Abstract

Can massive online retailers such as Amazon and Alibaba issue digital tokens that potentially compete with bank debit accounts? There is a long history of trading stamps and loyalty points, but new technologies are poised to sharply raise the significance of redeemable assets as a store of value. Here we develop a simple stylized model of redeemable tokens that can be used to study sales and pricing strategies for issuing tokens, including ICOs. Our central finding is that platforms can potentially earn higher revenues by making tokens non-tradable unless they can generate a sufficiently high outside-platform convenience yield.

JEL code: E42, G23, L51

Keywords: Digital currencies, platforms, tokens, initial coin offering
1 Introduction

As technology blurs the lines between finance and tech firms, and as innovation in transaction technologies continues to disrupt markets, many large platforms are issuing, or considering issuing, their own digital credits or tokens. In principle, tech firms and chains with a large retail customer base have a natural advantage in creating liquidity by ensuring that their tokens/credits can be used for in-platform purchases. Although most early implementations have focused on in-platform payment convenience (e.g., Uber and Lyft cash), some began as in-platform and have expanded to more general usage (e.g., Alipay and WePay). On a smaller scale, but collectively significant, many apps and games offer forms of virtual currency.

Here we present a simple, tractable model of redeemable (for goods but not for cash) platform tokens, and proceed to analyze a number of issues related to token supply policy and feature design. Our central result is if the market consists entirely of platform consumers, it will not necessarily be optimal to make tokens tradable. That is, unless a platform anticipates significant outside adoption – which due to network effects and regulatory constraints is not a realistic consideration for most issuers – there can be a strong case for making tokens non-tradable.\(^1\)

How is it possible that consumers might ever be willing to pay more for non-tradable tokens than for the same quantity of tradable ones? The intuition is that whereas an individual consumer generally benefits from having the right to sell tokens she already holds, the prospect of being able to buy tokens as needed from other consumers in the future undercuts her willingness to hold large quantities. The latter effect dominates in the canonical case where tokens are issued at a uniform individual price in a one-time ICO. Moreover, making tokens tradable constrains the ability of a platform to offer richer token price/quantity menus (e.g., selling bundles of discounted tokens) or to incorporate memory features. Thus, although issuing tokens that compete widely with fiat currencies may be a worthwhile goal for a few platforms — should regulatory authorities ultimately permit that — it is not necessarily the best strategy for most.

To put the problem we study in context, the first part of the paper presents a brief history of the evolution of different generations of redeemable platform assets from Green Stamps to Ethereum. Even within each generation of redeemable platform assets, there are a wide array of bundling and sales techniques. Although our highly stylized model links most closely to tokens/cash on today’s large retailer platforms (such as Amazon, Uber, Alibaba), we argue that some of the insights have links both to earlier versions and to potential new generations of

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\(^1\)For an analysis of general-purpose currencies, see the literature building on Kiyotaki and Wright (1989), for example, discussion in Bethune et al. (2020). Cong et al. (2021a) develop a model of token-based platform finance in which blockchain technology can mitigate underinvestment by addressing the owners time consistency problem.
redeemable assets. Anticipating our main result, we note that, in practice, many digital assets and their predecessors contain significant limitations on tradability.

The second part of the paper presents our simple partial equilibrium model of platform tokens and their liquidity. The critical wedge that creates possible gains from trade in tokens is our assumption that the platform can earn a higher rate of return on its outside investments than its customers are able to do on their retail bank accounts. We begin by using the model to explore simple strategies where all tokens are sold for the same price in an initial one-time auction, examining both the case of non-tradable and tradable tokens. A central result is that in any given size ICO, the non-tradable tokens generally can be sold at a higher price and yield higher profits to the platform than do tradable tokens. Essentially, tradability forces the issuer to compete with future resale markets and limits the power to charge a high price upfront.

The next part of the paper explores more sophisticated issuance strategies in which platforms use a price menu approach to bundle tokens in the initial coin offering, offering a larger discount to consumers who buy larger quantities of tokens. The advantage of a price menu is that the platform can potentially exploit all the potential gains from inter-temporal trade. But such an approach can only work if the token is non-tradable. Indeed, for tradable tokens, introducing a price menu adds nothing to the platform’s options, since resale trade undermines its ability to offer a discount to large purchasers.

We next explore outcomes when, in addition to the ICO, platforms also offer ongoing token sales. Despite the apparent simplicity of our canonical model, the range of strategies becomes significantly more complex in this case, albeit the core considerations illustrated in the one-time ICO case remain central. In particular, the expectation of future token sales reduces the quantity of tokens consumers are willing to purchase at any given price. We fully characterize the equilibrium when a platform aims to keep a constant steady-state supply of tokens outstanding, and show that in this case, consumers are never willing to hold more than one token at a time. Moreover, the traded and non-traded equilibria collapse to the same “token-in-advance” allocation, which again is dominated by a non-tradable ICO when the frequency of platform use is sufficiently high.

We go on to briefly discuss a generalization to the case where the probability of platform consumption varies across consumers and show that our main result on ICOs still goes through. (The core proofs are in Appendix B with further extensions in Online Appendix ??.) The final section includes a discussion of potential extensions.
Different Generations of Redeemable Platform Tokens

Although modern redeemable tokens are linked to rapid advances in payment technologies, the general idea is hardly new. Trading stamps such as Sperry and Hutchinson (S&H) Green Stamps were prominent from the 1930s to the 1980s. S&H sold their stamps in negotiated wholesale transactions to retailers, who in turn gave them as loyalty rewards to consumers (essentially proportional to purchases). Consumers would then paste the stamps into books and redeem them for a wide variety of consumer goods either at S&H stores or via its widely distributed catalog (Pollack (1988); Lonto (2013)). Whereas S&H was the largest vendor, there were numerous other versions worldwide, for example, the United Kingdom’s Green Shield stamps.\(^2\) It is difficult to determine the exact scale of trading stamps, but during the peak of trading stamps in the 1960s, S&H alone claimed to have printed tens of millions of catalogs per year. Interestingly, although trading stamps were essentially as anonymous and fungible as currency, many of the trading stamp companies attempted to prohibit trade to the extent they could, perhaps, in an effort to prevent a liquid market from emerging.

Improvements in data processing helped enable modern customer loyalty programs. Although airline and hotel programs are the most prominent, there are today a plethora of such programs today around the world in different retail industries. Since American Airlines launched the first airline frequent flyer program in 1981, the space has grown exponentially albeit with a significant pause during the Covid-19 pandemic.\(^3\) Because loyalty programs (in their current guise) are tabulated through a centralized system; the issuer has far more scope to limit tradability. Indeed most do, with enforcement over black markets improving over time, especially as government identity protocols have hardened. The size and scale of loyalty programs are enormous, and publicly traded companies are required to report points outstanding and the likely cost of these liabilities. At the end of 2019, the value of the outstanding 20 trillion in airline points in the US alone was worth in excess of $200 billion dollars.\(^4\)

American Airlines, for example, reports that it foregoes over 7% of annual passenger revenue due to redemption of frequent flyer miles, and the hangover of existing points has averaged over 20% of gross revenues in years prior to Covid-19.\(^5\) In Table 1, we refer to trading stamps as first-generation redeemable platform assets and loyalty points as the second generation.\(^6\)

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\(^2\)One of the two authors of this paper can claim to have collected and redeemed both the US and UK version of green stamps.

\(^3\)See De Boer and Gudmundsson (2012), who also discuss how the range and types of airline frequent flyer programs have evolved.


\(^5\)The 2019 annual 10-K reports Aadvantage balance is $8.615 billion and $3.362 billion redeemed in 2019, while the total operating revenue is $45.8 billion; net income and operating income are $1.686 billion and $3.065 billion, respectively.

\(^6\)See Brunnermeier et al. (2021) for further investigation of digitalization and money functions.
<table>
<thead>
<tr>
<th>Generation</th>
<th>Technology</th>
<th>Circulation Mechanism</th>
<th>Leading Examples</th>
<th>Redeemability</th>
<th>Tradability</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Generation:</td>
<td>TRADING STAMPS</td>
<td>Physical stamps</td>
<td>S&amp;H Green stamps (US), Green Shield stamps (UK)</td>
<td>Convertible to awards at trading stamp stores, or by mail, after filling savers’ books</td>
<td>Often legally prohibited, but black market</td>
<td>Peaked in the mid-60s when S&amp;H was printed 32 million catalogs and 140 million savers books</td>
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<td></td>
<td>(anonymous)</td>
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<td>Second Generation:</td>
<td>LOYALTY POINTS</td>
<td>Centralized Accounting</td>
<td>Airline and hotel loyalty points, also supermarket and drug stores</td>
<td>Convertible for services of issuing company or partners</td>
<td>Typically constrained with limited black markets</td>
<td>American Airline awards 7-8% of yearly revenue miles for points (8.5 billion outstanding in 2018). Stock equals 20% one year’s gross revenue</td>
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<td>Third Generation:</td>
<td>PLATFORM CASH</td>
<td>Centralized Accounting</td>
<td>Uber &amp; Lyft cash, Starbucks stored-value cards, Amazon gift cards, Q-Coin, Gash+</td>
<td>Generally redeemable only on platform but still evolving</td>
<td>Generally none or highly constrained</td>
<td>Uber cash, Starbucks balance, 3.3 billion Amazon gift cards unredeemed in 2019</td>
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<td>Fourth Generation:</td>
<td>TOKENS</td>
<td>Cryptography, Blockchain, Cybersecurity Technologies</td>
<td>Ethereum, Telegram, Diem, Alipay, WeChat Pay</td>
<td>Can be redeemable for platform services, but usually aspires to universal usage</td>
<td>Tradable or tradable with minor restrictions</td>
<td>Ether can be used for smart contract services, Ether topped over $500 billion in May, 2021</td>
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“Stable coins are a special case also redeemable for cash.”
As Table 1 illustrates, third-generation redeemable assets include Uber and Lyft cash, Amazon gift cards, Starbucks stored-value cards, Q-coins, and Gash+ (game cash issued by Gamania) appear to be in a very rapidly growing phase. A critical distinction between third-generation and second-generation programs is that third-generation tokens are most typically sold for cash, and often at a discount. This feature is in contrast to 2nd generation assets which are mostly given as a reward, linked to purchases with better terms for more frequent users. Importantly, third-generation platform tokens are very different from first and second generation in that they are extremely easy to use, often involving smaller frictions than any other payment mechanism for in-platform purchases. The potential scale of generation-three platform currencies, which are closest in spirit to what we model in this paper, is vastly larger than earlier loyalty programs simply because the technology has been made much more attractive to consumers, and easier to maintain for suppliers.

Finally, Table 1 lists the fourth-generation of redeemable platform tokens. A critical difference is that unlike earlier generations, these tokens can, in principle, be transacted on a shared infrastructure and seek usage beyond the issuer’s platform. Massive technological innovations attempt to provide credibility for the fourth-generation assets, for example, public ledgers created from a blockchain without the need for a centralized intermediary. A prominent example is Ether, which can be used (indeed, is required) for smart contracts on the Ethereum platform. Stable coins such as Facebook’s Diem proposal are a somewhat different animal, in that in principle, they are not only redeemable for platform services but can be redeemed for fiat currency. Finally, we note that the lines between generations 3 and 4 are blurred; Amazon, for example, has shown an interest in extending its financial reach outside of its platform through deals with Western Union and others through Amazon cash. Some voluntary adoption might also happen if a digital currency provides sufficient convenience. For example Q-coin, first-issued in 2002, initially designed to facilitate financial transactions within Tencent’s platform, ended up being broadly adopted by many online vendors and gaming companies as a payment method before the rise of WeChat Pay and Alipay.

3 A Simple Model

In this section, we develop a partial-equilibrium model to capture how consumers value a token that is underpinned by future claims on platform consumption. The aim is to develop a

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7 Needless to say, the legal issues behind these new payments mechanisms are still being sorted out; see Middlebrook and Hughes (2016) for a discussion of how case law relating to earlier tokens and currencies might impact on today’s versions.

A stylized framework that gives some general insights into how platform tokens might be designed and sold in a modern context. As the first stab, we only analyze the case of a monopoly platform, and all consumers are price takers and cannot cooperate.

### 3.1 Consumer Demand

We assume that one unit of the (perishable) platform commodity costs one dollar (there is no inflation in the fiat currency) and provides one unit of consumption. In any given period $t$, the representative consumer demands one unit of the platform commodity with probability $p$, and zero units with probability $1 - p$. All infinitely-lived consumers are identical with the time discount factor $\beta$. The fact that $p < 1$ captures that the individual consumer may not need platform consumption every period. The normalization of a single period’s consumption to 1 captures limits to the consumer’s period demand but can be varied to study platforms that involve large lumpy expenditures; indeed, all the main results here will go through.

The consumer’s wealth dynamic is the following:

$$W_{t+1} = (1 + r) \times (W_t - \Gamma_t - \Phi_t)$$

where $W_t$ is wealth at the start of period $t$. $\Gamma_t$ is fiat-money spending for platform consumption, and $\Phi_t$ is fiat money spent on token purchases. The the rate of return they earn on any wealth held outside the platform (which we interpret to be retail bank accounts) is $r$, and $\beta(1+r) = 1$.

As platform consumption is exogenous, consumers solve the cost minimization problem:

$$\min_{\Gamma_t, \Phi_t} \sum_{t=0}^{\infty} \beta^t (\Gamma_t + \Phi_t)$$

The underlying assumption is that the consumer’s utility is linear in platform consumption and wealth so that a consumer aims to minimize the present value of total expected expenditure for her exogenous platform consumption flow.

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9 We assume a stable consumer base. For issues related to dynamic adoption by users of general purpose (“generation 4” in our terminology) cryptocurrencies/tokens on digital platforms, see Cong et al. (2021b); see also Catalini and Gans (2018) and Athey et al. (2016).

10 Define $\Pi$ as the fiat currency price of a platform good. The scale of a platform depends on $p\Pi$. In our analysis, the price of a unit good is normalized to one. But one can envision a platform with low-frequency consumption (low $p$) but a high fiat currency price. One can easily show that all results go through with an arbitrary price of $\Pi$ for the platform good. That is, platform scale is irrelevant; only the consumption probability matters.

11 We have suppressed ‘$i$’ subscripts for individuals for notational convenience.
3.2 Valuing the Marginal Claim

In all that follows, a critical issue is how a consumer values a credit that pays for her $M^{th}$ unit of platform consumption, which will occur at some future date $N \geq M$, depending on the exact timing of the consumer’s needs for the platform good. The probability that the consumer will use the $M^{th}$ token in period $N$ is given by

$$X_{N,M} = \binom{N-1}{M-1} p^M (1-p)^{N-M}$$

where $\binom{N-1}{M-1}$ is the binomial coefficient $\frac{(N-1)!}{(N-M)!(M-1)!}$. Given consumers’ linear utility implied by eq. (1), expression (2) governs the value of the marginal claim which is a central input to how much a consumer is willing to pay for tokens.

3.3 Platform Currency and Issuance

We now introduce the possibility that the platform can issue a “currency” in the form of non-interest-bearing tokens that can be converted to one unit of the platform commodity. This paper only considers issuance plans where the platform cannot interfere with the token market once the plan is publicly announced (say, in a white paper). Of course, given the assumed utility function, the consumer will never need more than one token in any given period. Importantly, the consumer is not required to use the platform token and can always pay one dollar of fiat currency (that is, one dollar). As with the consumer, the platform is risk-neutral. Importantly, whereas the platform guarantees the purchasing power of its tokens on its platform, it does not offer to redeem them for face value in fiat currency. We assume that consumers buy tokens before observing their individual consumption shock. (We will later discuss alternative timing.)

The platform discounts the future at $\beta^* < \beta (= \frac{1}{1+r})$, to capture that as a large platform, it has better outside investment opportunities than do small consumers. This wedge is the sole source of gains from inter-temporal trade to justify token issuance in our baseline model. It immediately follows that in an efficient equilibrium, with no other liquidity, capital constraints or credibility issues, the consumers would purchase the entire present value of future platform consumption in the initial period, with the allocation of the welfare surplus from trade depending on the relative bargaining power of the two parties, for example depending on consumers’ outside options. As noted in the introduction, there may be many other reasons for gains from token

\[12\] The assumption that a large diversified firm or intermediary can earn a higher rate of return than small retail banking consumers is a conventional one. Similar to banks, the net interest margin for tech companies can be shaped by regulation. As Demirguc-Kunt and Huizinga (2000) show, using bank data across 80 countries, the net interest margin banks are able to earn (the difference between their deposit and lending rates) depends on an enormous range of factors, including both explicit and implicit taxation, leverage market concentration, deposit insurance regulation, macroeconomic conditions and many other factors.
issuance, but for the moment, we will focus exclusively on the discount wedge.

One critical issue is the extent to which the platform currency yields a flow of convenience services for transactions inside the platform, and potentially for trade outside, an assumption that is widely used to rationalize demand for currency that pays below the short-term market rate of interest. For now, we assume the convenience yield is zero in all transactions, that the token is only used for platform purchases, and that it does not effectively compete with fiat currency for trade outside the platform. We return to the convenience yield issue later.

3.4 Assumptions

Before proceeding to study token offerings, we initially make a number of assumptions to simplify the analysis, and later discuss what happens when we relax them.

1. Token issuance does not affect consumer demand for platform consumption. This abstracts from a number of possible benefits, for example, if currency issuance increases consumer time spent on the platform.

2. Zero production cost. This assumption not only abstracts from the cost of producing platform intermediation services, but also from the cost the platform pays in purchasing commodities to sell to consumers.

3. No platform failure or bankruptcy (otherwise a default premium is built into the token). Relatedly, if the platform issues tokens, these are assumed senior to any other debt the platform may issue.

4. The platform can make credible commitments to its future token issuance policy and to redeemability.

5. Any token issued by the platform is like a “stable coin” in the sense that its platform-use value is fixed in terms of fiat money, and we assume no inflation. However, unlike a stable coin, the consumer can only convert it to fiat money by selling to another consumer, and only if tokens are tradable.

6. Platform tokens are tradable among consumers only if the platform allows it.\textsuperscript{14}

These assumptions can be modified to get more general results, including allowing for a convenience yield, as well as for a proportional cost of goods production.\textsuperscript{13}

\textsuperscript{13}For a rich model of a platform’s choice between issuing tokens and being a cash-only business, see Chiu and Wong (2021).

\textsuperscript{14}We allow short-selling if tokens are tradable. They can only borrow from other consumers, but not the issuer.
4 Introduction of Platform Currency through ICOs

We now proceed to study the pricing and issuance strategies for a platform that introduces tokens through a once and for all “initial coin offering” (ICO).\(^{15}\) Note that if the platform did not engage in any financial offerings, its value (per consumer) would simply be the expected present value of sales:

\[
\frac{\beta^*}{1 - \beta^* p}
\]

The first-best is that consumers transfer their entire willingness to pay to the platform in the initial period. It is achievable by issuing a life-long membership which enables consumers to pay once and enjoy the free service for all time. The first-best discounted revenue is:

\[
\frac{\beta}{1 - \beta p}
\]

The present value of revenue after token issuance is bounded by \(\left[\frac{\beta^*}{1 - \beta^* p}, \frac{\beta}{1 - \beta p}\right]\). We consider a range of issuance policies and compare policies from the standpoint of the issuer.

4.1 Non-tradable Initial Coin Offering

We first consider the case where the tokens issued by the platform are not tradable, and where the platform announces a fixed (per capita) quantity of tokens that it is going to sell, \(M\). Importantly, in order to sell the full quantity of tokens the platform has put up for sale, all the tokens must be priced at the value of marginal token, which is the last to be spent (\(M^{th}\) token).

Making use of equation (2), we can solve for the ICO token price in the non-traded case \(P_{I,N}\)\(^{16}\)

\[
\sum_{k \geq 0} \binom{M - 1 + k}{k} x^k = \left(\frac{1}{1 - x}\right)^M
\]

An alternative and perhaps more intuitive approach to derive eq. (3) is through induction. Note that with one token, the present value to the consumer is \(V(1) = \beta p(1 + \beta(1 - p) + \beta^2(1 - p)^2 + ...) = \frac{\beta p}{1 - \beta(1 - p)}\). Then, we can solve \(V(M)\) with an iterative process. In a period when the \(X^{th}\) token is spent, the expected value of the \((X + 1)^{th}\) token is always \(\frac{\beta p}{1 - \beta(1 - p)}\) where the value of the \(X^{th}\) token is one. Then, we have

\[
V(X + 1) = \frac{\beta p}{1 - \beta(1 - p)} V(X)
\]

Thus, the token price of \(M\)-token issuance is \(V(M) = \left(\frac{\beta p}{1 - \beta(1 - p)}\right)^M\).
\[ P_{I,N} = \sum_{N \geq M} \beta^N X_{N,M} = \sum_{N \geq M} \beta^N \left( \frac{N - 1}{M - 1} \right) p^M (1 - p)^{N-M} = \left[ \frac{\beta p}{1 - \beta(1-p)} \right]^M \] (3)

One may view \( \frac{\beta p}{1 - \beta(1-p)} \) as the effective discount rate when the platform aims to issue an extra token. To sell an additional token, all tokens sold must depreciate \( \frac{\beta p}{1 - \beta(1-p)} \) which yields higher surplus for consumers. Note that we have assumed the platform sets the issuance quantity \( M \), but here it could equivalently set the token price \( P^{17,18} \).

Define \( R_{I,N} \) as the total revenue for non-tradable ICO token issuance. Then, we can rewrite the firm’s profit of token issuance as

\[ R_{I,N} = \text{Revenue from Token Issuance} + \sum_{i=M+1}^{\infty} \left[ \frac{\beta^i p}{1 - \beta^i(1-p)} \right] \text{Revenue in Fiat Money} \]

\[ = M \left[ \frac{\beta p}{1 - \beta(1-p)} \right]^M + \left( \frac{\beta^M p}{1 - \beta^M(1-p)} \right) \frac{\beta^M p}{1 - \beta^M} \] (4)

where the first term is the total revenue from token issuance and the second term in eq.(4) is fiat sales after all tokens are used.

### 4.2 Tradable ICO

We now solve for the price formula if tokens can be traded. We begin by noting that once all individuals are holding at most one token, the token price is governed by the willingness to pay of individuals who hold zero tokens. If the price is higher than their willingness to pay, no one wants to buy, and selling pressure pushes the token price down. If the price is lower than the willingness to pay, every consumer without a token wants to buy one and this bids up the price. Thus, the token price is unique when all individuals are holding at most one coin. Let \( \hat{P} \) denote this unique price.

\[ \hat{P} = \beta p + \beta (1 - p) \hat{P} \]

\[ \hat{P} = \frac{\beta p}{1 - \beta(1-p)} \]

\[ \hat{P} = \frac{\beta p}{1 - \beta(1-p)} \]

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17 The idea that the value and liquidity of a token depend on an underlying matching probability is reminiscent of Kiyotaki and Wright (1989)’s classic general equilibrium search-theoretic model of commodity money.

18 Our timing assumes that consumers that purchase a token in period \( t \) cannot use it until period \( t + 1 \). It is a trivial variant to allow consumers to use tokens in the same period they purchase them, \( t \), if the tokens are purchased before they observe their preference shock; it only affects discounting. The case where they can purchase tokens after observing their individual preference shock is more complex. There would then be two types of agents who value the tokens differently. We take this case up in Section 5.1.
The first term on the right-hand side denotes the present value of being able to consume the coin in the next period, and second term denotes the present value of being able to sell it. But this equation can be rearranged to yield

\[ \hat{P} = \frac{\beta p}{1 - \beta (1 - p)} \]

which is exactly the same as in the non-tradable case. Once all individuals have either zero or one token, there are no longer any gains from inter-consumer trade; a token has the same value to an individual whether she sells it or holds on until she has the first opportunity to use it. Inducing individuals to hold more than one coin, however, requires that they expect the price to appreciate at the rate \( \beta^{-1} \) every period, again assuming as we have so far that the convenience yield is zero.

Now suppose the platform wants to sell \( M \) tokens in an ICO, but where tokens are tradable; what is the price? The key observation is that if there are \( M \) tokens, it will take \( M - 1 \) periods for the first \( M - 1 \) coins (per capita) to be depleted. (This is much faster than would be the case without trade.) In period \( 1 + \frac{M - 1}{p} \), the price must reach its steady-state value of \( \hat{P} \). The ICO price for \( M \) tradable tokens must be given by

\[ P_{1,T} = \beta \frac{M - 1}{p} \left( \frac{\beta p}{1 - \beta (1 - p)} \right) \]  

(5)

To compare the gross revenue from a non-traded ICO of \( M \) tokens with a traded ICO of the same size, we first observe that when \( M = 1 \), tradability does not matter since all agents are homogeneous. We then note from equation (3) that to issue one extra non-tradable token, the platform needs to discount token prices by \( \frac{\beta p}{1 - \beta (1 - p)} \), while in the case of tradable tokens, equation (5) implies it would need to discount its price by \( \beta \frac{1}{p} \). Thus to compare the price of tradable tokens with that of non-tradable tokens (for any equivalent-size ICO), we need only to compare the two discount factors. Lemma 1 answers this question.

**Lemma 1 (Effective Discount Factor Dominance):** The effective discount factor is higher (closer to 1) for non-tradable ICO tokens than for tradable ICO tokens. (Proof: See Appendix A)

\[ \beta \frac{1}{p} < \frac{\beta p}{1 - \beta (1 - p)} \]

Comments: Lemma 1 states that for any sale of \( M > 1 \) tokens in an ICO, the price will be higher if the tokens are non-tradable. What is the intuition for this result, given that the
expected time to redemption of the marginal token is greater in the case of non-tradability? We have already given some intuition in the introduction and we will return to that shortly. But it is important to understand that at a formal level, the answer has to do with the fact that the consumer's utility function is convex in time of consumption. While commonly known that utility is concave in consumption within any given period, it is less known that the utility is convex in the time of consumption. To illustrate this convexity, consider the following two lotteries in time with the identical expected payoff.

**Lottery 1** (Price $P_C$): One dollar in period $M + 2$.

**Lottery 2** (Price $P_D$): One dollar in period $M + 1$ with 50% probability, and one dollar in period $M + 3$ with 50% probability.

Lottery 1, sold at a price $P_C$, delivers payoff one dollar with a “compressed” distribution in time — with 100% certainty in period $M + 2$. Lottery 2, sold at a price $P_D$, delivers one dollar payoff with a “dispersed” distribution in time — with 50% probability in either period $M + 1$ or $M + 3$. As shown in Figure 1 if one dollar yields the same utility $u(1)$ for any period, then convexity implies that for a given expected cash flow, the more dispersed the distribution in time, the higher it will be priced:

$$P_C = \frac{1}{2}u(1)(\beta^{M+1} + \beta^{M+3}) > u(1)\beta^{M+2} = P_D$$

The initial ICO token price, in both the tradable and non-tradable case, depends on the willingness to pay for the marginal token. Tradability compresses the distribution of the time required to spend the marginal token. In Figure 2 we plot the probability distribution function of the period in which the marginal token is spent with $M = 10$ tokens and consumption shock probability $p = 0.5$. All non-tradable tokens might be spent in as few as 10 periods if consumption shocks arrive in every period ex-post, but will typically take a much longer time for most consumers. For tradable tokens, all consumers always use tokens in the first $\frac{M-1}{p} = 18$ periods. As shown in Figure 2, tokens start to deplete in the Period 19 ($= \frac{M-1}{p} + 1$) with probability $p = 0.5$. The time distribution of expenditure for tradable tokens is compressed compared to non-tradable tokens. Given the convexity of the utility function in time, tradability thus lowers the token price.

An alternative interpretation is that tradability creates a resale market that pushes the platform to compete with itself. (This interpretation has a loose analogy to the Coase conjecture, albeit here consumers are homogeneous.) With the monopolist issuer, the resale market

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19 The fact that additive utility functions are convex in time has been previously noted and studied experimentally by DeJarnette et al. [2020].
Notes: The figure plots the convex $\beta$-discounting function and shows the prices of the following two lotteries: Lottery 1 (price $P_C$): One dollar in period $M + 2$; Lottery 2 (price $P_D$): One dollar in period $M + 1$ with 50% probability, and one dollar in period $M + 3$ with 50% probability. Convexity implies $P_C > P_D$.

introduces competition with the future, reduces the token price, and dampens the platform’s monopoly power. One might ask how a tradable token can sell for less than a non-tradable token when the consumer always has the option of not trading it. The answer is that with tradability, the platform cannot command as high a price precisely because the consumer knows there is always an option of buying on the outside market in the future, and this drives the requirement that the market price of a tradable token must rise faster than the shadow price of a non-tradable token, as we have just proven. As we shall see in later sections where we look at richer pricing strategies and memory functions, the potential advantages of non-tradable tokens run much deeper than just this point.

Of course, the proceeds from the ICO do not capture the entire story, since whenever a con-
FIGURE 2: Probability Distribution of the Last Token Spent

Notes: The figure plots the probability the period of the last token spent for tradable and non-tradable ICO respectively. We pick $M = 10$ tokens issued and the probability of consumption is $p = 0.5$. The solid line plots the probability distribution function of tradable tokens. The dashed line plots the probability distribution function of non-tradable tokens.

sumer tenders a token for a later purchase, the platform has to forego fiat currency revenue that it would have enjoyed absent any token issuance. But, as we next demonstrate in Proposition 1, the present value of future fiat revenue sales is also higher when tokens are non-traded, so a non-traded token ICO is unambiguously more profitable than a traded token ICO.

Recall that $R_{I,N}$ is the total revenue from non-tradable ICO token issuance, given above eq.(4). Define $R_{I,T}$ analogously.

**Proposition 1 (Non-tradability Revenue Dominance):** Tradability reduces the discounted revenue of the issuer for any $M$.

$$R_{I,N} > R_{I,T}$$

**Proof of Proposition 1**
The present value of platform revenue from a one-time tradable token ICO, including both the initial token sales revenue, and revenue from future fiat currency sales is given by\(^{20}\)

\[
R_{I,T} = M \times P_{I,T} + \beta^* \frac{M-1}{p} \frac{\beta^* p}{1 - \beta^*(1-p)} \left( \frac{\beta^* p}{1 - \beta^*} \right)
\]

\[
< M \times P_{I,N} + \left( \frac{\beta^* p}{1 - \beta^*(1-p)} \right)^M \left( \frac{\beta^* p}{1 - \beta^*} \right) = R_{I,N}
\]

This inequality \(\beta^* \frac{1}{p} < \frac{\beta^* p}{1 - \beta^*(1-p)}\) holds as the \(\beta^*\) version of the condition proved in Lemma 1. (That is, simply substitute \(\beta^*\) for \(\beta\) and the proof follows.) The only case where tradability does not affect the discounted revenue is where \(p = 1\) or \(\beta^* = 1\).

Comments: The logic is simple: The issuer starts to earn revenue in fiat money earlier with non-tradable tokens than with tradable tokens\(^{21}\). For the first \(M\) periods, all agents under both types of ICOs have at least one token, and \(p\) percent of them use it each period. But starting after period \(M\), a rising fraction of agents in the non-tradable ICO have no coins, and thus need to use fiat money for platform consumption. Under a tradable ICO, agents who hit zero coins can buy coins from agents who have two tokens or more; in fact, all agents have at least one coin for the first \(M-1\) periods. Thus revenue from fiat money is more backward loaded with tradable issuance than non-tradable issuance. The proof of Proposition 1 formalizes the speed of token spending, and shows that cash revenue always has a lower present value if tokens are tradable\(^{22}\).

For only in-platform use when \(M \geq 1\), tradable tokens are strictly dominated by non-tradable tokens in both revenues from token issuance and subsequent revenues from fiat money sales. To justify tradability, there must be additional benefits outside our model. Of course, it is true that in our setup, we are neglecting some very important potential merits of tradability.

First, we have been assuming risk neutrality; if agents are risk-averse in period utility, there could again be gains to tradability. Second, and more fundamentally, tradability makes the

\(^{20}\)The cash revenue for tradable ICO can be easily calculated. Consumers use tokens for the first \(\frac{M-1}{p}\) periods. Thus, the cash revenue with \(M\) tradable tokens equals to the cash revenue when \(M = 1\) but discounted by \(\beta^* \frac{M-1}{p}\). Recalling the cash revenue in eq.\(\text{(4)}\), we can get : \(\beta^* \frac{M-1}{p} \left( \frac{\beta^* p}{1 - \beta^*(1-p)} \right)^M \frac{\beta^* p}{1 - \beta^*}\).

\(^{21}\)With tradable tokens, no consumer would pay fiat money to the platform until period \(\frac{M-1}{p}\). If tokens are not tradable, a “lucky” consumer can spend all \(M\) tokens before period \(\frac{M-1}{p}\) and thereafter pay fiat money for the platform consumption.

\(^{22}\)Note that for the non-traded ICO, the revenue-maximizing choice of \(M\) must be an integer, given symmetry among consumers. However, the optimal traded ICO can be in units of \(p < 1\), since (in equilibrium) the \(p\) consumers who extinguish a token in any given period become buyers on the resale market. As we illustrate in Appendix\(^\Delta\), however, accounting for this infra-marginal advantage of divisibility (for traded tokens) is second-order and, for our baseline timing, does not affect Proposition 1. In section 5.1, we consider an alternative timeline that can lead to a case where the tradeable ICO yields higher revenue, but only for the particular low-issuance case where the optimal non-tradable ICO has \(M = 1\), and the optimal tradable ICO has \(M < 1\).
tokens liquid and potentially generates a liquidity premium. Relatedly, in our model, we have eliminated the possibility that tradable tokens can be exchanged for very similar tokens usable on other platforms. Third, we have eliminated the possibility that the tradable token can directly be used on other platforms or in peer-to-peer transfer. Therefore, the platform has a degree of monopoly power over its customer base. If tradability allows broader use of the token, this might be translated into a higher \( p \) in our model, which could reverse the results. Many crypto-exchanges provide services for token trading, for example, Coinbase. These, in essence, are the fundamental reasons why our result differs from the canonical result in the literature following Kiyotaki and Wright (1989), that the restrictions on the future resale of an asset lower its price (see, for example, Bethune et al. (2020)). As we have emphasized throughout, if a “generation 3” token can break through to become a general-purpose “generation 4” token that enters widespread use, higher profit is potentially possible. However, due to network effects, only a very few tokens can succeed in this way, and only as long as regulators permit them to issue general-use tokens.\(^{23}\)

### 4.3 Optimal issuance

Of course, Proposition 1 implies that non-tradable tokens earn higher revenue for any given issuance quantity.\(^{24}\) In this section, we characterize optimal issuance and further discuss the tradeoff between the gain from the ICO and the present value of foregone fiat money sales. The full maximization problem for the firm involves taking into account that if a consumer purchases \( M \) tokens, then she will use tokens for her first \( M \) purchases instead of paying in fiat currency. Thus the platform’s complete maximization problem is given by

\[
\max_M \left\{ M \left[ \frac{\beta p}{1 - \beta(1 - p)} \right]^M - \sum_{i=1}^{M} \left[ \frac{\beta^* p}{1 - \beta^*(1 - p)} \right]^i \right\}
\]

\( M \) is a local optimum if

\[
\left[ \frac{\beta p}{1 - \beta(1 - p)} \right]^M \geq (M - 1) \left[ \frac{\beta p}{1 - \beta(1 - p)} \right]^{M-1} - \left[ \frac{\beta p}{1 - \beta(1 - p)} \right]^M + \left[ \frac{\beta^* p}{1 - \beta^*(1 - p)} \right]^M (6)
\]

and

\(^{23}\)We are also abstracting from some possible downsides to tradability. In Gans et al. (2015), tokens can be a device to induce consumers to spend more time on the platform, and transferability can weaken this effect by making it easier for some token holders to opt out more easily.

\(^{24}\)Proposition 1 implies that the maximum revenue of tradable tokens is smaller than the revenue of non-tradable tokens at the optimal quantity of tradable tokens. By the definition of optimality, the maximum revenue of non-tradable tokens is larger than the revenue of non-tradable tokens at the optimal amount of tradable tokens.
Notes: The figure shows the gain and loss by reducing one token (from three to two). The issuer gains from the price increase, but loses from revenue from the marginal token. Given three tokens are optimal in this example, this figure corresponds to optimal issuance constraint, eq. 6.
\[
\left( \frac{\beta p}{1 - \beta(1 - p)} \right)^{M+1} < M \left( \left( \frac{\beta p}{1 - \beta(1 - p)} \right)^M - \left( \frac{\beta^* p}{1 - \beta^*(1 - p)} \right)^{M+1} \right)
\]

FIGURE 4: Optimal Issuance Quantity: From Three to Four

Notes: The figure shows the gain and loss from increasing tokens issued from three (optimal) to four. The issuer loses from the price decrease, but gains revenue from the extra token. Given three tokens are optimal, this figure corresponds to optimal issuance constraint, eq. (7).

Figures 3 and 4 illustrate inequalities (6) and (7). The gray areas represent the net gain and loss from issuing one less (more) token (assuming the optimal number is 3). The blue bars represent the present value of the foregone fiat money revenue from the \( M^{th} \) token issued \( \left( \frac{\beta^* p}{1 - \beta^*(1 - p)} \right)^M \). The dashed bars at the top represent the token price when \( M \) tokens are issued \( \left( \frac{\beta p}{1 - \beta(1 - p)} \right)^M \). (Here \( \beta \) appears instead of \( \beta^* \).) For example, Figure 2 shows how if the platform issues an extra token above the optimum of 3, it gains additional revenue from the extra token issued, but suffers from the price decline on other tokens, as well as the present value of the foregone fiat revenue on the marginal redemption.²⁵

²⁵Online Appendix ?? shows that eq. (6) and eq. (7) are necessary and sufficient conditions for the optimal
With the one-time ICO considered here, the firm is committed to selling all coins at the same price (perhaps because of regulation.) It is this very constraint that leaves consumers some surplus when \( M > 1 \), and allows them to enjoy some of the gains from token issuance. These gains, of course, must at least match the rate of return consumers can earn on savings held outside the platform, \( r \left( = \frac{1-\beta}{\beta} \right) \).

### 4.4 Consumer Surplus

Token issuance yields gains from trade given that the platform’s rate of return exceeds that of retail bank consumers. However, the quantity-price tradeoff implies that the issuer is unable to claim the entire surplus, and must share part of surplus from token issuance with consumers. For completeness, and since the issue might be of significance to regulators, here we explicitly derive consumer surplus.

Consumers derive utility \( \frac{\beta p}{1-\beta} \) from consumption but can pay for it in either tokens or fiat money. The consumer’s spending consists of two parts: token spending today, and the expected expense in fiat money after depletion of tokens. For the non-tradable tokens, we can also write the consumer surplus as the total willingness to pay for the first \( M \) tokens minus the cost of purchasing the tokens.

For non-tradable ICO tokens,

\[
CS_{I,N}(M) = \sum_{i=1}^{M} \left[ \frac{\beta p}{1-\beta(1-p)} \right]^{i} - M \times P_{I,N} - \left( \frac{\beta p}{1-\beta(1-p)} \right)^{M} \left( \frac{\beta p}{1-\beta} \right)
\]

Consumption Utility from First M Goods

Token Spending

Fiat Money Spending

For tradable ICO tokens,

\[
CS_{I,T}(M) = \frac{\beta p}{1-\beta} - M \times P_{I,T} - \beta^{M-1} \left( \frac{\beta p}{1-\beta(1-p)} \right) \left( \frac{\beta p}{1-\beta} \right)
\]

Almost parallel to the Proposition 1, we can easily show that tradable tokens are preferred by consumers: \( CS_{I,T}(M) > CS_{I,N}(M) \).\(^26\) Consumers benefit from tradability from both the issuance level. In another words, these two inequalities pin down a unique \( M \) as the revenue-maximizing issuance quantity. Also, we can show that optimal issuance is monotonic in the key parameters \( \beta \) as long as the optimal issuance level \( M \) is larger than one. A low-\( \beta \) firm values present resources more and prefers to issue more tokens.

\(^26\)The only difference is that consumers discount the fiat-money spending with \( \beta \), rather than \( \beta^{*} \) on the issuer’s side. \( \left( \frac{\beta p}{1-\beta(1-p)} \right)^{M} \left( \frac{\beta p}{1-\beta} \right) > \beta^{M-1} \left( \frac{\beta p}{1-\beta(1-p)} \right) \left( \frac{\beta p}{1-\beta} \right) \) holds by Lemma 1.
lower token price paid today and the lower expected fiat-money spending in the future. In our model, consumers always prefer a free market while the issuer benefits from more restrictions on consumers for a more favorable split of welfare gain. \( CS_{I,N}(M) \) and \( CS_{I,N}(M) \) fully incorporate the \( \beta \) convexity effect we discussed earlier, implying the price effect dominates for consumers.

4.5 Non-tradable ICO with a Price Menu

We now consider the possibility that instead of selling all tokens at the same price, the platform is allowed to offer a menu that relates the total price paid to the number of tokens sold. Consumers are able to get a lower average price, the more tokens they buy. In this case, it is easy to show that the firm can garner all the gains from trade and leave zero consumer surplus.

To derive the its optimal price menu, for issuing \( M \) tokens, we make use of equation (3) to arrive at

\[
\sum_{i=1}^{M} \left[ \frac{\beta p}{1 - \beta (1 - p)} \right]^i
\]

which essentially charges the consumer their marginal utility for each token. In this case, as \( M \to \infty \), the platform gets the maximum possible discounted revenue (first-best) out of consumers

\[
\lim_{M \to \infty} \sum_{i=1}^{M} \left[ \frac{\beta p}{1 - \beta (1 - p)} \right]^i = \frac{\beta p}{1 - \beta}
\]

Consumers get zero surplus since the platform can design a price menu so that consumers are indifferent along the menu. Thus, the design of price menu pushes up the average token price and therefore total platform revenue corresponding to any given token issuance\(^{27, 28}\)

\[
P_{I,PM} = \frac{\beta p}{1 - \beta} \left[ 1 - \left( \frac{\beta p}{1 - \beta (1 - p)} \right)^M \right] \frac{1}{M}
\]

It is quite straightforward to prove that a price menu approach adds nothing when the tokens are tradable: if the platform sells at a lower average price to bulk token buyers, it cannot prevent the arbitragers from making a profit through resale. (Nor can it stop coalitions of

\(^{27}\)If consumers can share information efficiently and ship products at a low cost, one customer can arbitrage along the price menu by aggregating demand from other consumers, buying a large quantity from the issuer at a low price, and shipping products to others. In reality, customers are willing to accept price discrimination if it is costly to arbitrage in the product market or if the purchased service is not transferable at all. For example, users have to link their Uber accounts to their cell phone numbers. Thus the nature of some businesses can make a price menu approach feasible in reality. However, for some durable goods, say an iPhone, the shipping cost is quite small compared with the product value.

\(^{28}\)\( \frac{\beta p}{1 - \beta (1 - p)} \) \( \frac{M}{M} > \left( \frac{\beta p}{1 - \beta (1 - p)} \right)^M = P_{I,N} \)
consumers from buying in bulk to get a lower average price.

4.6 Non-Tradable/Tradable ICO+SCO (Price Only)

Until now, we have considered one-time token issuance strategies that, over time, lead to a shrinking supply of tokens as consumers redeem them for in-platform purchases. A true prototype currency would not self-extinguish, particularly if the issuer wants to maintain the possibility of eventual use outside the platform. In this section, we introduce the possibility that after the initial ICO, the platform commits to subsequently engaging in routine “SCO” (seasoned coin offerings) sufficient to maintain a constant steady-state supply of tokens. Although we continue to assume that the platform can credibly commit to its issuance strategy, understanding how the expectation of ongoing sales affects the price of the initial ICO is also relevant to understanding how lack of credibility might affect initial issuance and price.

The introduction of SCOs turns out to change the calculus of token issuance quite fundamentally. In particular, we will demonstrate here the strong result that if SCOs are used to maintain a constant supply of tokens, then the maximum number of coins consumers will hold is one per person. This result is the same whether tokens are tradable or not, and in fact the tradable and non-tradable cases become equivalent.

We begin with the case of non-tradable tokens. One new question in this scenario is how to set the price in the SCO. In principle, there are two issuance strategies. (1) A no information policy, in which all consumers are offered the same price in every SCO regardless of their history in purchasing tokens and spending them. (2) A history-dependent policy where the platform can charge a price for SCO tokens that is a function of the consumer’s history with the platform. In Section 5.2, we consider SCO issuance with memory.

In this section, we will focus on the conventional “no information” policy that is perhaps the least likely to run afoul of privacy concerns. The no-information SCO strategy has very stark implications. The basic problem the platform faces is that for any steady-state $M$ it tries to sustain, consumers will only be willing to hold excess coins – more tokens that can be spent in one period – if they anticipate that the price will be rising over time at $r$, the rate of return the consumer earns on savings not invested in tokens. But this is only possible in equilibrium if the quantity of tokens is falling over time – which is a contradiction unless the excess tokens yield

\footnote{Our “Seasoned Coin Offering” terminology follows the commonly used “Initial Coin Offering” drawn from the digital currency/token literature, for example: Howell et al. 2020.}

\footnote{Importantly, this result applies only to the kind of memoryless tokens we have been considering so far; later we will introduce the possibility that platform can condition future token sales to individuals on past purchases.}

\footnote{The equivalence between traded and non-traded ICO+SCO can also break down with more complex SCO policies that do not attempt to maintain a constant supply of tokens.}
sufficient transactions convenience services, which we are abstracting from throughout most of this paper. Thus, the only steady-state token holding has every consumer entering each period with exactly one token. At the end of the period, the platform will offer \( p \) tokens per person at price

\[
P_S = \frac{\beta p}{1 - \beta(1 - p)}
\]

which of course corresponds to the ICO price at \( M = 1 \). In a sense, we might refer to this equilibrium as a “token-in-advance” model, since the consumer is always using tokens for platform purchases. The same result holds with tradable tokens.

**Proposition 2 (Token-in-advance Theorem):** In any equilibrium with a constant supply \( M \) of tokens, and with memoryless issuance strategy, \( M = 1 \) regardless of tradability.

**Proof of Proposition 2:**

\( M \leq 1 \): Recall that we assume the token price needs to be constant in every period. If the token price is expected to appreciate indefinitely at the interest rate, one token will eventually be worth more than the market value of the platform. However, the token price cannot exceed one because the value of underlying consumption is bounded (at one by assumption). Thus with a constant token supply, the price must be a constant. Therefore consumers cannot get capital gains to substitute for explicit interest payments, and the equilibrium token supply (per capita) \( M \) cannot exceed one in both tradable and non-tradable issuance.\(^{32}\)

\( M \geq 1 \): We further show the platform always wants to issue at least one token, not zero. Denote \( R_S \) as the total revenue of the ICO + SCO (“token-in-advance”). The issuer nevertheless gains a higher discounted revenue from issuing one token than with no tokens.

\[
R_S = \frac{\beta p}{1 - \beta(1 - p)} + \frac{\beta^* p}{1 - \beta^*} \frac{\beta p}{1 - \beta(1 - p)} = \frac{\beta^* p}{1 - \beta^*} + \frac{(\beta - \beta^*)p}{(1 - \beta + \beta p)(1 - \beta^*)} > \frac{\beta^* p}{1 - \beta^*}
\]

Revenue without token

Comments: Despite being able to earn ongoing revenues, the platform can only sell one token in the first period, and only \( p < 1 \) tokens per period thereafter. We note that from the point of view of credibility, this ICO+SCO equilibrium might be easier to implement than

\(^{32}\)No consumer would buy a second token since any consumer would prefer to invest in the risk-free asset and wait to buy a new token from the market or from the issuer after the next consumption shock arrives.
the ICO. Also, the “token in advance” model might be more viable in an environment where consumers face liquidity constraints.

4.7 Comparison of ICO+SCO with Non-tradable ICO

Does the “token-in-advance” constraint we have demonstrated necessarily imply that a one-time ICO dominates a policy of maintaining a constant steady-state supply of tokens in this environment? Not necessarily. From a revenue perspective, an ICO+SCO allows the issuer to secure ongoing token revenue from all future SCOs, rather than only from the one-time initial ICO. Also important is the fact that by releasing tokens more slowly, the platform will be able to get a higher (un-discounted) average price, that is, it garners a larger share of the gains from trade. The disadvantage, of course, is that the expectations of future SCO issuance limits how much the platform can front-load revenue into the initial ICO. A natural question is whether a non-tradable ICO issuance mechanism can beat the simple ICO+SCO with the “token-in-advance” constraint. This involves comparing $R_s$ with the discounted revenue of the non-tradable ICO:

$$R_{I,N} = M \left( \frac{\beta p}{1 - \beta(1 - p)} \right)^M + \frac{\beta^* p}{1 - \beta^*(1 - p)} \left( \frac{\beta^* p}{1 - \beta^*(1 - p)} \right)^M$$

**Proposition 3 (ICO versus ICO+SCO Dominance):** Under optimal issuance, the non-tradable ICO dominates the ICO+SCO if $\beta^*$ is sufficiently low ($\beta^* \to 0$). When the consumption probability $p$ is low ($p \to 0$) or $\beta^*$ is high ($\beta^* \to \beta$), an ICO+SCO dominates the non-tradable ICO.\footnote{See Online Appendix ?? for further discussion of Proposition 3.} (Proof: See Appendix A)

Comments: The tradeoff between ICO and ICO+SCO essentially depends on the size of gains from ICO token issuance and the rate at which the platform discounts future revenue. For any parameters, it is easy to compare numerically the closed-form expression for $R_{I,N}$ and $R_S$. Proposition 3 studies the dominance in three extreme cases: When $p$ is small or the issuer’s discount factor $\beta^*$ is close to the consumer’s discount factor $\beta$, the issuer may prefer the ICO+SCO issuance mechanism because the issuer can benefit from “token-in-advance” revenue in every future period while the issuer cannot benefit a lot from the large-quantity issuance in the ICO. For example, one can show that the ICO+SCO strictly dominates the non-tradable...
ICO in the parameter space where the optimal issuance quantity is 2 (two) under the non-tradable issuance. The non-tradable ICO will dominate the ICO+SCO when the issuer is impatient enough ($\beta^*$ is small). The benefit of the front-loading cash flow can be sufficiently large to offset the loss of future SCO revenue.

Certainly, the ICO + SCO with a constant steady supply of tokens and a constant steady-state price is much simpler than the optimal one-time ICO. It is also straightforward to show that for reasonable parameters, it can be supported as a trigger strategy equilibrium if we relax the no commitment assumption. Indeed, if the platform lacks credibility, the equilibrium can devolve to the ICO + SCO case ("token-in-advance"). However, absent credibility issues and when $p$ and $\beta$ are both close to one, the one-time non-tradable token ICO is much more profitable.

5 Assumptions Revisited

5.1 Alternative Timeline

In the benchmark model, token trading must happen before consumers know whether they need to spend tokens or not. This implies that there is no incentive for the platform to issue less than one token per customer as all consumers are equivalent, and tokens are not divisible. Suppose, however, that token sales in period $t$ occur after the period $t$ when consumption shock is observed. Then even if only $M < 1$ tokens per capita are issued, if tradable, they can efficiently be sold to consumers who can use them. More generally, the alternative timeline enables "hyper-tradable" tokens to be spent faster, boosts the token price, and thus creates more value from allowing tradability. Thus, the re-sale option induces consumers to pay a higher price in the tradable ICO.

We can also interpret the alternative timeline as one way of introducing heterogeneity among consumers, though we will explore a more general case in the next section. As we shall see, tradability turns out to be particularly advantageous when $p$ is low, where consumers have to wait a long time for their redemption.

**Example:** Consider the limiting case $\beta^* = 0$ and $beta < 0.5$. When $\beta^* = 0$, the issuer only needs to maximize the ICO token revenue. Later fiat money sales do not matter. Recall that

\[ R_{I,N} - R_S = 2a^2 + \frac{a^*}{1-a^*}a^{*2} - a - \frac{a^*}{1-a^*}a = a^{*2} + \frac{a^*}{1-a^*}(a^{*2} - a) = \frac{a^*}{1-a^*}(a^* - a) < 0 \]

\[ \text{Denote } a = \frac{\beta^* p}{1-\beta^*(1-p)} \text{ and } a^* = \frac{\beta p}{1-(1-p)} \text{. Conditional on optimal } M = 2, \text{ we can show that } R_{I,N} < R_S: \]
for the timeline where re-sales occur before individual consumption shocks are observed. Under the alternative timeline, $p$ consumers are willing to buy one token for immediate redemption; thus the issuer would issue at least $1+p$ as shown in Appendix A.3. Issuing $2+p$ tokens yields a lower revenue because the issuer always prefers to issue only $1+p$ tokens.

\[
(2+p)\left[\frac{\beta p}{1-\beta(1-p)}\right]^2 = (2+p)\frac{0.5p}{1-0.5(1-p)} \frac{\beta p}{1-\beta(1-p)} = p(2+p) \frac{\beta p}{1+p} \frac{1}{1-\beta(1-p)} < (1+p) \frac{\beta p}{1-\beta(1-p)}
\]

Thus, the issuer does not want to issue more than $1+p$ token per consumer. The issuer also has no incentive to issue less as they cannot charge a higher price. The non-tradable token revenue is $\mathcal{R}_{I,N} = (1+p)\frac{\beta p}{1-\beta(1-p)}$.

The alternative timeline changes the optimal tradable token issuance results because tokens can be sold to individuals who are able to spend them immediately and without delay. Suppose, the platform issues $M = kp$ tradable tokens. Note that all tokens will be used in the first $k$ periods. In the last period, $p$ consumers need tokens while only $p$ tokens are outstanding. The token price must equal one — the present value working backwards as before the ICO — token price would be $P_{I,T} = \beta^k$. Thus, the tradable token revenue is $\mathcal{R}_{I,T} = \beta^k kp$. In effect, the “hyper-tradable” token becomes cash like (albeit indivisible units).

In this case, tradability can possibly generate both higher token price and revenue in contrast to the timing embodied in Lemma 1.

\[
\max \mathcal{R}_{I,T} = \max_k \beta^k kp = -\frac{1}{\log(\beta)} e^{-1} p
\]

The optimal issuance quantity is $k = -\frac{1}{\log(\beta)}$ and the optimal token price is $P_{I,T} = e^{-1}$. Lemma 1 is violated when $p < e^{-1}$ — the tradable token price is higher:

\[
P_{I,T} = e^{-1} > p > \frac{\beta p}{1-\beta(1-p)} = P_{N,T}
\]

Proposition 1 can also be violated — the tradable tokens can earn higher profit if the following inequality holds\textsuperscript{35}:

\[
\mathcal{R}_{I,N} = (\frac{\beta p}{1-\beta(1-p)}) \times (1+p) = \frac{\beta + \beta p}{1-\beta + \beta p} p < \frac{\beta}{1-\frac{1}{2}\beta} p < -\frac{1}{\log(\beta)} e^{-1} p = \max \mathcal{R}_{I,T}
\]

which can be shown to hold if $\beta < 0.102$.

Note in the above example, the parameters are such the optimal per capita non-traded

\textsuperscript{35}Appendix A further discusses the token prices for $M \geq 1$ for the alternative timeline and the scenario for a generalized ICO +SCO.
issuance is at a minimum \((1 + p)\) with the alternative timeline, and optimal traded issuance is less than 1. However, for any tradable issuance greater than or equal to 1, lemma 1 still holds, and the non-traded price is higher; the proof is essentially the same. Proposition 1 also continues to hold (overall revenue including from fiat money sales is higher for a non-traded ICO), as we confirm in Appendices A.3 and A.4 So even with the alternative timeline, making tokens tradable can only yield higher revenue when optimal issuance for either approach is small. Thus, the advantages of being able to issue traded tokens in smaller (per capita) quantities are inframarginal, and for larger quantities, the convexity advantage of non-tradability we have illustrated dominates, even for the alternative timeline.

5.2 Money Memory

An important potential feature of a platform-backed currency – and in principle for any digital asset – is memory. How does the memory affect our results? A platform can fully observe a consumer’s account information, the full history of the account, and even the entire transaction history for each token. Making use of this information can enhance the advantage of non-tradability: for example, an issuer can only offer the SCO to consumers who just redeemed for consumption, rather than making the SCO available to all consumers. A platform can design a mechanism for the SCO that induces consumers to hold more than one token despite knowing that there will be future SCOs to replenish their stock in every period. Once tokens can be traded, memory incorporation gets harder to enforce as consumers can re-sell tokens in SCO to other consumers who are not targeted by the issuer. Online Appendix offers two possible issuance strategies with money memory to increase the profit from non-tradable token issuance.

5.3 Other Discussion on Assumptions

Convenience Yield

Memory has been pervasively used in business practice. CVS prints coupons for different products once consumers check out after scanning their CVS cards. Starbucks and McDonald’s load customized special offers to their mobile apps based on the analysis of customers’ past consumption data. Platforms can easily incorporate memory into tokens as a critical feature.

In the extreme, the platform can sell one token in the initial ICO for \(\frac{\beta p}{1 - \beta}\), which is the entire present value of future platform consumption to the consumer, but then commit to distributing free tokens to any agent who tenders their token for consumption in any given period. This is, of course, tantamount a membership system where the lifetime dues are paid once and for all upfront.

Formally, consider a specific class of issuance policies where in any future SCO, a platform only issues tokens to consumers with \(M - 1\) tokens. Denote \(a\) as the ICO token price, \(b\) as the SCO token price, and define token issuance mechanism \((X, Y, a, b)\) where \(X\) is the amount of tokens in the account, \(Y\) is amount of tokens to buy. The extreme case discussed above can be written as follows: The price scheme of the ICO is \((0, M, \frac{\beta p}{1 - \beta}, b)\), \((0, x, \infty, b)\) if \(x \neq M\). The price scheme of the SCO is \((M - 1, 1, a, 0)\), \((x, y, a, \infty)\) if \(x \neq M - 1\) or \(y \neq 1\). It is easy to check that “Buy \(M\) tokens in the ICO, and buy one token after a consumption shock in SCO” is an equilibrium strategy for consumers.
One important potential merit of platform tokens is in providing a convenience yield for the token holders. In the money-in-the-utility model (Sidrauskis (1967)), utility is increasing and strictly concave in real money balances. In our model, a convenience yield might be able to justify the issuance of tradable tokens if tradability brings greater convenience for transactions. For example, suppose convenience would directly affect the token prices by changing the effective discount rate:

\[ \beta(M, N) < \beta(M, T) \]

When tokens are more convenient to use, token holders are effectively more patient when holding tokens and willing to pay more fiat money in exchange for them. Where the government allows it, a platform currency, in principle, could yield a significant enough convenience yield to compete with a government currency.

**Non-zero Cost of Input Goods**

Assumption 2 posits zero cost of goods so that the entire revenue converts into platform profit. We can relax this assumption by allowing \( X \) proportion of platform sales to be attributable to the input cost of goods. In this case, the pricing equations and issuance policy results remain the same as before when we relax the zero cost of goods assumption, except that token prices are proportional to gross sales, not net platform profit.

The logic is straightforward: Token issuance adds financial income at the scale of gross revenue, which can be much larger than the size of profit from net platform revenue. For example, if an online retailer platform has a profit margin of 5% (where \( X = 95\% \)); the platform can issue tokens with denomination twenty dollars for each one-dollar profit. If the platform can create an interest return wedge of 3%, the value-added from token issuance will be 0.6 dollars, which implies a 60% increase in the platform profitability. Online Appendix ?? gives a rigorous formula to show how non-zero input costs leverage up the profit from tokenization.

**Platform Runs**

As the model is constructed, the platform tokens are not subject to runs because agents tender their tokens if and only if a consumption shock hits, and the good is assumed non-storable. In reality, the offerings of platforms such as Alibaba and Amazon cover a wide range

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\( ^{39} \)For example, payment with Amazon credit can be settled immediately, rather than going through credit card verification. See Kahn et al. (2020) for furthermore discussion about convenience.
of durable goods, which opens the possibility of having a panic with say, consumers using their
tokens to buy durables they do not yet need, despite storage costs. The platform can deal with
runs in standard fashion, for example, by reserving the right to suspend sales temporarily, but
the point is that even commodity-backed platform currencies are not immune to runs absent
a fully-credible outside guarantee.\footnote{The classic reference on pure multiple equilibrium bank runs is Diamond and Dybvig (1983).} Of course, in principle, the proceeds from token sales can
be deposited in low return, but highly liquid, government securities. The platform could have
a guaranteed refund in fiat currency if it were to temporarily stock out of goods in any given
period; then, however, it would enjoy much smaller profits from token issuance.\footnote{The platform can also adopt a policy of suspending service in a stockout to discourage runs.}

**Interest Payments**

In principle, platform tokens can pay interest “in-kind” (in tokens). In particular, suppose
tokens pay interest equal to $\frac{1-\beta}{\beta}$ on an ICO of $\frac{\beta p}{1-\beta}$ tokens, which could be tradable. This
policy is sustainable since it involves paying out $p$ tokens per period, exactly enough to replace
tendered coins, assuming no runs. Another important interpretation of the interest-bearing
tokens we have just detailed is as a “security token” where effectively the consumer owns a
share of the platform and the interest payments as a form of dividends. We leave “security
tokens” for future research.

### 6 Heterogeneous Consumers

In our framework, the consumption probability $p$ is the cornerstone for the token price.
In this section, we relax the assumption of homogeneity across consumers and ask whether
introducing heterogeneity overturns our conclusion that platforms may still find the issuance of
non-tradable tokens more profitable. Heterogeneity raises a number of subtle and interesting
issues, for example, how a platform can price discriminate and how heterogeneity affects token
prices. Here we limit ourselves to showing that, in principle, heterogeneity does not overturn
our core result that non-tradable token issuance can sometimes yield higher revenue, even with
memoryless tokens sold at a uniform price in an ICO.

For simplicity, we assume a society consisting of half frequent buyers $p_H$ and half infrequent
buyers $p_L$. A platform aims to issue $M$ tokens in total to all consumers, $M_L$ per infrequent
consumer at price $P_L$, and $M_H$ per frequent consumer at $P_H$ respectively. We only study the
ICO and here do not allow platform to price discriminate, that is $P_H = P_L$. The issuance
quantity and consumption frequency follows\footnote{The average token per capita and consumption frequency are population-weighted of different types of consumers.}

\[
\frac{M_L + M_H}{2} = M
\]

\[
\frac{p_L + p_H}{2} = p
\]

### 6.1 Non-tradable ICO

If a platform cannot price discriminate, all consumers coordinate in a pooling equilibrium where price \( \bar{P}_{I,N} \) is the same for everyone. Applying eq.\ref{eq:3}, the willingness to pay for the last token equals to the

\[
(\frac{\beta p_i}{1 - \beta (1 - p_i)})^{M_i} = \bar{P}_{I,N} \quad i \in \{H, L\}
\]

Given that half the population is of each type, then to issue \( M \) tokens per capita, the corresponding price \( \bar{P}_{I,N} \) must satisfy:

\[
\frac{\log(\bar{P}_{I,N})}{\log(\frac{\beta p_L}{1 - \beta (1 - p_L)})} + \frac{\log(\bar{P}_{I,N})}{\log(\frac{\beta p_H}{1 - \beta (1 - p_H)})} = 2M
\]

To simplify notation, we define function \( f(p) = \frac{1}{\log(\frac{\beta p}{1 - \beta (1 - p)})} \) in which case the non-tradable token price can be written as:

\[
\bar{P}_{I,N} = e^{\frac{2}{f(p_L) + f(p_H)}}^{M}
\]

Thus, after introducing heterogeneity, the effective discount factor of non-tradable tokens becomes \( e^{\frac{2}{f(p_L) + f(p_H)}} \).

### 6.2 Tradable ICO

With tradability, all consumers must receive the same token price in the ICO. Moreover, the token price must be expected to appreciate to generate the risk-free return required to induce agents of either type to hold more than one token. Frequent consumers gain more welfare surplus since they are more likely to use the tokens. The token price under heterogeneity is given by (See Online Appendix ?? for the derivation of the closed-form solution.):
\[ \tilde{P}_{I,T} = \beta \frac{M-1}{p} \left[ (1 - \beta)(1 - p_L)^\gamma \right] \frac{\beta p_L}{1 - \beta(1 - p_L)} + \beta\gamma(1 - p_L)^\gamma \frac{\beta p_H}{1 - \beta(1 - p_H)} \] (8)

where

\[ \gamma = -\left\lfloor \frac{\log(1 + \frac{p_L}{2p_H})}{\log(1 - \frac{1}{2}p_L)} \right\rfloor \]

The effective discount factor under tradability is still \( \beta \frac{1}{p} \). The second component in equation (8) is the price when only one token per capita remains. With tradability, the token price must appreciate at the rate of interest \( r \) regardless of the distribution of consumption probabilities. However, in contrast to the homogeneous case, consumers still have incentive to trade when \( M = 1 \), as frequent consumers are willing to buy tokens from infrequent buyers. To compare token prices, we need the following two lemmas,

**Lemma 2 (Effective Discount Factor Dominance with Heterogeneity):** Under heterogeneity, the effective discount rate of non-tradable ICO tokens is still higher than that of tradable ICO tokens. (Proof: See Appendix B)

\[ \beta \frac{1}{p} < e^{(\gamma\frac{p_L}{2p_H})} \]

**Lemma 3 (ICO Price Dominance with Heterogeneity):** When \( M = 1 \), the token price with tradability is lower than the non-tradable token price under heterogeneity. (Proof: See Appendix B)

\[ \tilde{P}_{I,N} > \tilde{P}_{I,T} \]

Comments: Lemma 2 is the enhanced version of Lemma 1 under the agent heterogeneity, implying that the tradable ICO price discounts faster than the non-tradable ICO price as the quantity of tokens issued increases. It should be noted that in the non-traded case, the discount rates are lower with heterogeneity than in the homogeneous case. However, the underlying deeper rationale for the result is the same as before; both types of agents have a utility that is convex in time.

Lemma 3 is something completely new. It proves that the tradable token price is lower than the non-tradable token price when \( M = 1 \). With heterogeneity, trading still occurs when less than one token circulates in the economy; thus, the two cases are no longer identical for \( M = 1 \). The underlying economic logic is the same as Lemma 2 and derives from the convexity of utility in time. Thus, as in the homogeneity case, the tradable token price is lower for any
possible quantity of token issuance, and our core tradability result is robust to the heterogeneity of consumption probabilities.

**Proposition 4 (Revenue Dominance with Heterogeneity):** The total revenue of tradable tokens is lower than the non-tradable token revenue under heterogeneity. (Proof: See Appendix B)

\[ \tilde{R}_{I,N} > \tilde{R}_{I,T} \]

Comments: Proposition 4 illustrates our core result is robust to the consumer heterogeneity. Both token price and cash revenue are higher if the platform prefers to ban tradability in the ICO. The core logic is the same as is the simpler case in Proposition 1. Lemmas 2 and 3 prove that the ICO itself yields higher token revenue for any given quantity of token issuance (parallel with Lemma 1). Non-tradability locks some tokens in consumers who are less likely to spend tokens soon. The platform can continue to benefit from it even when the per capita token supply falls below one. Proposition 4 proves that non-traded tokens also yield sales in fiat money earlier on, thus provides a higher discounted value. Under heterogeneity, tradable tokens limit the issuer’s profitability but allow consumers to spend tokens faster and efficiently.

### 7 Conclusion

In this paper, we have studied to what extent large retailer platforms might have an advantage in issuing non-interest bearing digital tokens (currencies) by leveraging the fact that there are many consumers who are regular buyers, and who might find in-platform tokens appealing and convenient, thereby allowing the platform to generate extra revenues through a net interest margin.

Our core finding is that in many cases, it may be advantageous to the platform to issue non-tradable tokens rather than tradable ones, even if that means foregoing ideas of creating a prototype currency, unless the prototype currency can create a significant convenience yield. For ICOs, when tokens are only used in-platform, this result holds even for the most favorable case to tradability (among platform consumers), which is when tokens must be sold at a uniform price. A qualification to our core result is that when we allow for an alternative timeline that makes the tokens tradable within the same period that idiosyncratic consumption shocks are revealed, a tradable ICO can lead to higher revenue. However, this happens only when the parameters are such that optimal issuance for a non-tradable ICO is at a minimum and the benefits to token issuance are infra-marginal.
Suppose one goes beyond the constraint of uniform price issuance. In that case, non-traded tokens can give the platform the ability to implement more sophisticated pricing strategies (for example, a price menu approach) and to incorporate memory features, all of which greatly expand their advantages. Of course, as we also emphasize, a more general model could also integrate other benefits to tradability, including, for example, the ability to use the tokens on other platforms.

It is important to recognize that at the end of the day, a great deal depends on regulation. For example, regulators could require within-platform tradability in order to give consumers a greater share of the surplus (as we have shown), or they could place financial reporting requirements on tradable tokens to reduce use in tax evasion and money laundering.

Our analysis has focused mainly on non-interest bearing tokens; if tokens can pay market interest, this can solve many of the problems we have analyzed. However, as discussed in the text, a pledge to pay market interest has its own issues, with implications for taxation, regulation, credibility, governance, and implementation.

The model presented here allows one to analyze a hierarchy of platforms depending on the frequency with which the consumer accesses them, and potentially also the size of transactions, and therefore how such differences might affect platform strategies when it comes to token/coin issuance. Our analysis aims to better understand token issuance mechanisms but does not intend to horse-race token issuance with traditional financing approaches, like bonds or equity. The huge range of crypto-currencies that have been issued to date, with ties to everything from social networking to real estate, provide fertile ground for empirical analysis. In principle, it is also possible to exploit data on related token devices from the pre-digital commerce era, including green stamps, loyalty points, etc., although as we have previously noted, the technology of this era did not allow for using data in the same way as today, and the issues around token offerings are new.

Loyalty programs and gift cards have been around for a long time and are already economically significant, but new generations of redeemable platform tokens are in an explosive growth phase and are set to play an increasing role in payments and the monetary economics of the future. Our analysis here suggests that a new redeemable platform token issuer needs to ask: Just how much outside usage does the platform need to generate to compensate for the higher rents it could gain in-platform by making its redeemable tokens non-tradable?
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Data Availability Statement

No new data were generated, analysed or associated in support of this research. The code underlying this research is available at https://doi.org/10.5281/zenodo.6418911
References


A APPENDIX: Proofs for Homogeneous Agent Case

A.1 Proof of Lemma 1: Effective Discount Factor Dominance

To show $\beta p < \frac{\beta p}{1 - \beta(1 - p)}$, we rewrite the inequality linearly as

$$\iff \beta p > \beta \frac{1}{p} - \beta^{1+\frac{1}{p}}(1-p)$$

Then, we define a function $\omega(\beta)$ and show $\omega(\beta) > 0$ in the range of $\beta \in (0, 1)$:

$$\omega(\beta) = \beta p - \beta^{\frac{1}{p}} + \beta^{1+\frac{1}{p}}(1-p)$$

First, it is easy to find that $\omega(0) = 0$ and $\omega(1) = 0$. Then, we characterize $\omega(\beta)$ with the first-order and second-order derivatives:

$$\omega'(\beta) = p - \frac{1}{p}\beta^{\frac{1}{p}-1} + (1 + \frac{1}{p})\beta^{\frac{1}{p}}(1-p)$$

$$\omega''(\beta) = \frac{1}{p}(\frac{1}{p} - 1)\beta^{\frac{1}{p}-2} + (1 + \frac{1}{p})\beta^{\frac{1}{p}-1}(1-p) = \beta^{\frac{1}{p}-2}\frac{1}{p}\frac{1}{p} \frac{1}{p}(1-1)(1-(1+p))beta$$

Note that $\omega'(0) = p > 0$ and $\omega'(1) = 0$. From the second-order derivative ($\omega'' = 0 \iff \beta = \frac{1}{1+p}$), we find that $\omega'(\beta)$ is monotonically decreasing when $\beta < \frac{1}{1+p}$ but increasing when $\beta > \frac{1}{1+p}$.

Then, we show the existence of a unique $\beta$ such that $\omega'(\beta) = 0$. $\omega'(1) = 0$ and $\omega''(1) < 0$ imply that $\omega'(1 - \epsilon) < 0$ where $\epsilon$ is a positive infinitesimal. By continuity, there must exist a $\beta$, $\omega'(\beta) = 0$. Uniqueness: If there is more than one root, say $0 < \beta_1 < \beta_2 < 1$, then $\omega'(\beta_1) = \omega'(\beta_2) = \omega'(1) = 0$. By continuity, there must exist $\tilde{\beta}_1$ and $\tilde{\beta}_2$ so that $\omega''(\tilde{\beta}_1) = \omega''(\tilde{\beta}_2) = 0$ and $\beta_1 < \tilde{\beta}_1 < \beta_2 < \tilde{\beta}_2 < 1$. However, we know that $\beta = \frac{1}{1+p}$ is the only root for $\omega''(\beta) = 0$ in the range of $(0, 1)$. This violation implies a unique solution to $\omega'(\beta) = 0$.

Last, we show that $\omega(\beta) > 0$ holds when $\beta \in (0, 1)$. $\omega'(0) = p > 0$ implies $\omega(\epsilon) > 0$ for a positive infinitesimal $\epsilon$. If there is a $\beta_3$ where $\omega(\beta_3) \leq 0$, we can find a $\beta_4 \in (\epsilon, \beta_3)$ so that $\omega(\beta_4) = 0$. $\omega(0) = \omega(\beta_4) = \omega(1) = 0$ implies at least two roots for $\omega'(\beta) = 0$. This violates the uniqueness of the solution to $\omega'(\beta) = 0$.

To give a graphical illustration, we plot the gap between the two effective discount factors as a function of $\beta$ with $p = 0.9$ as in Figure A.1.
A.2 Proof of Proposition 3: Non-tradable ICO dominance over ICO+SCO

We consider three corner cases $\beta^* \to 0$, $p \to 0$, and $\beta^* \to \beta$:

Case 1: $\beta^* \to 0$: $\max M R_{1,N} - R_S = \max M \left( \frac{\beta p}{1 - \beta (1-p)} M - \frac{\beta p}{1 - \beta (1-p)} \right) \geq [M(\frac{\beta p}{1 - \beta (1-p)})^M](M = 1) - \frac{\beta p}{1 - \beta (1-p)} = 0$

When $\beta^* \to 0$, ICO dominates because the issuer does not value future revenue at all. "Token-in-advance" limits the issuer’s ability to collect token revenue in the ICO period.

Case 2: $p \to 0$: The gain to issuing the second token is only second-order (a constant multiplies $p^2$):

$$\left( \frac{\beta p}{1 - \beta (1-p)} \right)^2 - \left( \frac{\beta^* p}{1 - \beta^* (1-p)} \right)^2 \approx \left[ \frac{\beta}{1 - \beta} \right]^2 - \left( \frac{\beta^*}{1 - \beta^*} \right)^2 p^2$$

But the loss is first order (a constant multiplies $p$):

$$\left( \frac{\beta p}{1 - \beta (1-p)} \right)^2 - \left( \frac{\beta p}{1 - \beta (1-p)} \right) \approx \frac{\beta}{1 - \beta} p$$

The first-order loss is larger than the second-order gain. Thus, the optimal non-tradable ICO token issuance is one. ICO+SCO strictly dominates non-tradable ICO with one token.
outstanding because SCO can frontload cash flow and increase revenue by \( p \left( \frac{\beta p}{1 - \beta (1 - p)} - \frac{\beta^* p}{1 - \beta^* (1 - p)} \right) \) in every future period.

Case 3: \( \beta^* \to \beta \): The gain from the second token issuance is close to zero. The loss is
\[
(\frac{\beta p}{1 - \beta (1 - p)})^2 - (\frac{\beta p}{1 - \beta (1 - p)}) \approx -\frac{\beta}{1 - \beta} p < 0.
\]
Thus, the optimal non-tradable ICO token issuance is also one. Similarly, the ICO+SCO dominates the non-tradable ICO in this case.

In Appendix ??, we construct a non-empty interval for \( \beta^* \) where ICO dominates ICO+SCO.

A.3 Alternative Timeline \( (M \geq 1) \)

This section evaluates the case \( M \geq 1 \) under the alternative timeline that consumers can buy tokens after the consumption shocks reveal. A platform announces token issuance plan in Period 0. For each period \( t \), consumption shocks reveal first. Each consumer has a probability \( p \) to consume on the platform. Then, consumers trade tokens and redeem tokens for consumption.

First, the issuer’s optimal token issuance plan will take the form of \( M + p \). The new timeline essentially generates heterogeneity in any given period. The heterogeneity is only about the consumption shock in the current Let’s consider any token issuance quantity \( M' \), where \( M' \in (M + p, M + 1 + p) \). To make \( M' \) feasible, the issuance plan is financially acceptable for (at least) one consumer with no consumption shock in the first period to purchase the \((M + 1)^{th}\) token. Symmetrically, all consumers with no consumption shock would accept to purchase \( M + 1 \) tokens. \( p \) consumers, who know consumption shock arrive in period 1, would be happy to purchase \( M + 2 \) tokens. The token price must be below one; thus, they enjoy pure surplus for consumption in period 1. Then, these consumers are happy to buy \( M + 1 \) tokens for all consumption shocks realized since period 2. The issuer will gain extra financial benefits to sell \( M + 1 + p \) tokens in total if they find it more profitable to issue \( M' > M + p \).

Then, we only consider plans with \( M + p \) tokens issuance plan. Consumers without initial shock are the marginal buyers in the market. The price must be good enough for consumers to enter the token economy. Token prices remain the same for tradable and non-tradable ICOs. But under the new timeline, consumers enjoy more surplus as \( p \) consumers can redeem their tokens immediately.

Non-tradable ICO:
\[
P_{I,N} = \left( \frac{\beta p}{1 - \beta (1 - p)} \right)^M
\]

 Tradable ICO:
\[
P_{I,T} = \beta^M
\]

When \( M > 1 \), non-tradable token price is still higher than tradable tokens as Lemma 1
applies in the alternative timeline. We further study the revenue dominance. For \( M + p \) non-tradable tokens, all consumers hold \( M \) tokens after the first round of redemption. Thus, the token redemption and cash revenue are the same as \( M \) tokens the original timeline. For \( M + p \) tradable tokens, the alternative timeline still helps tokens redeemed faster, but the benefit is not large enough.\(^{43}\)

\[
R_{I,T} = (M + p) \times P_{I,T} + \beta^* \frac{\beta^* p}{1 - \beta^*(1 - p)} \left( \frac{\beta^* p}{1 - \beta^*} \right)
< (M + p) \times P_{I,N} + \left( \frac{\beta^* p}{1 - \beta^*(1 - p)} \right)^M \left( \frac{\beta^* p}{1 - \beta^*} \right) = R_{I,N}
\]

The token price hinges on the willingness to pay for the marginal token. The alternative timeline generates less impact when the platform tries to issue more than one token. The alternative timeline cannot overturn our core result when \( M \geq 1 \) — the discount rate dominance is still first-order under the alternative timeline.

### A.4 Integer Problem

For tradable tokens, the optimal issuance ICO policy \( M \) might not be an integer. Consumers do not need to hold the same amount of tokens in the initial period, as the token price appreciates by interest rate. For example, for non-tradable tokens, \( M = 1 \) is optimal as the issuer cannot generate more profit if they go \( M = 2 \). However, if they choose to issue tradable tokens, they can essentially issue \( 1 + p \) tokens per capita — \( p \) consumers hold two tokens, and \( 1 - p \) consumers buy one token. Tradability allows a wider set of token quantities to choose for optimality. The tradable token price, of course, needs to further discount by \( \beta \).

Under the optimal policy, is the integer problem possible to change Proposition 1? We show that \( 1 + p \) tradable tokens cannot generate a higher profit than \( M = 1 \) non-tradable tokens; that is, the following inequality cannot hold:

\[
Rev_{I,N}^{M=2} < Rev_{I,N}^{M=1} = Rev_{I,T}^{M=1} < Rev_{I,T}^{M=1+p}
\]

For \( M = 1 \), tradable and non-tradable tokens generate the same revenue:

\[
Rev_{I,N}^{M=1} = Rev_{I,T}^{M=1} = \frac{\beta p}{1 - \beta(1 - p)} + \frac{\beta^* p}{1 - \beta^*(1 - p)}
\]

First, \( Rev_{I,N}^{M=2} < Rev_{I,N}^{M=1} \) implies that

\[^{43}\text{We still neglect the integer problem here, that is, we assume } \frac{M}{p} \text{ is an integer.}\]
\[2\left(\frac{\beta p}{1-\beta(1-p)}\right)^2 + \frac{\beta^* p}{1-\beta^*(1-p)}\left(\frac{\beta^* p}{1-\beta^*(1-p)}\right)^2 < \frac{\beta p}{1-\beta(1-p)} + \frac{\beta^* p}{1-\beta^*} \frac{\beta^* p}{1-\beta^*(1-p)}\]

\[\iff 2\frac{\beta p}{1-\beta(1-p)} - 1 \frac{\beta^* p}{1-\beta^*(1-p)} < \frac{\beta^* p}{1-\beta^*} \frac{\beta^* p}{1-\beta^*(1-p)}\]  \hspace{1cm} (A.1)

Second, \(\text{Rev}_{I,T}^{M=1} < \text{Rev}_{I,T}^{M=1+p}\) implies

\[\frac{\beta^* p}{1-\beta^*} \frac{\beta^* p}{1-\beta^*(1-p)} < (1+p)\beta^* \frac{\beta^* p}{1-\beta^*} + \frac{\beta^* p}{1-\beta^*} \frac{\beta^* p}{1-\beta^*(1-p)}\]

\[\iff \frac{\beta^* p}{1-\beta^*} \frac{\beta^* p}{1-\beta^*(1-p)} < \frac{(1+p)\beta - 1}{1-\beta^*}\]  \hspace{1cm} (A.2)

Equations (A.1) and (A.2) jointly imply the following inequality must hold:

\[2\frac{\beta p}{1-\beta(1-p)} - 1 \frac{\beta^* p}{1-\beta^*(1-p)} < \frac{(1+p)\beta - 1}{1-\beta^*}\]

However, the above inequality cannot hold as

\[2\frac{\beta p}{1-\beta(1-p)} - 1 \frac{\beta^* p}{1-\beta^*(1-p)} = \frac{2\beta p - 1 + \beta p - 1}{1-\beta(1-p)} = \frac{(1+p)\beta - 1}{1-\beta(1-p)} < \frac{(1+p)\beta - 1}{1-\beta^*} \frac{1-\beta^*(1-p)}{1-\beta(1-p)} > \frac{(1+p)\beta - 1}{1-\beta^*}\]

The contradiction implies that \(1+p\) tradable tokens cannot generate a higher revenue than one non-tradable token under optimality.
B APPENDIX: Proofs for Section 6, Extension to Heterogeneous Agent Case

B.1 Proof of Lemma 2: Effective Discount Factor Dominance with Heterogeneity

Recall that \( f(p) = \frac{1}{\log(\frac{\beta p}{1 - \beta(1-p)})} \). With heterogeneity, the effective discount factor for non-tradable tokens is \( e^{\frac{2}{f(p_L)} + \frac{2}{f(p_H)}} \) while the discount factor of tradable tokens remains the same \( \beta \) as in Lemma 1.

Case 1: \( p < 0.5 \): the convexity of \( f(x) \) implies that the non-tradable discount factor is higher than the case where \( p_L = 0, p_H = 2p \).

\[
e^{\frac{2}{f(p_L)} + \frac{2}{f(p_H)}} \geq e^{\frac{2}{f(0)} + \frac{2}{f(2p)}} = e^{2\log(\frac{2\beta p}{1 - \beta(1-2p)})} = \left(\frac{2\beta p}{1 - \beta(1-2p)}\right)^2
\]

Applying the formula in Lemma 1,

\[
\left(\frac{2\beta p}{1 - \beta(1-2p)}\right)^2 > (\beta^{\frac{1}{\beta}})^2 = \beta \frac{1}{p}
\]

Case 2: \( p \geq 0.5 \): the convexity of \( f(x) \) implies that the non-tradable discount factor is higher than the case where \( p_L = 2p - 1, p_H = 1 \):

\[
e^{\frac{2}{f(p_L)} + \frac{2}{f(p_H)}} \geq e^{\frac{2}{f(2p-1)} + \frac{2}{f(1)}}
\]

After taking logs, we need to prove:

\[
\frac{2}{f(2p-1) + f(1)} > \frac{1}{p} \log \beta
\]

\[
\iff (2p-1) \log(\frac{\beta(2p-1)}{1 - \beta + \beta(2p-1)}) > \log \beta
\]

Similarly, applying Lemma 1,

\[
(2p-1) \log(\frac{\beta(2p-1)}{1 - \beta + \beta(2p-1)}) > (2p-1) \log(\beta^{\frac{1}{\beta-1}}) = \log \beta
\]

For \( p_L, p_H \in [0,1] \) and \( p = \frac{p_L + p_H}{2} \), the effective discount factor of non-tradable tokens still dominates that of tradable tokens in the heterogeneity case, that is

\[
\beta^{\frac{1}{p}} < e^{\frac{2}{f(p_L)} + \frac{2}{f(p_H)}}
\]
B.2 Proof of Lemma 3: ICO Price Dominance with Heterogeneity

With only one token outstanding \((M = 1)\), the non-tradable token price is higher than the tradable token price if and only if:

\[
(1 - \beta(1 - p_L)) \frac{\log(\frac{1 + \frac{p_L}{\beta}}{1 - \frac{p_L}{\beta}})}{\log(\frac{1 - \frac{p_L}{\beta}}{1 - \frac{p_L}{\beta}})} + \beta(1 - p_L) \frac{\log(\frac{1 + \frac{p_H}{\beta}}{1 - \frac{p_H}{\beta}})}{\log(\frac{1 - \frac{p_H}{\beta}}{1 - \frac{p_H}{\beta}})} < e^{\frac{2}{f(p_L) + f(p_H)}}
\]

Define \(x = \frac{\beta p_L}{1 - \beta(1 - p_L)}\) and \(y = \frac{\beta p_H}{1 - \beta(1 - p_H)}\). It is easy to show that \(0 < x < y < \beta < 1\).

We aim to rewrite the inequality as a function of \(x, y, \beta\). Thus, we express \(p_L\) and \(p_H\) in \(x\) and \(y\):

\[
p_L = \frac{1 - \beta}{1 - x}, \quad p_H = \frac{1 - \beta}{1 - y}, \quad \frac{p_L}{p_H} = \frac{x(1 - y)}{(1 - x)y}.
\]

The non-tradable token price is rewritten in \(x\) and \(y\) as the following:

\[
x + \beta(1 - p_L) \frac{\log(\frac{1 + \frac{p_L}{\beta}}{1 - \frac{p_L}{\beta}})}{\log(\frac{1 - \frac{p_L}{\beta}}{1 - \frac{p_L}{\beta}})} (y - x) < e^{\frac{2}{f(p_L) + f(p_H)}}
\]

We subtract \(x\) from both side, take logs, and replace \(p_L, p_H\) with \(x, y\). Then, we move the two terms with \(x\) only to the left-hand side, and the other two terms depending on both \(x\) and \(y\) to the right-hand side:

\[
\LHS \frac{\log(\beta(1 - \frac{1 - \beta}{1 - x}))}{\log(1 - \frac{1 - \beta}{1 - x})} > \RHS \frac{\log(y - x) - \log(e^{\frac{2}{f(x) + f(y)}} - x)}{\log(1 + \frac{1}{2 \frac{x(1 - y)}{(1 - x)y}})}
\]

The \(LHS\) (left-hand side of the above inequality) is a function of \((\beta, x)\) and the \(RHS\) (right-hand side of the above inequality) is a function of \((x, y)\). We first show the \(RHS\) is monotonically increasing in \(y\). We use a graphical proof with the three-dimensional surface in Appendix Figure B.1. Appendix Figure B.2 picks parameters for \(x\) and plots the \(RHS\) as a function of \(y\) to illustrate the monotonicity.

The upper bound of \(RHS\) is reached with \(y = \beta\). Then, it is sufficient to prove the following inequality:

\[
\LHS \frac{\log(\beta(1 - \frac{1 - \beta}{1 - x}))}{\log(1 - \frac{1 - \beta}{1 - x})} > \RHS \frac{\log(\beta - x) - \log(e^{\frac{2}{f(x) + f(y)}} - x)}{\log(1 + \frac{1}{2 \frac{x(1 - y)}{(1 - x)y}})}
\]

It is equivalent to show that function \(G(\beta, x)\) is positive.
\[ \left[ \log(\beta - x) - \log(e^{\log(x)} + e^{\log(y)} - x) \right] \left[ \log(1 - \frac{1}{2} \frac{1 - \beta}{1 - y} x) - \log(1 + \frac{1}{2} \frac{1 - \beta}{1 - y} x) \right] \log(\frac{1 - \beta}{\beta} - \frac{1}{1 - y}) > 0 \]

Similarly, we show a graphical proof with the three-dimensional surface of \( G(\beta, x) \) in Appendix Figure B.3. Appendix Figure B.4 picks parameters for \( \beta \) and plots the \( G(\beta, x) \) as a function of \( x \). Thus, \( G(\beta, x) \) is positive for all parameters \( (\beta, x) \), and it implies that non-tradable token price is higher than tradable token price when \( M = 1 \).

FIGURE B.1: Three-dimensional surface of RHS monotonicity

Notes: This figure plots the three-dimensional surface of the RHS as a function of \((x,y)\).
FIGURE B.2: Two-dimensional curve of RHS monotonicity

Notes: This figure plots the RHS as a function of $y$, given $x = 0.05, 0.1, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$ and 0.9.

FIGURE B.3: Three-dimensional surface of $G(\beta, x)$ function value

Notes: This figure plots the three-dimensional surface of the $G(\beta, x)$.
FIGURE B.4: Two-dimensional curve of $G(\beta, x)$ function value

**Notes:** This figure plots the $G(\beta, x)$ as a function of $x$, given $\beta = 0.05, 0.1, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$ and $0.9$. The red dashed horizontal line refers to zero.

### B.3 Proof of Proposition 4: Revenue Dominance with Heterogeneity

Denote $C_i(n)$ is the average number of tokens redeemed in period $n$. $i$ represents the issuance mechanism: $i = T$ refers to tradable ICO, and $i = N$ represents non-tradable ICO in this proof.

We first prove the following result: Tradable tokens are spent faster than non-tradable tokens, formally defined as, for any $n$, the remaining tradable tokens are less than non-tradable tokens.

$$\kappa_T(n) = M - \sum_{j=1}^{j=n} C_T(j) \leq M - \sum_{j=1}^{j=n} C_N(j) = \kappa_N(n)$$

We use math induction method to prove $\kappa_N(n) \geq \kappa_T(n)$ for each $n$. Denote $p_i(n)$ is the share of tokens are redeemed in period $n$.

$$\kappa_i(n + 1) = \kappa_i(n) \times (1 - p_i(n))$$

For $N = 1$, it is obvious, $\kappa_T(1) = \kappa_N(1) = M - p$ and $p_T(1) = p_N(1) = \frac{p}{M_T}$.

For all $n \leq \frac{M-1}{p} + 1$, $\kappa_T(n - 1) \leq \kappa_N(n - 1)$ holds. Under tradable regime, consumers still have at least one token at hands and $p$ tokens can be redeemed in period $n$. Thus, we
can compute \( p_T(n) = \frac{p}{\kappa_T(n-1)} \). For non-tradable tokens, it is possible that some consumption shocks can be not be filled if some consumers run out of tokens.

\[
p_N(n) \leq \frac{p}{\kappa_N(n-1)} \leq \frac{p}{\kappa_T(n-1)} = p_T(n)
\]

The remaining tokens after the redemption in period \( N \)

\[
\kappa_N(n) = \kappa_N(n-1)(1 - p_N(n)) \geq \kappa_T(n-1)(1 - p_N(n)) \geq \kappa_T(n-1)(1 - p_T(n)) = \kappa_T(n)
\]

Thus, \( \kappa_N(n) \geq \kappa_T(n) \) and \( p_T(n) \geq p_N(n) \) still hold for period \( n \).

Then, we consider the case where \( n > \frac{M-1}{p} + 1 \), consumers start to run out of tradable tokens. We know \( \kappa_T(n-1) < \kappa_N(n-1) \).

For tradable tokens, we know \( \kappa_T(n-1) \leq 1 \). First, no consumer will own more than one token (otherwise, consumers trade). Moreover, remaining tokens are distributed to frequent consumers first as they are willing to pay a higher price for the token more than infrequent consumers.\(^{44}\) If \( \kappa_T(n-1) > 0.5 \), all frequent consumers hold one token. The rest \( \kappa_T(n-1) - 0.5 \) tokens will be owned by infrequent consumers. Then, we can calculate the \( p_T(N) \):

\[
p_T(n) = \begin{cases} 
p_H & \kappa_T(n-1) \leq \frac{1}{2} \\
\frac{0.5p_H + (\kappa_T(n-1) - 0.5)p_L}{\kappa_T(n-1)} & \kappa_T(n-1) > \frac{1}{2}
\end{cases}
\]

For non-tradable tokens, we cannot determine the token distribution in period \( n-1 \). However, we can write down the upper bound \( p_N(n) \). \( p_N(n) \) is maximized when all tokens allocated to frequent consumers if \( \kappa_N(n-1) < \frac{1}{2} \) all frequent consumers hold tokens. If \( \kappa_N(n-1) > \frac{1}{2} \), \( p_N(n) \) is maximized when all frequent consumers hold at least one token, the rest of tokens are hold by infrequent consumers (so that infrequent consumers can spend tokens as many as possible in period \( n \)).

\[
p_N(n) = \begin{cases} 
p_H & \kappa_N(n-1) \leq \frac{1}{2} \\
\frac{0.5p_H + (\min\{0.5,\kappa_N(n-1) - 0.5\})p_L}{\kappa_N(n-1)} & \kappa_N(n-1) > \frac{1}{2}
\end{cases}
\]

Then, we show \( p_T(n) \geq p_N(n) \) when \( n > \frac{M-1}{p} + 1 \):

**Case 1:** \( \kappa_T(n-1) \leq \frac{1}{2} \)

\( p_T(n) = p_H \geq p_N(n) \) holds as the \( p_H \) is the global maximum redemption rate.

**Case 2:** \( \kappa_T(n-1) > \frac{1}{2} \). We know that \( \kappa_N(n-1) > \kappa_T(n-1) > \frac{1}{2} \).

\(^{44}\)Frequent consumers are more likely to spend the token than infrequent consumers, thus they would prefer to pay more.
\[
p_T(n) = \frac{0.5p_H + (\kappa_T(n-1) - 0.5)p_L}{\kappa_T(n-1)} = p_L + 0.5 \frac{p_H - p_L}{\kappa_T(n-1)} \geq p_L + 0.5 \frac{p_H - p_L}{\kappa_N(n-1)} = p_N(n)
\]

Similarly, the quantity of remaining tokens is

\[
\kappa_N(n) = \kappa_N(n-1)(1 - p_N(n)) \geq \kappa_T(n-1)(1 - p_T(n)) = \kappa_T(n)
\]

By math induction, we know tradable tokens are spent faster than non-tradable tokens, for any \(n\):

\[
\kappa_N(n) \geq \kappa_T(n) \quad (B.1)
\]

We have already known that token revenue is higher if the token is non-tradable. The only remaining question is to show the cash revenue is also higher for non-tradable tokens. We can express the present value of cash revenue with \(C_i(n)\):

\[
\tilde{R}_{I,N}^{\text{Cash}} = \sum_{j=1}^{\infty} \beta^j [p - C_N(j)] \geq \sum_{j=1}^{\infty} \beta^j [p - C_T(j)] = \tilde{R}_{I,T}^{\text{Cash}}
\]

We can rewrite

\[
M - \frac{p\beta^*}{1 - \beta^*} - \tilde{R}_{I,N}^{\text{Cash}} = \sum_{j=1}^{\infty} C_N(j) - \frac{p\beta^*}{1 - \beta^*} - \sum_{n=1}^{\infty} \beta^n C_N(j) + p \sum_{n=1}^{\infty} \beta^n = \sum_{j=1}^{\infty} (1 - \beta^j)C_N(j)
\]

\[
= (1 - \beta^* ) \sum_{j=1}^{\infty} C_N(j) \left( \sum_{n=0}^{j-1} \beta^n \right) = (1 - \beta^* ) \sum_{n=0}^{\infty} \sum_{k=n}^{\infty} C_N(k+1)
\]

\[
= (1 - \beta^* ) \sum_{n=0}^{\infty} \beta^n (M - \kappa_N(n))
\]

We apply the inequality \([B.1]\):

\[
M - \frac{p\beta^*}{1 - \beta^*} - \tilde{R}_{I,N}^{\text{Cash}} = (1 - \beta^* ) \sum_{n=0}^{\infty} \beta^n (M - \kappa_N(n)) \leq (1 - \beta^* ) \sum_{n=0}^{\infty} \beta^n (M - \kappa_T(n)) = M - \frac{p\beta^*}{1 - \beta^*} - \tilde{R}_{I,T}^{\text{Cash}}
\]

Thus, non-tradable tokens gain a higher cash revenue \(\tilde{R}_{I,N}^{\text{Cash}} \geq \tilde{R}_{I,T}^{\text{Cash}}\). Recall Lemma 3 shows non-tradable token price is higher, thus

\[
\tilde{R}_{I,N} = M \times \tilde{P}_{I,N} + \tilde{R}_{I,N}^{\text{Cash}} \geq M \times \tilde{P}_{I,T} + \tilde{R}_{I,T}^{\text{Cash}} = \tilde{R}_{I,T}
\]

48