply change ΔL into a proportional component and a component that points along the isoquant. In particular, we write $\Delta L = \widetilde{\Delta L} + \lambda L$ for some $\lambda \in \mathbb{R}$. Since output is strictly increasing in labor, we can always find a λ such that the remainder $\widetilde{\Delta L}$ leaves output unchanged, that is, $\nabla_L F(L, \theta^*(L))\widetilde{\Delta L} = 0$. Since ΔL is an increase in relative skill supply, this implies

$$F_{L_2}(L, \theta^*(L))\widetilde{\Delta L}_2 = -F_{L_1}(L, \theta^*(L))\widetilde{\Delta L}_1 \geq 0 . \tag{3}$$

The proportional component of ΔL, λL, induces technical change that leaves the skill premium unchanged by Assumption 2. Thus, the induced technical change effect on the skill premium will be the same for the change ΔL as for its component $\widetilde{\Delta L}$, so we can restrict attention to $\widetilde{\Delta L}$ in the following.

Consider now the second-order effects of $\widetilde{\Delta L}$ on output with and without technology adjustment. With technology adjustment, the first-order effect is, by the envelope theorem,

$$\frac{d}{d\mu} F^*(L + \mu\widetilde{\Delta L})\bigg|_{\mu=0} \equiv \nabla_L F(L, \theta^*(L))\widetilde{\Delta L} \, .$$

Hence, we obtain the second-order effect as

$$\frac{d^2}{(d\mu)^2} F^*(L + \mu\widetilde{\Delta L})\bigg|_{\mu=0} = \widetilde{\Delta L}^T \nabla^2_{LL} F(L, \theta^*(L))\widetilde{\Delta L} + \widetilde{\Delta\theta}^{*T} \nabla^2_{L\theta} F(L, \theta^*(L))\widetilde{\Delta L} \, ,$$

where $\widetilde{\Delta\theta}^* = \nabla_L \theta^*(L)\widetilde{\Delta L}$. Without technology adjustment, we obtain

$$\frac{d^2}{(d\mu)^2} F(L + \mu\widetilde{\Delta L}, \theta^*(L))\bigg|_{\mu=0} = \widetilde{\Delta L}^T \nabla^2_{LL} F(L, \theta^*(L))\widetilde{\Delta L} \, .$$

The two functions $F^*(.)$ and $F(., \theta^*(L))$ coincide at L but F^* is greater everywhere else. So, F^* must be less concave than F at L (e.g. Dixit, 1990, pp. 113–114) and, hence,

$$\widetilde{\Delta\theta}^{*T} \nabla^2_{L\theta} F(L, \theta^*(L))\widetilde{\Delta L} \geq 0 \, .$$

This situation is illustrated in Figure 1. Rearranging the cross-derivative, we obtain

$$\nabla_\theta \left(\nabla_L F(L, \theta^*(L))\widetilde{\Delta L} \right) \widetilde{\Delta\theta}^* \geq 0 \, . \tag{4}$$

Equation (4) is the key to my results on weak relative bias. It states that technology will always adjust in a way that increases the output gains from the initial change in labor supply. Put differently, the induced technical change will always be complementary to the labor supply change. Extensions of equation (4) will also appear in the generalizations in Sections 4 and 5.

It is now easy to see that, for an increase in relative skill supply, the complementarity logic of equation (4) implies that the induced technical change must raise the skill premium. In particular, rearrange equation (4),

$$\nabla_\theta F_{L_2}(L, \theta^*(L))\widetilde{\Delta L}_2 \widetilde{\Delta\theta}^* \geq -\nabla_\theta F_{L_1}(L, \theta^*(L))\widetilde{\Delta L}_1 \widetilde{\Delta\theta}^* \, ,$$

and divide this by equation (3), assuming $\widetilde{\Delta L} \neq 0$ (for $\widetilde{\Delta L} = 0$ the proposition is trivially true), to obtain

$$\frac{\nabla_\theta F_{L_2}(L, \theta^*(L))\widetilde{\Delta\theta}^*}{F_{L_2}(L, \theta^*(L))} \geq \frac{\nabla_\theta F_{L_1}(L, \theta^*(L))\widetilde{\Delta\theta}^*}{F_{L_1}(L, \theta^*(L))} \, ,$$

which implies

$$\nabla_\theta \frac{F_{L_2}(L, \theta^*(L))}{F_{L_1}(L, \theta^*(L))} \widetilde{\Delta\theta}^* \geq 0 \, .$$

□

By establishing weak relative bias, Proposition 1 essentially puts forward a Le Chatelier principle for relative, instead of absolute, demand curves. The original Le Chatelier principle as proposed by Samuelson (1947) states that long-run demand (when all inputs can adjust) is

Weak Relative Bias

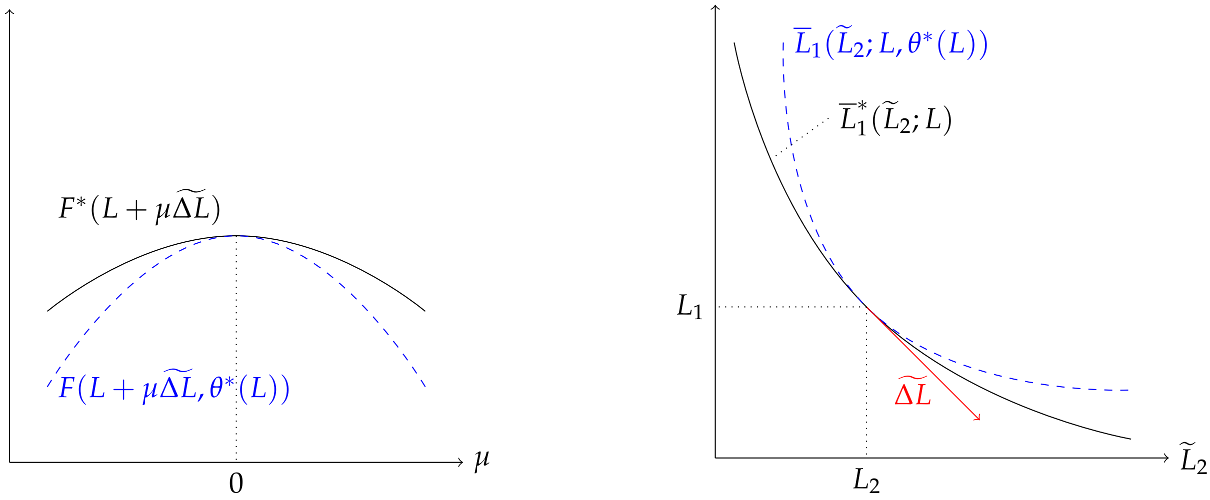

Figure 1. Illustration of the proof of Proposition 1. The left panel shows output along the line $L + \mu\widetilde{\Delta L}$, which passes through the initial labor supply L (at $\mu = 0$) and is tangent to the isoquant at L. The marginal output gain when moving away from L declines more slowly if technology adjusts (the solid curve) than if it is fixed (the dashed curve), leading to weak relative bias. The right panel shows the isoquants through L when technology adjusts (solid curve) and when it is fixed (dashed curve). When moving in direction of $\widetilde{\Delta L}$, the slope of the isoquant increases more slowly when technology adjusts, which provides an alternative way to prove weak relative bias.

more elastic than short-run demand (when some inputs are fixed). Transferred to a general equilibrium environment with inelastic labor supply, this implies that an increase in some type of labor reduces its (absolute) wage by more in the short run than in the long run, that is, the long-run adjustment of inputs raises the wage (see the absolute bias results in Acemoglu, 2007). Proposition 1 now establishes this result for the relative supply and the relative wage between two different types of labor.

The proof uses the same type of envelope argument that can also be used to prove the original Le Chatelier principle (e.g. Dixit, 1990, pp. 115–116), but applies it along the isoquant instead of in direction of a single input factor.

An alternative perspective on the proof is provided in the right panel of Figure 1. If technology adjusts, the relative wage is given by the negative of the slope of the isoquant of $F^*(.)$ through L (the solid curve); if technology is fixed, the relative wage is the negative of the slope of the isoquant of $F(., \theta^*(L))$ through L (the dashed curve). The endogenous-technology isoquant (the one of F^*), however, is the lower envelope of a family of fixed-technology isoquants, including that of $F(., \theta^*(L))$. Hence, at the point of tangency L, the endogenous-technology isoquant is less convex than the fixed-technology isoquant. So, when moving from L in direction of L_2 along the isoquants, the skill premium falls by more when technology is fixed (i.e., along the fixed-technology isoquant) than when it is allowed to adjust (along the endogenous-technology isoquant). It follows that the adjustment of technology must raise the skill premium.

The intuition behind weak relative bias is readily derived from the proof and especially the key equation (4). An increase in relative skill supply raises the return to complementary changes in technology. Since complementarity is a symmetric relationship, such changes in technology in turn raise the marginal output gain from increasing relative skill supply, which is given by the skill premium.

While Proposition 1 establishes that Assumption 2 is sufficient for weak relative bias, the assumption is clearly also necessary if we look for a result applying to increases in relative skill supply in all directions. In particular, if there is a proportional change in labor supply inducing strictly skill-biased technical change (i.e., Assumption 2 is violated), we can always find both an increase in relative skill supply that induces skill-biased technical change (sufficiently close to the proportional change) and one that induces the opposite type of technical change (sufficiently close to the negative of the proportional change). Even in such cases, however, the second part of the proof of Proposition 1 remains valid and we still have weak relative bias for changes in labor supply along the isoquant.

Under what conditions on aggregate production F is Assumption 2 satisfied? A simple condition is that F is homogeneous in L. In this case, it is easy to check that the equilibrium technology $\theta^*(L)$ is homogeneous of degree zero, which clearly satisfies Assumption 2.[9]

Homogeneity of F in L seems natural in settings where technology development comes at a multiplicative cost such that aggregate production takes the form $F(L, \theta) = G(L, \theta)(1 - C(\theta))$. It also seems like a weak restriction in settings where technology is restrained only by the boundaries of the set of feasible technologies Θ. This case arises naturally in settings of pure technology adoption or when technology development is constrained by some factor that is in fixed supply; more generally, it captures any kind of situation where an inelastically supplied resource is allocated optimally. Examples for such settings are provided in Section 6 and in Online Appendix G.

Yet, homogeneity of F in L is largely incompatible with the additive cost structure considered in Corollaries 1 and 2. In particular, if aggregate production takes the form $G(L, \theta) - C(\theta)$, homogeneity of G in L does not yield homogeneity of aggregate production F in L. Without homogeneity in L, however, technology will generally change in response to proportional changes in labor supply. Assumption 2 then requires that these changes in technology have no effect on the skill premium. As a first step, note that this requirement is satisfied for the labor-augmenting technology in Corollaries 1 and 2. There, a proportional change in labor inputs leads to a proportional change in the technologies θ_1 and θ_2 (due to the homotheticity restrictions on G and C), which in turn leaves the skill premium unaffected. So, the setup of previous results in covered by Assumption 2. But, even within the class of settings with additive technology cost, Assumption 2 can accommodate more general forms of technology. In particular, suppose that we complement the labor-augmenting technology (θ_1, θ_2) by another technology $\theta_3 \in \mathbb{R}^{M-2}$ to arrive at a production function of the form $G(\theta_1 L_1, \theta_2 L_2, \theta_3) - C(\theta_1, \theta_2, \theta_3)$. Assuming that G and C are homogeneous in the first two arguments, one can show that a proportional change in labor inputs causes a proportional change in the labor-augmenting technology (θ_1, θ_2) while leaving technology θ_3 unaffected; this in turn leaves the skill premium unchanged.[10] Hence, intuitively, the labor-augmenting technology absorbs the effects

[9] In fact, for θ^* to be homogeneous of degree zero, it is sufficient to have a production function of the form $F(L, \theta) = f(g(L, \theta), L)$ for some real-valued function g that is linear homogeneous in L and a function f that is strictly increasing in its first argument. See Online Appendix D for a proof.

[10] This holds if we impose strict concavity of $F(L, \theta)$ in θ, such that first-order conditions uniquely identify the

of proportional changes in labor inputs, whereas the more general part θ_3 only responds to changes in relative labor inputs.

As a concrete example, consider the case of a CES production function where not only the labor-augmenting terms but also the elasticity of substitution itself are endogenous:

$$F(L, \theta_1, \theta_2, \theta_3) = \left[(\theta_1 L_1)^{\theta_3} + (\theta_2 L_2)^{\theta_3}\right]^{\frac{1}{\theta_3}} - C(\theta_1, \theta_2, \theta_3)$$

with $(\theta_1, \theta_2) \in \mathbb{R}^2_+$ and $\theta_3 \leq 1$. If the cost function C is homogeneous of degree $k > 1$ in (θ_1, θ_2) (and F is strictly concave in θ), a change in labor inputs by the factor λ results in a change in $(\theta_1^*(L), \theta_2^*(L))$ by the factor $\lambda^{1/(k-1)}$ while $\theta_3^*(L)$ is unaffected. This leaves the skill premium unchanged and, thereby, satisfies Assumption 2.[11]

To summarize, Assumption 2 covers settings where aggregate production is homogeneous in labor, which seems most applicable to settings with a multiplicative cost of technology or an inelastic supply of inputs into technology production. In these cases, no restrictions on the form in which technology enters the production function are needed (beyond those implied by homogeneity of production in labor). Moreover, Assumption 2 is satisfied in settings with an additive cost of technology as long as production includes labor-augmenting technologies and satisfies certain homotheticity-like restrictions. Importantly, besides labor-augmenting technologies, these settings can include other technologies of arbitrary forms.

3.2. Strong Relative Bias

The strong bias condition from existing work (Corollary 2) is restricted to environments with purely labor-augmenting technology. Since weak relative bias holds much more generally, one might expect that also the strong bias condition has an insightful generalization. I provide such a generalization in the following.

The only restriction needed is that aggregate production is homothetic when accounting for the endogenous adjustment of technology.

Assumption 3. *The endogenous-technology production function F^* is homothetic.*

Assumption 3 holds under essentially the same restrictions on aggregate production as Assumption 2. In particular, all the settings discussed above in which Assumption 2 holds satisfy Assumption 3 as well.

Moreover, just like Assumption 2 for weak relative bias, Assumption 3 is indispensable if we seek a condition for strong bias that pertains to all increases or decreases in relative skill supply instead of only to those pointing in a certain direction. If Assumption 3 is violated, we can always find both an increase in relative skill supply that raises the skill premium after adjustment of technology and one that lowers it.[12]

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equilibrium technology. If this is the case, it is in fact sufficient for Assumption 2 that G and C satisfy the following homotheticity-like conditions: G can be written as $f(g(\theta_1 L_1, \theta_2 L_2, \theta_3))$ for some real-valued function g that is linear homogeneous in the first two arguments and a strictly increasing function f; and C can be written analogously. See Online Appendix D for the proof.

[11] See again Online Appendix D for a proof.

[12] Without Assumption 3, Proposition 2 below would still hold for changes in relative skill supply that point along the isoquant of F^*.

If Assumption 3 holds, however, all increases in relative skill supply have the same effect on the skill premium, irrespective of their direction in the L_1-L_2-plane. I show in the following that this effect is positive – that is, there is strong relative bias – if and only if the aggregate production function is locally quasiconvex.[13]

Definition 1. A function $g : \mathbb{R}^2 \to \mathbb{R}$, $(x, y) \mapsto g(x, y)$, is locally quasiconvex at (x, y) if and only if $d^2\overline{x}(\tilde{y}; x, y)/(d\tilde{y})^2|_{\tilde{y}=y} \leq 0$, where $\overline{x}(.; x, y) : Y \to \mathbb{R}$ is the isoquant given by $g(\overline{x}(\tilde{y}; x, y), \tilde{y}) = g(x, y)$ for all $\tilde{y} \in Y$ and some suitably defined domain Y.

In words, a function is locally quasiconvex if and only if its isoquant is locally concave in the sense of having a negative second derivative.

Proposition 2 (Local Strong Bias with Two Skills)**.** *Suppose that $N = 2$, $\Theta \subset \mathbb{R}^M$ (for arbitrary $M \in \mathbb{N}$), F is twice continuously differentiable, and the equilibrium technology θ^* is differentiable in labor supply L. Moreover, suppose that Assumption 3 holds.*

Then, for any initial labor supply L and any change in labor supply ΔL with $\Delta L_2/L_2 \geq \Delta L_1/L_1$, the skill premium locally increases after adjustment of technology,

$$\nabla_L \frac{w_2^*(L)}{w_1^*(L)} \Delta L \geq 0 \,,$$

if and only if the aggregate production function F^* is locally quasiconvex at L.

Proof. As in the proof of Proposition 1, we can decompose any increase in relative skill supply ΔL into a proportional component λL and a component $\widetilde{\Delta L}$ that points along the isoquant of F^*, that is, $\nabla_L F^*(L)\widetilde{\Delta L} = 0$. By Assumption 3, the proportional component has no effect on the skill premium after adjustment of technology, so we can restrict attention to the isoquant component $\widetilde{\Delta L}$.

Denote the isoquant of F^* through L by $\overline{L}_1^*(.; L)$. Its slope is equal to the negative of the skill premium:

$$\left.\frac{d\overline{L}_1^*(\tilde{L}_2; L)}{d\tilde{L}_2}\right|_{\tilde{L}_2 = L_2} = -\frac{F_{L_2}^*(\overline{L}_1^*(L_2; L), L_2)}{F_{L_1}^*(\overline{L}_1^*(L_2; L), L_2)} \,.$$

Consequently, the second derivative is the marginal change in the skill premium when moving along the isoquant:

$$\begin{aligned}
\left.\frac{d^2\overline{L}_1^*(\tilde{L}_2; L)}{(d\tilde{L}_2)^2}\right|_{\tilde{L}_2 = L_2} &= \frac{d}{d\tilde{L}_2}\left(-\frac{F_{L_2}^*(\overline{L}_1^*(\tilde{L}_2; L), \tilde{L}_2)}{F_{L_1}^*(\overline{L}_1^*(\tilde{L}_2; L), \tilde{L}_2)}\right)\Bigg|_{\tilde{L}_2 = L_2} \\
&= -\delta \nabla_L \frac{F_{L_2}^*(L)}{F_{L_1}^*(L)} \widetilde{\Delta L}
\end{aligned}$$

for some $\delta \in \mathbb{R}_{++}$, where the second line follows from the fact that $\widetilde{\Delta L}$ is proportional to $\left(d\overline{L}_1^*(\tilde{L}_2; L)/d\tilde{L}_2\big|_{\tilde{L}_2 = L_2}, 1 \right)$ by construction. This yields the equivalence between a locally

[13] The connection between strong relative bias and (quasi-)convexity is already partially anticipated in footnote 22 in Acemoglu (2007). There, Acemoglu (2007) notes that, in the case of labor-augmenting technology, strong relative bias arises if and only if a particular "modified production function" becomes convex. Here, I show that strong relative bias is indeed equivalent to quasiconvexity of the original (i.e., non-"modified") aggregate production function in much more general environments.

Strong Relative Bias

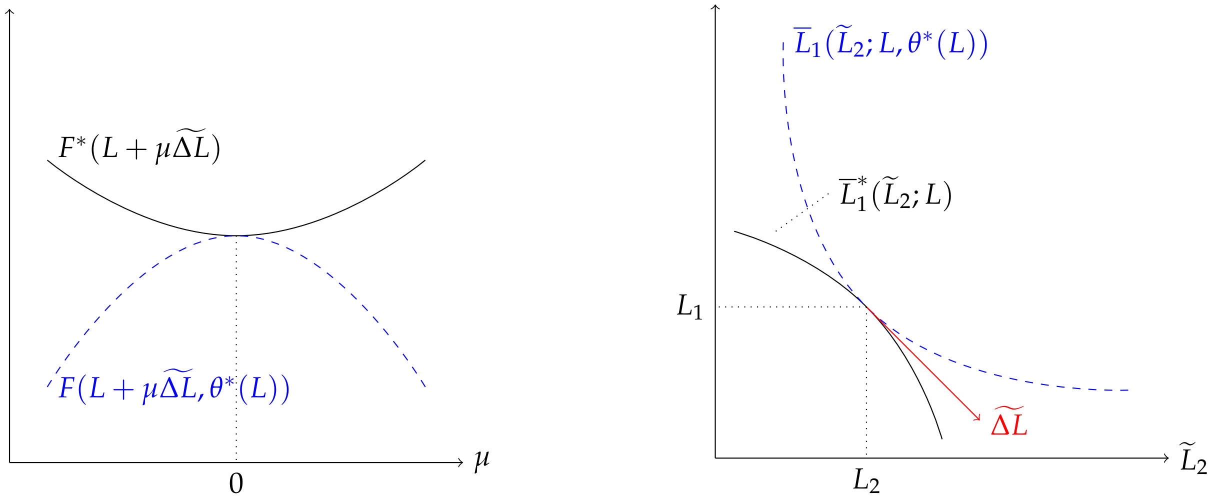

Figure 2. Illustration of strong relative bias. The panels reproduce the graphs from Figure 1, but now for the case of strong relative bias. In the left panel, output is convex if technology adjusts (the solid curve). Hence, when moving from L in direction $\widetilde{\Delta L}$, the output gain in this direction, and thereby the skill premium, increases. In the right panel, the isoquant is concave if technology adjusts (the solid curve). Thus, when moving in direction $\widetilde{\Delta L}$, the negative of the slope of the isoquant, and hence the skill premium, increases.

concave isoquant of F^* and strong relative bias. □

Strong relative bias is illustrated in Figure 2. The figure basically shows the same graphs as Figure 1, but, in contrast to Figure 1, aggregate production F^* is convex along the isoquant (left panel) and, consequently, the endogenous-technology isoquant $\overline{L}_1^*$ is concave (right panel).

By establishing a link between convexity properties of aggregate production and strong relative bias, Proposition 2 reveals a parallel to endogenous growth theory. There, increasing returns to scale in aggregate production are necessary for persistent growth in a large class of models (cf. Romer, 1994; Acemoglu, 2009). While increasing returns to scale constitute a failure of concavity along lines through the origin, the failure of concavity required for strong relative bias concerns the isoquants of F^* and is in this sense orthogonal to returns to scale.

From a modeling perspective, Proposition 2 is informative about how (not) to set up a model that features strong relative bias. For example, in a setting where identical firms choose their technologies independently of each other, production functions must be concave in labor and technology around the equilibrium. By Proposition 2, strong relative bias cannot occur in such a model. In Online Appendix F, I discuss two ways to introduce strong relative bias, both of which create some form of interdependence between firms' technology choices. In the first approach, there are ad hoc spillovers between firms' technologies, reminiscent of learning-by-doing models of endogenous growth (e.g. Romer, 1986; Lucas, 1988). In the second approach, interdependence occurs via the market for technologies, where technology firms supply their innovations to final good firms, and non-rivalry of innovations implies that technology firms sell their ideas to all active final good firms at once. This specification follows monopolistic competition based models of endogenous growth such as Romer (1990) and Aghion and Howitt (1992). In both approaches, interdependence between firms' technologies breaks the requirement that production functions are jointly concave in labor and technology, enabling quasiconvexity and strong relative bias.

The baseline model in Online Appendix F.1, without strong relative bias, may also be interpreted as describing a process of pure technology adoption, whereas the other models incorporate some features of true innovation (such as spillovers from imperfect protection of an individual firms' knowledge, or imperfect competition from the partial protection of intellectual property). With this interpretation, Proposition 2 admits the conclusion that strong relative bias cannot arise from technology adoption alone but requires some portion of innovation.

4. Directed Technical Change with Global Labor Supply Changes

The above results move beyond labor-augmenting technology but still maintain the focus of previous results on local labor supply changes. In the following, I show how both results on weak and strong relative bias extend to global changes in labor supply.

4.1. Weak Relative Bias

With local labor supply changes, weak relative bias originates from the complementarity encapsulated in equation (4): the induced technical change is such that it raises the return to the underlying change in labor supply. This complementarity also arises with global labor supply changes: let L and L' denote the initial and the eventual labor supply, respectively; then, by definition of the equilibrium technology, the output gain of moving from L to L' must be greater under the new than under the old technology:

$$F(L', \theta^*(L')) - F(L, \theta^*(L')) \geq F(L', \theta^*(L)) - F(L, \theta^*(L)) \, . \tag{5}$$

The skill premium, however, is determined by the output gain from a local increase in relative skill supply, not from a global one. Thus, to relate the induced technical change to the skill premium, we need complementarity with local instead of global labor supply changes. While complementarity with the global change, as formalized in equation (5), implies complementarity with at least some local changes on the path from L to L', it does not necessarily do so for all of these local changes.[14] In particular, complementarity relationships may change on the way from L to L', such that the induced technical change is skill biased when evaluated at the initial labor supply but not at the new one.

Milgrom and Roberts (1996) face the same problem in their global extension of the Le Chatelier principle (for absolute instead of relative demand). They preclude reversals of complementarity by imposing quasisupermodularity and a single-crossing property on the production function.

[14]Formally, with $\Delta L := L' - L$, equation (5) implies that

$$\int_0^1 \left(\nabla_L F(L + \mu \Delta L, \theta^*(L')) - \nabla_L F(L + \mu \Delta L, \theta^*(L)) \right) \Delta L \, d\mu \geq 0 \, .$$

The integrand must be positive at some but does not have to be positive at all μ. Thereby, we obtain a close counterpart to equation (4) at some but not necessarily all points between L and L'.

I follow the same path here, with the difference that I can dispense with the single-crossing property. In particular, I first order technologies according to their skill bias as follows.

Definition 2 (Skill Bias)**.** For any two technologies $\theta, \theta' \in \Theta$, θ is skill biased relative to θ' if and only if

$$\frac{F_{L_2}(L,\theta)}{F_{L_1}(L,\theta)} \geq \frac{F_{L_2}(L,\theta')}{F_{L_1}(L,\theta')}$$

for all L. We write $\theta \succeq^{sb} \theta'$.

Using the resulting order relation, I can define a lattice and quasisupermodularity.

Definition 3 (Lattice)**.** The set $(\Theta, \succeq^{sb})$ is a lattice if and only if any two technologies $\theta, \theta' \in \Theta$ have a supremum and an infimum in Θ.[15]

Definition 4 (Quasisupermodularity)**.** The function $F(L,\theta)$ is quasisupermodular in θ under $\succeq^{sb}$ if, for any L and $\theta, \theta' \in \Theta$,

$$F(L,\underline{\theta}) \leq F(L,\theta) \text{ for all } \underline{\theta} \in \inf(\theta,\theta') \Rightarrow F(L,\theta') \leq F(L,\overline{\theta}) \text{ for some } \overline{\theta} \in \sup(\theta,\theta') \, ,$$

where $\inf(\theta,\theta')$ denotes the set of infima of θ and θ', and $\sup(\theta,\theta')$ denotes the set of suprema.[16]

With the skill bias order $\succeq^{sb}$, unlike Milgrom and Roberts (1996), I do not have to impose single crossing in labor and technology for my results. This is because the skill bias order already implies a form of complementarity between an increase in relative skill supply and a technical change in direction of $\succeq^{sb}$: under a more skill-biased technology, the return to an increase in relative skill supply is larger. Essentially, by ordering technologies according to their skill bias, I get the effects of single crossing 'for free'.[17] Indeed, the following proposition shows that quasisupermodularity alone is sufficient to extend weak relative bias to global labor supply changes.

Proposition 3 (Global Weak Bias with Two Skills)**.** *Suppose that $N = 2$, $(\Theta, \succeq^{sb})$ is a lattice, and F is quasisupermodular in θ under $\succeq^{sb}$. Moreover, suppose that Assumption 2 holds.*

Then, for any two labor supply vectors L and L' with $L_2'/L_1' \geq L_2/L_1$, technology is more skill biased under L' than under L, that is, $\theta^*(L') \succeq^{sb} \theta^*(L)$.

Proof. As for local weak relative bias, we first use Assumption 2 to focus on labor supply changes along the isoquant. In particular, by Assumption 2, we can scale the new labor supply L' up or down without changing the skill bias of the corresponding equilibrium technology.

[15] Note that I have not excluded the case where two distinct technologies $\theta, \theta' \in \Theta$ lead to the same skill bias, that is, $\theta \succeq^{sb} \theta'$ and $\theta' \succeq^{sb} \theta$. In such a case, $\succeq^{sb}$ is not a partial order but a preorder. Then, Definition 3 defines what is conventionally called a prelattice, the extension of the lattice concept from partially ordered to preordered sets. The difference is that, in a preordered set, supremum and infimum may not be unique.

[16] This definition of quasisupermodularity is an extension of the original definition of Milgrom and Shannon (1994) to the case where infima and suprema are not necessarily unique. This case arises when $\succeq^{sb}$ is not a partial order but a preorder, see footnote 15. Whenever $\succeq^{sb}$ is a partial order (and, hence, infima and suprema are unique), Definition 4 collapses to the definition in Milgrom and Shannon (1994).

[17] Yet, the skill bias order, together with quasisupermodularity, does not imply single crossing between relative skill supply and technology. See Online Appendix E for a more detailed comparison with standard results from monotone comparative statics.

Hence, without loss of generality, we can assume that L' is on the same exogenous-technology isoquant as L, that is, $F(L, \theta^*(L)) = F(L', \theta^*(L))$.

The second step is to use quasisupermodularity and the 'natural' complementarity between increases in relative skill supply and skill-biased technical change to show that the new equilibrium technology must be a supremum of the new and the old technology. For that, let $\overline{L}_1(\widetilde{L}_2; L, \theta^*(L))$ denote the isoquant through L at the exogenous technology $\theta^*(L)$, such that $F(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L)) = F(L, \theta^*(L))$ for all $\widetilde{L}_2$. Note that this isoquant passes through both L and L', with $L'_2 \geq L_2$.

Now, take any infimum $\underline{\theta} \in \inf(\theta^*(L), \theta^*(L'))$. Since $\theta^*(L)$ is skill biased relative to $\underline{\theta}$, we have

$$\frac{F_{L_2}(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L))}{F_{L_1}(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L))} \geq \frac{F_{L_2}(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta})}{F_{L_1}(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta})}$$

for all $\widetilde{L}_2$ in the support of the isoquant. Rearranging the inequality, we obtain:

$$\begin{aligned} 0 &\geq F_{L_2}(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta}) - F_{L_1}(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta}) \frac{F_{L_2}(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L))}{F_{L_1}(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \theta^*(L))} \\ &= \frac{d}{d\widetilde{L}_2} F(\overline{L}_1(\widetilde{L}_2; L, \theta^*(L)), \widetilde{L}_2, \underline{\theta}) \,. \end{aligned}$$

Hence, when moving along the isoquant from L to L', output under technology $\underline{\theta}$ declines. We thus obtain the following order of output levels,

$$F(L', \underline{\theta}) \leq F(L, \underline{\theta}) \leq F(L, \theta^*(L)) = F(L', \theta^*(L)) \,,$$

where the second inequality stems from the equilibrium technology condition (2). Since these inequalities apply to all infima of $\theta^*(L)$ and $\theta^*(L')$, the outer (in)equalities and quasisupermodularity imply that there must be a supremum $\overline{\theta} \in \sup(\theta^*(L), \theta^*(L'))$ with

$$F(L', \overline{\theta}) \geq F(L', \theta^*(L')) \,.$$

So, $\overline{\theta}$ is an element of the maximizer set in the equilibrium technology condition (2). Since F is continuous and quasisupermodular in θ, the maximizer set in (2) is a complete sublattice of Θ.[18] For this case, we assumed that the equilibrium technology $\theta^*(L')$ is selected from the supremum of the maximizer set. By transitivity, $\theta^*(L')$ must then also be contained in the supremum of $\theta^*(L)$ and $\theta^*(L')$, which completes the proof. □

4.2. Strong Relative Bias

When extending strong relative bias to global labor supply changes, there is no need to prevent changes in complementarity relationships via quasisupermodularity. Instead, if the endogenous-technology wage function w^* is differentiable almost everywhere, we can simply integrate over all marginal changes in the skill premium when moving from L to L' and apply the local results from Proposition 2 along the path.

[18] See, for example, Corollary 2 and the subsequent discussion in Milgrom and Shannon (1994).

We thus obtain that any (global) increase in relative skill supply raises the skill premium after adjustment of technology (i.e., there is global strong bias everywhere) if and only if the aggregate production function F^* is quasiconvex. Analogously, any increase in relative skill supply reduces the skill premium after adjustment of technology (i.e., there is global strong bias nowhere) if and only if F^* is quasiconcave. Given that conditions for the existence of strong bias, rather than for its absence, are of particular interest, the contrapositive of the latter statement seems more interesting: there exists an increase in relative skill supply that raises the skill premium after adjustment of technology if and only if F^* is not quasiconcave.

Proposition 4 establishes that these insights hold even if wages are not differentiable.

Proposition 4 (Global Strong Bias with Two Skills)**.** *Suppose that $N = 2$ and Assumption 3 is satisfied. Then, the following holds.*

1. *There exists an increase in relative skill supply that strictly raises the skill premium after adjustment of technology if and only if F^* is not quasiconcave.*

2. *Any increase in relative skill supply raises the skill premium after adjustment of technology if and only if F^* is quasiconvex.*

Proof. As described above, for the case where the wage function w^* is differentiable almost everywhere, the proposition follows immediately from path integration of relative wage changes and application of Proposition 2. The general case is covered by the even more general Theorem 2 presented in the next section. □

Proposition 4 basically provides the same insights as the local strong bias condition in Proposition 2. It corroborates that a failure of quasiconcavity of aggregate production is key for strong relative bias, with all the implications discussed above.

5. Directed Technical Change with Many Skill Levels

In environments with more than two different types of labor, the results for two skill levels can in principle be applied to any pair of labor types, holding all other types' supply levels constant. When applied to each pair from a potentially large set of labor types, however, Assumptions 2 and 3 become increasingly restrictive. So, in the following, I present extensions of the two-skills results without imposing Assumptions 2 and 3 to hold pairwise. The main insights from the two-skills case generalize in fairly natural ways.

5.1. Weak Relative Bias

I first impose a many-skills equivalent of Assumption 2, saying that a proportional change in all labor types shall not induce technical change that affects relative wages.

Assumption 4. *For any labor supply L, any $\lambda \in \mathbb{R}_{++}$, and any two skill levels s and $\tilde{s}$,*

$$\frac{F_{L_s}(L, \theta^*(L))}{F_{L_{\tilde{s}}}(L, \theta^*(L))} = \frac{F_{L_s}(L, \theta^*(\lambda L))}{F_{L_{\tilde{s}}}(L, \theta^*(\lambda L))} .$$

Using this assumption, I provide two complementary results on weak relative bias for many skills. The first extends the results from the two-skills case to the level of two different groups of skills. The drawback is that this approach does not yield any predictions about how relative wages within the two groups will change. The second result makes predictions about relative wages for every pair of skills, but requires substantially stronger restrictions on the production function.

The most general version of the first result is presented in Theorem 3 in Appendix A. It essentially states that a labor supply change that raises the effective relative labor supply of some group of skills – in the sense that the (suitably transformed) labor supply change of this group has a positive effect on output – induces technical change that raises the (suitably weighted) average wage of this group relative to the remaining skills. Here, I focus on the version of this result in which the labor supply changes within the two groups of skills are proportional; that is, relative labor supply remains constant within each group but changes across groups.

Proposition 5 (Local Weak Bias with Many Skills)**.** *Suppose that $\Theta \subset \mathbb{R}^M$ (for arbitrary $M \in \mathbb{N}$), F is twice continuously differentiable, and the equilibrium technology θ^* is differentiable in labor supply L. Moreover, suppose that Assumption 4 holds.*

Consider a partition of the skill set $\{S^{(1)}, S^{(2)}\}$, an initial labor supply L, and a change in labor supply ΔL such that $\Delta L_s = \lambda_1 L_s$ for all $s \in S^{(1)}$ and $\Delta L_s = \lambda_2 L_s$ for all $s \in S^{(2)}$. Without loss of generality, let $\lambda_2 \geq \lambda_1$.

Then, the induced technical change $\Delta\theta^* = \nabla_L \theta^*(L)\Delta L$ locally raises the average wage in group $S^{(2)}$ relative to the average wage in group $S^{(1)}$,

$$\nabla_\theta \frac{\overline{w}_{S^{(2)}}(L, \theta^*(L))}{\overline{w}_{S^{(1)}}(L, \theta^*(L))} \Delta\theta^* \geq 0 \, ,$$

where the average wages are given by, for $i = 1, 2$,

$$\overline{w}_{S^{(i)}}(L, \theta^*(L)) = \frac{\sum_{s \in S^{(i)}} w_s(L, \theta^*(L)) L_s}{\sum_{\tilde{s} \in S^{(i)}} L_{\tilde{s}}} \, .$$

Proof. The proposition follows as a special case from Theorem 3 in Appendix A. □

Proposition 5 provides a group-level version of weak relative bias: an increase in the supply of some group of skills induces technical change that raises the average wage of this group relative to the average wage among the remaining skills. Theorem 3 in Appendix A dispenses with the restriction to labor supply changes that are proportional within groups. It provides an extension of Proposition 5 to labor supply changes in arbitrary directions.

A noteworthy special case of Proposition 5 is obtained when group $S^{(2)}$ is a singleton and $\lambda_1 = 0$; that is, labor supply of a single skill level $\tilde{s}$ increases, while all other supply levels remain constant. For this case, Proposition 5 predicts that the induced technical change raises the wage of $\tilde{s}$ relative to the average wage of all other skills.

To move beyond changes in average wages and provide a more detailed account of how the induced technical change affects the wage distribution, further restrictions are needed. I present now a set of conditions under which any pervasive increase in relative skill supply induces technical change that leads to a pervasive increase in skill premia.

A pervasive increase in relative skill supply is defined, following Costinot and Vogel (2010), as a labor supply change such that the supply of more skilled workers increases relative to the supply of less skilled workers for any pair of skill levels. Similarly, a pervasive increase in skill premia is a change in skill premia such that the wage of more skilled workers increases relative to the wage of less skilled workers for any pair of skills. Accordingly, I define pervasively skill-biased technical change as follows.

Definition 5 (Pervasive Skill Bias)**.** For any two technologies $\theta, \theta' \in \Theta$, θ is pervasively skill biased relative to θ' if and only if

$$\frac{F_{L_s}(L,\theta)}{F_{L_{s'}}(L,\theta)} \geq \frac{F_{L_s}(L,\theta')}{F_{L_{s'}}(L,\theta')}$$

for all $s \geq s'$ and for all L. We write $\theta \succeq^{psb} \theta'$.

With this definition of skill bias, we obtain an extension of the global weak bias result of Proposition 3.[19]

Theorem 1 (Global Weak Bias with Many Skills)**.** *Suppose that $(\Theta, \succeq^{psb})$ is a lattice, F is quasisupermodular in θ under $\succeq^{psb}$, and Assumption 4 holds.*

Then, for any two labor supply vectors L and L' with $L'_s/L'_{s'} \geq L_s/L_{s'}$ for all $s \geq s'$, the technology under L' is pervasively skill biased relative to the technology under L, that is, $\theta^*(L') \succeq^{psb} \theta^*(L)$.

Sketch of proof. The proof uses basically the same reasoning as the proof of the two-skill case (i.e., Proposition 3). All details are presented in Appendix C.1. The basic steps are as follows.

First, by the usual arguments, Assumption 4 allows to focus on L' on the exogenous-technology isoquant that passes through L.

Next, we take an infimum $\underline{\theta}$ of $\theta^*(L)$ and $\theta^*(L')$ and note that $F(L,\underline{\theta}) \leq F(L,\theta^*(L))$. We then show that, since $\theta^*(L)$ is pervasively skill biased relative to $\underline{\theta}$, the output gain when moving from L to L' is greater under $\theta^*(L)$ than under $\underline{\theta}$. This yields $F(L',\underline{\theta}) \leq F(L',\theta^*(L))$.

By quasisupermodularity, it follows that there must be a supremum $\overline{\theta}$ of $\theta^*(L)$ and $\theta^*(L')$ such that $F(L',\overline{\theta}) \geq F(L',\theta^*(L'))$. This implies, by the same arguments as in the proof of Proposition 3, that $\theta^*(L')$ itself must be a supremum and, hence, $\theta^*(L') \succeq^{psb} \theta^*(L)$. □

Theorem 1 provides a direct extension of the result for two skills in Proposition 3. It is important, however, to note that the requirement of quasisupermodularity becomes substantially more restrictive when applied to environments with many skills. To see this, note that quasisupermodularity is automatically satisfied in situations where any two technologies can

[19] At first glance, one may think that a local version of Theorem 1 follows immediately from iterated application of Proposition 5: first increase labor supply of all skills $s > 1$ proportionately such as to match the increase in the supply of $s = 2$ relative to $s = 1$. Repeat this for skills $s > 2$ (to match the increase of $s = 3$ relative to $s = 2$) and so on. Applying Proposition 5 at any step should imply that the induced technical change raises all skill premia. The problem is that Proposition 5 does not say anything about the change in relative wages within groups. Thus, when proportionately raising all supply levels for $s > 1$, we may get declining skill premia within the group of skills with $s > 1$ (for example). So, Theorem 1 does not follow from Proposition 5. It is, in fact, a much stronger result, which requires stronger restrictions.

be ordered according to their skill bias. So, quasisupermodularity becomes restrictive only when the skill-bias order is incomplete.

With two skills, the only reason why the skill-bias order can be incomplete is that relative wage effects are reversed when moving through the labor supply space: a given technical change raises the skill premium at some labor input but lowers it at another input. Thus, quasisupermodularity is satisfied as soon as the relative wage effects of technical change are stable (in terms of sign, not magnitude) over the labor supply space.

With many skills, this is not enough: there may be changes in technology that do not affect skill premia uniformly but, instead, raise them in some part and lower them in another part of the wage distribution. Such technologies cannot be ranked by skill bias, even if technical change effects are stable in the above sense. In this case, quasisupermodularity requires that technical changes that raise skill premia in different parts of the wage distribution must not be substitutes. For illustration, imagine two technical changes, one that raises skill premia in the upper part and one that raises skill premia in the lower part of the wage distribution. Quasisupermodularity requires that, if one of these changes raises output absent the other, it must still raise output when the other change has been implemented.

While these restrictions are substantial, Section 6 and Online Appendix G provide natural examples where they are satisfied.

As a final extension, I show that Theorem 1 also holds when the skill set is not finite but a continuum. The arguments from the finite case are essentially unchanged with a continuum of skills, but their transfer requires some mathematical clarification. I provide this transfer in Appendix B.

5.2. Strong Relative Bias

For strong relative bias, the transfer from local to global labor supply changes does not require any additional restrictions (see the two-skills case above). Therefore, I directly extend the global strong bias result for two skills, Proposition 4, to the case with an arbitrary number of skills. Indeed, Theorem 2 provides a straightforward generalization of Proposition 4.

Theorem 2 (Global Strong Bias with Many Skills)**.** *Suppose Assumption 3 holds. Then, the following is true.*

1. *If there exists a pervasive increase in relative skill supply that strictly raises all skill premia after adjustment of technology, then F^* is not quasiconcave.*

   Moreover, if F^* is not quasiconcave on some line along which relative skill supply increases pervasively, then there exists a pervasive increase in relative skill supply that does not lower all skill premia after adjustment of technology.

2. *If every pervasive increase in relative skill supply raises all skill premia after adjustment of technology, then F^* is quasiconvex on all lines along which relative skill supply increases pervasively.*

   Moreover, if F^* is quasiconvex, then no pervasive increase in relative skill supply strictly lowers all skill premia after adjustment of technology.

Proof. See Appendix C.2. □

Following Proposition 4, the first part of Theorem 2 connects strong relative bias – here in the sense that a pervasive increase in relative skill supply leads to a pervasive increase in skill premia – to a failure of quasiconcavity of the aggregate production function F^*. As in the two-skills case, a failure of quasiconcavity is necessary for strong relative bias. Unlike with two skills, however, it is not generally sufficient. This is for two reasons. First, with many skill levels, a pervasive increase in relative skill supply may increase some and decrease other skill premia. Second, quasiconcavity may fail only in directions, which are not considered in the theorem, that is, directions along which some skill supply ratios rise while others fall. In consequence, there is only a partial converse: without quasiconcavity on a line along which relative skill supply rises, some pervasive increase in relative skill supply does not lower all skill premia. As soon as there are only two skill levels, this becomes a full converse: with two skills there is only one skill premium and if this does not fall, it must rise; moreover, any direction in the L_1-L_2-plane is one along which relative skill supply either (weakly) falls or rises.

Essentially the same adjustments are necessary in the second part of Theorem 2, connecting strong relative bias to quasiconvexity of F^*. Again, if there are only two skill levels, the statement collapses to the second part of Proposition 4.

6. Endogenous Automation Technology

I now demonstrate how to apply the results of the previous sections by analyzing directed technical change in assignment models where different types of labor and capital are assigned endogenously to a range of tasks. Such models provide a natural formalization of automation, whereby labor is replaced by capital in the production of certain tasks.

Important technology variables in these models, such as the productivity of capital, do not have the labor-augmenting form required by existing results on directed technical change. Hence, the generality of my results is indispensable in the following analysis.

Relative to settings with labor-augmenting technology, a directed technical change analysis in assignment models with automation technology has a number of benefits.[20] First, the results align well with intuitive notions of technical change. Second, they can be tested directly in empirical work, as labor-replacing technology variables can be identified with empirical measures of concrete automation technologies.[21] Third, they make statements about a form of technical change that is widely perceived to be among the most important determinants of future changes in the employment and wage structure. Finally, the literature on assignment models of the type analyzed here is growing rapidly, with applications in labor (e.g. Acemoglu and Autor, 2011), trade (e.g. Costinot and Vogel, 2010), growth (e.g. Acemoglu and Restrepo, 2018a), and public economics (e.g. Rothschild and Scheuer, 2013). Bringing results on directed

[20]See Acemoglu and Restrepo (2018b) for a complementary list of advantages of the approach with labor-replacing technology.

[21]See for example the use of counts of industrial robots as a measure for automation technology by Graetz and Michaels (2018); Acemoglu and Restrepo (2020); Dauth et al. (2019); Abeliansky and Prettner (2017); Acemoglu and Restrepo (2019), the use of survey data on the adoption of various automation technologies in manufacturing by Lewis (2011), and the use of data on harvesting machines in agriculture by Clemens, Lewis and Postel (2018).

technical change to the assignment environment keeps them connected to the newest strand of the theoretical literature on wage and income inequality.

In the main text, I focus on the endogenous determination of capital productivity in a setting where firms choose their technologies from a predetermined set. Online Appendix G provides a more extensive treatment of the assignment framework and shows that the results also hold in a setting where technologies are embodied in intermediate inputs produced by an R&D sector that controls the quality of the intermediate inputs. Moreover, in Online Appendix G.1, I show that my directed technical change results can also be used to study the role of the assignment of factors to tasks itself, demonstrating how general the notion of technology used in the previous sections is.

6.1. Setup

The analysis builds on the assignment model by Teulings (1995), augmented to incorporate capital as an additional production factor. There is a continuum of tasks (or intermediate goods), indexed by $x \in X = [0,1]$, and a single final good. A continuum of competitive firms produces the final good out of tasks according to

$$Y = \beta \left(\int_0^1 Y_x^{\frac{\epsilon-1}{\epsilon}} \, dx \right)^{\frac{\epsilon}{\epsilon-1}}$$

with $\beta > 0$, and $\epsilon > 0$ being the elasticity of substitution across tasks. Tasks in turn are produced linearly from capital and labor,

$$Y_x = \alpha K_x + \int_0^1 \gamma(s,x) L_{s,x} \, ds \,,$$

where K_x denotes the amount (or density) of capital assigned to task x, $L_{s,x}$ is the amount of labor of skill s assigned to task x (or the joint density of labor over skills and tasks), and $\alpha > 0$ and $\gamma(s,x) > 0$ are (task-specific) productivities of capital and the differentially skilled types of labor. The wage of one unit of labor of skill s is denoted by w_s.

There is a continuum of skills, indexed by $s \in S = [0,1]$, and labor supply $L = \{L_s\}_{s \in S}$ (or the marginal density of labor over skills) is exogenous. The total amount of capital is denoted by $K = \int_0^1 K_x \, dx$. Capital is produced at marginal cost r from final good. This mimics the steady state of dynamic models in which capital is accumulated over time and the long-run interest rate is fixed by preferences and depreciation.

The productivities of capital, α, and final good production, β, are endogenous. In particular, firms choose their productivity levels subject to the frontier

$$g(\alpha, \beta) \leq \overline{g} \,, \tag{6}$$

where g is continuous, strictly increasing, and strictly convex. For later reference, denote the combination of productivities that maximizes their product $\alpha\beta$ by

$$(\overline{\alpha}, \overline{\beta}) \in \underset{(\alpha,\beta) \in \mathbb{R}^2_{++}}{\operatorname{argmax}} \left\{ \alpha\beta \mid g(\alpha,\beta) \leq \overline{g} \right\} \,. \tag{7}$$

I show below that we can neglect all productivity pairs with $\alpha \geq \overline{\alpha}$, as those are never chosen by firms.[22]

An exogenous-technology equilibrium consists of wages for each skill type, a joint distribution of labor over tasks and skills, and a distribution of capital over tasks, such that labor markets clear and firms maximize profits, given exogenous productivities α and β.[23] An endogenous-technology equilibrium is an exogenous-technology equilibrium for productivities α and β that maximize firm profits subject to the productivity frontier (6).

In equilibrium, factors are assigned to tasks according to their comparative advantage, which is determined by the labor productivity schedule $\gamma(s, x)$. I assume that γ is twice differentiable and satisfies

$$\frac{\partial^2 \log \gamma(s,x)}{\partial s \partial x} > 0 \quad \text{and} \quad \frac{\partial \log \gamma(s,x)}{\partial x} > 0 \tag{8}$$

for all s, x. The first part ensures that more skilled workers have comparative advantage in tasks with a higher index x (more complex tasks, henceforth). Since the productivity of capital α is constant across tasks (a normalization without material impact on the results), the second part implies that every type of labor has comparative advantage versus capital in more complex tasks.[24]

The pattern of comparative advantage induces a simple assignment rule for factors to tasks. In particular, for given labor inputs L and productivities α, β, the profit-maximizing assignment is characterized by (i) a threshold task $\tilde{x}$ such that all tasks below $\tilde{x}$ are produced by capital and all tasks above by labor; (ii) a strictly increasing and onto matching function $m : S \to [\tilde{x}, 1]$ such that $m(s)$ denotes the task assigned to workers of skill s. This characterization follows from a direct extension of the arguments in Costinot and Vogel (2010).[25]

Now, let $F(L, \alpha, \beta)$ denote net output $Y - r\int_0^1 K_x \, dx$ under the profit-maximizing assignment rules $\tilde{x}$, $\{K_x\}_{x \in [0,\tilde{x}]}$, and m, given labor inputs L and productivities α, β. In any symmetric exogenous-technology equilibrium, wages must satisfy, for all s,

$$w_s(L, \alpha, \beta) = F_s(L, \alpha, \beta) \ , \tag{9}$$

where $F_s(L, \alpha, \beta)$ denotes the Gateaux derivative of F in direction of L_s. This derivative corresponds to the notion of the marginal product of labor of skill s in the case with a continuum of skills.[26]

In a symmetric endogenous-technology equilibrium, where firms choose productivities to maximize profits, we must have

$$(\alpha^*(L), \beta^*(L)) \in \underset{\alpha,\beta}{\operatorname{argmax}} \{F(L, \alpha, \beta) \mid g(\alpha, \beta) \leq \overline{g}\} \tag{10}$$

[22] A further restriction needed is that the cost of capital respects the lower bound $r > \overline{\alpha}\overline{\beta}$. This is because final good and task production are linear in capital while capital production is linear in final good. Such linearity in circular production may enable infinite output, analogously to unbounded growth of the AK-type in a dynamic model, if the marginal cost of capital is too low.

[23] I focus on exogenous-technology equilibria in which the productivities α and β are symmetric across firms.

[24] See Online Appendix G.1 for a discussion of these assumptions.

[25] See Lemma 4 in Online Appendix G.1 for details.

[26] Appendix B provides a detailed explanation of derivative concepts in the continuum case.

and, for all s,

$$w_s^*(L) = w_s(L, \alpha^*(L), \beta^*(L)) = F_s(L, \alpha^*(L), \beta^*(L)) \ .$$

Symmetric exogenous- and endogenous-technology equilibria exist for any labor input L if and only if the endogenous-technology production function $F^*(L)$ is concave, which I assume henceforth.[27]

Assumption 5. *The endogenous-technology production function $F^*(L) := F(L, \alpha^*(L), \beta^*(L))$ is concave.*

At exogenous productivities, F is always concave in L in the present model. For concavity of the envelope F^*, we need that the productivity frontier g is sufficiently convex. Moreover, if g is sufficiently convex, the equilibrium technology $(\alpha^*(L), \beta^*(L))$ will be unique, which I also assume to be the case from here on.[28]

6.2. Weak Relative Bias of Automation Technology

By equations (9) and (10), the model fits into the general framework analyzed in the previous sections.[29] To apply the results on weak relative bias, it remains to verify that the particular conditions of Theorem 1′ (the continuum counterpart of Theorem 1 for weak bias with many skills) are satisfied. We can establish the following.

Lemma 1.

1. *The aggregate production function $F(L, \alpha, \beta)$ is linear homogeneous in L at all feasible α, β.*

2. *Technologies are ordered according to their pervasive skill bias as follows:*

$$(\alpha', \beta') \succeq^{psb} (\alpha, \beta) \quad \Leftrightarrow \quad \alpha' \geq \alpha$$

   for all (α', β') and (α, β) in the set

$$\Theta = \left\{(\alpha, \beta) \in \mathbb{R}_+^2 \mid g(\alpha, \beta) = \overline{g},\, \alpha \leq \overline{\alpha}\right\} \ ,$$

   where $\overline{\alpha}$ is uniquely determined by equation (7).

3. *For all L, $(\alpha^*(L), \beta^*(L)) \in \Theta$.*

Proof. See Appendix C.3. □

Linear homogeneity of aggregate production immediately implies that the equilibrium technology $(\alpha^*(L), \beta^*(L))$ is unaffected by the scale of labor supply and, thus, Assumption 4 (scale invariance of skill bias) is satisfied. The remaining parts of Lemma 1 show that we can order any two technologies according to their pervasive skill bias once we restrict attention to technologies in the set Θ. This set in turn is large enough to contain all possible equilibrium

[27] The proof of this statement uses standard arguments and is provided in Online Appendix F (Observation 1) for a general class of directed technical change models.

[28] Alternatively, we could always select the equilibrium technology with the highest $\alpha^*(L)$, in line with the selection rule imposed in Section 2.

[29] More precisely, it fits into the extension to a continuum skill space in Appendix B.

technologies. Hence, restricting attention to Θ is without loss of generality. This has two important consequences: (i) the ordered set $(\Theta, \succeq^{psb})$ is a chain and hence a lattice, (ii) aggregate production F is quasisupermodular in (α, β) on $(\Theta, \succeq^{psb})$, because any function on a chain is quasisupermodular.

Thus, all conditions of Theorem 1′ are satisfied and we obtain the following application of weak relative bias.

Corollary 3 (Weak Bias of Automation)**.** *Any pervasive increase in relative skill supply from L to L', where $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$, induces improvements in capital productivity: $\alpha^*(L') \geq \alpha^*(L)$.*

Proof. By Theorem 1′, we have $(\alpha^*(L'), \beta^*(L')) \succeq^{psb} (\alpha^*(L), \beta^*(L))$. By Lemma 1, this implies $\alpha^*(L') \geq \alpha^*(L)$. □

The increase in capital productivity must come at the cost of decreased final good productivity. Moreover, the proof of Corollary 3 shows that the shift from final good to capital productivity is pervasively skill biased, raising all skill premia. Finally, Lemma 3 (provided in Appendix C.3) reveals that the shift towards capital productivity is accompanied by a reallocation of tasks from labor to capital, the epitome of automation.

It is worth to note at this point that the present assignment model is a challenging environment for comparative statics. Labor supply has infinite dimension and there is generally no closed-form solution for the aggregate production function. Direct computations based on the implicit function theorem would seem cumbersome at best. Yet, after establishing two simple properties in Lemma 1, my generalized directed technical change theorems deliver clear-cut results.

6.3. Strong Relative Bias of Automation Technology

By Theorem 2′ (the extension of Theorem 2 to a continuum of skills), strong relative bias requires that the aggregate production function fails to be quasiconcave. In the present model, this is ruled out by Assumption 5, which is necessary for a symmetric equilibrium to generally exist. Hence, we can never have a situation where the induced improvement in capital productivity is so strong that a pervasive increase in relative skill supply leads to a pervasive increase in skill premia.

Corollary 4 (No Strong Bias of Automation)**.** *There is no pervasive increase in relative skill supply that strictly raises all skill premia.*

The impossibility of strong relative bias is due to the fact that, in the present model, firms choose their technologies independently of each other. This precludes (quasi-)convexity in the aggregate endogenous-technology production function, because individual and aggregate production functions are identical.

I now introduce spillover effects between firms' technology choices into the model. This creates a separation between individual and aggregate production functions, enabling convexity in the aggregate and, thereby, strong relative bias. In Online Appendix G.2, I show that the same effects can be achieved with a setup in which an R&D sector supplies technology-embodying intermediate inputs to the aggregate of all final good firms. This separates the choice of labor demand from technology choices, allowing for convexity in labor and technology jointly while maintaining concavity in each factor individually.

I introduce spillover effects via the productivity frontier. In particular, let an individual firm i's productivity frontier be given by

$$\widetilde{g}(\alpha_i, \beta_i, \alpha_{-i}, \beta_{-i}) \leq \overline{g} \,,$$

where α_{-i} and β_{-i} collect the productivities of all other firms. Denote by $\widehat{g}(\alpha, \beta)$ the symmetric technology frontier, that is,

$$\widehat{g}(\alpha, \beta) \equiv \widetilde{g}(\alpha_i, \beta_i, \alpha_{-i}, \beta_{-i}) \quad \text{if} \quad \alpha_j = \alpha \text{ and } \beta_j = \beta \text{ for all } j \,.$$

I assume that there are positive knowledge spillovers in the sense that an individual firm's 'technology cost' is minimized when all other firms choose the same technology:

$$\widehat{g}(\alpha, \beta) \leq \widetilde{g}(\alpha, \beta, \alpha_{-i}, \beta_{-i}) \quad \text{for all feasible} \quad \alpha, \beta, \alpha_{-i}, \beta_{-i} \,.$$

Clearly, such spillover effects do not change the structure of symmetric exogenous-technology equilibria. With endogenous technology, however, they create a distinction between individual and aggregate production functions. In particular, for a given labor input L, an individual firm chooses its technology to achieve

$$\widetilde{F}^*(L, \alpha_{-i}, \beta_{-i}) := \max_{\alpha, \beta} \left\{ F(L, \alpha, \beta) \mid \widetilde{g}(\alpha, \beta, \alpha_{-i}, \beta_{-i}) \leq \overline{g} \right\} \,. \tag{11}$$

The symmetric technology $(\alpha^*(L), \beta^*(L))$ that maximizes aggregate output, however, yields

$$\widehat{F}^*(L) := \max_{\alpha, \beta} \left\{ F(L, \alpha, \beta) \mid \widehat{g}(\alpha, \beta) \leq \overline{g} \right\} \,. \tag{12}$$

It can be verified that the technology $(\alpha^*(L), \beta^*(L))$, when chosen by all firms, also solves the individual technology choice problem (11).

For $(\alpha^*(L), \beta^*(L))$ to be an equilibrium technology, we additionally need that the wages given by $w_s^*(L) = F_s(L, \alpha^*(L), \beta^*(L))$ support the symmetric labor input choices L by individual firms, that is,

$$L \in \operatorname*{argmax}_{\widetilde{L}} \widetilde{F}^*(\widetilde{L}, \alpha^*_{-i}(L), \beta^*_{-i}(L)) - \int_0^1 w_s^*(L) \widetilde{L}_s \, ds \,,$$

where $\alpha^*_{-i}(L)$ and $\beta^*_{-i}(L)$ denote the collections with $\alpha_j = \alpha^*(L)$ and $\beta_j = \beta^*(L)$ for all firms $j \neq i$. This is guaranteed if the individual endogenous-technology production function $\widetilde{F}^*(L, \alpha_{-i}, \beta_{-i})$ is concave in L.[30]

Assumption 6. *The individual endogenous-technology production function $\widetilde{F}^*(L, \alpha_{-i}, \beta_{-i})$ is concave in L for all α_{-i}, β_{-i}.*

[30]The formal argument is provided in Online Appendix F (Observation 2) for a general class of models with spillover effects.

So, under Assumption 6, the technology $(\alpha^*(L), \beta^*(L))$ is part of an endogenous-technology equilibrium for any feasible labor supply L.[31]

The key distinction to the case without spillovers is that we only have to restrict the curvature of the individual endogenous-technology production function $\widetilde{F}^*$, not the aggregate function $\widehat{F}^*$. Indeed, it is easy to see that the aggregate endogenous-technology function is the upper envelope of the individual functions:

$$\widehat{F}^*(L) = \max_{\alpha,\beta} \left\{ \widetilde{F}^*(L, \alpha_{-i}, \beta_{-i}) \mid \alpha_j = \alpha \, \forall j, \, \beta_j = \beta \, \forall j, \, \widehat{g}(\alpha, \beta) \leq \overline{g} \right\} .$$

Thus, on any line $\tau \mapsto l(\tau) = L + \tau(L' - L)$, aggregate production must be less concave than individual production:

$$\frac{d^2}{(d\tau)^2} \widehat{F}^*(l(\tau)) \geq \frac{d^2}{(d\tau)^2} \widetilde{F}^*(l(\tau), \alpha^*_{-i}(L), \beta^*_{-i}(L)) .$$

Assumption 6 now commands that the individual production function $\widetilde{F}^*$ is concave on the line, but this leaves the possibility that

$$\frac{d^2}{(d\tau)^2} \widehat{F}^*(l(\tau)) > 0 \geq \frac{d^2}{(d\tau)^2} \widetilde{F}^*(l(\tau), \alpha^*_{-i}(L), \beta^*_{-i}(L)) . \tag{13}$$

If this happens on a line along which relative skill supply increases pervasively, we have strong relative bias according to Theorem 2′.

Corollary 5 (Strong Bias of Automation with Spillovers)**.** *Consider a line $\tau \mapsto l(\tau) = L + \tau(L' - L)$ with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$. Suppose (13) holds everywhere on this line. Then,*

$$\frac{w^*_{s'}(L')}{w^*_s(L')} > \frac{w^*_{s'}(L)}{w^*_s(L)}$$

for a strictly positive mass of (and potentially all) skill pairs with $s' > s$.

Proof. First, construct the auxiliary line $\widetilde{l}(\tau) = L + \tau(\widetilde{L}' - L)$ with $\widetilde{L}' = \lambda L'$ and λ such that

$$\widehat{F}^*(L) = \widehat{F}^*(\widetilde{L}') .$$

This line can also be written as $\widetilde{l}(\tau) = \lambda l(\tau) + (1 - \lambda)(1 - \tau)L$. Together with linear homogeneity of $\widehat{F}^*$, this implies that, at all $\tau \in [0, 1]$,

$$\frac{d^2}{(d\tau)^2} \widehat{F}^*(\widetilde{l}(\tau)) = \lambda^2 \frac{d^2}{(d\tau)^2} \widehat{F}^*(l(\tau)) > 0 .$$

Thus, the restriction of $\widehat{F}^*$ to $\widetilde{l}(\tau)$ must have a local minimum at some $\tau \in (0, 1)$, so it is not quasiconcave. Moreover, by construction, relative skill supply increases along $\widetilde{l}(\tau)$. So, part 1 of Theorem 2′ implies that some skill premia must increase strictly when moving from L to

[31] Other equilibria may exist but are inefficient, as technology would not maximize output. I focus on the efficient equilibrium technologies $(\alpha^*(L), \beta^*(L))$ here.

L'.[32] Since wages are continuous in skill, this must hold for a strictly positive mass of skill pairs. □

The conditions of Corollary 5 ensure that a pervasive increase in relative skill supply strictly raises a non-zero mass of skill premia. This includes the case where all skill premia increase in response to a pervasive increase in relative skill supply. Online Appendix G.2 provides such an example.

To summarize, spillover effects between firms' technology choices allow for a distinction between individual and aggregate endogenous-technology production functions, which creates the potential for convexity in the aggregate production function and, thus, strong relative bias.

6.4. Automation and International Trade

The previous sections have analyzed how automation technology responds to changes in the structure of labor supply. Such changes can occur for institutional, demographic, or cultural reasons but also because of international trade, which makes foreign labor accessible to domestic firms. I now demonstrate that the previous results are easily applicable to study the impact of international trade on automation. The key insight is that, in the present model, international trade affects the economy as if it were a change in labor supply.

I consider two countries, North and South. The Northern economy is modeled by the spillover model of the previous subsection. The South differs from the North in three aspects. First, the South has pervasively smaller relative skill supply, that is, $L^S_{s'}/L^S_s \leq L^N_{s'}/L^N_s$ for all skill pairs $s' \geq s$, where L^S denotes labor supply in the South and L^N is labor supply in the North. Second, Southern firms do not choose their productivities themselves but copy them from Northern firms at some loss. In particular, Southern firms use productivities $\delta\alpha^N$ and $\delta\beta^N$ with $\delta \in (0,1)$, where α^N and β^N are the equilibrium technologies in the North. Finally, labor productivity can be lower in the South, $\gamma^S(s,x) = \Delta\gamma^N(s,x)$ with $\Delta \in (0,1]$ and γ^S and γ^N denoting the labor productivity schedules in the South and North, respectively. The difference in labor productivities is largely irrelevant for the following analysis but realistically allows for differences in wage levels between the two countries.

I compare autarky with a free-trade situation, where countries can freely trade tasks and final goods, but workers are immobile.[33] Capital and final good productivities are higher in the North, so, under free trade, both will only be produced in the North. Thereby, we obtain the following pattern of trade: the North imports tasks performed by labor from the South, combines them with the tasks performed by Northern labor and capital to produce final goods, and ships part of the final output back to the South in exchange for the Southern labor tasks.

It follows that free-trade equilibria are equivalent to autarky equilibria in the North where Northern labor supply is extended by the effective units of Southern labor to $L^N + \Delta L^S$.[34] Since

[32] Part 1 of Theorem 2′ implies that there will be some pervasive increase in relative skill supply that does not generate a pervasive fall in skill premia. The proof of the theorem clarifies that this pervasive increase in relative skill supply is located on the line along which quasiconcavity fails.

[33] Intermediate cases with partial trade opening or trade costs can be treated in a similar way.

[34] For a formal derivation of this result, see Online Appendix G.3.

the South has pervasively smaller relative skill supply, the effective world labor supply $L^N + \Delta L^S$ also features pervasively smaller relative skill supply than the Northern labor supply L^N.

We thus obtain that trade opening has the same effects in the North as a pervasive fall in relative skill supply from L^N to $L^N + \Delta L^S$. Corollary 3 then immediately implies the following weak relative bias result.

Corollary 6 (Weak Bias of Automation from Trade). *Relative to autarky, free trade leads to a fall in Northern capital productivity: $\alpha^{N*} \geq \alpha^{NT*}$, where α^{N*} and α^{NT*} are Northern capital productivities in autarky and with free trade, respectively.*

The fall in Northern capital productivity (and simultaneous rise in final good productivity) induced by trade causes a pervasive fall in skill premia in the North, according to Lemma 1. This counteracts the usual Heckscher-Ohlin effects, which cause an increase in skill premia in the skill-abundant North. If there is strong relative bias, the effects of the induced technical change are strong enough to outweigh the Heckscher-Ohlin effects, leading to a situation where trade opening causes a fall in Northern skill premia after adjustment of technology.

In particular, let $\widehat{F}^{N*}$ denote the endogenous-technology aggregate production function in the North under autarky, equivalent to the endogenous-technology production function $\widehat{F}^*$ in Section 6.3. Suppose that $\widehat{F}^{N*}$ is convex along the line between L^N and $L^N + \Delta L^S$. Then, by Corollary 5, directed technical change effects dominate and trade leads to a fall in at least some skill premia in the North.

Corollary 7 (Strong Bias of Automation from Trade). *Suppose that $\widehat{F}^{N*}$ is strictly convex on the line between L^N and $L^N + \Delta L^S$. Then, relative to autarky, free trade leads to a fall in Northern skill premia in some parts of the wage distribution:*

$$\frac{w_{s'}^{N*}}{w_s^{N*}} \geq \frac{w_{s'}^{NT*}}{w_s^{NT*}}$$

for a strictly positive mass of (and potentially all) skill pairs with $s' > s$, where w^{N*} and w^{NT*} denote Northern wages in autarky and with free trade, respectively.

So, if aggregate production fails to be quasiconcave, directed technical change overwrites the usual Heckscher-Ohlin effects and trade with a skill-scarce country reduces wage inequality in some parts of the skill-abundant country's wage distribution.[35]

The results are in contrast to those from Acemoglu (2003), who studies the interaction between trade and directed technical change in a setting with labor-augmenting technology. The main difference is that, in Acemoglu (2003), the tasks produced by Southern labor cannot be traded directly but only after they have been combined with technology-embodying intermediate inputs in the South. Northern technology firms thus do not direct their R&D activities

[35] In the South, Heckscher-Ohlin effects call for a reduction in inequality, but again, there is a countervailing force: trade integrates Southern labor into the Northern final good production chain and exposes it to competition from Northern capital, which disproportionately affects low-skilled workers. If the productivity difference between Northern and Southern capital under autarky is large enough (i.e., δ is small enough), this latter effect dominates and trade leads to an increase in wage inequality in the South, again counter to the usual Heckscher-Ohlin effects. These results, however, do not depend on the endogeneity of technology and are thus not discussed in further detail.

towards world labor supply but exclusively towards labor supply in the North even when there is trade. In such a setting, trade opening is not equivalent to a decrease in relative skill supply but operates solely through an increase in the relative price of skill-intensive goods in the North, which leads to skill-biased technical change.[36]

Relatedly, Sampson (2014) and Yeaple (2005) study the effect of trade on wage inequality with endogenous technology in models of intra-industry trade. In their models, trade does not stem from differences in endowments and technology but from product differentiation and economies of scale as in Krugman (1980). To export, firms must incur a fixed cost, which incentivizes them to invest in more productive technologies. Complementarity between firm productivity and worker skill then gives rise to a positive effect of trade on skill premia.[37] Hence, as in Acemoglu (2003), these models predict that trade induces skill-biased technical change, opposite to the model presented above.

7. Conclusion

The first part of the paper develops general results, based on simple concepts, about the effects of the supply of skills on the skill bias of technical change. The results are independent of the functional form of aggregate production, hold for a variety of different microfoundations of endogenous technology choices, for settings with more than two and potentially infinitely many different levels of skill, and apply to both discrete and infinitesimal changes in the supply of skills. They show that, under a scale-invariance restriction on the skill bias of technology, any increase in the relative supply of skills induces skill-biased technical change. Moreover, the total effect of an increase in relative skill supply on skill premia, accounting both for the induced technical change effect and the direct effect, can be positive only if aggregate production fails to be quasiconcave. This generalizes upon existing results on weak and strong relative equilibrium bias of technology, which are limited to the special case of purely labor-augmenting technology, two skill levels, and infinitesimal changes in the supply of skills.

The second part uses the developed theory to derive novel predictions on endogenous automation technology in assignment models of the type proposed by Teulings (1995). In the model investigated, a continuum of differentially skilled workers and capital, taking the form of machines that perfectly substitute for labor in the production of tasks, are assigned to a continuum of tasks, which in turn are combined to produce a single final good. Three results stand out. First, any increase in relative skill supply induces an improvement in capital productivity, which in turn leads to a rise in skill premia. Second, when there are external effects between firms' technologies, the effects of the induced improvement in capital productivity can be strong enough to outweigh the negative direct effect on skill premia, such that skill premia increase in relative skill supply. Third, in a two-country version of the model, trade

[36] This result also depends on the assumption that intellectual property rights are enforced only in the North and this does not change with trade opening. If property rights were enforced in the South starting with the opening of trade, results would likely be more similar to those obtained in my analysis.

[37] A similar effect is present in Burstein and Vogel (2017). While individual firms' technologies are exogenous in their model, trade induces a reallocation of market shares towards more productive firms, making technology more skill biased in the aggregate.

with a skill-scarce country reduces capital productivity and, thereby, lowers skill premia in a skill-abundant country. This effect may overturn the usual Heckscher-Ohlin effects, such that trade causes a fall in inequality in the skill-abundant country after adjustment of technology.

There are several starting points for future research. First, the results of the first part and the results on the effects of skill supply on automation may serve as a starting point for future explorations of the implications of directed technical change in general and endogenous automation in particular for the design of redistributive policies, such as redistributive labor income taxation. The results on the interaction of international trade and automation may as well be the starting point for an analysis of optimal trade policy along the lines of Costinot, Donaldson, Vogel and Werning (2015). Second, the predictions on determinants of automation technology from the second part should be of interest for empirical work. Especially the predictions on the effects of trade on automation are testable once a suitable source of exogenous variation across observational units in the exposure to trade is found. Finally, moving beyond the analysis of low-skill automation by relaxing the assumption that machines always have comparative advantage versus workers in less complex tasks seems an important goal for future theory.

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A. Local Weak Bias with Many Skills: A Generalization

Here, I present a generalized version of Proposition 5, the group-level version of local weak relative bias presented in Section 5 of the main text. While Proposition 5 holds for labor supply changes that are proportional within the two groups of skills considered, the following theorem extends the results to labor supply changes in arbitrary directions.

Theorem 3 (Local Weak Bias with Many Skills)**.** *Suppose that $\Theta \subset \mathbb{R}^M$ (for arbitrary $M \in \mathbb{N}$), F is twice continuously differentiable, and the equilibrium technology θ^* is differentiable in labor supply L. Moreover, suppose that Assumption 4 holds.*

Consider a partition of the skill set $\{S^{(1)}, S^{(2)}\}$, an initial labor supply L, and a change in labor supply ΔL. Let $\widetilde{\Delta L}$ denote the labor supply change that leads to equivalent changes in relative labor supply but leaves output unchanged, that is, $\widetilde{\Delta L} = \Delta L + \lambda L$ for $\lambda \in \mathbb{R}$ such that $\nabla_L F(L, \theta^*(L))\widetilde{\Delta L} = 0$. Suppose that $\sum_{s \in S^{(i)}} \widetilde{\Delta L}_s \neq 0$ for $i = 1, 2$.

Then, if $\sum_{s \in S^{(2)}} F_{L_s}(L, \theta^*(L))\widetilde{\Delta L}_s > 0$, the induced technical change $\Delta\theta^* = \nabla_L \theta^*(L)\Delta L$ locally raises the weighted average wage in group $S^{(2)}$ relative to the weighted average wage in group $S^{(1)}$,

$$\nabla_\theta \frac{\overline{w}_{S^{(2)},\widetilde{\Delta L}}(L, \theta^*(L))}{\overline{w}_{S^{(1)},\widetilde{\Delta L}}(L, \theta^*(L))} \Delta\theta^* \geq 0 \, ,$$

where the weighted average wages are given by, for $i = 1, 2$,

$$\overline{w}_{S^{(i)},\widetilde{\Delta L}}(L, \theta^*(L)) = \left| \sum_{s \in S^{(i)}} w_s(L, \theta^*(L)) \frac{\widetilde{\Delta L}_s}{\sum_{\tilde{s} \in S^{(i)}} \widetilde{\Delta L}_{\tilde{s}}} \right| \, .$$

Proof. The proof follows closely the proof of the local weak bias result for two skills (Proposition 1), replacing individual wages by weighted average wages within the two groups.

First, by the same arguments as for Proposition 1, we can focus on the technical change $\widetilde{\Delta\theta}^* = \nabla_L \theta^*(L)\widetilde{\Delta L}$ induced by the re-scaled labor supply change $\widetilde{\Delta L}$ instead of the technical change $\Delta\theta^*$ induced by ΔL. (The relative wage effects of $\widetilde{\Delta\theta}^*$ and $\Delta\theta^*$ are the same by Assumption 4.)

Next, the envelope arguments leading to equation (4) in the proof of Proposition 1 are independent of the dimensionality of labor supply. So, equation (4), showcasing the complementarity between labor supply change and induced technical change, holds here as well:

$$\nabla_\theta \left(\nabla_L F(L, \theta^*(L))\widetilde{\Delta L} \right) \widetilde{\Delta\theta}^* \geq 0 \,.$$

Separating the marginal products of the two skill groups $S^{(1)}$ and $S^{(2)}$, we obtain:

$$\nabla_\theta \left(\nabla_{L_{S^{(2)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(2)}} \right) \widetilde{\Delta\theta}^* \geq \nabla_\theta \left(-\nabla_{L_{S^{(1)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(1)}} \right) \widetilde{\Delta\theta}^* \,, \tag{14}$$

where the subscript $S^{(i)}$, for $i = 1, 2$, indicates that the corresponding vector contains the entries for all skills in $S^{(i)}$, and only those. Moreover, the labor supply change $\widetilde{\Delta L}$ satisfies, by construction and by hypothesis,

$$\nabla_{L_{S^{(2)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(2)}} = -\nabla_{L_{S^{(1)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(1)}} > 0 \,. \tag{15}$$

Dividing (15) by (14) yields

$$\frac{\nabla_\theta \left(\nabla_{L_{S^{(2)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(2)}} \right) \widetilde{\Delta\theta}^*}{\nabla_{L_{S^{(2)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(2)}}} \geq \frac{\nabla_\theta \left(-\nabla_{L_{S^{(1)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(1)}} \right) \widetilde{\Delta\theta}^*}{-\nabla_{L_{S^{(1)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(1)}}} \,,$$

which implies that

$$\nabla_\theta \frac{\nabla_{L_{S^{(2)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(2)}}}{-\nabla_{L_{S^{(1)}}} F(L, \theta^*(L))\widetilde{\Delta L}_{S^{(1)}}} \widetilde{\Delta\theta}^* \geq 0 \,.$$

Multiplying by $|\sum_{s \in S^{(1)}} (-\widetilde{\Delta L}_{S^{(1)}})| / |\sum_{s \in S^{(2)}} (-\widetilde{\Delta L}_{S^{(2)}})|$ yields the desired result. □

Theorem 3 shows that a change ΔL that raises the effective relative labor supply of some group of skills $S^{(2)}$ – in the sense that the transformed labor supply change $\widetilde{\Delta L}_{S^{(2)}}$ of this group has a positive effect on output – induces technical change that raises the weighted average wage of this group relative to the remaining skills. The weights in the average wages are determined by the transformed labor supply change $\widetilde{\Delta L}$.

Due to the central role of the transformed labor supply change, Theorem 3 is somewhat difficult to interpret in full generality.[38] The special cases considered in Proposition 5 and the subsequent discussion contain the central insights in a more transparent form.

[38] If the initial labor supply change ΔL does not vary systematically between the two groups, or if the two groups have very unequal sizes in terms of initial income shares, the transformed change $\widetilde{\Delta L}_s$ may even switch its sign within a group and, consequently, some weights in the average wages may be negative.

B. Global Weak and Strong Bias With a Continuum of Skills

Here, I consider the case where the skill set S is given by the continuum $[0,1]$. I show that the results from Section 5 (for an arbitrary but finite number of skills) are essentially unchanged with a continuum of skills.

Labor supply is given by $L \in \mathcal{L}^2_{++}([0,1])$, the space of strictly positive, square-(Lebesgue)-integrable functions on the unit interval. Thus, for each $\theta \in \Theta$, the aggregate production function $F(\,.\,,\theta)$ is a functional on $\mathcal{L}^2_{++}([0,1])$.

Assumption 1 on Θ and F remains valid: Θ is compact, F is continuous in θ, continuously differentiable in L, and features strictly positive marginal products of labor. The differentiability assumptions, however, require some clarification. In particular, I assume that F is continuously Fréchet differentiable in L and – to distinguish it from the finite-dimensional gradient in the main text – denote the Fréchet derivative of F in L at (L,θ) by $D_L F(L,\theta)$.

For every (L,θ), the Fréchet derivative is a bounded linear functional from $\mathcal{L}^2([0,1])$ to $\mathbb{R}$. By the Riesz representation theorem, it can be represented by a square-integrable function $s \mapsto F_s(L,\theta)$, such that

$$D_L F(L,\theta)[\Delta L] = \int_0^1 F_s(L,\theta) \Delta L_s \, ds \, ,$$

for every $\Delta L \in \mathcal{L}^2([0,1])$. The function $F_s(L,\theta)$ is determined only up to a subset of skills of measure zero. To avoid this indeterminacy, I assume that, at every (L,θ), there exists a continuous representing function $F_s(L,\theta)$.[39] This continuous function is then necessarily unique. I call the value of the unique continuous function $F_s(L,\theta)$ at s the marginal product of labor type s. The assumption of strictly positive marginal products of labor now translates into $F_s(L,\theta)$ being strictly positive for all s and at all feasible (L,θ).

I summarize the continuum version of Assumption 1 as follows.

Assumption 1′. *The set of feasible technologies Θ is a compact topological space. The aggregate production function F is continuous in technology θ and continuously Fréchet differentiable in labor supply L. At every (L,θ), the Fréchet derivative of F with respect to L can be represented (as described above) by a continuous function $s \mapsto F_s(L,\theta) \in \mathbb{R}_{++}$.*

With these clarifications, the equilibrium conditions for technology and wages are the same as in the finite-skills case. Technology satisfies

$$\theta^*(L) \in \max_{\theta \in \Theta} F(L,\theta)$$

with the same qualifications regarding uniqueness and selection as with finite skills. Wages equal marginal products of labor,

$$w_s(L,\theta) = F_s(L,\theta) \quad \text{for all } s \, ,$$

where $F_s(L,\theta)$, here and henceforth always, denotes the unique representing function mentioned in Assumption 1′.

[39] Instead, we could accept the economically irrelevant degree of indeterminacy in marginal products of labor and formulate the results below for almost all, instead of all, relative wages. This qualification would be the only adjustment needed, the remainder of the results would be unchanged.

We can now extend the weak and strong relative bias results for many skills to the continuum setting. I focus on extensions of the global results, Theorem 1 for weak relative bias and Theorem 2 for strong relative bias. Both theorems are unchanged, except that the skill set is a continuum and, for clarity, we replace Assumption 1 by Assumption 1′.

Theorem 1′ (Global Weak Bias with a Continuum of Skills)**.** *Let $S = [0,1]$ and impose Assumption 1′. Suppose that $(\Theta, \succeq^{psb})$ is a lattice, F is quasisupermodular in θ under $\succeq^{psb}$, and Assumption 4 holds.*

Then, for any two labor supply vectors L and L' with $L'_s/L'_{s'} \geq L_s/L_{s'}$ for all $s \geq s'$, the technology under L' is pervasively skill biased relative to the technology under L, that is, $\theta^*(L') \succeq^{psb} \theta^*(L)$.

Proof. The proof of Theorem 1 goes through, with the following adjustments.

First, the path $\widetilde{L}(\tau)$ in Lemma 2 now is a function from $[0,1]$ to $\mathcal{L}^2_{++}([0,1])$ instead of a function from $[0,1]$ to $\mathbb{R}^N_{++}$. In the proof of Lemma 2, the differential equation that yields $g(\tau)$ must be adjusted to

$$\int_{S^{up}} w_s(\widetilde{L}(\tau), \theta^*(L))(L'_s - L_s)\, ds + \int_{S^{down}} w_{s'}(\widetilde{L}(\tau), \theta^*(L))(L'_{s'} - L_{s'})\, ds' \frac{dg(\tau)}{d\tau} = 0\,.$$

The remainder of the proof of Lemma 2 is unchanged.

Next, in the proof of Theorem 1, equation (20) for the change of $F(.,\underline{\theta})$ along the path $\widetilde{L}(\tau)$ now becomes

$$\frac{d}{d\tau}F(\widetilde{L}(\tau),\underline{\theta}) = \int_0^1 \left[w_s(\widetilde{L}(\tau),\underline{\theta}) - \frac{w_{\tilde{s}}(\widetilde{L}(\tau),\underline{\theta})}{w_{\tilde{s}}(\widetilde{L}(\tau),\theta^*(L))} w_s(\widetilde{L}(\tau),\theta^*(L)) \right] \frac{d\widetilde{L}_s(\tau)}{d\tau}\, ds\,.$$

All the remaining parts of the proof are unchanged. □

Theorem 2′ (Global Strong Bias with a Continuum of Skills)**.** *Let $S = [0,1]$ and impose Assumption 1′. Suppose Assumption 3 holds. Then, the following is true.*

1. *If there exists a pervasive increase in relative skill supply that strictly raises all skill premia after adjustment of technology, then F^* is not quasiconcave.*

   Moreover, if F^* is not quasiconcave on some line along which relative skill supply increases pervasively, then there exists a pervasive increase in relative skill supply that does not lower all skill premia after adjustment of technology.

2. *If every pervasive increase in relative skill supply raises all skill premia after adjustment of technology, then F^* is quasiconvex on all lines along which relative skill supply increases pervasively.*

   Moreover, if F^* is quasiconvex, then no pervasive increase in relative skill supply strictly lowers all skill premia after adjustment of technology.

Proof. The structure of the proof is the same as for Theorem 2, but the adjustments needed to accommodate a continuum of skills are numerous enough to warrant a separate exposition here. I only prove part 1 of the theorem, however. As for Theorem 2, part 2 is proven analogously to part 1. So, from part 1 and the proof of Theorem 2, the proof of part 2 can be grasped immediately.

Step 1. I start by showing that, if there are labor supply vectors L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ (a pervasive increase in relative skill supply) such that $w^*_{s'}(L')/w^*_s(L') > w^*_{s'}(L)/w^*_s(L)$ for all $s' \geq s$ (all skill premia increase), then F^* cannot be quasiconcave.

Let $H(L) = \{\widetilde{L} \in \mathcal{L}^2_{++}([0,1]) \mid D_L F(L, \theta^*(L))[\widetilde{L} - L] = 0\}$ be the hyperplane tangent to the isoquant of F at L, holding technology fixed at $\theta^*(L)$. By Assumption 3, we can restrict attention to cases where $L' \in H(L)$. Let $\widetilde{L}(\tau)$ parameterize the line through L and L', such that $\widetilde{L}(0) = L$ and $\widetilde{L}(1) = L'$.

Now, to derive a contradiction, suppose that F^* is quasiconcave. Then, $H(L)$ must be tangent to the (convex) upper contour set of F^* at L. Hence, the restriction of F^* to the line $\widetilde{L}(\tau)$ must attain its maximum at $\tau = 0$ (i.e., at L). For univariate functions, quasiconcavity is equivalent to unimodality. So, $F^*(\widetilde{L}(\tau))$ must decrease in τ for $\tau \geq 0$.

Next, consider the derivative of $F(\widetilde{L}(\tau), \theta^*(L'))$ with respect to τ at $\tau = 1$, which is given by

$$\frac{d}{d\tau} F(\widetilde{L}(1), \theta^*(L')) = \int_0^1 F_s(L', \theta^*(L')) \frac{d\widetilde{L}_s(1)}{d\tau} \, ds \, ,$$

where we used that $\widetilde{L}(1) = L'$. Since both $F(\widetilde{L}(\tau), \theta^*(L))$ and the derivative $d\widetilde{L}(\tau)/d\tau$ are constant in τ (the latter because $\widetilde{L}(\tau)$ is a line), we have

$$\frac{d}{d\tau} F(\widetilde{L}(0), \theta^*(L)) = \int_0^1 F_s(L, \theta^*(L)) \frac{d\widetilde{L}_s(0)}{d\tau} \, ds = \int_0^1 F_s(L, \theta^*(L)) \frac{d\widetilde{L}_s(1)}{d\tau} \, ds = 0 \, .$$

Combining the previous two equations, we can write

$$\frac{d}{d\tau} F(\widetilde{L}(1), \theta^*(L')) = \int_0^1 \left[F_s(L', \theta^*(L')) - \lambda F_s(L, \theta^*(L)) \right] \frac{d\widetilde{L}(1)}{d\tau} \, ds \tag{16}$$

for any scalar λ.

Let $\widetilde{s} \in (0,1)$ denote a skill level such that $d\widetilde{L}_s(1)/d\tau \leq 0$ for all $s < \widetilde{s}$ and $d\widetilde{L}_s(1)/d\tau > 0$ for all $s > \widetilde{s}$. Such a skill exists for two reasons: first, we consider a pervasive increase in relative skill supply; second, the vectors L and L' must disagree on a subset of skills with positive measure. The latter is true because otherwise, L and L' were the same element in $\mathcal{L}^2([0,1])$ (elements in this space are only identified up to a subset of skills of measure zero). This in turn would imply that the Fréchet derivative and, thus, wages were the same at L and L', contradicting the fact that all skill premia increase strictly when moving from L to L'.

Given such a skill $\widetilde{s}$, set

$$\lambda = \frac{F_{\widetilde{s}}(L', \theta^*(L'))}{F_{\widetilde{s}}(L, \theta^*(L))} \, .$$

Inserting this into equation (16), we obtain

$$\frac{d}{d\tau} F(\widetilde{L}(1), \theta^*(L')) = \int_0^1 \left[F_s(L', \theta^*(L')) - \frac{F_{\widetilde{s}}(L', \theta^*(L'))}{F_{\widetilde{s}}(L, \theta^*(L))} F_s(L, \theta^*(L)) \right] \frac{d\widetilde{L}(1)}{d\tau} \, ds \, . \tag{17}$$

By hypothesis, we have

$$\frac{F_s(L', \theta^*(L'))}{F_{\widetilde{s}}(L', \theta^*(L'))} > (<) \frac{F_s(L, \theta^*(L))}{F_{\widetilde{s}}(L, \theta^*(L))}$$

for all $s > (<)\widetilde{s}$. Rearranging yields

$$F_s(L', \theta^*(L')) - \frac{F_{\widetilde{s}}(L', \theta^*(L'))}{F_{\widetilde{s}}(L, \theta^*(L))} F_s(L, \theta^*(L)) > (<) 0$$

for all $s > (<)\widetilde{s}$. By construction, the derivative $d\widetilde{L}_s(1)/d\tau$ has exactly the same sign, that is, $d\widetilde{L}_s(1)/d\tau > (\leq) 0$ for all $s > (<)\widetilde{s}$. Thus, from (17) we obtain that $dF(\widetilde{L}(1), \theta^*(L'))/d\tau > 0$, which contradicts our previous result that $F^*(\widetilde{L}(\tau))$ is decreasing in τ for $\tau \geq 0$.

So, the initial claim that F^* is quasiconcave must be false, which completes the first part of the proof.

Step 2. In the second part, I show that, if F^* is not quasiconcave along some line $\widetilde{L}(\tau)$ with $d \log \widetilde{L}_s(\tau)/d\tau$ increasing in s, then there exist L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ (a pervasive increase in relative skill supply) such that $w^*_{s'}(L')/w^*_s(L') > w^*_{s'}(L)/w^*_s(L)$ for at least one pair $s' > s$ (at least one skill premium increases strictly).

First, if F^* is not quasiconcave on $\widetilde{L}(\tau)$, it must be possible to parameterize the line such that

$$F^*(\widetilde{L}(0)) = F^*(\widetilde{L}(1)) > F^*(\widetilde{L}(\widetilde{\tau}))$$

for some $\widetilde{\tau} \in (0, 1)$. (This again follows from the fact that, for a univariate function, quasiconcavity is equivalent to unimodality.)

By the envelope theorem in Corollary 4 of Milgrom and Segal (2002), this implies

$$\begin{aligned} F^*(\widetilde{L}(\widetilde{\tau})) - F^*(\widetilde{L}(0)) &= \int_0^{\widetilde{\tau}} \int_0^1 F_s(\widetilde{L}(\tau), \theta^*(\widetilde{L}(\tau))) \frac{d\widetilde{L}_s(\tau)}{d\tau} \, ds \, d\tau < 0 \\ F^*(\widetilde{L}(1)) - F^*(\widetilde{L}(\widetilde{\tau})) &= \int_{\widetilde{\tau}}^1 \int_0^1 F_s(\widetilde{L}(\tau), \theta^*(\widetilde{L}(\tau))) \frac{d\widetilde{L}_s(\tau)}{d\tau} \, ds \, d\tau > 0 \, . \end{aligned}$$

It follows that there exist $\tau_1 < \tau_2$ such that

$$\int_0^1 F_s(\widetilde{L}(\tau_1), \theta^*(\widetilde{L}(\tau_1))) \frac{d\widetilde{L}_s(\tau_1)}{d\tau} \, ds < 0 < \lambda \int_0^1 F_s(\widetilde{L}(\tau_2), \theta^*(\widetilde{L}(\tau_2))) \frac{d\widetilde{L}_s(\tau_2)}{d\tau} \, ds \, ,$$

for any scalar $\lambda > 0$. Since $\widetilde{L}(\tau)$ is a line, $d\widetilde{L}(\tau_1)/d\tau$ and $d\widetilde{L}(\tau_2)/d\tau$ are equal. Thus, the two inequalities imply

$$\int_0^1 \left[F_s(\widetilde{L}(\tau_1), \theta^*(\widetilde{L}(\tau_1))) - \lambda F_s(\widetilde{L}(\tau_2), \theta^*(\widetilde{L}(\tau_2))) \right] \frac{d\widetilde{L}_s(\tau_1)}{d\tau} \, ds < 0 \, .$$

Let $\widetilde{s} \in (0, 1)$ denote a skill such that $d\widetilde{L}_s(\tau_1)/d\tau \geq 0$ if $s > \widetilde{s}$ and $d\widetilde{L}_s(\tau_1)/d\tau \leq 0$ if $s < \widetilde{s}$. Such a skill level must exist because we consider an increase in relative skill supply. Then, replace the constant λ as in Step 1 to obtain

$$\int_0^1 \left[F_s(\widetilde{L}(\tau_1), \theta^*(\widetilde{L}(\tau_1))) - \frac{F_{\widetilde{s}}(\widetilde{L}(\tau_1), \theta^*(\widetilde{L}(\tau_1)))}{F_{\widetilde{s}}(\widetilde{L}(\tau_2), \theta^*(\widetilde{L}(\tau_2)))} F_s(\widetilde{L}(\tau_2), \theta^*(\widetilde{L}(\tau_2))) \right] \frac{d\widetilde{L}_s(\tau_1)}{d\tau} \, ds < 0 \, . \quad (18)$$

Next suppose, to derive a contradiction, that all skill premia decrease weakly from τ_1 to τ_2, that is,

$$\frac{F_s(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1)))}{F_{\widetilde{s}}(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1)))} \gtreqless \frac{F_s(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))}{F_{\widetilde{s}}(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))} \quad \text{if} \quad s \gtreqless \widetilde{s}\,.$$

It follows that the first vector in the inner product on the left-hand side of inequality (18) has positive (negative) entries for s above (below) $\widetilde{s}$. But by construction of $\widetilde{s}$, the same holds for the second vector $d\widetilde{L}(\tau_1)/d\tau$. Their product must hence be positive, in contradiction to inequality (18).

Thus, when moving from $\widetilde{L}(\tau_1)$ to $\widetilde{L}(\tau_2)$ (a pervasive increase in relative skill supply), at least one skill premium must increase strictly. □

C. Omitted Proofs

This section contains all proofs omitted from the main text.

C.1. Proof of Theorem 1

The proof of Theorem 1 uses the following lemma.

Lemma 2. *Consider any L and L' such that $F(L,\theta^*(L)) = F(L',\theta^*(L))$. Then, there exists a differentiable path $\widetilde{L} : [0,1] \to \mathbb{R}^N_{++}$, $\tau \mapsto \widetilde{L}(\tau)$, such that*

(i) $\widetilde{L}(0) = L$ and $\widetilde{L}(1) = L'$,

(ii) every component $\widetilde{L}_s(\tau)$, for $s \in S$, is monotonic,

(iii) $F(\widetilde{L}(\tau),\theta^*(L)) = F(L,\theta^*(L))$ for every $\tau \in [0,1]$.

Proof. For $L = L'$ the lemma is trivial, so I focus on the case where $L \neq L'$.Ie construct a path with the desired properties. Let S^{up} denote the set of skills with $L'_s > L_s$ and $S^{down} = S \setminus S^{up}$. Then, for all $s \in S^{up}$, let

$$\widetilde{L}_s(\tau) = L_s + \tau(L'_s - L_s)\,.$$

For all $s \in S^{down}$, let

$$\widetilde{L}_s(\tau) = L_s + g(\tau)(L'_s - L_s)$$

for a real-valued, differentiable function g.

Finally, choose g such that it solves the differential equation

$$\sum_{s\in S^{up}} F_{L_s}(\widetilde{L}(\tau),\theta^*(L))(L'_s - L_s) + \sum_{s'\in S^{down}} F_{L_{s'}}(\widetilde{L}(\tau),\theta^*(L))(L'_{s'} - L_{s'})\frac{dg(\tau)}{d\tau} = 0$$

with the initial condition $g(0) = 0$. Such a solution exists by the Peano existence theorem. Moreover, by construction, we have that $dg(\tau)/d\tau \geq 0$ for all $\tau \in (0,1)$ and $g(1) = 1$. To see the latter, suppose that $g(1) < 1$ (the case where $g(1) > 1$ can be excluded analogously). Then, we have $\widetilde{L}_s(1) = L'_s$ for $s \in S^{up}$ but $\widetilde{L}_s(1) \geq L'_s$ for $s \in S^{up}$ with strict inequality for at least some $s \in S^{up}$. So, $F(\widetilde{L}(1),\theta^*(L)) > F(L',\theta^*(L)) = F(L,\theta^*(L))$, whereas the differential equation implies that $F(\widetilde{L}(1),\theta^*(L)) = F(L,\theta^*(L))$ (i.e., we remain on the isoquant when following the path characterized by the differential equation): a contradiction.

Thus, the constructed path $\widetilde{L}$ satisfies all properties required by the lemma. □

With Lemma 2, we can prove Theorem 1. First, by Assumption 4, we can scale L' up or down without inducing technical change that affects relative wages. So, we can restrict attention to L' such that $F(L,\theta^*(L)) = F(L',\theta^*(L))$.

Next, take some infimum $\underline{\theta} \in \inf(\theta^*(L),\theta^*(L'))$ and consider a monotonic and differentiable path $\widetilde{L}(\tau)$ from L to L' as described by Lemma 2. How does $F(\widetilde{L}(\tau),\underline{\theta})$ vary along this path? As $F(\widetilde{L}(\tau),\theta^*(L))$ is constant in τ, we can write, at any point $\tau \in (0,1)$:

$$\frac{d}{d\tau}F(\widetilde{L}(\tau),\underline{\theta}) = \frac{d}{d\tau}F(\widetilde{L}(\tau),\underline{\theta}) - \lambda\frac{d}{d\tau}F(\widetilde{L}(\tau),\theta^*(L)) \tag{19}$$

for any scalar λ.

Now, fix an arbitrary τ and let $\widetilde{s}$ denote a skill level such that $d\widetilde{L}_s(\tau)/d\tau \leq 0$ for all $s \leq \widetilde{s}$ and $d\widetilde{L}_s(\tau)/d\tau \geq 0$ for all $s > \widetilde{s}$. Such a skill exists because we consider a pervasive increase in relative skill supply. Then, set

$$\lambda = \frac{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau),\underline{\theta})}{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau),\theta^*(L))} \; .$$

Inserting this into equation (19) and writing the right-hand side more extensively, we obtain

$$\frac{d}{d\tau}F(\widetilde{L}(\tau),\underline{\theta}) = \left[\nabla_L F(\widetilde{L}(\tau),\underline{\theta}) - \frac{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau),\underline{\theta})}{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau),\theta^*(L))}\nabla_L F(\widetilde{L}(\tau),\theta^*(L))\right]\frac{d\widetilde{L}(\tau)}{d\tau} \; . \tag{20}$$

Since $\theta^*(L)$ is pervasively skill biased relative to $\underline{\theta}$, we have

$$\frac{F_{L_s}(\widetilde{L}(\tau),\theta^*(L))}{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau),\theta^*(L))} \geq (\leq) \frac{F_{L_s}(\widetilde{L}(\tau),\underline{\theta})}{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau),\underline{\theta})}$$

for all $s \geq (\leq) \widetilde{s}$. Rearranging yields

$$F_{L_s}(\widetilde{L}(\tau),\underline{\theta}) - \frac{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau),\underline{\theta})}{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau),\theta^*(L))}F_{L_s}(\widetilde{L}(\tau),\theta^*(L)) \leq (\geq) 0$$

for all $s \geq (\leq) \widetilde{s}$. By construction, the entries of the vector $d\widetilde{L}(\tau)/d\tau$ have exactly the opposite signs. Thus, from (20) we obtain that $dF(\widetilde{L}(\tau),\underline{\theta})/d\tau \leq 0$.

Since this holds for all $\tau \in (0,1)$, it must hold that

$$F(L',\underline{\theta}) \leq F(L,\underline{\theta}) \leq F(L,\theta^*(L)) = F(L',\theta^*(L)) \; ,$$

where the second inequality stems from the equilibrium technology condition (2). This holds for any infimum $\underline{\theta}$. Thus, by quasisupermodularity, there must exist a supremum $\overline{\theta}$ of $\theta^*(L)$ and $\theta^*(L')$ such that

$$F(L',\overline{\theta}) \geq F(L',\theta^*(L')) \; .$$

This, by the same arguments as in the proof of Proposition 3, implies that $\theta^*(L')$ itself must be a supremum of $\theta^*(L)$ and $\theta^*(L')$. Hence, $\theta^*(L') \succeq^{psb} \theta^*(L)$.

C.2. Proof of Theorem 2

Part 1 of Theorem 2 I start by proving part 1 of Theorem 2. The proof is divided into two steps.

Step 1. I first show that, if there are labor supply vectors L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ (a pervasive increase in relative skill supply) such that $w^*_{s'}(L')/w^*_s(L') > w^*_{s'}(L)/w^*_s(L)$ for all $s' \geq s$ (all skill premia increase), then F^* cannot be quasiconcave.

Let $H(L) = \{\widetilde{L} \in \mathbb{R}^N_{++} \mid \nabla_L F(L, \theta^*(L))(\widetilde{L} - L) = 0\}$ be the hyperplane tangent to the isoquant of F at L, holding technology fixed at $\theta^*(L)$. By Assumption 3, we can restrict attention to cases where $L' \in H(L)$. Let $\widetilde{L}(\tau)$ parameterize the line through L and L', such that $\widetilde{L}(0) = L$ and $\widetilde{L}(1) = L'$.

Now, to derive a contradiction, suppose that F^* is quasiconcave. Then, $H(L)$ must be tangent to the (convex) upper contour set of F^* at L. Hence, the restriction of F^* to the line $\widetilde{L}(\tau)$ must attain its maximum at $\tau = 0$ (i.e., at L). For univariate functions, quasiconcavity is equivalent to unimodality. So, $F^*(\widetilde{L}(\tau))$ must decrease in τ for $\tau \geq 0$.

Next, consider the derivative of $F(\widetilde{L}(\tau), \theta^*(L'))$ with respect to τ at $\tau = 1$:

$$\frac{d}{d\tau} F(\widetilde{L}(1), \theta^*(L')) = \nabla_L F(L', \theta^*(L')) \frac{d\widetilde{L}(1)}{d\tau},$$

where we used that $\widetilde{L}(1) = L'$. Since both $F(\widetilde{L}(\tau), \theta^*(L))$ and the derivative $d\widetilde{L}(\tau)/d\tau$ are constant in τ (the latter because $\widetilde{L}(\tau)$ is a line), we have

$$\frac{d}{d\tau} F(\widetilde{L}(0), \theta^*(L)) = \nabla_L F(L, \theta^*(L)) \frac{d\widetilde{L}(0)}{d\tau} = \nabla_L F(L, \theta^*(L)) \frac{d\widetilde{L}(1)}{d\tau} = 0\,.$$

Combining the previous two equations, we can write

$$\frac{d}{d\tau} F(\widetilde{L}(1), \theta^*(L')) = \nabla_L F(L', \theta^*(L')) \frac{d\widetilde{L}(1)}{d\tau} - \lambda \nabla_L F(L, \theta^*(L)) \frac{d\widetilde{L}(1)}{d\tau} \tag{21}$$

for any scalar λ.

Let $\tilde{s}$ denote a skill level such that $d\widetilde{L}_s(1)/d\tau \leq 0$ for all $s \leq \tilde{s}$ and $d\widetilde{L}_s(1)/d\tau \geq 0$ for all $s > \tilde{s}$. Such a skill exists because we consider a pervasive increase in relative skill supply. Then, set

$$\lambda = \frac{F_{L_{\tilde{s}}}(L', \theta^*(L'))}{F_{L_{\tilde{s}}}(L, \theta^*(L))}\,.$$

Inserting this into equation (21), we obtain

$$\frac{d}{d\tau} F(\widetilde{L}(1), \theta^*(L')) = \left[\nabla_L F(L', \theta^*(L')) - \frac{F_{L_{\tilde{s}}}(L', \theta^*(L'))}{F_{L_{\tilde{s}}}(L, \theta^*(L))} \nabla_L F(L, \theta^*(L))\right] \frac{d\widetilde{L}(1)}{d\tau}\,. \tag{22}$$

By hypothesis, we have

$$\frac{F_{L_s}(L', \theta^*(L'))}{F_{L_{\tilde{s}}}(L', \theta^*(L'))} > (<) \frac{F_{L_s}(L, \theta^*(L))}{F_{L_{\tilde{s}}}(L, \theta^*(L))}$$

for all $s > (<)\tilde{s}$. Rearranging yields

$$F_{L_s}(L', \theta^*(L')) - \frac{F_{L_{\tilde{s}}}(L', \theta^*(L'))}{F_{L_{\tilde{s}}}(L, \theta^*(L))} F_{L_s}(L, \theta^*(L)) > (<) 0$$

for all $s > (<)\tilde{s}$. By construction, the entries of the vector $d\tilde{L}(\tau)/d\tau$ have exactly the same signs, that is, $d\tilde{L}_s(1)/d\tau \geq (\leq) 0$ for all $s > (<)\tilde{s}$, with strict inequalities at least for $s = 1$ and $s = N$ (otherwise, L would be proportional to L', which, by Assumption 3, is incompatible with a strict increase in all skill premia). Thus, from (22) we obtain that $dF(\tilde{L}(1), \theta^*(L'))/d\tau > 0$, which is incompatible with our previous result that $F^*(\tilde{L}(\tau))$ is decreasing in τ for $\tau \geq 0$.

So, the initial claim that F^* is quasiconcave must be false, which completes the first step of the proof.

Step 2. In the second step, I show that, if F^* is not quasiconcave along some line $\tilde{L}(\tau)$ with $d \log \tilde{L}_s(\tau)/d\tau$ increasing in s, then there exist L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ (a pervasive increase in relative skill supply) such that $w^*_{s'}(L')/w^*_s(L') > w^*_{s'}(L)/w^*_s(L)$ for at least one pair $s' > s$ (at least one skill premium increases strictly).

First, if F^* is not quasiconcave on $\tilde{L}(\tau)$, it must be possible to parameterize the line such that

$$F^*(\tilde{L}(0)) = F^*(\tilde{L}(1)) > F^*(\tilde{L}(\tilde{\tau}))$$

for some $\tilde{\tau} \in (0, 1)$. (This again follows from the fact that, for a univariate function, quasiconcavity is equivalent to unimodality.)

By the envelope theorem in Corollary 4 of Milgrom and Segal (2002), this implies

$$F^*(\tilde{L}(\tilde{\tau})) - F^*(\tilde{L}(0)) = \int_0^{\tilde{\tau}} \nabla_L F(\tilde{L}(\tau), \theta^*(\tilde{L}(\tau))) \frac{d\tilde{L}(\tau)}{d\tau} \, d\tau < 0$$
$$F^*(\tilde{L}(1)) - F^*(\tilde{L}(\tilde{\tau})) = \int_{\tilde{\tau}}^1 \nabla_L F(\tilde{L}(\tau), \theta^*(\tilde{L}(\tau))) \frac{d\tilde{L}(\tau)}{d\tau} \, d\tau > 0 \, .$$

It follows that there exist $\tau_1 < \tau_2$ such that

$$\nabla_L F(\tilde{L}(\tau_1), \theta^*(\tilde{L}(\tau_1))) \frac{d\tilde{L}(\tau_1)}{d\tau} < 0 < \lambda \nabla_L F(\tilde{L}(\tau_2), \theta^*(\tilde{L}(\tau_2))) \frac{d\tilde{L}(\tau_2)}{d\tau} \, ,$$

for any scalar $\lambda > 0$. Since $\tilde{L}(\tau)$ is a line, $d\tilde{L}(\tau_1)/d\tau$ and $d\tilde{L}(\tau_2)/d\tau$ are equal. Thus, the two inequalities imply

$$\left[\nabla_L F(\tilde{L}(\tau_1), \theta^*(\tilde{L}(\tau_1))) - \lambda \nabla_L F(\tilde{L}(\tau_2), \theta^*(\tilde{L}(\tau_2))) \right] \frac{d\tilde{L}(\tau_1)}{d\tau} < 0 \, .$$

As in Step 1, let $\tilde{s}$ denote a skill such that $d\tilde{L}_s(\tau_1)/d\tau$ is greater (smaller) zero if s is greater (smaller) $\tilde{s}$. Then replace the constant λ to obtain

$$\left[\nabla_L F(\tilde{L}(\tau_1), \theta^*(\tilde{L}(\tau_1))) - \frac{F_{L_{\tilde{s}}}(\tilde{L}(\tau_1), \theta^*(\tilde{L}(\tau_1)))}{F_{L_{\tilde{s}}}(\tilde{L}(\tau_2), \theta^*(\tilde{L}(\tau_2)))} \nabla_L F(\tilde{L}(\tau_2), \theta^*(\tilde{L}(\tau_2))) \right] \frac{d\tilde{L}(\tau_1)}{d\tau} < 0 \, . \tag{23}$$

Next suppose, to derive a contradiction, that all skill premia decrease from τ_1 to τ_2, that is,

$$\frac{F_{L_s}(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1)))}{F_{L_{\tilde{s}}}(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1)))} \gtreqless \frac{F_{L_s}(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))}{F_{L_{\tilde{s}}}(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))} \quad \text{if} \quad s \gtreqless \tilde{s}\,.$$

It follows that the first vector in the inner product on the left-hand side of inequality (23) has positive (negative) entries for s above (below) $\tilde{s}$. But by construction of $\tilde{s}$, the same holds for the second vector $d\widetilde{L}(\tau_1)/d\tau$. Their product must hence be positive, in contradiction to inequality (23).

Thus, when moving from $\widetilde{L}(\tau_1)$ to $\widetilde{L}(\tau_2)$ (a pervasive increase in relative skill supply), at least one skill premium must increase strictly.

Part 2 of Theorem 2 The proof of the second part of Theorem 2 is again divided into two steps. Step 1(2) follows closely step 2(1) in the proof of part 1 above.

Step 1. I first show that, if every pervasive increase in relative skill supply raises all skill premia, then F^* must be quasiconvex on all lines along which relative skill supply increases pervasively. The proof is by contradiction.

For that purpose, suppose that F^* is not quasiconvex on some line $\widetilde{L}(\tau)$ with $d\log\widetilde{L}_s(\tau)/d\tau$ increasing in s. Then, it must be possible to parameterize the line such that

$$F^*(\widetilde{L}(0)) = F^*(\widetilde{L}(1)) < F^*(\widetilde{L}(\tilde{\tau}))$$

for some $\tilde{\tau} \in (0,1)$.

By the envelope theorem in Corollary 4 of Milgrom and Segal (2002), this implies

$$F^*(\widetilde{L}(\tilde{\tau})) - F^*(\widetilde{L}(0)) = \int_0^{\tilde{\tau}} \nabla_L F(\widetilde{L}(\tau),\theta^*(\widetilde{L}(\tau)))\frac{d\widetilde{L}(\tau)}{d\tau}\,d\tau > 0$$
$$F^*(\widetilde{L}(1)) - F^*(\widetilde{L}(\tilde{\tau})) = \int_{\tilde{\tau}}^{1} \nabla_L F(\widetilde{L}(\tau),\theta^*(\widetilde{L}(\tau)))\frac{d\widetilde{L}(\tau)}{d\tau}\,d\tau < 0\,.$$

It follows that there exist $\tau_1 < \tau_2$ such that

$$\nabla_L F(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1)))\frac{d\widetilde{L}(\tau_1)}{d\tau} > 0 > \lambda\nabla_L F(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))\frac{d\widetilde{L}(\tau_2)}{d\tau}\,,$$

for any scalar $\lambda > 0$. Since $\widetilde{L}(\tau)$ is a line, $d\widetilde{L}(\tau_1)/d\tau$ and $d\widetilde{L}(\tau_2)/d\tau$ are equal. Thus, the two inequalities imply

$$\left[\nabla_L F(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1))) - \lambda\nabla_L F(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))\right]\frac{d\widetilde{L}(\tau_1)}{d\tau} > 0\,.$$

Denote again by $\tilde{s}$ a skill such that $d\widetilde{L}_s(\tau_1)/d\tau$ is greater (smaller) zero if s is greater (smaller) $\tilde{s}$. Then replace the constant λ to obtain

$$\left[\nabla_L F(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1))) - \frac{F_{L_{\tilde{s}}}(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1)))}{F_{L_{\tilde{s}}}(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))}\nabla_L F(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))\right]\frac{d\widetilde{L}(\tau_1)}{d\tau} > 0\,. \quad (24)$$

We know by hypothesis that all skill premia increase from τ_1 to τ_2, that is,

$$\frac{F_{L_s}(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1)))}{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau_1),\theta^*(\widetilde{L}(\tau_1)))} \lesseqgtr \frac{F_{L_s}(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))}{F_{L_{\widetilde{s}}}(\widetilde{L}(\tau_2),\theta^*(\widetilde{L}(\tau_2)))} \quad \text{if} \quad s \gtreqless \widetilde{s}\,.$$

It follows that the first vector in the inner product on the left-hand side of inequality (24) has negative (positive) entries for s above (below) $\widetilde{s}$. But by construction of $\widetilde{s}$, the entries of the second vector $d\widetilde{L}(\tau_1)/d\tau$ have the opposite signs. Their product must hence be negative, in contradiction to inequality (24).

Thus, the initial claim that F^* is not quasiconvex on $\widetilde{L}(\tau)$ must be false. Since this reasoning holds for any line along which relative skill supply increases pervasively, F^* must be quasiconvex on all such lines.

Step 2. Here, I show that, if F^* is quasiconvex, then no pervasive increase in relative skill supply can strictly reduce all skill premia. The proof is again by contradiction.

To this end, suppose that there exist L and L' with $L'_{s'}/L'_s \geq L_{s'}/L_s$ for all $s' \geq s$ such that $w^*_{s'}(L')/w^*_s(L') < w^*_{s'}(L)/w^*_s(L)$ for all $s' \geq s$.

Let $H(L) = \{\widetilde{L} \in \mathbb{R}^N_{++} \mid \nabla_L F(L,\theta^*(L))(\widetilde{L} - L) = 0\}$ be the hyperplane tangent to the isoquant of F at L, holding technology fixed at $\theta^*(L)$. By Assumption 3, we can restrict attention to cases where $L' \in H(L)$. Let $\widetilde{L}(\tau)$ parameterize the line through L and L', such that $\widetilde{L}(0) = L$ and $\widetilde{L}(1) = L'$.

Since F^* is quasiconvex, $H(L)$ must be tangent to the (convex) lower contour set of F^* at L. Hence, the restriction of F^* to the line $\widetilde{L}(\tau)$ must attain its minimum at $\tau = 0$ (i.e., at L). Moreover, the equivalence between quasiconcavity and unimodality for univariate functions (see step 1 of part 1 above) implies that $F^*(\widetilde{L}(\tau))$ must increase in τ for $\tau \geq 0$.

Next, consider the derivative of $F(\widetilde{L}(\tau),\theta^*(L'))$ with respect to τ at $\tau = 1$, which is given by

$$\frac{d}{d\tau}F(\widetilde{L}(1),\theta^*(L')) = \nabla_L F(L',\theta^*(L'))\frac{d\widetilde{L}(1)}{d\tau}\,,$$

where we used that $\widetilde{L}(1) = L'$. Since both $F(\widetilde{L}(\tau),\theta^*(L))$ and the derivative $d\widetilde{L}(\tau)/d\tau$ are constant in τ (the latter because $\widetilde{L}(\tau)$ is a line), we have

$$\frac{d}{d\tau}F(\widetilde{L}(0),\theta^*(L)) = \nabla_L F(L,\theta^*(L))\frac{d\widetilde{L}(0)}{d\tau} = \nabla_L F(L,\theta^*(L))\frac{d\widetilde{L}(1)}{d\tau} = 0\,.$$

Combining the previous two equations, we can write

$$\frac{d}{d\tau}F(\widetilde{L}(1),\theta^*(L')) = \nabla_L F(L',\theta^*(L'))\frac{d\widetilde{L}(1)}{d\tau} - \lambda\nabla_L F(L,\theta^*(L))\frac{d\widetilde{L}(1)}{d\tau} \tag{25}$$

for any scalar λ.

Let $\widetilde{s}$ denote a skill level such that $d\widetilde{L}_s(1)/d\tau \leq 0$ for all $s \leq \widetilde{s}$ and $\widetilde{L}_s(1)/d\tau \geq 0$ for all $s > \widetilde{s}$. Such a skill exists because we consider a pervasive increase in relative skill supply. Then, set

$$\lambda = \frac{F_{L_{\widetilde{s}}}(L',\theta^*(L'))}{F_{L_{\widetilde{s}}}(L,\theta^*(L))}\,.$$

Inserting this into equation (25), we obtain

$$\frac{d}{d\tau}F(\widetilde{L}(1),\theta^*(L')) = \left[\nabla_L F(L',\theta^*(L')) - \frac{F_{L_{\widetilde{s}}}(L',\theta^*(L'))}{F_{L_{\widetilde{s}}}(L,\theta^*(L))}\nabla_L F(L,\theta^*(L))\right]\frac{d\widetilde{L}(1)}{d\tau}\ . \tag{26}$$

By hypothesis, we have

$$\frac{F_{L_s}(L',\theta^*(L'))}{F_{L_{\widetilde{s}}}(L',\theta^*(L'))} < (>)\frac{F_{L_s}(L,\theta^*(L))}{F_{L_{\widetilde{s}}}(L,\theta^*(L))}$$

for all $s > (<)\widetilde{s}$. Rearranging yields

$$F_{L_s}(L',\theta^*(L')) - \frac{F_{L_{\widetilde{s}}}(L',\theta^*(L'))}{F_{L_{\widetilde{s}}}(L,\theta^*(L))}F_{L_s}(L,\theta^*(L)) < (>)0$$

for all $s > (<)\widetilde{s}$. By construction, the entries of the vector $d\widetilde{L}(\tau)/d\tau$ have exactly the opposite signs, that is, $d\widetilde{L}_s(1)/d\tau \geq (\leq)0$ for all $s > (<)\widetilde{s}$, with strict inequalities at least for $s = 1$ and $s = N$ (otherwise, L and L' would be proportional, which, by Assumption 3 is incompatible with strictly decreasing skill premia). Thus, from (26) we obtain that $dF(\widetilde{L}(1),\theta^*(L'))/d\tau < 0$, which is contradicts our previous result that $F^*(\widetilde{L}(\tau))$ is increasing in τ for $\tau \geq 0$.

So, the initial claim must be false: if F^* is quasiconvex, there cannot exist a pervasive increase in relative skill supply that strictly lowers all skill premia.

C.3. Proof of Lemma 1

It is useful to first establish the following lemma.

Lemma 3. *For any feasible technology (α,β) and labor supply L, let $\widetilde{x}(L,\alpha,\beta)$ denote the assignment threshold for capital in the exogenous-technology equilibrium, that is, let $\widetilde{x}(L,\alpha,\beta)$ be such that all tasks below it are produced by capital and all tasks above it by labor.*

Then, for any feasible (α,β) and (α',β'), if $\widetilde{x}(L,\alpha,\beta) \leq \widetilde{x}(L,\alpha',\beta')$,

$$\frac{w_{s'}(L,\alpha,\beta)}{w_s(L,\alpha,\beta)} \leq \frac{w_{s'}(L,\alpha',\beta')}{w_s(L,\alpha',\beta')} \quad \textit{for all } s \leq s'\ .$$

If $\widetilde{x}(L,\alpha,\beta) < \widetilde{x}(L,\alpha',\beta')$, the inequality for skill premia holds strictly for at least a set of skills of strictly positive measure.

Proof. Given $\widetilde{x}(L,\alpha,\beta)$, a profit-maximizing assignment of workers to tasks requires that

$$w_s(L,\alpha,\beta) = \max_{x\in[\widetilde{x}(L,\alpha,\beta),1]}\frac{\partial Y}{\partial Y_x}\gamma(s,x)\ ,$$

where $\partial Y/\partial Y_x$ denotes the Gateaux derivative of the final good production function in direction of Y_x. Taking logs and differentiating with respect to s yields, by an envelope argument,

$$\frac{d\log w_s(L,\alpha,\beta)}{ds} = \frac{\partial \log\gamma(s,m(s))}{\partial s}\ ,$$

where $m(s)$ is the matching function that returns the task assigned to each skill s.

Now, consider an increase in the threshold task from $\tilde{x}(L, \alpha, \beta)$ to $\tilde{x}(L, \alpha', \beta')$. The same arguments as in Lemma 5 in Costinot and Vogel (2010) imply that this shifts the matching function upwards everywhere: $m'(s) \geq m(s)$ for all s, where $m'(s)$ denotes the matching function under (L, α', β'). By log supermodularity of $\gamma(s, x)$, as assumed in equation (8), this implies

$$\frac{d \log w_s(L, \alpha, \beta)}{ds} = \frac{\partial \log \gamma(s, m(s))}{\partial s} \leq \frac{\partial \log \gamma(s, m'(s))}{\partial s} = \frac{d \log w_s(L, \alpha', \beta')}{ds} \,.$$

For the strict version, note that the matching function must shift upwards strictly at least for those skills matched to tasks on $(\tilde{x}(L, \alpha, \beta), \tilde{x}(L, \alpha', \beta'))$, which, by the above reasoning, implies that skill premia for this segment of skills increase strictly. □

I proceed with the proof of Lemma 1.

Part 1. For linear homogeneity of the aggregate production function, note that firm profits are linear homogeneous in labor and capital. Thus, the optimal capital input scales with labor supply, the optimal assignment is independent of the scale of labor supply, and aggregate output in equilibrium is linear homogeneous in labor (see Lemma 5 in Online Appendix G.1 for more details).

Part 3. For the restriction of the set of feasible technologies, note that we can write net output as

$$\beta \left[\left(\int_0^{\tilde{x}} \tilde{K}_x^{\frac{\epsilon-1}{\epsilon}} \, dx + \int_{\tilde{x}}^1 \left(\gamma(m^{-1}(x), x) L_{m^{-1}(x)} \frac{dm^{-1}(x)}{dx} \right)^{\frac{\epsilon-1}{\epsilon}} dx \right)^{\frac{\epsilon}{\epsilon-1}} - \frac{r}{\alpha\beta} \int_0^{\tilde{x}} \tilde{K}_x \, dx \right] , \quad (27)$$

where $\tilde{K}_x := \alpha K_x$ for all x.

Consider now a feasible technology (α, β) (i.e., one that satisfies the productivity frontier) with $\alpha > \overline{\alpha}$. By definition of $\overline{\alpha}$, we have that $\alpha\beta \leq \overline{\alpha}\overline{\beta}$ and $\beta < \overline{\beta}$. With the previous representation of net output, it is clear that (α, β) must lead to strictly lower net output than $(\overline{\alpha}, \overline{\beta})$, irrespective of labor supply L, and will hence never be chosen in equilibrium.

Finally, it is obvious that firms will never choose a technology that doesn't satisfy the productivity frontier with equality. Thus, the only potential equilibrium technologies are those in Θ.

Part 2. To establish the ordering of technologies, consider any $(\alpha, \beta), (\alpha', \beta') \in \Theta$ with $\alpha' > \alpha$. At given technology and labor input, the assignment rules $\tilde{x}$, m, and $\{K_x\}_{x \in [0, \tilde{x}]}$ are chosen such as to maximize net output (27), which is equivalent to the maximization

$$\tilde{Y}(L, \alpha, \beta) := \max_{\tilde{x}, m, \{\tilde{K}_x\}_{x \in [0, \tilde{x}]}} \left(\int_0^{\tilde{x}} \tilde{K}_x^{\frac{\epsilon-1}{\epsilon}} \, dx + \int_{\tilde{x}}^1 \left(\gamma(m^{-1}(x), x) L_{m^{-1}(x)} \frac{dm^{-1}(x)}{dx} \right)^{\frac{\epsilon-1}{\epsilon}} dx \right)^{\frac{\epsilon}{\epsilon-1}} - \frac{r}{\alpha\beta} \int_0^{\tilde{x}} \tilde{K}_x \, dx \,.$$

Technology enters $\tilde{Y}$ only via the product $\alpha\beta$. The product $\alpha g^{-1}(\alpha, \overline{g})$, where $g^{-1}(\alpha, \overline{g})$ is the inverse of the productivity frontier g with respect to β, is strictly concave in α. Thus, $\overline{\alpha} \geq \alpha' > \alpha$ implies that $\alpha'\beta' > \alpha\beta$. It follows that technology (α', β') leads to a strictly higher 'pseudo net output' $\tilde{Y}$ than technology (α, β). Moreover, since $\tilde{Y}(L, \alpha, \beta) \equiv F(L, \alpha, \beta)/\beta$, $\tilde{Y}$ is linear homogeneous in L. We thus obtain:

$$\widetilde{Y}(L,\alpha',\beta') = \int_0^1 \widetilde{w}_s(L,\alpha',\beta')L_s\,ds > \int_0^1 \widetilde{w}_s(L,\alpha,\beta)L_s\,ds = \widetilde{Y}(L,\alpha,\beta)\,, \tag{28}$$

where $\widetilde{w}_s(L,\alpha,\beta) := w_s(L,\alpha,\beta)/\beta$ (for all s) is the marginal product of labor type s in the pseudo net output function $\widetilde{Y}$.

Next, the optimal assignment threshold $\widetilde{x}$ must satisfy the condition

$$\widetilde{w}_{\underline{s}}(L,\alpha,\beta) = \frac{\gamma(\underline{s},\widetilde{x}(L,\alpha,\beta))r}{\alpha\beta}\,.$$

Suppose now, to derive a contradiction, that $\widetilde{x}(L,\alpha',\beta') \leq \widetilde{x}(L,\alpha,\beta)$. Then, the threshold condition implies that $\widetilde{w}_{\underline{s}}(L,\alpha',\beta') < \widetilde{w}_{\underline{s}}(L,\alpha,\beta)$. Moreover, by Lemma 3, skill premia are lower under (α',β') than under (α,β). Thus, we obtain $\widetilde{w}_s(L,\alpha',\beta') < \widetilde{w}_s(L,\alpha,\beta)$ for all s, a contradiction to (28).

So, we must have $\widetilde{x}(L,\alpha',\beta') > \widetilde{x}(L,\alpha,\beta)$. By Lemma 3, this implies that all skill premia are greater under (α',β') than under (α,β), and strictly greater on a skill set of non-zero measure. Thus, $(\alpha',\beta') \succeq^{psb} (\alpha,\beta)$ and $(\alpha,\beta) \not\succeq^{psb} (\alpha',\beta')$.

This establishes the equivalence between increases in α and pervasive skill bias given by part 2 of Lemma 1.