

Firm Quality Dynamics and the Slippery Slope of Credit Intervention

Wenhao Li* Ye Li†

October 25, 2025

Abstract

A salient trend in crisis intervention has emerged in recent decades: government and central banks have offered funding directly to nonfinancial firms, bypassing banks and other credit intermediaries. We analyze the long-term consequences of such policies by focusing on firm quality dynamics. In a laissez-faire economy, firms with high productivity are more likely to survive crises than those with low productivity. The government funding support saves more firms but cannot be customized based on firm productivity, dampening the cleansing effect of crises. The policy distortion is self-perpetuating: a downward bias in the firm quality distribution necessitates larger interventions in future crises. Our mechanism is quantitatively important: we show that if policymakers ignore such distortionary effects on firm quality dynamics, the resultant credit intervention would almost double the optimal amount.

We are grateful to four anonymous referees and the editor, Nir Jaimovich, for their constructive comments. We thank Vladimir Asriyan, Itzhak Ben-David, Chun Chang, Hui Chen, Nicolas Crouzet, Eduardo Dávila, Jason Rodrick Donaldson, Ehsan Ebrahimi, Isil Erel, Maryam Farboodi, Deeksha Gupta, Zhiguo He, Qiushi Huang, Erica Jiang, Mete Kilic, Arvind Krishnamurthy, Jun Li, Simon Mayer, Konstantin Milbradt, Martin Oehmke, Guillermo L. Ordoñez, Giorgia Piacentino, Vincenzo Quadrini, Adriano Rampini, Uday Rajan, Linda Schilling, Gunjan Seth, René M. Stulz, Neng Wang, David Zeke, Chao Zi, Miao Ben Zhang, Piotr Zoch, and seminar and conference participants at Barcelona School of Economics Summer Forum, China Macro-Finance Workshop, Colorado Finance Summit, Finance Theory Group Yale Meeting, HEC-McGill Winter Finance Workshop with Goethe University, IMF Macro-Financial Division, Liquidity in Macroeconomics Workshop, Midwest Finance Association, Paris December Finance Meeting, SFS Cavalcade North America, Shanghai Advanced Institute of Finance (SAIF), PHBS Workshop in Macroeconomics and Finance, and USC Marshall. The paper was previously circulating under the title “The Distortionary Effects of Central Bank Direct Lending on Firm Quality Dynamics”.

*University of Southern California, Marshall School of Business and NBER. E-mail: liwenhao@marshall.usc.edu

†University of Washington, Foster School of Business. E-mail: liye@uw.edu

1 Introduction

Since the Global Financial Crisis (GFC), credit intervention has grown in size and become more direct. The programs during the GFC injected liquidity through the banking sector, with only a few exceptions involving direct funding for nonfinancial firms, such as the bond purchase programs by the Bank of Japan and the European Central Bank. During the COVID-19 crisis, not only were the GFC-era programs promptly reinstated, but central banks and governments across the world also initiated new programs that directly provided liquidity support to nonfinancial firms, such as the Primary and Secondary Market Corporate Credit Facilities (PMCCF and SMCCF), Main Street Lending Program (MSLP), and Paycheck Protection Program (PPP) in the U.S.

This observation underscores a trend in which credit intervention has been ambitiously designed under the mantra of doing “whatever it takes” to prevent crises. In light of this trend, it is important to consider the following question: what could be the long-term effects of credit intervention, and in particular, what causes intervention to grow in scale from one crisis to the next?

We answer this question from the perspective of firm quality dynamics. In our model, firms accumulate capital and produce, but they differ in productivity. High-quality firms have larger financing capacity because, relative to low-quality firms, they are less likely to hold up creditors through strategic default and more likely to continue operations. Therefore, high-quality firms have better access to liquidity and thus are more likely to survive in crises.¹ Crises exhibit a cleansing effect: the economy emerges from crises with a higher fraction of firms being high-quality.

Funding support from the government helps firms survive crises but dampens the cleansing effect because, unlike private-sector creditors, the government is unable to differentiate firms by their productivity. Therefore, credit intervention alleviates the decline of aggregate output but reduces average firm productivity. As a result, a slippery slope of intervention emerges: the economy enters into the next crisis with lower productivity and thus requires a greater scale of intervention to sustain output. In our calibrated model, increasing the scale of intervention in the current crisis by one dollar per unit of firm capital leads to an increase of 4 cents per unit of capital in the next intervention should another crisis happen in ten years. Given that the capital stock grows over time, the inter-crisis pass-through rate in the total dollar amount is even larger.

A key feature of our model is the trade-off between quantity and quality. Optimal intervention

¹A larger financing capacity leads to a higher survival probability but does not imply that high-quality firms are less financially constrained, because the tightness of financial constraint is measured by the gap between funding and the targeted level of spending. High-quality firms may also have a higher spending target as we shall explain later.

in crises strikes a balance between distorting firm quality dynamics and preserving overall production capacity. It is important for policymakers to recognize our mechanism when deciding on the intervention scale. We show that ignoring the distortionary effects on firm quality dynamics results in a more aggressive intervention that almost doubles the welfare-maximizing amount.

In the following, we summarize the key modeling ingredients and provide more details on our main results. We follow the continuous-time formulation of the multi-sector models (Eberly and Wang, 2011) but simplify households' preferences to be risk-neutral. There are two types of firms that produce generic goods for consumption and investment using their productive capital with a constant return-to-scale technology. Type- H firms have higher productivity than type- L firms. In normal times, firms are not financially constrained and invest to grow their capital. The forward-looking valuation of capital (i.e., Tobin's q) is the present value of production flows and drives the optimal investment (Hayashi, 1982; Abel and Eberly, 1994; Brunnermeier and Sannikov, 2014).

The arrival of a crisis follows a Poisson process (Gourio, 2012; Wachter, 2013). We model a liquidity crisis in the spirit of Holmström and Tirole (1998). Firms differ in their crisis exposure: each firm draws a baseline level of survival probability from a common distribution unrelated to its type. Firms can raise debt financing to increase survival probability. Type- H firms have a higher targeted level of spending on survival as their capital is more valuable than that of type- L firms. The first form of the cleansing effect emerges: type- H firms want to spend more, which may translate into a higher survival probability. Our emphasis shall be on the second form of cleansing effect: type- H firms can borrow more than type- L firms.² It is the interference with this force that causes policy intervention to distort firm quality dynamics and exhibit intertemporal dependence.

Firms face a debt limit imposed by private-sector creditors as they may hold up creditors via strategic default (e.g., Bolton and Scharfstein, 1990; Hart and Moore, 1998). In strategic default, firm owners lose capital to the creditors but can extract value from the creditors through renegotiation.³ Therefore, type- L firms have a stronger incentive to strategically default than type- H firms because, due to lower productivity, their capital is less valuable. The fact that type- L firms face a tighter debt limit than type- H firms causes the aforementioned second form of cleansing effect.

²The cleansing effects of crises in our model are distinct from those via creative destruction or cyclical reallocation (e.g., Eisfeldt and Rampini, 2006; Kogan et al., 2017; Acemoglu et al., 2018), and the efficiency gain is from creditors' disciplining firms against overspending rather than weeding out unproductive firms that crowd out productive peers in product or input markets (e.g., Caballero and Hammour, 1994; Acharya et al., 2021).

³Firm owners' bargaining power may originate from their threat to divert resources, engage in wasteful spending, or withhold human capital (Aghion and Bolton, 1992; Hart and Moore, 1994).

This cleansing effect arises from the link between firm type and financing capacity. This link is quite general and can be modeled differently, for example, through a collateral constraint, in which case type- H firms pledge their more valuable capital as collateral and can raise more funds.⁴ Our modeling choice is motivated by the empirical literature on credit intervention.⁵

The government extends credit to firms in order to help more firms survive the crisis, effectively acting as a financial intermediary (Lucas, 2016). It finances lending with taxes on households and transfers the repayments in lump sums, as in the models of credit policy and unconventional monetary policy (e.g., Gertler and Karadi, 2011; Gertler et al., 2012; Araújo et al., 2015; Del Negro et al., 2017). When setting interest rates, the government follows private-sector creditors as in practice.⁶ However, when setting the size of credit support, the government imposes a uniform limit on firms of both types: a firm can borrow up to a multiple of its capital stock. This reflects the policy design in reality—credit limit is set proportional to accounting measures of operation scale, which map to the capital units in our model, such as the programs during the COVID-19 crisis.⁷

Firms can choose any amount of borrowing from the credit facilities within the limit set by the government. The impact of credit intervention cannot be solely judged by the take-up. In our model, credit intervention enlarges firms' financing capacity both directly through the liquidity available and indirectly by crowding in private-sector funding. By improving the survival probability, credit intervention also affects the pricing of credit by private-sector creditors. Any change in the scale of credit intervention results in a new equilibrium of financing capacity and costs.

Credit intervention benefits type- L firms more than type- H firms, dampening the cleansing effect of crises. Intuitively, since type- L firms obtain less funds from private-sector creditors, the marginal impact of more funds from the government is greater for type- L firms. While private-sector investors properly account for type- H and type- L firms' difference in incentive to hold up creditors, the government does not, and therefore, its funding support generates distortionary ef-

⁴Debt capacity depends on collateral value under limited commitment (Kehoe and Levine, 1993; Kiyotaki and Moore, 1997; Geanakoplos, 2010; Rampini and Viswanathan, 2010; Li, Whited, and Wu, 2016).

⁵The recent evidence documented in the literature suggests a significant conflict of interests between borrowers and lenders in crises (Hanson et al., 2020b; Lynch, 2021; Griffin et al., 2023).

⁶For publicly traded bonds, the government can rely on market prices, e.g., PMCCF and SMCCF in the U.S., and if a firm's debt is not publicly traded, the government can lend alongside informed banks and rely on banks to screen out and exclude firms that are riskier for a given level of interest rate, e.g., MSLP during the COVID-19 pandemic.

⁷MSLP sets a limit to six times the borrower's EBITDA, a measure of operating income rather than profitability per unit of resources deployed. The limit in PPP is tied to payroll rather than labor productivity. PMCCF and SMCCF impose limits tied to an issuer's existing debts (liability size rather than productivity). In addition to the uniform credit limit, if the government forgoes differentiation on interest rates, our mechanism is amplified (Appendix B.3).

fects. Policy makers have pointed out the lack of firm differentiation in credit facilities (English and Liang, 2020). There are several explanations. The government lacks information on firm productivity.⁸ Political considerations may go against treating firms differently in crises. And, customizing credit support for individual firms is infeasible when speedy implementation is required.

In summary, the productivity difference between type- H and L firms translates into a difference in private-sector financing capacity. A cleansing effect of crises emerges from the fact that type- H firms have a greater financing capacity and thus a higher survival probability. Credit intervention dampens such cleansing effect. Since both types of firms can obtain funding from the government, our analysis focuses on policy distortions among firms that can access the facility but have different levels of productivity. In practice, credit facilities may exclude certain firms, but the criterion is rarely productivity-based and thus unlikely to help avoid distortions in firm quality distribution.⁹

In our model, the government faces a trade-off between quantity and quality in line with the empirical evidence.¹⁰ Consider increasing credit support from zero to 30% of GDP.¹¹ In our calibrated model, the policy saves 8% of production capacity (i.e., capital units) but reduces the quality improvement (i.e., the increase in the fraction of firms being type- H) from about 9% to 4%. Optimal intervention strikes a balance between allowing more firms to survive and distorting the quality distribution. Reducing funding support strengthens the cleansing effect, so the economy has higher productivity post-crisis but has to climb out of a deeper decline in total output. If the government supplies more funding, the output drop is contained at the expense of lowering productivity.

Accordingly, the welfare is a bivariate function of the state variables, the total number of firms and the fraction of firms being type- H . When solving the constrained-efficient scale of intervention, we first consider a policymaker that ignores the distortionary impact on quality dynamics and is only aware of the positive impact on capital quantity. Here, the trade-off is standard—investing

⁸In our model, firms differ in both type (productivity) and crisis exposure (survival probability), so the government cannot infer the type from credit pricing in the market and thus cannot properly account for firms' incentive to hold up creditors. Credit risk and productivity are correlated but not perfectly aligned in our model and data (Appendix C.1).

⁹For example, PMCCF and SMCCF introduced in the U.S. during the COVID-19 pandemic excluded firms with high credit risk. In Appendix C, we show that credit risk and productivity are far from being perfectly correlated.

¹⁰Intervention was effective in preserving production capacity during the COVID-19 crisis (Bartik et al., 2020; Bartlett and Morse, 2020; Hubbard and Strain, 2020; Denes et al., 2021; Kawaguchi et al., 2021). On the cleansing effect of the COVID-19 crisis, Muzi et al. (2023) find less productive firms were more likely to cease operations, and Bruhn et al. (2023) find economic activity was reallocated toward more productive firms beyond what is implied by cyclical variation. These two papers document that intervention dampens the cleansing effect. Moreover, Dörr et al. (2022) find that credit support disproportionately benefited firms that were financially vulnerable pre-COVID 19.

¹¹This is close in magnitude to the size of credit support in the U.S. during the COVID-19 crisis, including various credit programs such as PPP and MSLP. See Section 3 for more discussions on the magnitude of crisis interventions.

goods in firm survival vs. consuming goods—and the resultant intervention is almost twice the size of optimal intervention that properly accounts for the impact on both state variables, quantity and quality. Therefore, ignoring the impact on quality dynamics leads to excessive intervention. This exercise shows that recognizing the mechanism in this paper is quantitatively important.

Another perspective on welfare analysis is under- vs. over-spending. For both type- H and L firms, intervention enlarges the financing capacity of firms that under-spend facing a large crisis exposure but induces overspending among firms that are less liquidity-constrained and over-borrow. Note that over-spending on survival happens because the option to hold up creditors through strategic default is in-the-money only if the firm survives. Therefore, under a sufficiently large scale of intervention, overspending is guaranteed to happen. It can happen among both type- H and L firms, though more prominent among type- L firms whose incentive to strategically default is greater.

It is important to note that firms that overspend on survival engage in a negative NPV transaction but are not zombies. In crises, a firm has temporary liquidity needs; after surviving the crisis, it recovers, producing goods and making investments guided by Tobin’s q . Zombies are firms that are permanently impaired and continue operating only by relying on external funds (e.g., Caballero, Hoshi, and Kashyap, 2008; Acharya, Eisert, Eufinger, and Hirsch, 2019; Acharya, Lenzu, and Wang, 2021). In Appendix B.4, we discuss an extension where the presence of zombie firms amplifies the distortionary effects of credit intervention and makes the effects more persistent.

In spite of the temporary nature of firms’ liquidity problem, policy distortions persist over time. In our calibrated model, increasing credit support from zero to 30% of GDP reduces the fraction of firms being type- H by 4 percentage points even ten years after the crisis. When a subsequent crisis arrives, the economy requires an even greater scale of intervention, because a lower average firm productivity translates into a smaller private-sector financing capacity in aggregate. Therefore, a slippery slope of intervention emerges in our model: credit intervention in the current crisis begets interventions of greater scale in future crises. This slippery slope is robust even when we shut down agents’ expectations of ever-growing intervention. The main takeaway from our analysis is that credit intervention faces a quantity-quality trade-off, and any one-time deviation towards more aggressive intervention generates a ripple effect that permeates indefinitely into the future.

Literature. The role of governments and central banks constantly evolves throughout history in response to crises, political struggles, and technological innovations (Goodhart, 1998; Calomiris, Flandreau, and Laeven, 2016). Direct lending to nonfinancial firms is a meaningful addition to

the policy toolbox. During a credit market freeze (Stiglitz and Weiss, 1981; De Meza and Webb, 1987), the government can step in, effectively functioning as a financial intermediary (Bebchuk and Goldstein, 2011; Lucas, 2016).¹² The COVID-19 crisis normalized the use of direct liquidity support to nonfinancial firms and will have a long-lasting effect on firms' expectations and their investment and financing decisions (Elenev et al., 2020).

The models of unconventional monetary policy assume an exogenous deadweight loss of direct lending (Gertler and Kiyotaki, 2010; Cúrdia and Woodford, 2011; Gertler and Karadi, 2011; Gertler, Kiyotaki, and Queralto, 2012; Araújo, Schommer, and Woodford, 2015; Del Negro, Eggertsson, Ferrero, and Kiyotaki, 2017). We unpack the black box of costs of government lending to nonfinancial firms, or asset purchases in general, and emphasize the endogenous evolution of firm quality distribution and a novel dynamic mechanism that leads to a slippery slope of intervention.¹³

Broadly, our paper contributes to the literature on the costs of crisis intervention, such as risk exposure (Lucas, 2012), tax distortions as a form of financing costs (Hanson, Scharfstein, and Sunderam, 2018), feedback loop between sovereign and private-sector risk (Acharya et al., 2014; Brunnermeier et al., 2016), distortions in bank capital allocation (Antill and Clayton, 2021), and debt overhang and bankruptcy costs (Balloch et al., 2020; Brunnermeier and Krishnamurthy, 2020; Crouzet and Tourre, 2020; Greenwood et al., 2020; Wang et al., 2020).¹⁴

In our model, crises exhibit cleansing effects that emerge from type- H and L firms' differences in debt capacity, which is, in turn, due to creditors imposing discipline against overspending. By dampening the cleansing effect, government intervention weakens such discipline. The efficiency gain from the crisis cleansing effect in our model is different from that in the existing models of cleansing effects that emphasize unproductive firms crowding out productive firms in output (product) or input (factor) markets and weeding out unproductive firms improves efficiency (e.g., Caballero and Hammour, 1994). The mechanism in our model is solely built on the financial aspects of crises.¹⁵ Incorporating type- L firms crowding out type- H firms in product or factor

¹²Bassetto and Cui (2020) analyze tax/subsidy as an alternative to credit policy in addressing financial frictions.

¹³Beyond our emphasis of endogenous quality, our paper focuses on the intensive margin of policy intervention—among firms that receive credit support, the costs of capital of firms with different productivities are homogenized—while other studies emphasize the distortions from the extensive margin, i.e., a subset of firms receive a disproportionately large amount of credit support (Kurtzman and Zeke, 2020; Papoutsis, Piazzesi, and Schneider, 2021).

¹⁴The recent contributions on the benefits of credit-market intervention focus on the positive externalities that cannot be internalized by private lenders (e.g., Bebchuk and Goldstein, 2011; Philippon and Schnabl, 2013; Liu, 2016; Giannetti and Saidi, 2019; Hanson, Stein, Sunderam, and Zwick, 2020a).

¹⁵The cleansing effects of crises in our model are also distinct from those via creative destruction or cyclical reallocation (e.g., Eisfeldt and Rampini, 2006; Kogan et al., 2017; Acemoglu et al., 2018).

markets amplifies the inefficiency of credit intervention, dampening the cleansing effect of crises.

Firm quality distribution evolves endogenously in our model under a fixed information structure. Our paper focuses on the interaction between policy intervention and firm quality and differs from studies on endogenous asset quality under an evolving information structure and agents' incentives to alter asset quality, motivated by the GFC (Eisfeldt, 2004; Chari, Shourideh, and Zetlin-Jones, 2014; Chemla and Hennessy, 2014; Kurlat, 2013; Gorton and Ordoñez, 2014; Bigio, 2015; Zryumov, 2015; Bolton, Santos, and Scheinkman, 2016; Moreira and Savov, 2017; Caramp, 2017; Hu, 2017; Vanasco, 2017; Fukui, 2018; Neuhann, 2018; Asriyan, Fuchs, and Green, 2019; Daley, Green, and Vanasco, 2020; Lee and Neuhann, 2021; Farboodi and Kondor, 2021).¹⁶

Our model features misallocation. Unlike the literature that studies factor allocation (Ramey and Shapiro, 1998; Eisfeldt and Rampini, 2006, 2008; Jovanovic and Rousseau, 2008), our focus is on liquidity allocation between firms and households and between high- and low-quality firms. Intervention causes misallocation among firms but improves efficiency by channeling liquidity from households to firms. We analyze the trade-off, contributing to the literature on financial frictions and misallocation (Banerjee and Moll, 2010; Gilchrist, Sim, and Zakrajšek, 2013; Midrigan and Xu, 2014; Moll, 2014; Fuchs, Green, and Papanikolaou, 2016; Dou, Ji, Tian, and Wang, 2020; David and Zeke, 2021). As in David, Schmid, and Zeke (2018), there is a connection between firm productivity and risk exposure: type-*H* firms, by having larger financing capacity, are less affected by crises. Intervention distorts the firm quality dynamics by changing firms' crisis exposure.

2 The Model

2.1 Preferences and Technology

Consider a continuous-time economy with a unit measure of representative agents ("households") and a government. Households have risk-neutral utility with discount rate r :

$$\mathbb{E} \left[\int_{t=0}^{\infty} e^{-rt} dc_t \right], \quad (1)$$

¹⁶policy makers may actively alter the information structure, which in turn affects the optimal intervention (Goldstein and Sapra, 2014; Bouvard, Chaigneau, and Motta, 2015; Shapiro and Skeie, 2015; Williams, 2015; Faria-e-Castro, Martinez, and Philippon, 2016; Goldstein and Leitner, 2018).

where c_t is the cumulative consumption. Households can own and trade equity and debt of firms. Firms maximize shareholders' value by managing capital to produce non-durable numeraire goods.

There are two types of firms. Type- H firms' capital produces A^H units of goods per unit of time. The productivity of type- L capital is A^L ($A^H > A^L$). Capital depreciates at rate δ . We use superscripts for type and subscripts for time. Given the aggregate capital stocks, K_t^H and K_t^L , the total output of numeraire goods is

$$Y_t \equiv A^H K_t^H + A^L K_t^L = (A^H \omega_t + A^L (1 - \omega_t)) K_t, \quad (2)$$

where ω_t , the fraction of type- H capital, represents the firm quality distribution

$$\omega_t \equiv \frac{K_t^H}{K_t^H + K_t^L}, \quad (3)$$

and K_t is the total units of capital

$$K_t \equiv K_t^H + K_t^L. \quad (4)$$

The output in the economy depends on both the capital quality, ω_t , and capital quantity, K_t .

Firms can grow capital through investment. Let q_t^j , $j \in \{H, L\}$, denote the endogenous value of capital that will be solved in equilibrium. It plays an important role in our analysis, as it incorporates the expectation of the future growth path and disruptions in crises. Given the time- t value of capital, q_t^j , a type- j firm chooses the investment rate (or capital growth rate), ι_t^j :

$$\max_{\iota_t^j} q_t^j \iota_t^j k_t - \Phi(\iota_t^j, k_t), \quad (5)$$

where we adopt the cost function from the literature on Q-theory of investment (Hayashi, 1982):

$$\Phi(\iota_t^j, k_t) = \underbrace{\left(\iota_t^j + \frac{\theta}{2} \iota_t^{j2} \right)}_{\phi(\iota_t^j)} k_t \quad (6)$$

Under this functional form, we obtain the classic Q-theory formula of optimal investment:

$$\iota_t^j = \frac{q_t^j - 1}{\theta}. \quad (7)$$

The arrival of crises follows a Poisson process, N_t , with intensity λ (e.g., Gourio, 2012; Wachter, 2013). We model a liquidity crisis in the spirit of Holmström and Tirole (1998), where firms need to inject additional resources to preserve their production capacity. In a crisis (i.e., $dN_t = 1$), firms raise money for survival. Let $x_t \geq 0$ denote the amount of financing per unit of capital a firm obtains and spends on surviving the crisis. The firm then faces a survival probability $F(x_t + \zeta)$, where ζ is drawn from a common cumulative distribution function $H(\zeta)$ independently across firms and is realized before the firm makes its financing decision x_t . We assume that $F(\cdot)$ is strictly increasing and concave with $F(0) = 0$, so a higher ζ means a higher chance of survival (since $F'(\cdot) > 0$) and a lower marginal value of liquidity (since $F''(\cdot) < 0$). The random variable ζ captures different factors that affect the severity of a firm's liquidity crisis. For example, a low ζ reflects a severe mismatch between cash inflows and outflows, such as customers delaying payment and suppliers suspending trade credit. A firm's decision to raise external funds depends on its type $j \in \{H, L\}$ (through the capital value q_t^j) and the severity of its liquidity crisis given by ζ , so we denote the optimal amount of financing per unit of capital as $x_t^j(\zeta)$, $j \in \{H, L\}$.

Incorporating both normal-time growth through investment and crisis-time exit with probability $1 - F(x_t^j(\zeta) + \zeta)$, aggregate type- j capital stock has the following law of motion:

$$dK_t^H = K_t^H(\iota_t^H - \delta)dt - K_t^H \left[\int_{\zeta} (1 - F(x_t^H(\zeta) + \zeta)) dH(\zeta) \right] dN_t, \quad (8)$$

$$dK_t^L = K_t^L(\iota_t^L - \delta)dt - K_t^L \left[\int_{\zeta} (1 - F(x_t^L(\zeta) + \zeta)) dH(\zeta) \right] dN_t + \eta K_t dt. \quad (9)$$

We will show that in equilibrium, $q_t^H > q_t^L$, so type- H firms grow faster in normal times ($\iota_t^H > \iota_t^L$). Moreover, we will show that type- H firms have higher survival rates in crises. To maintain stationarity of the quality distribution, we introduce exogenous births of size $\eta K_t dt$ for the L type.

2.2 Financial Frictions and Government Intervention in Crises

First, we establish an efficiency benchmark. Consider a social planner's choice of $x_t^j(\zeta)$, the spending on survival by a firm of type- j , $j \in \{H, L\}$ with a realized ζ . When spending x per unit of capital, probability of preserving the capital is $F(x + \zeta)$ and the expected payoff is $F(x + \zeta)q_t^j$:

$$\max_x F(x + \zeta)q_t^j - x. \quad (10)$$

In equation (10), we consider a liquidity crisis in which firms spend real resources for survival, following Holmström and Tirole (1998). The planner balances the social benefit from increasing the firm's survival probability and the social cost of using up real resources (the generic goods).¹⁷ Under a strictly concave $F(\cdot)$, the optimal amount of spending is given by the first-order condition:

$$F'(x_t^{*j}(\zeta) + \zeta)q_t^j = 1 \quad (11)$$

for $x_t^{*j}(\zeta) > 0$, and $x_t^{*j}(\zeta) = 0$ if $F'(x_t^{*j}(\zeta) + \zeta)q_t^j = F'(\zeta)q_t^j < 1$ as the amount of financing cannot be negative. We use $x_t^{*j}(\zeta)$ to denote the socially optimal (first-best) level.

Next, we model debt financing offered by private-sector creditors and show that, without frictions, the credit market equilibrium generates the social optimum. The creditors charge an interest rate $r_t^j(\zeta, x)$ on a type- j firm with realized ζ and debt level x per unit of capital. The firm chooses the amount of debt financing to maximize the expected value:

$$x_t^j(\zeta) = \arg \max_{0 \leq x \leq \bar{d} + \bar{g}} F(x + \zeta) [q_t^j - (1 + r_t^j(\zeta, x))x] . \quad (12)$$

The firm survives with probability $F(x + \zeta)$, and after paying the principal and interest to creditors, the firm owners' value is $q_t^j - (1 + r_t^j(\zeta, x))x$. The private-sector credit supply is constrained at $\bar{d}$. Government intervention, $\bar{g}$, relaxes the constraint. Note that what distinguishes crises and normal times is the limited private-sector credit in crises. In normal times, firms' investment is unconstrained, given by (7). Therefore, the tightening of credit conditions in crises reminisces the concept a financial shock (Jermann and Quadrini, 2012). A credit freeze can be attributed to lenders' balance-sheet impairment, elevated uncertainty, or informational frictions.¹⁸

¹⁷During the COVID-19 pandemic, from the restaurant industry to software services, businesses altered products and production processes, and doing so entails real resources spent on human capital (e.g., redeploying and training workers to produce new products), and materials and real estate (e.g., e-commerce platforms building new warehouses to address heightened logistics demand). Spending such real resources is important for firms' survival. One may argue that certain liquidity crises do not entail spending real resources and survival only requires bridge financing. For example, firms experiencing delay in customer payments need external financing to bridge through liquidity shortage. In Appendix D.5, we consider an extended model to allow for this possibility.

¹⁸Credit freeze happens for various reasons, such as lenders' lack of capital (Bernanke and Lown, 1991), information decay in booms (Gorton and Ordoñez, 2014; Asriyan, Laeven, and Martin, 2018), foreigners' withdrawal (Van Nieuwerburgh and Veldkamp, 2009; Koijen, Koulischer, Nguyen, and Yogo, 2020), and ambiguity in risk evaluation (Boyarchenko (2012); Caballero and Simsek (2013); Drechsler (2013)). A market crash happened during the global financial crisis (Acharya, Schnabl, and Suarez, 2013; Brunnermeier, 2009; Gorton, Laarits, and Metrick, 2020; Kacperczyk and Schnabl, 2010; Krishnamurthy, 2010). Credit markets were under tremendous stress during the COVID-19 pandemic (Falato, Goldstein, and Hortaçsu, 2020; Haddad, Moreira, and Muir, 2020; Halling, Yu, and Zechner, 2020;

The interest rate, $r_t^j(\zeta, x)$, is set by the creditors' break-even condition:

$$F(x + \zeta)(1 + r_t^j(\zeta, x))x = x. \quad (13)$$

When lending to firms, the government also charges this rate, taking advantage of the market signals. There are different types of policy designs that allow the government to rely on market signals. Examples include the PMCCF and SMCCF in the U.S. during the COVID-19 pandemic.¹⁹

Using (13), we substitute out $r_t^j(\zeta, x)$ in the firm value in (12) and obtain a simplified objective function $F(x + \zeta)q_t^j - x$, which is exactly the planner's objective function given by (10). Therefore, without the funding constraints (i.e., with $\bar{d}$ set to infinity), the credit market equilibrium generates the socially optimal (first-best) level of spending on survival, $x_t^{*j}(\zeta)$ given by (11). To simplify the discussion below, we introduce the notation, $\bar{\zeta}_t^j$, defined by

$$F'(\bar{\zeta}_t^j)q_t^j = 1. \quad (14)$$

Under $q_t^H > q_t^L$ and a strictly concave $F(\cdot)$, we have $\bar{\zeta}_t^H > \bar{\zeta}_t^L$ from (14). Type- H capital is naturally more valuable than type- L capital because a type- H firm has a higher productivity, i.e., $A^H > A^L$. We will later formally show $q_t^H > q_t^L$ after fully solving the endogenous capital values in equilibrium. The following lemma summarizes the first-best levels of financing.

Lemma 1 (First best) *The first-best level of financing for a type- j firm with a realized ζ is given by*

$$x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+, \quad (15)$$

where $j \in \{H, L\}$. Under $q_t^H > q_t^L$, we have $\bar{\zeta}_t^H > \bar{\zeta}_t^L$.

A firm's optimal level of financing is the minimum of the first-best level and the available funding:

$$x_t^j(\zeta) = \min\{x_t^{*j}(\zeta), \bar{d} + \bar{g}\}. \quad (16)$$

Government intervention improves efficiency by bringing $x_t^j(\zeta)$ closer to $x_t^{*j}(\zeta)$. Here the government acts as a financial intermediary in crises (Lucas, 2016). It finances lending with lump-sum taxes on deep-pocketed households and transfers repayments to households.

Kargar, Lester, Lindsay, Liu, Weill, and Zúñiga, 2020; Ma, Xiao, and Zeng, 2022).

¹⁹If a firm's debt is not publicly traded, the government can "free-ride" banks' information production in pricing (for example, the government relied on banks to screen borrowers in MSLP during the COVID-19 pandemic).

So far, the only source of inefficiency is the limited funding supply in the private sector, and the optimal intervention simply requires $\bar{g}$ to be sufficiently large so that $x_t^j(\zeta) = x_t^{*j}(\zeta)$. However, funding in crises is limited because of frictions not only on the credit supply side but also on the credit demand side. Introducing borrowers' moral hazard allows credit intervention to have potentially harmful effects: a large intervention can lead to over-spending on survival in crises.

We model borrowers' moral hazard in the form of strategic default. Up to this point, we have assumed that firms repay debts if they survive. However, as is well-known in the corporate finance literature (Bolton and Scharfstein, 1990; Hart and Moore, 1998), there are two types of default. First, a firm defaults because it cannot survive, which happens with probability $1 - F(x_t^j(\zeta) + \zeta)$. Second, a firm survives but its owners extract value from creditors through strategic default and renegotiation. The owners may threaten to divert resources, engage in wasteful spending, or withhold human capital (Aghion and Bolton, 1992; Hart and Moore, 1994). After renegotiation, the firm owners obtain β while the private-sector creditors seize the capital and obtain $q_t^j - \beta$ as the control right is typically transferred to creditors when default happens. The recovery value is zero for the government, which is in line with evidence of the prevalent abuse of government funding during crises (Hanson et al., 2020b; Lynch, 2021; Griffin et al., 2023). Also, it is rare for the government to engage in bankrupt firms' restructuring (seizing the capital and continuing operations). Thus, for firms that strategically default, government funding is essentially a subsidy on survival.

A firm that has survived the crisis compares the value per unit of capital from strategic default, β , with the value under debt repayment, $q_t^j - [1 + r_t^j(\zeta, x_t^j(\zeta))] x_t^j(\zeta)$. Firms will strategically default if and only if it is strictly better than repaying its debt. Let $\underline{\zeta}_t^j$ denote the solution to the following indifference condition defined over ζ ,

$$q_t^j - [1 + r_t^j(\underline{\zeta}_t^j, x_t^j(\underline{\zeta}_t^j))] x_t^j(\underline{\zeta}_t^j) = \beta, \quad (17)$$

if a solution exists. Note that if β is sufficiently high, for example, $\beta = q_t^j$, then all firms prefer strategic default over debt repayment and the indifference condition may never hold for any ζ . If $\beta = 0$, then all firms repay their debts. Therefore, the equation (17) may not have a solution. The following lemma shows that when the solution of equation (17) exists, it is unique, and there are fewer type- H firms that strategically default than type- L firms. The full proof is in the appendix.²⁰

²⁰The intuition behind the proof is as follows. Given q_t^j , we compute the value under debt repayment, $q_t^j - [1 + r_t^j(\zeta, x_t^j(\zeta))] x_t^j(\zeta)$ for all values of ζ , which we prove is a monotonic function of ζ . This function incorporates the interest-setting function $r_t^j(\zeta, x)$ given by (13) and a firm's optimal borrowing $x_t^j(\zeta)$ function given by (16). The

Lemma 2 (Strategic default) *A type- j firm with realized ζ strategically defaults if and only if $\zeta < \zeta_t^j$, where ζ_t^j as a solution to (17) is unique. Furthermore, under $q_t^H > q_t^L$, we have $\zeta_t^H < \zeta_t^L$.*

Firms with a lower ζ have more liquidity needs and borrow more, so they tend to strategically default. Intuitively, if a firm has more debt obligations, default saves a larger repayment. Default also causes the firm owners to lose capital ownership, so when capital is more valuable (q_t^j is higher), firms tend not to strategically default. Given that $q_t^L < q_t^H$ implies $\zeta_t^L > \zeta_t^H$, there are more type- L firms that strategically default ($\zeta \in (0, \zeta_t^L)$) than type- H firms.

For firms with $\zeta \in (0, \zeta_t^L)$, their objective function is $F(x_t^j(\zeta) + \zeta)\beta$ without considering repayment as they shall strategically default. Therefore, they would borrow as much as possible to maximize survival probability and their expected gain from strategic default. Private-sector creditors know q_t^j (and hence the strategic-default threshold ζ_t^j), and they know firms' draw of ζ , so private-sector creditors know which firm will repay debts ($\zeta \geq \zeta_t^L$) and which will strategically default ($\zeta < \zeta_t^L$). When lending to firms with $\zeta < \zeta_t^L$, private-sector creditors understand that such firms borrow as much as possible, so, beyond the interest rate $r_t^j(\zeta, x_t^j(\zeta))$, private-sector creditors also specify a debt limit, denoted by $\hat{d}_t^j(\zeta)$,

$$\hat{d}_t^j(\zeta) = F(\hat{d}_t^j(\zeta) + \zeta + \bar{g})(q_t^j - \beta), \quad (18)$$

to ensure that they break even.²¹ Note that the interest rate is relevant for firms that strategically default even though they do not repay their debts, because the decision to default or not is based on comparing the value in strategic default and the value after debt repayment, as shown in (17).

Lemma 3 (Endogenous debt capacity) *For any ζ , $\hat{d}_t^j(\zeta)$ in (18) exists and is unique. It is strictly increasing and concave in ζ . Furthermore, under $q_t^H > q_t^L$, we have $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$.*

We summarize how events unfold in a crisis. The competitive private-sector creditors are informed about firms' type j and crisis exposure that is inversely indexed by ζ . Therefore, they offer loan contracts indexed by j and ζ . The contractual interest rate is set by the creditors' break-even condition (13) so that a type- j firm with a realized ζ faces interest rate $r_t^j(\zeta, x_t^j(\zeta))$. Note that

threshold ζ_t^j is where this value under debt repayment as a function of ζ crosses β , the value under strategic default.

²¹Note that if $\hat{d}_t^j(\zeta) \geq \bar{d}$, $x_t^j(\zeta) = \bar{d} + \bar{g}$, in which case the creditors earn positive profits because, under a strictly increasing and concave $F(\cdot)$, $F(\bar{d} + \bar{g} + \zeta)(q_t^j - \beta) > \bar{d}$. The creditors may want to lend more but they are constrained by the amount of available funds, $\bar{d}$. Such funding shortage and the positive profits earned by those who can provide funding is a common feature of liquidity crises that we capture in our model.

the interest rate depends on the borrowing amount. For a type- j firm with ζ below $\underline{\zeta}_t^j$, the creditors also impose a borrowing limit, $\hat{d}_t^j(\zeta)$, recognizing the borrower's incentive of strategic default. As shown in (18), the borrowing limit ensures that the creditors break even when lending to these firms. For these firms, loan repayment is off-equilibrium, so one may wonder why the contracts specify interest rates according to (13). Interest rates are still important because the firm's decision to strategically default or not is made through the comparison between the value from repaying the loan and that from strategic default, as shown in the indifference condition (17). The interest rates, given by (13), imply that in the off-equilibrium scenario where these firms make repayment, the competitive creditors break even.²² In summary, a type- j firm's optimal choice of x is given by

$$x_t^j(\zeta) = \underbrace{\mathbf{1}_{\zeta \geq \underline{\zeta}_t^j} \min\{(\bar{\zeta}_t^j - \zeta)^+, \bar{d} + \bar{g}\}}_{\text{no strategic default}} + \underbrace{\mathbf{1}_{\zeta < \underline{\zeta}_t^j} \left(\min\{\hat{d}_t^j(\zeta), \bar{d}\} + \bar{g} \right)}_{\text{strategic default}}, \quad (19)$$

Firms that repay debts raise funding given by (16). Firms that strategically default max out borrowing. Based on Lemmas 1, 2, and 3, the proposition below summarizes firms' financing strategy.

Proposition 1 (Equilibrium financing) *The financing amount of a type- j firm, $j \in \{H, L\}$, with a realized ζ , $x_t^j(\zeta)$, is given by (19), with $\bar{\zeta}_t^H > \bar{\zeta}_t^L$, $\underline{\zeta}_t^H < \underline{\zeta}_t^L$, and $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ under $q_t^H > q_t^L$. It also has the following properties: (1) $x_t^j(\zeta)$ is increasing and concave in ζ for $\zeta < \underline{\zeta}_t^j$ and $\hat{d}_t^j(\zeta) \leq \bar{d}$; (2) $x_t^j(\zeta)$ is decreasing and linear in ζ for $\zeta \in [\underline{\zeta}_t^j, \bar{\zeta}_t^j]$ and $(\bar{\zeta}_t^j - \zeta)^+ \leq \bar{d} + \bar{g}$.*

For a type- j firm, $j \in \{H, L\}$ with ζ above the default threshold $\underline{\zeta}_t^j$, a lower ζ (i.e., a stronger liquidity need) leads to more borrowing. For this firm, the only source of potential inefficiency is the funding limit. For a type- j firm with $\zeta < \underline{\zeta}_t^j$, it faces an additional problem of endogenous debt limit due to its incentive to strategically default. As $\hat{d}_t^j(\zeta)$ increases in ζ , a firm with a stronger liquidity need (i.e., a lower ζ) actually has a smaller debt capacity $\hat{d}_t^j(\zeta)$ and thus can borrow less. Moreover, given the concavity of $\hat{d}_t^j(\zeta)$ in ζ (see Lemma 3), the further ζ decreases, the faster debt capacity shrinks. This is a classic insight from the corporate finance literature: a borrower's own lack of commitment against strategic default—which is more severe when ζ is lower and the

²²Note that for the interest rates given by equation (13), the creditors do not probability-weight scenarios of repayment conditional on survival vs. strategic default conditional on survival. The creditors know a firm's type, j , and ζ , so they know which firms repay and which firms strategically default conditional on surviving the crisis. Therefore, the interest rates do not “price in” strategic default. For firms with $\zeta < \underline{\zeta}_t^j$, the interest rates simply specify the off-equilibrium scenario of how much to pay if they do not strategically default and behave as firms with $\zeta \geq \underline{\zeta}_t^j$. The indifference condition (17), is an incentive compatibility condition that generates the ζ threshold for strategic default.

financing need is stronger (see Lemma 2)—compromises its financing capacity. The financing strategy characterized in Proposition 1 has a rich set of features that allow us to discuss different channels through which government intervention can affect firms' financing in crises and may alleviate or exacerbate inefficiencies.

Discussion: Productivity and credit risk. Firms differ in productivity, A^j (type) and ζ (a smaller ζ indicates stronger liquidity needs in crises). ζ drives a wedge between firm type and credit quality. A type- H firm may default by failing to survive, which happens with probability $1 - F(x_t^H(\zeta) + \zeta)$. A type- H firm may also strategically default. In contrast, a type- L firm that survives the liquidity crisis may choose to repay its debt if its ζ is above the strategic default threshold, $\underline{\zeta}_t^L$. This distinction between firm productivity (type) and credit risk is consistent with the evidence that we present in Appendix C.1. To summarize, in both data and the model, firm quality and credit risk are not perfectly aligned. Therefore, although the government may differentiate credit quality by following the private-sector pricing of credit risk, it cannot obtain a perfect indicator of firm productivity from credit markets. Moreover, some credit programs do not even differentiate firms by credit risk, for example, PPP during the COVID-19 crisis (see Appendix B.3 for our analysis).

2.3 Efficiency and Equilibrium

Intervention and efficiency. The efficiency criterion in our model is whether firms' spending on survival is at the first-best level given by (15). There are two frictions, and both contribute to the financial constraints on spending. The first friction is on the funding supply side, i.e., the limit on private-sector funding, denoted by $\bar{d}$. The second is on the funding demand side, i.e., borrowers' strategic default, which gives rise to the endogenous limit on private-sector debt capacity, $\hat{d}_t^j(\zeta)$.

For firms that repay debts (i.e., with $\zeta \geq \underline{\zeta}_t^j$), government intervention enlarges financing capacity and unequivocally improves efficiency by relaxing the funding constraint. These firms repay their debts after surviving the crisis, so, under the disciplinary effect of debt repayment, they do not over-spend. They spend up to $x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+$, a level that balances the marginal benefit of improving the survival probability and the marginal cost of resources (see Lemma 1).

For firms that strategically default (i.e., with $\zeta < \underline{\zeta}_t^j$), government intervention also enlarges financing capacity and increases the financing amount, $x_t^j(\zeta)$, but the efficiency implications are more complex. Government intervention enlarges firms' financing capacity through two channels,

direct liquidity injection and private funding crowding-in. First, an increase in $\bar{g}$ provides more funding to these firms, as shown in (19). This additional source of funds increases the survival probability and thus lowers the interest rate charged by private-sector creditors as determined by (13). Second, as shown in (18), an increase in $\bar{g}$ raises the survival probability, making the creditors more willing to lend, resulting in a higher $\hat{d}_t^j(\zeta)$ (a crowding-in effect).

Lemma 4 (Private funding crowding-in) *For any ζ and $j \in \{H, L\}$, and given q_t^j , government financing crowds in private lending, $\partial \hat{d}_t^j(\zeta) / \partial \bar{g} > 0$, and reduces the interest rate, $\partial r_t^j(\zeta) / \partial \bar{g} \leq 0$.*

To analyze the efficiency implication of credit intervention among firms that strategically default, first notice that the goal of these firms is to maximize the survival probability. After surviving the crisis, firm owners renege on debt repayment and extract rent by holding up the creditors in renegotiation. These firms over-spend if the amount they borrow, $\min\{\hat{d}_t^j(\zeta), \bar{d}\} + \bar{g}$, is above the first-best level, $x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+$, and any further increase in $\bar{g}$ exacerbates such inefficiency.

Among firms that strategically default, over-spending tends to happen to those with high ζ . Formally, if there exists a threshold $\zeta_t^{*j} < \underline{\zeta}_t^j$ such that $\min\{\hat{d}_t^j(\zeta_t^{*j}), \bar{d}\} + \bar{g} = (\bar{\zeta}_t^j - \zeta_t^{*j})^+$ then any firm with $\zeta \in (\zeta_t^{*j}, \underline{\zeta}_t^j)$ over-spends, because the endogenous debt capacity, $\hat{d}_t^j(\zeta)$, is increasing in ζ while the first-best level of financing, $(\bar{\zeta}_t^j - \zeta)^+$, is decreasing in ζ . For these firms, a higher $\bar{g}$ exacerbates over-spending on survival. For firms that have $\zeta < \zeta_t^{*j}$ and underspend, an increase in $\bar{g}$ improves efficiency by alleviating their under-spending, whether the firms strategically default or not. The next proposition summarizes the efficiency implications of credit intervention.

Proposition 2 (Intervention and efficiency) *Intervention improves efficiency by alleviating under-spending on survival for firms that repay debts (i.e., $\zeta \geq \underline{\zeta}_t^j$) and face a binding funding constraint (i.e., $\bar{d} + \bar{g} < \bar{\zeta}_t^j - \zeta$) and also for firms that strategically default (i.e., $\zeta < \underline{\zeta}_t^j$) and underspend (i.e., $\hat{d}_t^j(\zeta) + \bar{g} < \bar{\zeta}_t^j - \zeta$). Intervention reduces efficiency by exacerbating over-spending on survival among firms that strategically default (i.e., $\zeta < \underline{\zeta}_t^j$) and over-spend (i.e., $\hat{d}_t^j(\zeta) + \bar{g} > \bar{\zeta}_t^j - \zeta$).*

It is clear that the detrimental effects of credit intervention through over-spending depend on the scale of intervention. When the government increases $\bar{g}$, the spending of firms that strategically default increases through higher $\bar{g}$ and through a higher $\hat{d}_t^j(\zeta)$ due to the crowding-in effect in Lemma 4. This enlarges the over-spending region of ζ and thus exacerbates the inefficiency from over-spending. On the other hand, when $\bar{g}$ declines, the over-spending region shrinks.

Moreover, a sufficiently large scale of intervention can guarantee a net negative impact. There is a limit to efficiency improvement from relaxing the financial constraint on under-spending firms: when $\bar{g}$ is sufficiently large, all firms that repay debts have reached their first-best level of financing and would not spend more. However, there is no limit on over-borrowing and over-spending: the goal of firms that strategically default is to maximize their survival probability, so they always borrow as much as they can from the government. The following corollary summarizes this result.

Corollary 1 (Excessive credit intervention) *The cost of over-spending dominates the benefit from financing the under-spending firms when the scale of credit intervention is sufficiently large.*

So far, we have separately discussed the efficiency implications of credit intervention on firms with $\zeta \geq \underline{\zeta}_t^j$ and on firms with $\zeta < \underline{\zeta}_t^j$. Credit intervention may also affect the strategic default threshold, $\underline{\zeta}_t^j$. While a higher $\bar{g}$ does not affect β , the value that firm owners can extract through strategic default and renegotiation, it increases firm value in the debt repayment region by relaxing the financial constraint, allowing profitable investment to be funded.²³ This force lowers the default threshold, $\underline{\zeta}_t^j$ and reduces the measure of firms that strategically default. However, as we will show in the next section, this channel of efficiency improvement is not quantitatively significant.

Beyond our analysis of the efficiency implications of intervention within a firm type, Lemma 1, 2, and 3 also reveal the difference in the impact of intervention across firm types. In Proposition 2, credit intervention improves efficiency by relaxing the financial constraint on firms that repay debts. According to Lemma 1, among these firms, type- H firms have a higher first-best level of spending so this positive impact is more prominent among type- H firms. In Proposition 2, credit intervention exacerbates over-spending among firms that strategically default. According to Lemma 2, there are more type- L firms that strategically default than type- H firms. This suggests that the negative impact through over-spending tends to be more reflected among type- L firms.

By incorporating the funding supply- and demand-side frictions, our model allows both under- and over-spending to emerge in equilibrium. While under-spending can happen to both firms that repay debts and those that strategically default, over-spending happens to the latter. Credit intervention improves efficiency among firms that underspend and exacerbates inefficiency among those that over-spend. The goal of our model is to capture such intricate effects of credit intervention and, after calibration, to allow the dominant channels to emerge in the quantitative analysis.

²³Recall that once the firm survives the liquidity shock, a strategic default is a comparison between the firm value under debt repayment, which increases in $\bar{g}$, and the value from strategic default β .

In our model, credit intervention is size-dependent but does not depend on an individual firm's type, $j \in \{H, L\}$, or the severity of its liquidity crisis, ζ . A firm with k_t units of capital can borrow from the government up to $k_t \bar{g}$. There are several motivations behind this setup. First and foremost, the government may not have information on firms' types, in line with the tradition in economics that emphasizes the informational disadvantage of central authorities (Hayek, 1945). Importantly, our setup captures policy design in reality. The funding limit is set proportional to accounting measures of operation scale (k_t in our model) such as the programs during COVID-19 pandemic. MSLP set a borrowing limit of six times the borrower's operating income rather than profits or productivity per unit of resources deployed. Similarly, the limit in PPP was a multiple of the borrower's payroll rather than tied to labor productivity. Furthermore, the corporate bond purchase programs, i.e., PMCCF and SMCCF, imposed limits proportional to an issuer's existing debts (a measure of liability size rather than productivity).²⁴

In the next subsection, we discuss how the differential effects of credit intervention on type- L and type- H firms translate into channels that may affect the cleansing effect of crises and thereby generate intertemporal dependence in intervention scale across crises (i.e., the slippery slope of intervention). In the following, we close this subsection and complete the equilibrium characterization by defining the stationary equilibrium and solving the endogenous capital values.

Capital value and equilibrium. Capital value plays an important role. It is the key distinguishing factor that separates type- L and type- H firms. As shown in Lemma 1, capital values drive the first-best level of spending in crises. According to Lemma 2 and 3, capital values affect the strategic default thresholds and firms' endogenous debt capacity in the private funding market.

To simplify notation, we define the expected value per unit of type- j capital to the owner of a type- j firm with a realized ζ as follows

$$\pi_t^j(\zeta) = \underbrace{(F(x_t^j(\zeta) + \zeta)q_t^j - x_t^j(\zeta)) \mathbf{1}_{\zeta \geq \underline{\zeta}_t^j}}_{\text{no strategic default}} + \underbrace{F(x_t^j(\zeta) + \zeta)\beta \mathbf{1}_{\zeta < \underline{\zeta}_t^j}}_{\text{strategic default}}, \quad (20)$$

where $F(\cdot)$ is the survival probability as a function of crisis severity, ζ , and funds available $x_t^j(\zeta)$.

²⁴Primary market corporate bond purchase facility: The amount of outstanding bonds or loans of an eligible issuer that borrows from the Facility may not exceed 130 percent of the issuer's maximum outstanding bonds and loans on any day between March 22, 2019, and March 22, 2020. Secondary market corporate bond purchase facility: The amount of bonds that the Facility will purchase from the secondary market of any eligible issuer is capped at 10 percent of the issuer's maximum bonds outstanding on any day between March 22, 2019, and March 22, 2020.

Note that, in case of no strategic default, we substitute out the interest rate using the creditors' break-even condition (18). In equilibrium, capital value, q_t^j , satisfies the following equation that equates investors' required rate of return over dt , i.e., the discount rate, $r dt$, and the sum of expected return on holding capital from price appreciation, cash-flow yield net of investment costs in normal times, capital growth via investment net of depreciation, and return in crises calculated as the difference between the ζ -averaged capital value in a crisis and the pre-crisis capital value:

$$r = \mathbb{E}_t \left[\frac{dq_t^j/dt}{q_{t-}^j} \right] + \frac{A^j - \phi(\iota_{t-}^j)}{q_{t-}^j} + (\iota_{t-}^j - \delta) + \lambda \frac{\int_0^\infty \pi_t^j(\zeta) dH(\zeta) - q_{t-}^j}{q_{t-}^j}, \quad (21)$$

where we use the subscript $t-$ in q_{t-}^j to denote the pre-crisis capital value and accordingly ι_{t-}^j to denote the investment rate driven by q_{t-}^j (see (7)). The value $\pi_t^j(\zeta)$ is defined in (20), and as reminder, $H(\cdot)$ is the c.d.f. for ζ . The equilibrium can be characterized below.

Proposition 3 (Stationary equilibrium) *There exists a stationary equilibrium where firms' investment rate in normal times, ι^j , borrowing in crises, $x^j(\zeta)$, borrowing cost in crises, $r^j(\zeta)$, debt capacity under strategic default, $\hat{d}^j(\zeta)$, and capital value, q^j , are time-invariant. Furthermore, the equilibrium is unique with $q^H > q^L$, under the following conditions: (1) $\iota^j - \delta \leq r$; (2) $\frac{\beta \lambda_F}{4[1 - \lambda_F(q^j - \beta)]} < 1$, where λ_F is the intensity parameter of exponential distribution c.d.f. $F(\cdot)$.*

The first parameter condition is standard in the asset pricing literature: $\iota^j - \delta$, the growth rate from investment net of depreciation cannot be higher than the discount rate, r ; otherwise capital value is infinite. The second parameter condition limits the degree of moral hazard: Given λ_F in $F(x_t^j(\zeta) + \zeta) = 1 - e^{-\lambda_F(x_t^j(\zeta) + \zeta)}$, the firm owners' bargaining power in strategic default cannot be too large, i.e., β cannot be too large relative to $q^j - \beta$ (the creditors' recovery value). Moreover, given the degree of moral hazard, the survival probability cannot be too responsive to liquidity injection, i.e., λ_F cannot be too large. We will show that the conditions (1) and (2) do not bind in our calibration, that is, the interior values deliver a close match between model and data moments.

In our baseline model, the scale of credit intervention, $\bar{g}$, is time-invariant. We characterize an equilibrium with constant capital values as it provides a transparent presentation of key economic mechanisms. Later we will consider an economy where the government employs a dynamic strategy of credit intervention, in which case the value of capital varies over time.

2.4 The Cleansing Effect of Crises and Slippery Slope of Intervention

In this subsection, we characterize the cleansing effect of crises, which is key to the intertemporal linkage that generates the slippery slope of credit intervention. To characterize the aggregate dynamics, we use (8) and (9) to derive the law of motion for two state variables, capital quality ω_t defined in (3), and capital quantity K_t defined in (4):

$$\frac{dK_t}{K_{t-}} = \underbrace{\left[-\delta + (\omega_{t-}\iota_{t-}^H + (1 - \omega_{t-})\iota_{t-}^L) + \eta \right]}_{\mu_t^K(\omega_{t-})} dt + \underbrace{(\omega_{t-}\kappa_t^H + (1 - \omega_{t-})\kappa_t^L - 1)}_{\Delta_t^K(\omega_{t-})} dN_t, \quad (22)$$

where κ_t^j , the fraction of type- j capital that survives a crisis, is defined as

$$\kappa_t^j \equiv \int F(x_t^j(\zeta) + \zeta) dH(\zeta), \quad (23)$$

and

$$d\omega_t = \underbrace{\omega_{t-} (1 - \omega_{t-}) \left(\iota_{t-}^H - \iota_{t-}^L - \frac{\eta}{1 - \omega_{t-}} \right)}_{\mu_t^\omega(\omega_{t-})} dt + \underbrace{\left(\frac{\omega_{t-}\kappa_t^H}{\omega_{t-}\kappa_t^H + (1 - \omega_{t-})\kappa_t^L} - \omega_{t-} \right)}_{\Delta_t^\omega(\omega_{t-})} dN_t. \quad (24)$$

From Proposition 3, the normal-time capital growth rate, ι_{t-}^j , and capital surviving rate in crises, κ_t^j , are time-invariant in the stationary equilibrium. Given (7), we have $\iota^H > \iota^L$ under $q^H > q^L$, which then implies that ω_t tends to drift upward. As previously discussed, we preserve the stationarity of ω_t via the exogenous entry rate of type- L firms by the rate of η as introduced in (9).

The cleansing effect refers to an increase in ω_t in crises, i.e., $\Delta_t^\omega(\omega_{t-}) > 0$, which is equivalent to type- H firms having a higher survival rate than type- L firms, i.e., $\kappa_t^H > \kappa_t^L$. Therefore, the cleansing effect of crises is simply about which type of firms in aggregate spend more on survival. Consider the first-best scenario where firms are not financially constrained and spend on survival at the level given by (15). Here the cleansing effect emerges because type- H firms have a higher first-best level of spending than type- L firms, i.e., $x^{H*}(\zeta) > x^{L*}(\zeta)$ (see Lemma 1). Type- H firms want to borrow more and spend more on survival because their capital is more valuable.

Proposition 4 (The cleansing effect in the first-best economy) *In the first-best economy where firms spend at the level given by (15), crises have a cleansing effect, i.e., $\Delta_t^\omega(\omega_{t-}) > 0$.*

As previously discussed, once we introduce the funding supply- and demand-side frictions, firms may under- or over-spend on survival. Next, we first discuss how introducing only the funding-supply friction to the first-best economy affects the cleansing effect of crises. The funding supply-side friction—that is, the private-sector creditors cannot lend beyond $\bar{d}$ —causes firms of both types to underspend and thereby dampens the cleansing effect in Proposition 4. Intuitively, when this funding-supply constraint binds, i.e., $\bar{d} < x^{j*}(\zeta)$, both types of firms borrow and spend at the same level, $\bar{d}$. Intervention, by expanding funding supply and allowing more firms to spend at the first-best level, reverses this dampening effect. The next lemma summarizes this result.

Corollary 2 (Funding-supply friction and the cleansing effect) *Introducing a sufficiently tight funding constraint ($\bar{d} \rightarrow 0$) to the first-best economy weakens the cleansing effect of crises. Credit intervention, by expanding funding supply, reverses this force and strengthens the cleansing effect.*

Next, we discuss the impact of introducing funding demand-side friction on the cleansing effect of crises. The demand-side friction, i.e., firms' inability to commit against strategic default, gives rise to a new channel of cleansing effect. As shown in Lemma 3, firms that strategically default borrow as much as they can to maximize their survival probability, and as a result, the private-sector creditors impose a debt limit. Type- H firms face a higher endogenous debt limit, i.e., $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ and thus can borrow more than type- L firms. This is a new channel of cleansing effect.²⁵ While the cleansing effect in Proposition 4 arises from type- H firms wanting to spend more on survival than type- L firms, this new channel is about type- H firms being able to obtain more funding. Credit intervention dampens this channel of cleansing effect by offering funding to both types of firms. Intuitively, under a concave $F(\cdot)$, the marginal impact of $\bar{g}$ on survival probability is greater for type- L firms because type- H firms already have a higher level of private-sector funding.

Corollary 3 (Funding-demand friction and the cleansing effect) *For any ζ that causes both types of firms to strategically default, and when the private-sector credit limit binds before the private-sector funding constraint (i.e., $\hat{d}_t^j < \bar{d}$), a cleansing effect emerges in crises: the surviving probability of type- H firms is greater than that of type- L firms. Credit intervention weakens this channel.*

In sum, two channels of cleansing effects emerge in crises: (1) among firms that repay debts, type- H firms *want to* borrow more to preserve capital; (2) among firms that strategically default,

²⁵Appendix C.2 provides evidence that type- H firms have larger financing capacity than type- L .

type- H firms *can* borrow more. Credit intervention strengthens the first channel of cleansing effect but it can weaken the second channel. Moreover, there is a limit to the impact of credit intervention through the first channel: If $\bar{g}$ is sufficiently large to allow the first-best spending on survival for both types, then any further increase in $\bar{g}$ no longer changes the survival probability for firms that repay debts. In contrast, there is no limit to the impact of credit intervention on the second channel: no matter how large $\bar{g}$ is, any further increase in $\bar{g}$ always improves the survival probability for firms that strategically default as they maximize their borrowing. Therefore, for a sufficiently large $\bar{g}$, the weakening of cleansing effect by credit support for firms that strategically default dominates the strengthening of cleansing effect by credit support for firms that repay debts.

So far, we have discussed that a type- H firm has a higher survival probability than a type- L firm if either both firms repay their debts, in which case type- H firms *want to* borrow more under a higher spending target, or if both firms strategically default, in which case type- H firms *can* borrow more under a higher debt capacity. There is also the case where a type- H firm repays its debt while the type- L firm with the same ζ strategically defaults.²⁶ In the following, to sharpen the analytical results, we introduce a parameter restriction so that we can focus on characterizing the quantitatively relevant region of parameter values. The following restriction is not a binding constraint in our parameter calibration; in other words, we do not impose the restriction in calibration, but we find that the restriction is satisfied in our baseline calibration and holds in a much broader set of parameter values. We will discuss calibration in detail in the next section.

Condition 1 q^H is sufficiently large and q^L is sufficiently low so that the equilibrium has the following properties: (1) $\underline{\zeta}^H = 0$, (2) $\bar{\zeta}^L < \bar{d} + \bar{g}$, and (3) $\bar{\zeta}^H > \hat{d}^L(\underline{\zeta}^L) + \underline{\zeta}^L + \bar{g}$.

Condition 1 focuses on capital values. As discussed in Subsection 2.3, q^j drives $\bar{\zeta}^j$ and, given the severity of a firm's liquidity crisis, ζ , it determines the first-best level of spending through $\bar{\zeta}^j$. Moreover, q^j determines the strategic default threshold $\underline{\zeta}^j$ and the endogenous debt limit, $\hat{d}^j(\zeta)$. As we explain below, this condition is quite general because one can view the binary types in our model as a discretized version of a continuum of firm types: type- H firms represent the right tail of the most productive, while the firms with low productivity are represented by type L in the model.

Under Condition 1, type- H firms do not strategically default ($\underline{\zeta}^H = 0$) because the firm owners find it rather costly to lose control rights and capital value. Firms may still face a lack of funding

²⁶Note that from Lemma 2, type- H firms have a lower default threshold for ζ , so given the same ζ , we can rule out the case where the type- H firm strategically defaults while the type- L firm repays its debt.

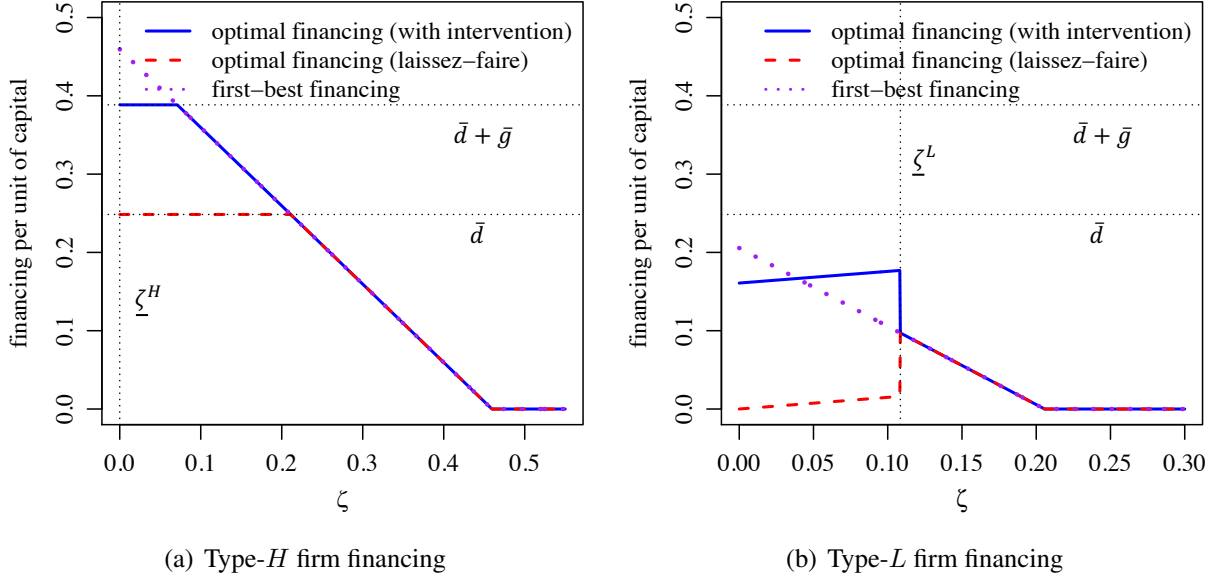


Figure 1: Optimal firm financing. This figure illustrates the optimal firm financing in different scenarios, under the calibrated model parameters that we discuss in Section 3. The solid line shows the optimal financing under credit intervention. The dashed line shows the optimal financing without credit intervention ($\bar{g} = 0$). The dotted line shows the first-best financing amount without funding supply- and demand-side frictions.

supply, so, for type H , the relevant form of inefficiency is under-spending. Panel A of Figure 1 illustrates the optimal amount of financing for a type- H firm with different values of ζ . This graph is based on the calibrated parameter values that will be discussed in the next section. When a firm has a sufficiently large ζ , the first-best level of financing is zero (see (15)). As ζ declines, the liquidity crisis becomes more severe, and the first-best level of financing increases (dotted line). Financing is constrained by funding supply, which is equal to $\bar{d}$ without government intervention (dashed line) and is equal to $\bar{d} + \bar{g}$ with government intervention (solid line).

The second property, $\bar{\zeta}^L < \bar{d} + \bar{g}$, implies that type- L firms do not underspend due to the lack of funding supply even when the liquidity shock is most severe ($\zeta = 0$ and the first-best spending is $\bar{\zeta}^L$). This is due to the fact that a low value of type- L capital leads to a low first-best spending level. As shown in Panel B of Figure 1, the first-best level (dotted line) is below $\bar{d} + \bar{g}$. Note that type- L firms still face the endogenous debt limit due to strategic default. The dashed line in Panel B of Figure 1 shows that when ζ is small and the liquidity needs are strong, the endogenous debt limit binds. As ζ increases from zero, the debt limit, $\hat{d}^L(\zeta)$, increases (see Lemma 3), so the firm is able to raise more funds. However, as long as ζ is below the strategic default threshold, $\underline{\zeta}^L$, the

financing level is below the first-best. The solid line shows the situation with credit intervention. The firm is able to raise more funds from the government and from the private-sector creditors due to the crowding-in effect in Lemma 4. However, over-spending emerges among firms with relatively high ζ (still below $\underline{\zeta}^L$), even though those with ζ close to zero still underspend.

As previously discussed, crises generate a cleansing effect among firms that repay debts and among firms that strategically default. However, given the same ζ , when type- H firms repay debts and type- L firms strategically default (i.e., $\zeta \in (\underline{\zeta}^H, \underline{\zeta}^L)$), it is ambiguous which type spends more. The last property delivered by Condition 1 resolves such ambiguity. For any $\zeta < \underline{\zeta}^L$ (type- L 's strategic default region), the last property implies $\bar{\zeta}^H - \zeta > \hat{d}^L(\underline{\zeta}^L) + \bar{g}$ so that type- H firms spend more on survival than type- L firms, which leads to the cleansing effect of crises in this region of ζ .

Proposition 5 (Capital destruction and the cleansing effect in crises) *Crises feature capital destruction, $\Delta_t^K < 0$, and under Condition 1, have a cleansing effect, $\Delta_t^\omega > 0$.*

Our focus on crises with cleansing effects is empirically motivated. For example, during the COVID-19 crisis, firms with lower productivity were more likely to cease operations permanently (Muzi et al., 2023). And, during the COVID-19 crisis, economic activity was reallocated toward firms with higher pre-crisis labor productivity, and such reallocation is stronger compared with pre-crisis times (Bruhn et al., 2023). These studies also document that credit intervention dampens the cleansing effect. Moreover, Dörr et al. (2022) find that credit intervention during the COVID-19 pandemic disproportionately benefited firms that were already financially vulnerable pre-crisis.²⁷ Consistent with the evidence, the proposition shows that credit intervention dampens the cleansing effect in our model. Moreover, credit intervention has a positive effect on firm survival and the total quantity of capital preserved, which is also consistent with the evidence (Bartik et al., 2020; Bartlett and Morse, 2020; Hubbard and Strain, 2020; Denes et al., 2021; Kawaguchi et al., 2021). Therefore, the government faces a trade-off between capital quantity, K_t , and quality ω_t .

Proposition 6 (The quantity-quality trade-off) *Credit intervention alleviates capital destruction*

$$\frac{\partial \Delta_t^K}{\partial \bar{g}} > 0,$$

²⁷Muzi et al. (2023) analyze data from firms in 34 countries. Bruhn et al. (2023) utilizes the World Bank's Enterprise Surveys COVID-19 Follow-up Surveys, encompassing around 8,000 firms in 23 emerging and developing countries across Europe and Asia. Dörr et al. (2022) examine 1.5 million German companies.

but dampens the cleansing effect

$$\frac{\partial \Delta_t^\omega}{\partial \bar{g}} < 0.$$

The slippery slope of intervention. The cleansing effect of crises, $\Delta_t^\omega > 0$, and the impact of government intervention, $\partial \Delta_t^\omega / \partial \bar{g} < 0$ are important for analyzing the intertemporal linkages. Firm quality, ω_t , is a key state variable. By dampening the cleansing effect of the current crisis, credit intervention begets an intervention of greater scale in the next crisis. The following proposition characterizes a critical connection between pre-crisis firm quality (ω_{t-}) and intervention scale.

Proposition 7 (Pre-crisis firm quality and intervention scale) *To contain the output drop at any given level, the required scale of intervention is larger if the pre-crisis firm quality is lower.*

Output drop in crises is caused by the decline of capital quantity. Since type- L firms have a lower capital productivity, they need more units of capital to produce the same level of output than type- H firms. Furthermore, type- L firms have less private-sector financing capacity so they need to rely more on support from the government. Therefore, when an economy enters into a crisis with more type- L firms and fewer type- H firms (i.e., a lower ω_{t-}), it requires the government to provide a larger scale of intervention if the goal is to contain the drop in output at a certain level.²⁸

This connection between the pre-crisis firm quality and intervention scale is key to establishing the slippery slope of intervention. In Proposition 6, we show that credit intervention dampens the cleansing effect and reduces the post-crisis average firm quality, ω_t . When the economy enters into the next crisis with a lower firm quality, the scale of intervention has to increase, if the policymaker aims to contain the output drop at any given level. In the next section, we calibrate our model and show that the slippery slope of intervention is quantitatively important.

Discussion: Low productivity firms vs. zombie firms. Zombie firms are permanently impaired firms whose operation relies on external financial resources (Caballero et al., 2008; Acharya et al., 2019, 2021). Type- L firms are not zombies. In a crisis, both type- H and L firms may over-spend and can thus be viewed as engaging in a negative-NPV transaction, but after they survive the crisis, these firms will spend at the efficient level in normal times, guided by their Tobin's q (capital value), and use their capital to produce. Firms' problem in crises is of a temporary nature. We

²⁸This mechanism is still valid when the policy goal is to contain capital destruction (see Appendix D.3).

model a liquidity crisis, not a solvency crisis. Relative to the literature on zombie firms, our paper offers a different and complementary perspective on the distortions from credit intervention. In Appendix B.4, we discuss how to incorporate zombie firms in our model and how the presence of such firms amplifies the distortionary effects of policy intervention. Finally, we emphasize that our mechanism does not rely on zombies crowding out normal firms in product or factor markets, which is a key ingredient in models on crisis cleansing effect, zombie firms, and the efficiency implications of policy intervention (e.g., Caballero and Hammour, 1994; Acharya et al., 2021).

3 Quantitative Analysis

3.1 Model Calibration

For simulation analysis, we initiate ω_t at $\bar{\omega}$, which is long-run average implied by the stationary distribution of ω_t , determined by both the drift and Poisson components of its law of motion.

Before calibrating the model, we parameterize the survival probability function as $F(x) = 1 - \exp(-\lambda_F x)$ and, for the cumulative distribution function of ζ that indexes the baseline level of survival probability, we use $H(\zeta) = 1 - \exp(-\zeta/l_\zeta)$ (with an average ζ equal to l_ζ). Including λ_F and l_ζ , we have a total of twelve parameters. The other ten parameters include the investment cost parameter θ , crisis frequency λ , firm owners' rent from debt renegotiation β , capital depreciation rate δ , discount rate r , capital productivities A^H and A^L , entry rate of new firms η , the private-sector credit-supply limit in crises $\bar{d}$, and a baseline value for government credit support $\bar{g}$. The parameter values and moment conditions are summarized in Table 1 and Table 2, respectively.

The investment cost function controls the average investment rate of the economy, and we set the parameter θ to generate an average annual investment-to-capital ratio of 10% following the literature (e.g., Gertler et al. (2020) target a 2.5% quarterly investment-to-capital ratio).

We set the parameter governing the survival probability function, λ_F to generate an average output drop in crises that matches the average GDP decline in crises. According to Reinhart and Rogoff (2009), the peak-to-trough decline in GDP across a large sample of crises is 9.3%.

The parameter l_ζ in the distribution of ζ affects the sensitivity of firm survival to credit intervention. When l_ζ is small, there is a large density of type- L firms that rely on government funding support, so the impact of intervention on firm survival is stronger. Bartlett and Morse (2020) used a survey of 1,000 small businesses to study how PPP loans impacted their survival likelihoods. They

Table 1: Parameter values. This table shows the parameter values in our quantitative analysis and the corresponding calibration targets. Five model parameters are directly set by the observed data, including λ , δ , r , η , and $\bar{g}$. The rest of parameters, θ , λ_F , l_ζ , β , A^H , A^L , and $\bar{d}$ are solved to match the moment targets.

Parameter	Description	Value	Data Source or Targeted Moment
θ	Investment cost	9.2	Average investment/capital ratio
λ_F	Survival rate	5.8	Average GDP drop in crises
l_ζ	Average ζ	0.16	Impact of credit intervention on firm survival
λ	Crisis frequency	0.07	Crisis frequency in the data
β	Debt restructuring rent	0.43	Average creditor recovery rate
δ	Capital depreciation rate	0.2	Capital depreciation and firm exit rate
r	Real discount rate	0.066	Average real bond return plus equity premium
A^H	Productivity of type- H firms	0.57	Average output-to-capital ratio
A^L	Productivity of type- L firms	0.15	TFP inter-quartile ratio
η	Entry rate of new firms	0.062	Firm entry rate
$\bar{d}$	Private-sector credit availability	0.25	Private-sector debt/GDP ratio
$\bar{g}$	Government credit support	0.14	COVID-19 credit support in the U.S.

Table 2: Moment matching. We report the moment matching results for calibrating θ , λ_F , l_ζ , β , A^H , A^L , and $\bar{d}$.

Moment Description	Model	Data
Average investment-to-capital ratio	0.1	0.1
Average GDP drop in crises	-9.3%	-9.3%
Average impact of credit intervention on firm survival likelihood	10%	10%
Average creditor recovery rate	49%	49%
Average output-to-capital ratio	45%	45%
TFP ratio between 90% and 10% percentiles	3.7	3.7
Average private-sector debt/GDP ratio	36%	36%

find that PPP application success increased a firm’s medium-run survival probability by 20.5%, but only for microbusinesses (those with 1-5 employees). Kawaguchi et al. (2021) surveyed small businesses during the COVID-19 pandemic and found that lump-sum subsidies in Japan increased firms’ self-reported prospects of survival by 19%. However, there is also evidence that the impact is much smaller. For example, Hubbard and Strain (2020) used an intent-to-treat model with data on private firms from Dun & Bradstreet. They find that PPP eligibility reduced business closure odds by 0.22% with no significant effect for firms closer to the 500-employee cutoff. Given the mixed evidence, we target a 10% survival likelihood improvement.

Next, for crisis frequency λ , we map to the empirical counterpart in Taylor (2015), i.e., the frequency of crises is about 6% in a panel of 17 countries from 1800 to 2012.

The parameter β , which represents the borrowers' value after debt renegotiation and restructuring, governs the firm owners' incentive to strategically default. We set the parameter to generate a bankruptcy recovery rate for creditors that match the empirical counterpart. Below we explain how we calculate the creditors' recovery rate in our model. For type- j firm, $j \in \{H, L\}$, there are two cases of bankruptcy: (1) liquidity-induced bankruptcy, which happens with probability $1 - F(\zeta + x^j(\zeta))$, and the recovery rate is zero; (2) strategic default, which happens when a firm survives the crisis and has $\zeta < \underline{\zeta}^j$, generates a recovery rate for creditors equal to $(q^j - \beta)/x^j(\zeta)$. Therefore, the average creditors' recovery rate of type- j firms in bankruptcy (averaging over ζ) is

$$\mathcal{R}^j = \int \left((1 - F(\zeta + x^j(\zeta))) \cdot 0 + F(\zeta + x^j(\zeta)) 1_{\zeta < \underline{\zeta}^j} \frac{q^j - \beta}{x^j(\zeta)} \right) dH(\zeta)$$

with a conditional probability mass

$$\mathcal{M}^j = \int \left((1 - F(\zeta + x^j(\zeta))) + F(\zeta + x^j(\zeta)) 1_{\zeta < \underline{\zeta}^j} \right) dH(\zeta)$$

Then total (cross-type) average creditor recovery rate is

$$\bar{R} = \frac{\bar{w}\mathcal{R}^H + (1 - \bar{w})\mathcal{R}^L}{\bar{w}\mathcal{M}^H + (1 - \bar{w})\mathcal{M}^L}$$

The moment target for $\bar{R}$ is the empirical average creditor recovery in Chapter 11 bankruptcies. According to Antill (2022), average recovery rate from Moody's Ultimate Recovery database is 49%, and according to Dou et al. (2021), the average recovery rates from across different classes of creditors is 46%.²⁹ We use 49% recovery rate as our targeted value.

The parameter, δ in the model can be interpreted as the depreciation rate of capital plus exits of businesses in normal times. According to Business Dynamics Statistics, the average firm establishment exit annual shutdown rate is about 10% from 1982 to 2020. With a quarterly depreciation of 2.5% (Gertler and Karadi, 2011), we set $\delta = 0.025 * 4 + 0.1 = 0.2$.³⁰

The discount rate r reflects how the firm owners discount cash flows. Since productive capital is a long-term asset, we consider the long-term real rate plus an unlevered equity premium. In the U.S., the historical average real return of government bond is 2.85% and the historical average real

²⁹See Row 1 of Table 1 in Antill (2022). Table 1 Panel B in Dou et al. (2021) reports junior and senior creditor recovery rates and their fractions of total debt. The weighted average is $(0.559)*0.204 + (1-0.559)*0.788 = 0.46$.

³⁰We note that a larger depreciation parameter δ in the literature is not uncommon, especially when it incorporates the business exit rate. For example, Gertler et al. (2020) set $\delta = 0.33$ to match the investment to capital ratio.

return of equity is 8.46%.³¹ The proper discount rate on capital is the unlevered cost of equity, which is between the two discount rates, depending on average leverage. With a historical average debt/equity ratio of 0.5, the unlevered real return of equity is 6.6%.³² Therefore, we set $r = 0.066$.

We choose the productivity parameters A^H and A^L jointly to generate a ratio A^H/A^L that matches the ratio between 90% and 10% percentiles of TFP distribution³³ and an average output-to-capital ratio close to 1/2.24 in Elenov et al. (2021).³⁴ And, we calibrate the firm entry rate to the data counterpart in Clementi and Palazzo (2016), setting $\eta = 0.062$.³⁵ For $\bar{d}$ that determines private-sector credit availability, we target the ratio of total (non-financial) private-sector debt to GDP in the data. This ratio is on average 0.36 in the historical sample of 1952–2023.³⁶

Finally, we set the baseline case $\bar{g}$ according to the size of intervention (not total takeover, which is smaller) during COVID-19, including programs initiated by both the central bank and government in the U.S. The Fed set up around 2.3 trillion credit support for firms ($\sim 11\%$ of 2020 GDP) decomposed into 0.6 trillion in MSLP, 0.75 trillion in PMCCF and SMCCF, and 0.95 trillion in PPP Liquidity Facility. The total fiscal response that supports businesses is 4 trillion.³⁷ Taken together, the amount is about 30% of the GDP in 2020. Thus, we set the ratio of credit support-to-capital stock $\bar{g} = 0.45 * 30\% \approx 0.14$ where, as previously discussed, 0.45 is the output-to-capital ratio.

3.2 The Trade-Off between Quantity and Quality

In Proposition 6, we show that an increase in the scale of credit intervention preserves more units of capital in crises, i.e., $\partial \Delta_t^K / \partial \bar{g} > 0$, but dampens the cleansing effect of crises, i.e., $\partial \Delta_t^\omega / \partial \bar{g} < 0$. Capital quantity, K_t , and quality, ω_t are the two state variables that drive aggregate dynamics. Figure 2 illustrates the quantity-quality trade-off. We fix the pre-crisis ω_t at the average value $\bar{\omega}$

³¹See Tables VII and X of Jordà et al. (2019). Note that the world-average real return of long-term government bond is 2.61%, close to the U.S. average. The world-average real equity return is 7.12%, also similar to the U.S. average.

³²We download the flow-of-funds data on non-financial corporate business debt (ticker “BCNSDODNS”) and equity (ticker “NCBEILQ027S”), and the average debt/equity ratio from 1951–2022 is about 0.5. We ignore tax and calculate the unlevered required return on equity as $(0.0846 + 0.5 \times 0.0285)/(1 + 0.5) = 6.6\%$.

³³See Appendix C.1 for details of how we measure the TFP distribution and Appendix D.2 for our solution of an extended model where a firm’s productivity switches between A^H and A^L at idiosyncratic Poisson times.

³⁴An alternative target is the Kaldor facts in growth models, i.e., an average capital to output ratio of 2.5.

³⁵In Appendix D.1, we conduct sensitivity analysis for this parameter.

³⁶See Table “Debt of Nonfinancial Sectors, 1952–2023” under “Z.1-Financial Accounts” issued by the Fed.

³⁷For the decomposition of the programs offered by the Fed, see this Brookings article, “[What did the Fed do in response to the COVID-19 crisis?](#)”, and for the decomposition of fiscal response, see this IMF report, “[Fiscal Monitor Database of Country Fiscal Measures in Response to the COVID-19 Pandemic](#)”.

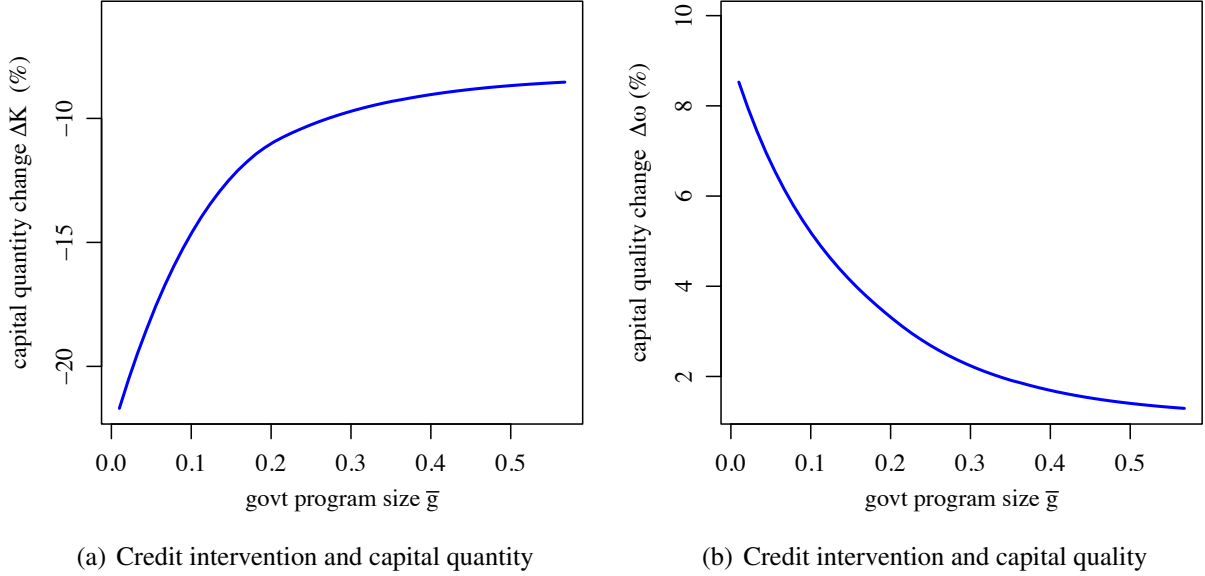


Figure 2: Credit intervention impact: Capital quantity vs. quality. This figure illustrates how $\bar{g}$ affects ΔK_t and $\Delta\omega_t$ in a crisis. The calculation requires the pre-crisis ω_{t-} which we set to the average value $\bar{\omega}$ defined in Section 3.1.

defined in Section 3.1. In Panel A, we plot the percentage change of K_t against $\bar{g}$. The figure shows that as intervention changes from $\bar{g} = 0$ to $\bar{g} = 0.14$ (i.e., 30% of GDP as discussed in Section 3.1), the percentage destruction of capital shrinks from -22% to -13% . In Panel B, as intervention changes from $\bar{g} = 0$ to $\bar{g} = 0.14$, the cleansing effect $\Delta\omega_t$ falls from about 9% to 4%.

An increase in the scale of intervention, $\bar{g}$, improves efficiency by reducing the gap between financially constrained firms' spending and the first-best level. A higher $\bar{g}$ can also lead to over-spending among firms that strategically default. While both forces preserve capital quantity, their impact on capital quality differs. Lemma 2 shows that there are more type- L firms that choose to strategically default than type- H firms. The calibrated parameter values satisfy Condition 1 in Section 2: type- H firms do not strategically default, while strategic default and over-spending can happen among type- L firms. Therefore, when $\bar{g}$ increases, its positive impact on type- H firms' survival is limited, as type- H firms do not spend beyond the first-best level. In contrast, among type- L firms that strategically default, spending on survival always increases in $\bar{g}$. Thus, as the size of intervention increases, type- L firms benefit more. This dampens the cleansing effect of crises.

Panel A of Figure 3 illustrates this mechanism. Fixing a value of ζ , we plot the optimal financing and spending of a type- H firm (dashed line) against $\bar{g}$, which flattens out at the first-best

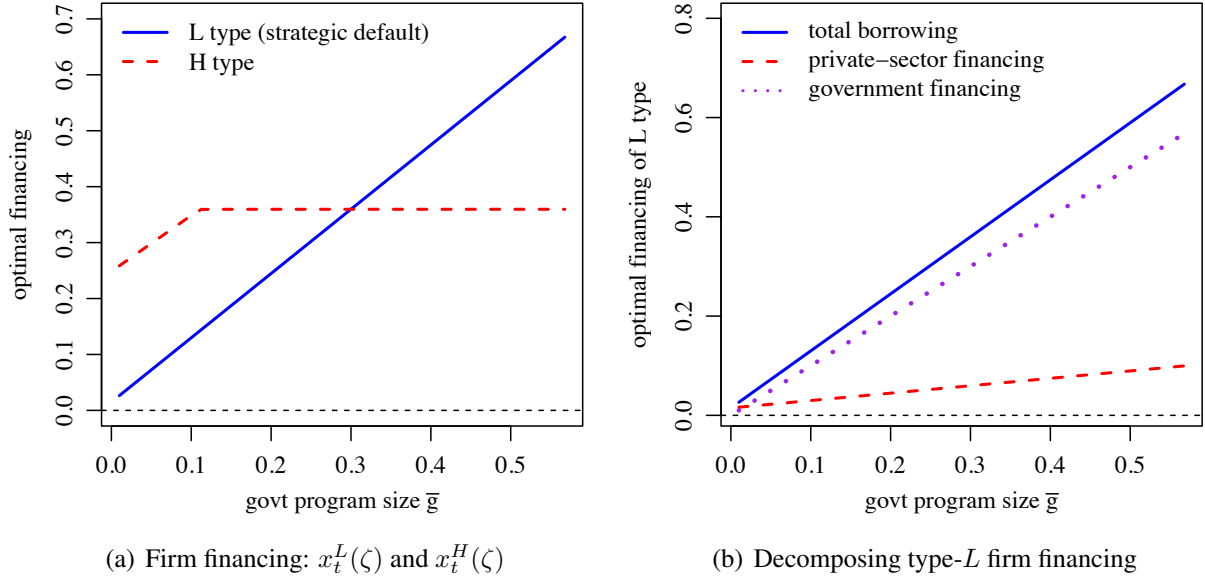


Figure 3: Credit intervention and firm financing. This figure illustrates how intervention affects the optimal amount of financing by H and L type firms, accounting for the endogenous responses of q^H and q^L . In both panels, we choose $\zeta = 0.1$, which is a case of strategic default for L-type firms. In panel (b), we decompose the actual borrowing of L-type firms into private-sector financing (which reaches the endogenous limit $\hat{d}^L$), and government financing (which reaches the limit $\bar{g}$). As $\bar{g}$ changes, all equilibrium variables change accordingly, including capital values.

level, and that of a type- L firm that strategically defaults, which is always increasing in $\bar{g}$. In Panel B, we decompose the type- L firm's spending into funding from the government (dashed line) and the private sector (dotted line). A higher $\bar{g}$ allows the type- L firm to spend more through both the direct liquidity provision and crowding in the private-sector funding (see Lemma 4 in Section 2).

So far, our discussion of the welfare and efficiency implications of credit intervention has focused on the impact in crises, i.e., the quantity-quality trade-off, $\partial\Delta_t^K/\partial\bar{g} > 0$ and $\partial\Delta_t^\omega/\partial\bar{g} < 0$. The impact of credit intervention also spills over to normal times. By improving survival probability for both types, credit intervention increases capital values, q^L and q^H , and thereby raises normal-time investment rates, ι^H and ι^L (see (7)), lifting upward the growth trajectory of capital quantity, K_t . Moreover, as previously discussed, the positive impact on survival probability is greater for type- L firms (which is why credit intervention dampens the cleansing effect of crises), so a higher $\bar{g}$ increases q^L more than q^H , introducing a downward bias in the drift of ω_t .

Next, we introduce the welfare function as a criterion for the overall impact of credit intervention. At time t , the social welfare is defined as the present value of life-time consumption flows

and is a function of the two state variables, K_t and ω_t . Since the economy is scalable with respect to capital, we conjecture the welfare at time t as $W(\omega_t)K_t$. It can be written as follows:

$$\mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} (\omega_s A^H + (1 - \omega_s) A^L) K_s ds - (\omega_s \iota_s^H + (1 - \omega_s) \iota_s^L) K_s ds - I_s K_{s-} dN_s \right], \quad (25)$$

where, in the integral, we record the consumption flow as the aggregate output net of goods invested in normal times and crisis times, and the spending in a crisis at time s , I_s , is given by

$$I_s \equiv \omega_{s-} \int_{\zeta} x_s^H(\zeta) dH(\zeta) + (1 - \omega_{s-}) \int_{\zeta} x_s^L(\zeta) dH(\zeta). \quad (26)$$

In the stationary equilibrium, the K_t -scaled welfare function $W(\omega)$ satisfies the following ordinary differential equation:

$$\begin{aligned} rW(\omega) = & \omega A^H + (1 - \omega) A^L - (\omega \iota^H + (1 - \omega) \iota^L) + W(\omega) \mu_K(\omega) + W'(\omega) \mu_\omega(\omega) \\ & - \lambda I(\omega) + \lambda [W(\omega + \Delta^\omega(\omega)) (1 + \Delta^K(\omega)) - W(\omega)] \end{aligned} \quad (27)$$

We numerically solve for the welfare function $W(\omega)$ as determined by equation (27) and illustrate the impact of $\bar{g}$ on welfare in Figure 4. To highlight the dependence of welfare on $\bar{g}$, we write the welfare as $W(\omega; \bar{g})$. Figure 4(a) shows the impact of $\bar{g}$ on the average firm quality $\bar{\omega}(\bar{g})$, where the average is calculated based on simulated path of the economy. We find that as the government expands its scale of intervention, the average firm quality declines.

Figure 4(b) plots the percentage improvement in welfare from the laissez-faire economy to the intervened economy, $W(\bar{\omega}(\bar{g}); \bar{g})/W(\bar{\omega}(0); 0) - 1$, which increases in $\bar{g}$ for small $\bar{g}$, but decreases once $\bar{g}$ passes a threshold. The welfare curve is upward-sloping when the scale of intervention is low. The marginal improvement of welfare due to type- H firms' efficient spending dominates the loss from wasteful spending by type- L firms that strategically default. Therefore, a timid intervention almost guarantees a positive contribution to welfare at the margin. In Corollary 1 in Section 2, we show that excessive credit intervention can destroy welfare, which is on the downward-sloping segment of the welfare curve. The over-spending by type- L firms comes at the expense of aggregate consumption, so even though the total capital stock, K_t , grows faster, the social welfare, which is the present value of households' life-time consumption, declines. Intuitively, the optimal intervention balances the benefit from relaxing the financial constraint on under-spending firms

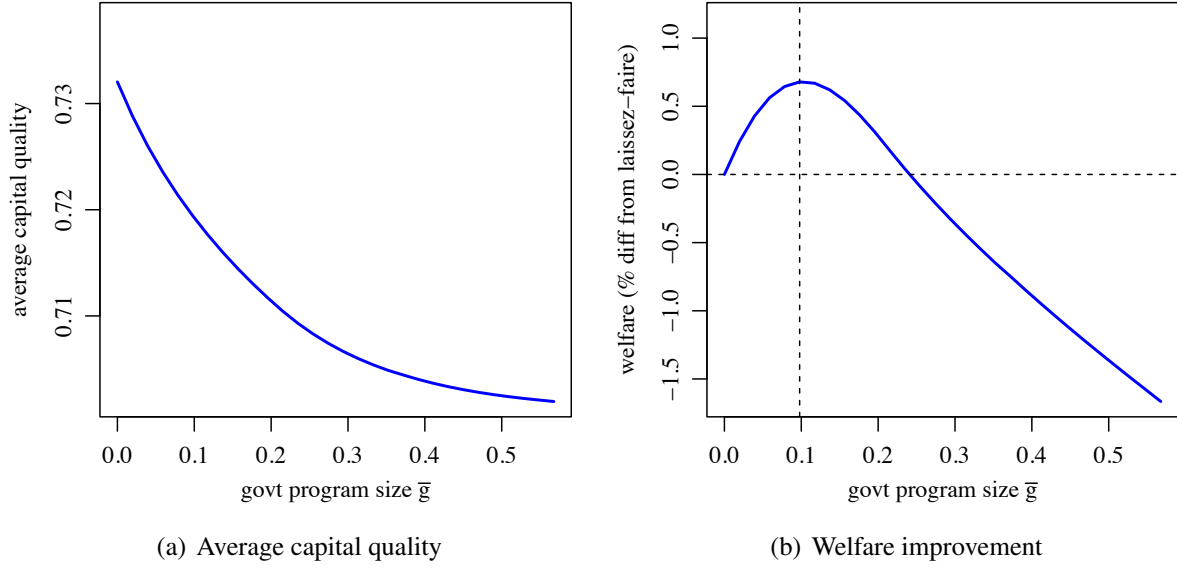


Figure 4: Credit intervention and welfare. In panel (a), we plot the average firm quality $\bar{\omega}$ as a function of government intervention $\bar{g}$. For each $\bar{g}$, we solve the model again and calculate the average of simulated ω_t as $\bar{\omega}(\bar{g})$. In panel (b), we show the welfare difference $W(\bar{\omega}(\bar{g}); \bar{g})/W(\bar{\omega}(0); 0) - 1$ as a function of government intervention $\bar{g}$.

and the cost of exacerbating over-spending among type- L firms that strategically default. The vertical dotted line marks the optimal intervention size, which is equivalent to 22% of GDP. For comparison, in Section 3.1, we document that the total scale of credit support from the government and central bank in the U.S. during the COVID-19 crisis is about 30% of GDP.

In the stationary equilibrium, the scale of intervention, $\bar{g}$, is constant, chosen at $t = 0$. In Panel A of Figure 5, we allow the government to optimize $\bar{g}$ at $t = 0$, i.e.,

$$g^*(\omega_0) = \arg \max_{\bar{g}} W(\omega_0; \bar{g})$$

and plot the optimal $\bar{g}$ against ω_0 (solid line). The curve is upward-sloping. Intuitively, at the left end where the economy is dominated with type- L firms, the government optimally restricts funding support because a large scale intervention is likely to result in type- L firms' over-spending. In contrast, near the right end where type- H firms dominate, the optimal scale of intervention is high. When the economy starts with a larger fraction of firms being type- H (i.e., ω_0 is higher), the planner can focus more on the efficiency gain from addressing type- H firms' under-spending.

The mechanism in our model has quantitatively important implications for policy making. The

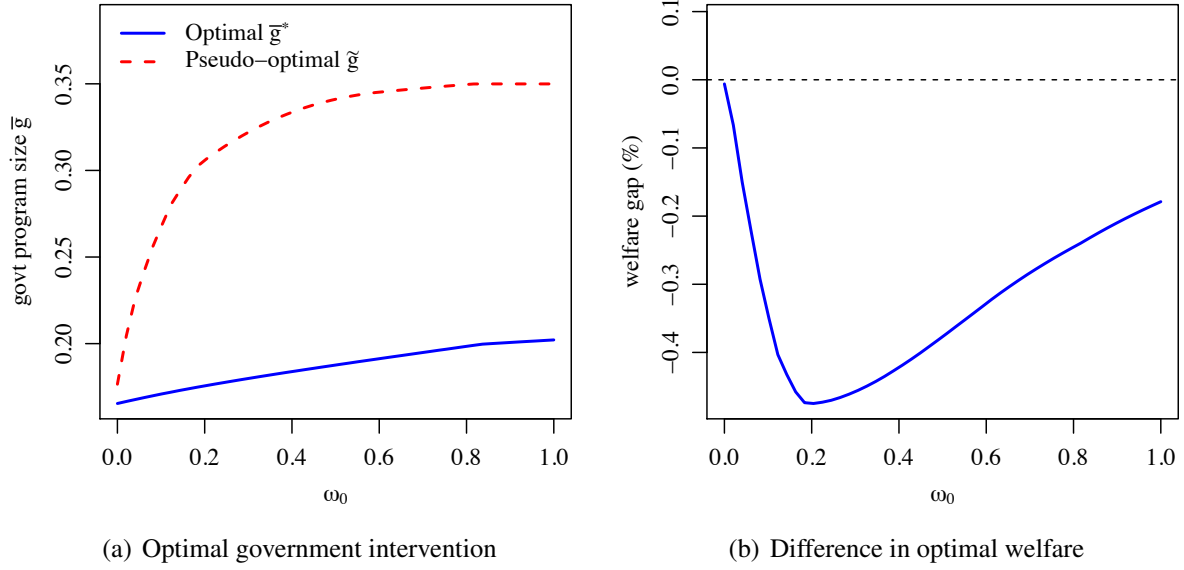


Figure 5: Optimal intervention and welfare. In panel (a), we plot the optimal intervention $\bar{g}^*(\omega_0)$ as a function of the initial state ω_0 , and the pseudo-optimal policy $\tilde{g}(\omega_0)$ that ignores the impact of intervention on firm quality change, treating $\Delta^\omega(\omega; \bar{g})$ as $\Delta^\omega(\omega; 0)$ for all $\bar{g}$. In panel (b), we show the percentage decline of social welfare due to using the pseudo-optimal policy instead of the optimal policy.

dashed line in Panel A of Figure 5 plots a “pseudo-optimal” $\bar{g}$ that is chosen at $t = 0$ without considering the negative impact of credit intervention on capital quality, ω_t . Specifically, when solving the welfare function given by (27), the planner mistakenly replaces $\Delta^\omega(\omega; \bar{g})$ with $\Delta^\omega(\omega; \bar{g} = 0)$. The resultant “pseudo welfare function”, denoted by $\tilde{W}(\omega; \bar{g})$, represents a policy-making criterion that ignores the key mechanism in our paper—credit intervention dampens the cleansing effect of crises—and thus only focuses on the positive impact on capital quantity. Formally, the associated “pseudo-optimal scale of intervention” is given by

$$\tilde{g}^*(\omega_0) = \arg \max_{\bar{g}} \tilde{W}(\omega_0; \bar{g})$$

As shown in Panel A of Figure 5, ignoring the policy impact on capital quality leads to intervention (dashed line) that is almost double the size of optimal intervention (solid line). In Panel B of Figure 5, we plot the percentage decline of welfare from using $\tilde{g}^*(\omega_0)$ rather than the optimal policy $g^*(\omega_0)$.³⁸ Ignoring the impact of credit intervention on capital quality translates into a sizeable

³⁸The curve starts at $\omega_0 = 0$ as it is an absorbing state where the economy is populated by only type- L firms and

welfare loss. The welfare cost is largest at the intermediate values of ω_0 .

3.3 The Slippery Slope of Credit Intervention

The downward bias in firm quality brought by credit intervention generates a slippery slope of intervention. We first show that the impact of credit intervention on ω_t persists over time. Therefore, intervention in the current crisis leads to a lower ω_{t-} entering into the next crisis. According to Proposition 7, a lower ω_{t-} translates into a greater scale of intervention if the policymaker's goal is to contain the output drop to a certain level. Therefore, in equilibrium, credit intervention in the current crisis begets interventions of greater scales in future crises.

As our focus shifts towards the dynamics of intervention, a key issue to address is agents' expectation of the policy plan. To clearly illustrate the mechanism, we focus on the *forward propagation* of intervention impact: In the current crisis, intervention dampens the cleansing effect, reducing ω_t , and such reduction affects the scale of intervention in future crises. Agents' expectation of the dynamic policy plan confounds the mechanism by introducing a *backward propagation* of intervention impact: agents' current behavior varies with their expectation of intervention in future crises. Agents' expectation enters into the equilibrium conditions only through the capital values, q^H and q^L . We shut down this expectation channel by solving q^H and q^L under $\bar{g} = 0$. In Appendix B.1, we consider a fully dynamic model where the government optimally adjusts the scale of intervention in every crisis, and agents, under rational expectation, have perfect knowledge of the policy plan so both the forward and backward propagation are active.

The distortionary effects of credit intervention on firm quality distribution are very persistent. In Panel A of Figure 6, we plot ω_t ten years ($t = 10$) after the current crisis ($t = 0$) against the intervention size $\bar{g}$ at $t = 0$. As in Section 3.2, the government sets a constant scale of intervention at $t = 0$. We fix the starting firm quality ω_0 at its deterministic steady state in the economy with $\bar{g} = 0$. An increase of credit intervention $\bar{g}$ from 0 to 0.14 (intervention/GDP about 30%, which is our baseline calibration) causes ω_t to decline by around 0.04 ten years later, which is a significant decline in the percentage of firms being type- H . Note that since we shut down agents' expectation of intervention in q^H and q^L , the persistent impact of intervention on ω_t does not come from policy distortions in agents' normal-time investments and the drift of ω_t . The persistent impact on ω_t is purely generated by the reduction of ω_t in the current crisis at $t = 0$.

$\omega_t = \omega_0 = 0$ so credit intervention cannot change ω_t . Note that the other extreme, i.e., $\omega_0 = 1$ where the economy has only type- H firms, is not an absorbing state due to the entry of type- L firms.

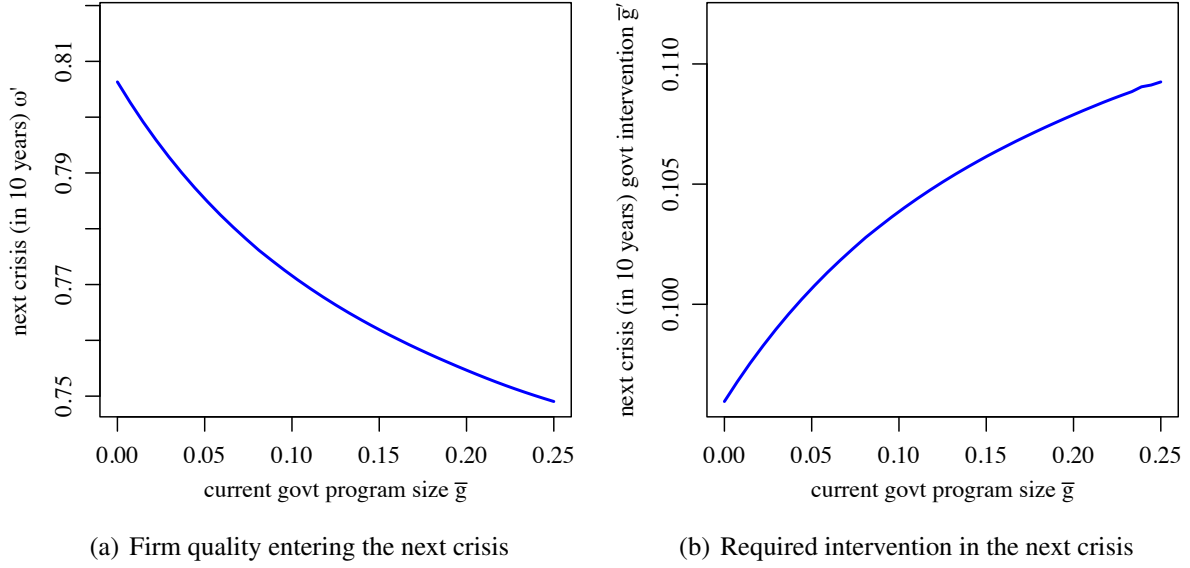


Figure 6: Intervention pass-through across crises. We show how $\bar{g}$ in the current crisis affects capital quality entering the next crisis, ω' , and intervention needed, $\bar{g}'$, in the next crisis to contain output drop within -10% . The next crisis happens ten years after the current one. Agents expect no intervention (pass-through is only due to forward propagation). The current crisis happens at ω equal to the average value of ω_t in the laissez-faire economy.

Accordingly to Proposition 7, such deterioration of capital quality in the future translates into a greater scale of intervention that is necessary for containing the output drop to a certain level. In Panel B of Figure 6, we consider a crisis that happens ten years from now and plot the necessary scale of intervention against the current scale of intervention. An increase of current intervention $\bar{g}$ from 0 to 0.14 leads to an increase of intervention scale with a pass-through rate of about 7%, i.e., each one dollar of intervention per unit of capital in the current crisis generates 7 cents extra intervention per unit of capital in the next crisis should it happen ten years later. If we take into account the growth of capital stock over the ten-year period, the inter-crisis pass-through rate in the dollar amount is even greater.

We further illustrate the slippery slope of credit intervention with a quarterly simulation of two crises, one in Q1 of the first year and the other in Q1 of the tenth year with $\bar{g}$ equal to 0.14 following our baseline calibration in Section 3.1. In Panel A of Figure 7, we compare the paths of ω_t in the simulation and in an economy without intervention, both starting from the average value of ω_t in the economy without intervention. After the first crisis, ω_t jumps upward due to the cleansing effect, but the quality wedge between the laissez-faire economy and intervened economy

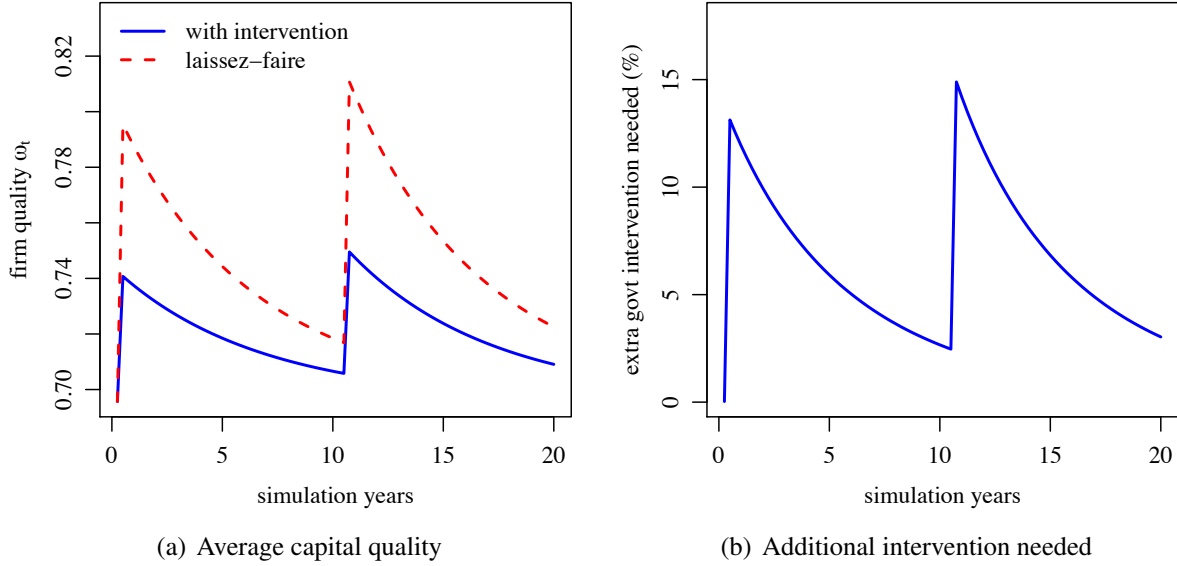


Figure 7: The slippery slope of intervention: Simulated paths. We compare the quarterly simulation of two economies, the laissez-faire economy and the intervened economy without agents' expectation of intervention, where one crisis happens in Q1 of the first year and another crisis in Q1 of the tenth year. Both simulations start at the average ω_t in the laissez-faire economy. In Panel (a), we plot the path of capital quality, ω_t . In Panel (b), for both economies, we calculate the amount of intervention required to contain output drop within -10% if a crisis happens over the next instant, and then we plot the percentage increase from the laissez-faire economy to the intervened economy.

widens as intervention dampens the cleansing effect. Because we shut down agents' expectation of intervention, firms in both economies have the same normal-time investment rates driven by the same capital values, and ω_t in both economies converge to the same steady state over time. In spite of the normal-time convergence, the impact of intervention on ω_t is persistent, evidently shown by the sizable wedge ten years later when the second crisis hits the economy. In the second crisis, the cleansing effect increases ω_t , and the quality wedge widens again.

Next, we take as given the simulated paths of ω_t of the two economies in Panel A of Figure 7, and, at any point in time along the paths, we calculate the necessary scale of intervention were a crisis to happen in the very next instant. In Panel B of Figure 7, we show the extra amount of funding support in the intervened economy relative to that in the laissez-faire economy in percentage terms. Here the government's goal is to contain the output drop within -10% . Because the two economies have the same drift of ω_t , the quality wedge is widest right after a crisis and narrows in normal times. Immediately after the first crisis, if another crisis were to happen, the intervened

economy requires about 13% more funding support from the government than the laissez-faire economy. The wedge shrinks in normal times but still quantitatively significant. For example, five years after the first crisis (at $t = 5$), if a crisis were to happen over the next instant dt , the intervened economy requires a 7% more funding support than the laissez-faire economy.

In sum, credit intervention biases ω_t downwards in crises. As a result, the economy enters into future crises with a smaller share of firms being type- H than the laissez-faire benchmark, so the credit support needed to contain output drop is larger. Our model generates a slippery slope of intervention, a trap of policymakers' own making: the past interventions cause the government to spend more should a crisis occur in the future. However, this policy trap can be a necessary evil because by relaxing firms' financial constraints in crises, policy interventions can improve welfare. In Appendix B.1, we show that the same pattern emerges even when the government optimally adjusts $\bar{g}$ in a fully dynamic fashion to maximize the social welfare.

3.4 Extension: Alternative Policy Design

The scale of intervention is set proportional to capital stock (i.e., a firm's operational scale in our model), and the government charges market-based interest rates. As previously discussed, this specification follows the policy design in practice. Next, we consider an alternative policy design that improves the welfare. In our model, inefficiency from intervention is from the over-spending by firms that maximize their borrowing and strategically default. These firms do not repay the loans. The firms that actually make repayments, by internalizing the market-based debt costs, do not over-spend. In summary, only firms that efficiently spend the government funding make repayments, and those who abuse it do not repay. Therefore, we consider a new policy: the government eliminates repayment and injects liquidity in the form of subsidy rather than loans.

The new design improves welfare relative to the baseline policy for the following reasons. First, by improving firms' survival probabilities, the subsidy reduces the interest rates charged by private-sector creditors. By doing so, it makes strategic default less attractive and repaying loans to private-sector creditors more attractive, enlarging firms' capacity to borrow from private-sector creditors (crowding in "informed liquidity"). This is the extensive margin: the subsidy reduces the number of firms that strategically default and may over-borrow and over-spend. Second, consider the intensive margin: eliminating repayment helps firms that do not strategically default (note that these firms spend efficiently) but does not change the situation of firms that strategically default.

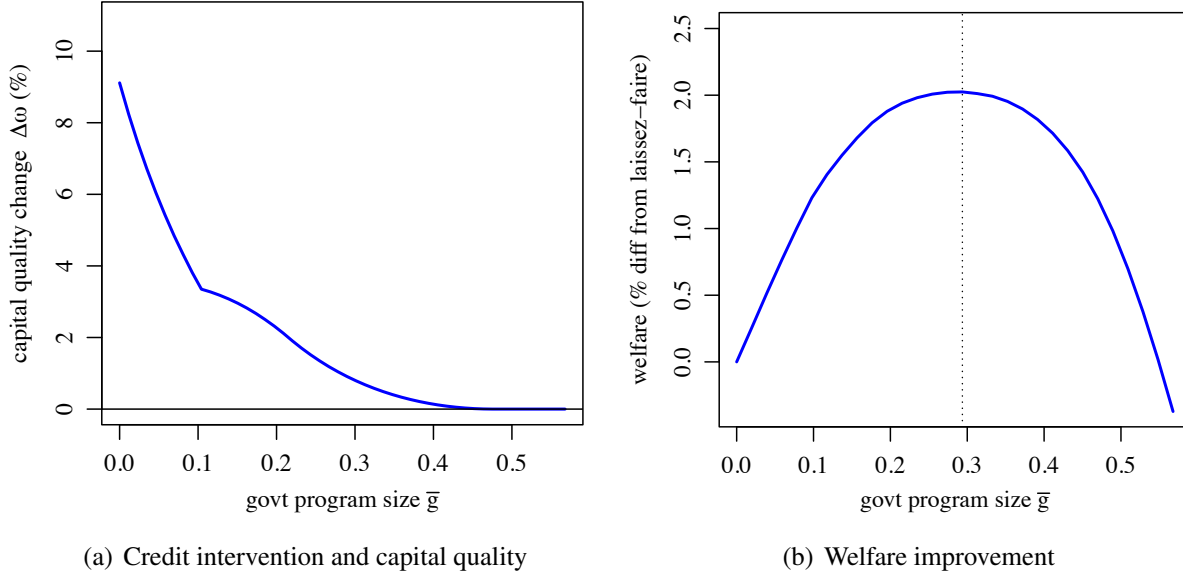


Figure 8: Impact of pure government subsidy: capital quality and welfare. This figure illustrates how $\bar{g}$ (government intervention is pure subsidy) affects $\Delta\omega_t$ and welfare. The calculation requires the pre-crisis ω_{t-} which we set to the average value $\bar{\omega}$. For each $\bar{g}$, we solve the model again and calculate the average of simulated ω_t as $\bar{\omega}(\bar{g})$. In panel B, we show the welfare difference $W(\bar{\omega}(\bar{g}); \bar{g})/W(\bar{\omega}(0); 0) - 1$ as a function of government intervention $\bar{g}$.

These firms do not repay the government anyway as their goal is to borrow as much as possible to increase survival probability so that their option to hold up creditors becomes in-the-money.

What we propose seems the opposite to the famous Bagehot’s Dictum—central banks should lend freely at high rates in crises. In Bagehot (1897), the condition behind this policy recommendation is that central banks only lend to solvent firms and only lend against good collateral. This condition requires central banks to be informed about firms’ solvency and collateral quality. In our paper, we study liquidity support provided by central banks and governments in general, and the starting point of our analysis is the lack of differentiation among firms of different qualities. Therefore, our model leads to a policy recommendation that differs from Bagehot’s. We acknowledge that this subsidy-based liquidity injection improves welfare relative to the baseline policy in our specific setting that may not reflect fully the complexity of realistic policy-making environments.

In Appendix D.4, we discuss details on the model solution. In Figure 8, we illustrate how government intervention in the form of a subsidy affects the change in firm quality in a crisis and welfare. In panel A, we find that increasing the scale of intervention still lowers firm quality in a crisis. Given $\bar{g}$, the impact seems stronger than that in the baseline model (see Figure 4) because,

without repayment, subsidy induces more take-up than loans. There is a kink point around $\bar{g} = 0.1$, above which the subsidy is so high that all firms choose not to strategically default on private-sector creditors. When firms can rely more on government subsidies, they borrow less from private-sector creditors so the benefit of strategic default diminishes.³⁹ Below this kink point, some firms still strategically default on private-sector creditors, and the inefficiency of over-borrowing exists; since type- L firms have stronger incentive to over-borrow and strategically default, intervention dampens the cleansing effect of crises more strongly below the kink point than above it.

In panel B of Figure 8, we find that there is a greater increase in welfare for a wider range of $\bar{g}$ than what Panel B of Figure 4 shows for the main model. Moreover, the optimal intervention scale, indicated by the peak of the welfare curve, is larger. Therefore, government intervention in the form of a subsidy is more efficient than loans in spite of the fact that intervention still distorts firm quality dynamics. In Appendix D.4, we show that intervention in the form of subsidy also features a slippery slope that is quantitatively similar to that of intervention in the form of loans.

4 Conclusion

To analyze the long-term consequences of credit intervention in crises, we develop a model of firm quality dynamics and highlight a trade-off between quantity and quality in determining the scale of intervention. Crises exhibit a cleansing effect: firms with high productivity *want to* spend more on surviving a crisis, and they *can* spend more than firms with low productivity because their financing capacity is larger. Credit intervention relaxes the financial constraint for all firms and preserves the total production capacity in the economy. However, by benefiting firms with low productivity more, credit intervention dampens the cleansing effect of crises. Our model generates a slippery slope of intervention. As the current intervention biases downward the firm quality distribution, the economy enters the next crisis with lower total productivity, and an intervention of a greater scale becomes necessary. Larger interventions lead to stronger distortions, which in turn call for even larger interventions in the future. However, we show that when carefully designed, credit intervention improves welfare relative to the laissez-faire benchmark.

³⁹Firm owners' value from strategic default, β , is below capital value, so if subsidy leads to less borrowing from (and repayment to) private-sector creditors, firms owners choose to keep capital (control rights) rather than go bankrupt.

Data Availability Statement. The data and code underlying this article are available at <https://doi.org/10.5281/zenodo.16895137>

References

- Abel, A. B. and J. C. Eberly (1994). A unified model of investment under uncertainty. *The American Economic Review* 84(5), 1369–1384.
- Acemoglu, D., U. Akcigit, H. Alp, N. Bloom, and W. Kerr (2018). Innovation, reallocation, and growth. *American Economic Review* 108(11), 3450–91.
- Acharya, V., I. Drechsler, and P. Schnabl (2014). A pyrrhic victory? bank bailouts and sovereign credit risk. *Journal of Finance* 69(6), 2689–2739.
- Acharya, V. V., M. Crosignani, T. Eisert, and S. Steffen (2022). Zombie lending: Theoretical, international, and historical perspectives. *Annual Review of Financial Economics* 14, 21–38.
- Acharya, V. V., T. Eisert, C. Eufinger, and C. Hirsch (2019). Whatever It Takes: The Real Effects of Unconventional Monetary Policy. *The Review of Financial Studies* 32(9), 3366–3411.
- Acharya, V. V., S. Lenzu, and O. Wang (2021). Zombie lending and policy traps.
- Acharya, V. V., P. Schnabl, and G. Suarez (2013). Securitization without risk transfer. *Journal of Financial Economics* 107(3), 515–536.
- Aghion, P. and P. Bolton (1992). An Incomplete Contracts Approach to Financial Contracting. *The Review of Economic Studies* 59(3), 473–494.
- Altman, E. I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *The Journal of Finance* 23(4), 589–609.
- Antill, S. (2022). Do the right firms survive bankruptcy? *Journal of Financial Economics* 144(2), 523–546.
- Antill, S. and C. Clayton (2021). Crisis interventions in corporate insolvency.
- Araújo, A., S. Schommer, and M. Woodford (2015). Conventional and unconventional monetary policy with endogenous collateral constraints. *American Economic Journal: Macroeconomics* 7(1), 1–43.
- Asriyan, V., W. Fuchs, and B. Green (2019). Liquidity sentiments. *American Economic Review* 109(11), 3813–48.
- Asriyan, V., L. Laeven, and A. Martin (2018). Collateral booms and information depletion. (DP13340).
- Bagehot, W. (1897). *Lombard Street: A Description of the Money Market*. New York: Charles Scribner's Sons.
- Balloch, C., S. Djankov, Juanita Gonzalez-Uribe, and D. Vayanos (2020). A restart procedure to deal with covid-19. Covid-19 in Developing Economies. CEPR E-Book.
- Banerjee, A. V. and B. Moll (2010). Why does misallocation persist? *American Economic Journal: Macroeconomics* 2(1), 189–206.

- Bartik, A. W., Z. B. Cullen, E. L. Glaeser, M. Luca, C. T. Stanton, and A. Sunderam (2020). When should public programs be privately administered? theory and evidence from the paycheck protection program. (NBER WP No. 27623).
- Bartlett, Robert P. I. and A. Morse (2020). Small business survival capabilities and policy effectiveness: Evidence from Oakland. (27629).
- Bassetto, M. and W. Cui (2020). A Ramsey Theory of Financial Distortions.
- Bebchuk, L. A. and I. Goldstein (2011). Self-fulfilling credit market freezes. *The Review of Financial Studies* 24(11), 3519–3555.
- Bernanke, B. and C. S. Lown (1991). The credit crunch. *Brookings Papers on Economic Activity* 22(2), 205–248.
- Bigio, S. (2015). Endogenous liquidity and the business cycle. *American Economic Review* 105(6), 1883–1927.
- Bolton, P., T. Santos, and J. A. Scheinkman (2016). Cream-skimming in financial markets. *The Journal of Finance* 71(2), 709–736.
- Bolton, P. and D. S. Scharfstein (1990). A theory of predation based on agency problems in financial contracting. *The American Economic Review* 80(1), 93–106.
- Bouvard, M., P. Chaigneau, and A. Motta (2015). Transparency in the financial system: Rollover risk and crises. *The Journal of Finance* 70(4), 1805–1837.
- Boyarchenko, N. (2012). Ambiguity shifts and the 2007–2008 financial crisis. *Journal of Monetary Economics* 59(5), 493 – 507. Carnegie-NYU-Rochester Conference Series on Public Policy - Robust Macroeconomic Policy at Carnegie Mellon University on November 11-12, 2011.
- Bruhn, M., A. Demirgüç-Kunt, and D. Singer (2023). Competition and firm recovery post-COVID-19. *Small Business Economics*, 1–32.
- Brunnermeier, M. and A. Krishnamurthy (2020). Corporate debt overhang and credit policy. BPEA conference.
- Brunnermeier, M. K. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic Perspectives* 23(1), 77–100.
- Brunnermeier, M. K., L. Garicano, P. R. Lane, M. Pagano, R. Reis, T. Santos, D. Thesmar, S. Van Nieuwerburgh, and D. Vayanos (2016). The sovereign-bank diabolic loop and esbies. *American Economic Review* 106(5), 508–12.
- Brunnermeier, M. K. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *The American Economic Review* 104(2), 379–421.
- Caballero, R. J. and M. L. Hammour (1994). The cleansing effect of recessions. *The American Economic Review* 84(5), 1350–1368.
- Caballero, R. J., T. Hoshi, and A. K. Kashyap (2008). Zombie lending and depressed restructuring in japan. *American Economic Review* 98(5), 1943–1977.
- Caballero, R. J. and A. Simsek (2013). Fire sales in a model of complexity. *The Journal of Finance* 68(6), 2549–2587.

- Calomiris, C. W., M. Flandreau, and L. Laeven (2016). Political foundations of the lender of last resort: A global historical narrative. *Journal of Financial Intermediation* 28, 48 – 65. Rules for the Lender of Last Resort.
- Caramp, N. (2017). Sowing the seeds of financial crises: Endogenous asset creation and adverse selection.
- Chari, V. V., A. Shourideh, and A. Zetlin-Jones (2014). Reputation and persistence of adverse selection in secondary loan markets. *American Economic Review* 104(12), 4027–70.
- Chemla, G. and C. A. Hennessy (2014). Skin in the game and moral hazard. *The Journal of Finance* 69(4), 1597–1641.
- Clementi, G. L. and B. Palazzo (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics* 8(3), 1–41.
- Cooper, R. W. and J. C. Haltiwanger (2006). On the Nature of Capital Adjustment Costs. *The Review of Economic Studies* 73(3), 611–633.
- Crouzet, N. and F. Tourre (2020). Can the cure kill the patient? Corporate credit interventions and debt overhang.
- Cúrdia, V. and M. Woodford (2011). The central-bank balance sheet as an instrument of monetary policy. *Journal of Monetary Economics* 58(1), 54 – 79. Carnegie-Rochester Conference Series on Public Policy: The Future of Central Banking April 16-17, 2010.
- Daley, B., B. Green, and V. Vanasco (2020). Securitization, ratings, and credit supply. *The Journal of Finance* 75(2), 1037–1082.
- David, J. M., L. Schmid, and D. Zeke (2018). Risk-adjusted capital allocation and misallocation.
- David, J. M. and D. Zeke (2021). Risk-taking, capital allocation and optimal monetary policy.
- De Meza, D. and D. C. Webb (1987). Too much investment: a problem of asymmetric information. *The Quarterly Journal of Economics* 102(2), 281–292.
- Del Negro, M., G. Eggertsson, A. Ferrero, and N. Kiyotaki (2017). The great escape? A quantitative evaluation of the Fed’s liquidity facilities. *The American Economic Review* 107(3), 824–857.
- Denes, M., S. Lagaras, and M. Tsoutsoura (2021). First come, first served: The timing of government support and its impact on firms. *SSRN Electronic Journal*.
- Dörr, J. O., G. Licht, and S. Murmann (2022). Small firms and the COVID-19 insolvency gap. *Small Business Economics* 58(2), 887–917.
- Dou, W., Y. Ji, D. Tian, and P. Wang (2020). Asset pricing with misallocation.
- Dou, W. W., L. A. Taylor, W. Wang, and W. Wang (2021). Dissecting bankruptcy frictions. *Journal of Financial Economics* 142(3), 975–1000.
- Drechsler, I. (2013). Uncertainty, time-varying fear, and asset prices. *The Journal of Finance* 68(5), 1843–1889.
- Eberly, J. and N. Wang (2011). Reallocating and pricing illiquid capital: Two productive trees. *International Economic Review*.
- Eisfeldt, A. (2004). Endogenous liquidity in asset markets. *The Journal of Finance* 59(1), 1–30.

- Eisfeldt, A. and A. Rampini (2006). Capital reallocation and liquidity. *Journal of Monetary Economics* 53(3), 369–399.
- Eisfeldt, A. and A. Rampini (2008). Managerial incentives, capital reallocation, and the business cycle. *Journal of Financial Economics* 87(1), 177–199.
- Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2020). Can the COVID bailouts save the economy? *Economic Policy*.
- Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2021). A macroeconomic model with financially constrained producers and intermediaries. *Econometrica* 89(3), 1361–1418.
- English, W. B. and J. N. Liang (2020). Designing the main street lending program: Challenges and options. (64).
- Falato, A., I. Goldstein, and A. Hortaçsu (2020). Financial fragility in the COVID-19 crisis: The case of investment funds in corporate bond markets. (27559).
- Farboodi, M. and P. Kondor (2021). Cleansing by tight credit: Rational cycles and endogenous lending standards.
- Faria-e-Castro, M., J. Martinez, and T. Philippon (2016). Runs versus Lemons: Information Disclosure and Fiscal Capacity. *The Review of Economic Studies* 84(4), 1683–1707.
- Foster, L., J. Haltiwanger, and C. Syverson (2008). Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *American Economic Review* 98(1), 394–425.
- Fuchs, W., B. Green, and D. Papanikolaou (2016). Adverse selection, slow-moving capital, and misallocation. *Journal of Financial Economics* 120(2), 286 – 308.
- Fukui, M. (2018). Asset quality cycles. *Journal of Monetary Economics* 95, 97–108.
- Gala, V. D., J. F. Gomes, and T. Liu (2022). Marginal q. *Jacobs Levy Equity Management Center for Quantitative Financial Research Paper*.
- Geanakoplos, J. (2010). *The Leverage Cycle*, pp. 1–65. University of Chicago Press.
- Gertler, M. and P. Karadi (2011). A model of unconventional monetary policy. *Journal of Monetary Economics* 58(1), 17–34.
- Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of Monetary Economics* 3(3), 547–599.
- Gertler, M., N. Kiyotaki, and A. Prestipino (2020). A macroeconomic model with financial panics. *The Review of Economic Studies* 87(1), 240–288.
- Gertler, M., N. Kiyotaki, and A. Queralto (2012). Financial crises, bank risk exposure and government financial policy. *Journal of Monetary Economics* 59, S17 – S34.
- Giannetti, M. and F. Saidi (2019). Shock propagation and banking structure. *The Review of Financial Studies* 32(7), 2499–2540.
- Gilchrist, S., J. W. Sim, and E. Zakrajšek (2013). Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs. *Review of Economic Dynamics* 16(1), 159–176. Special issue: Misallocation and Productivity.

- Goldstein, I. and Y. Leitner (2018). Stress tests and information disclosure. *Journal of Economic Theory* 177, 34–69.
- Goldstein, I. and H. Sapra (2014). Should banks’ stress test results be disclosed? an analysis of the costs and benefits. *Foundations and Trends® in Finance* 8(1), 1–54.
- Goodhart, C. A. (1998). *The Evolution of Central Banks*. Cambridge, MA.
- Gorton, G., T. Laarits, and A. Metrick (2020). The run on repo and the fed’s response. *Journal of Financial Stability* 48, 100744.
- Gorton, G. and G. Ordoñez (2014). Collateral crises. *American Economic Review* 104(2), 343–78.
- Gourio, F. (2012). Disaster risk and business cycles. *American Economic Review* 102(6), 2734–66.
- Greenwood, R., B. Iverson, and D. Thesmar (2020). Sizing up corporate restructuring in the COVID crisis. *Brookings Papers of Economic Activity*.
- Griffin, J. M., S. Kruger, and P. Mahajan (2023). Did fintech lenders facilitate ppp fraud? *The Journal of Finance* 78(3), 1777–1827.
- Haddad, V., A. Moreira, and T. Muir (2020). When selling becomes viral: Disruptions in debt markets in the COVID-19 crisis and the fed’s response. (NBER WP No. 27168).
- Halling, M., J. Yu, and J. Zechner (2020). How Did COVID-19 Affect Firms’ Access to Public Capital Markets? *The Review of Corporate Finance Studies* 9(3), 501–533.
- Hanson, S., J. Stein, A. Sunderam, and E. Zwick (2020a). Business credit programs in the pandemic era. *Brookings Papers of Economic Activity*.
- Hanson, S. G., D. S. Scharfstein, and A. Sunderam (2018). Social Risk, Fiscal Risk, and the Portfolio of Government Programs. *The Review of Financial Studies* 32(6), 2341–2382.
- Hanson, S. G., J. C. Stein, A. Sunderam, and E. Zwick (2020b). Business credit programs in the pandemic era. *Brookings Papers on Economic Activity Fall*, 3–60.
- Hart, O. and J. Moore (1994). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics* 109(4), 841–879.
- Hart, O. and J. Moore (1998). Default and Renegotiation: A Dynamic Model of Debt*. *The Quarterly Journal of Economics* 113(1), 1–41.
- Hayashi, F. (1982). Tobin’s marginal q and average q: A neoclassical interpretation. *Econometrica* 50(1), 213–224.
- Hayek, F. (1945). The use of knowledge in society. *American Economic Review* 35, 519–530.
- Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy* 106(1), 1–40.
- Hu, Y. (2017). A dynamic theory of bank lending, firm entry, and investment fluctuations.
- Hubbard, R. G. and M. R. Strain (2020). Has the paycheck protection program succeeded? (NBER WP No. 28032).

- İmrohoroğlu, A. and Ş. Tüzel (2014). Firm-level productivity, risk, and return. *Management Science* 60(8), 2073–2090.
- Jermann, U. and V. Quadrini (2012). Macroeconomic effects of financial shocks. *American Economic Review* 102(1), 238–71.
- Jordà, Ò., K. Knoll, D. Kuvshinov, M. Schularick, and A. M. Taylor (2019). The rate of return on everything, 1870–2015. *The Quarterly Journal of Economics* 134(3), 1225–1298.
- Jovanovic, B. and P. L. Rousseau (2008). Mergers as Reallocation. *The Review of Economics and Statistics* 90(4), 765–776.
- Kacperczyk, M. and P. Schnabl (2010). When safe proved risky: Commercial paper during the financial crisis of 2007–2009. *Journal of Economic Perspectives* 24(1), 29–50.
- Kaplan, S. N. and L. Zingales (1997). Do investment-cash flow sensitivities provide useful measures of financing constraints? *The Quarterly Journal of Economics* 112(1), 169–215.
- Kargar, M., B. Lester, D. Lindsay, S. Liu, P.-O. Weill, and D. Zúñiga (2020). Corporate bond liquidity during the COVID-19 crisis. (NBER WP No. 27355).
- Kawaguchi, K., N. Kodama, and M. Tanaka (2021). Small business under the COVID-19 crisis: Expected short- and medium-run effects of anti-contagion and economic policies. *Journal of the Japanese and International Economies* 61, 101138.
- Kehoe, T. J. and D. K. Levine (1993). Debt-constrained asset markets. *The Review of Economic Studies* 60(4), 865–888.
- Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy* 105(2), 211–248.
- Kogan, L., D. Papanikolaou, A. Seru, and N. Stoffman (2017). Technological Innovation, Resource Allocation, and Growth*. *The Quarterly Journal of Economics* 132(2), 665–712.
- Koijen, R. S., F. Koulischer, B. Nguyen, and M. Yogo (2020). Inspecting the mechanism of quantitative easing in the euro area. *Journal of Financial Economics* forthcoming.
- Krishnamurthy, A. (2010). How debt markets have malfunctioned in the crisis. *Journal of Economic Perspectives* 24(1), 3–28.
- Kurlat, P. (2013). Lemons markets and the transmission of aggregate shocks. *American Economic Review* 103(4), 1463–89.
- Kurtzman, R. and D. Zeke (2020). Misallocation costs of digging deeper into the central bank toolkit. *Review of Economic Dynamics* 38, 94–126.
- Lamont, O., C. Polk, and J. Saaá-Requejo (2001). Financial constraints and stock returns. *The Review of Financial Studies* 14(2), 529–554.
- Lee, M. J. and D. Neuhann (2021). A dynamic theory of collateral quality and long-term interventions.
- Li, S., T. M. Whited, and Y. Wu (2016). Collateral, Taxes, and Leverage. *The Review of Financial Studies* 29(6), 1453–1500.
- Liu, X. (2016). Interbank market freezes and creditor runs. *The Review of Financial Studies* 29(7), 1860–1910.

- Lucas, D. (2012). Valuation of government policies and projects. *Annual Review of Financial Economics* 4(1), 39–58.
- Lucas, D. (2016). Credit policy as fiscal policy. *Brookings Papers on Economic Activity* (1), 1–57.
- Lynch, J. (2021). Bounded support: Success and limitations of liquidity support during times of crisis.
- Ma, Y., K. Xiao, and Y. Zeng (2022). Mutual fund liquidity transformation and reverse flight to liquidity. *The Review of Financial Studies* 35(10), 4674–4711.
- Midrigan, V. and D. Y. Xu (2014). Finance and misallocation: Evidence from plant-level data. *American Economic Review* 104(2), 422–58.
- Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review* 104(10), 3186–3221.
- Moreira, A. and A. Savov (2017). The macroeconomics of shadow banking. *Journal of Finance* 72(6), 2381–2432.
- Muzi, S., F. Jolevski, K. Ueda, and D. Viganola (2023). Productivity and firm exit during the COVID-19 crisis: Cross-country evidence. *Small Business Economics* 60(4), 1719–1760.
- Neuhann, D. (2018). Inefficient asset price booms.
- Olley, S. and A. Pakes (1992). The dynamics of productivity in the telecommunications equipment industry.
- Papoutsis, M., M. Piazzesi, and M. Schneider (2021). How unconventional is green monetary policy?
- Peters, R. H. and L. A. Taylor (2017). Intangible capital and the investment-q relation. *Journal of Financial Economics* 123(2), 251–272.
- Philippon, T. and P. Schnabl (2013). Efficient recapitalization. *The Journal of Finance* 68(1), 1–42.
- Ramey, V. A. and M. D. Shapiro (1998). Costly capital reallocation and the effects of government spending. *Carnegie-Rochester Conference Series on Public Policy* 48, 145–194.
- Rampini, A. A. and S. Viswanathan (2010). Collateral, risk management, and the distribution of debt capacity. *The Journal of Finance* 65(6), 2293–2322.
- Reinhart, C. M. and K. S. Rogoff (2009). *This time is different: Eight centuries of financial folly*. Princeton University Press.
- Shapiro, J. and D. Skeie (2015). Information Management in Banking Crises. *Review of Financial Studies* 28(8), 2322–2363.
- Stiglitz, J. E. and A. Weiss (1981). Credit rationing in markets with imperfect information. *The American Economic Review* 71(3), 393–410.
- Taylor, A. M. (2015). Credit, financial stability, and the macroeconomy. *Annu. Rev. Econ.* 7(1), 309–339.
- Van Nieuwerburgh, S. and L. Veldkamp (2009). Information immobility and the home bias puzzle. *The Journal of Finance* 64(3), 1187–1215.
- Vanasco, V. (2017). The downside of asset screening for market liquidity. *The Journal of Finance* 72(5), 1937–1982.

- Wachter, J. A. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance* 68(3), 987–1035.
- Wang, J., J. Yang, B. C. Iverson, and R. Kluender (2020). Bankruptcy and the COVID-19 crisis.
- Whited, T. M. and G. Wu (2006). Financial constraints risk. *The Review of Financial Studies* 19(2), 531–559.
- Williams, B. (2015). Stress tests and bank portfolio choice.
- Zryumov, P. (2015). Dynamic adverse selection: Time varying market conditions and endogenous entry.