Market Power in Neoclassical Growth Models

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This paper examines the optimal accumulation of capital and the effects of government debt in neoclassical growth models in which firms have market power and therefore charge prices above marginal cost. In this environment, the real interest rate earned by savers is less than the net marginal product of capital. We establish a new method for evaluating dynamic efficiency that can be applied in such economies. A plausible calibration suggests that the wedge between the real interest rate and the marginal product of capital is about 4 percentage points and that the U.S. economy is dynamically efficient. In addition, government Ponzi schemes can have different implications for welfare than they do under competition. Even if the government can sustain a perpetual rollover of debt and accumulating interest, the policy may nonetheless reduce welfare by depressing steady-state capital and aggregate consumption. These findings suggest that even with low interest rates, as have been observed recently, fiscal policymakers should still be concerned about the crowding-out effects of government debt.
This paper explores the role of market power in otherwise neoclassical models of economic growth. Our analysis is based on a well-known principle: When firms with market power set their prices as a markup over marginal cost, they also choose a capital stock at which the marginal product of capital exceeds the cost of capital. Other things being equal, this wedge between the marginal product of capital and the cost of capital depresses the real interest rate. We examine the implications of this fact by extending two canonical models of capital accumulation: the Solow (1956) growth model and the Diamond (1965) overlapping-generations model.

The link between market power and the real interest may be especially important in the contemporary economy. Many observers have suggested that, over the past several decades, markets have become less competitive, and markups have increased. (See, for example, Barkai, 2020; De Loecker, Eeckhout, and Unger, 2020; and Philippon, 2020.) At about the same time, the U.S. economy, along with many other economies around the world, has experienced a dramatic decline in real interest rates. These developments may be connected. If so, understanding the role of market power is necessary to correctly interpret data on the real interest rate.

Neoclassical growth models with market power provide a new lens through which to view fiscal policy and optimal national saving and investment. Discussions of fiscal policy often emphasize that government budget deficits can impede economic growth by crowding out capital. Recently, however, some observers have pointed to low real interest rates to suggest that crowding out may now be of little concern. (See, for example, Blanchard, 2019, and Furman and Summers, 2020.) The existence of market power calls this inference into question. A real interest rate depressed by market power does not reflect the social value of additional capital.

To make these points as simply as possible, we start by building on Solow’s (1956) model of economic growth and Phelps’s (1961) analysis of optimal capital accumulation. The main change we make to this textbook model is to add firms with market power. We show that this amendment profoundly alters how the standard growth framework is applied to interpret the data, especially data on the real interest rate. Market power can push the real interest rate below the economy’s growth rate. Under competition, this condition is associated with the overaccumulation of capital—a condition known as dynamic inefficiency. But that is not the case in an economy with market power.

We produce a new method for evaluating dynamic efficiency in the presence of market power. The method starts with firms’ income per unit of capital, a common measure for rates of return in competitive economies. It then applies two corrections. An estimate of the typical markup in the economy corrects for the underpayment of capital that arises from market power. An estimate of Tobin’s $q$ corrects for the fact that the national accounts include the pure profits that firms receive in measured capital income.

To gauge the practical importance of these effects, we calibrate this extended Solow model to roughly match data for the U.S. economy. If typical prices are marked up 20% over marginal
cost, as some of the literature suggests, the real interest rate is about 4 percentage points below the social return on capital. Correcting for the effects of market power yields an estimate of the social return of more than 8%. Because this estimated social return on capital is much higher than the economy’s growth rate, it indicates that the U.S. economy is dynamically efficient.

After examining these effects in the Solow model, we consider the sustainability and welfare effects of government debt. To do so, we build on the classic work of Diamond (1965) on capital accumulation in an overlapping-generations model. Recall that, in competitive economies, dynamic efficiency is closely connected to the feasibility and desirability of government Ponzi schemes. (See, for example, O’Connell and Zeldes, 1988.) If the real interest rate is below the growth rate, indicating dynamic inefficiency, the government can run a permanent primary deficit: it can give a transfer to every generation and roll over the resulting government debt forever. The crowding out that results from the debt issuance increases welfare because the economy has more capital than is optimal.

This close connection between dynamic efficiency and Ponzi schemes breaks down in an economy with market power. We show that if market power pushes the real interest rate below the growth rate, the government can again run a Ponzi scheme. However, such a Ponzi scheme, though feasible, may not be a free lunch. Even a successful Ponzi scheme may reduce steady-state welfare.

The impact of a government Ponzi scheme on welfare depends on two effects. The first we call the aggregate effect. Because government debt crowds out capital, it depresses aggregate consumption if the net marginal product of capital exceeds the growth rate and increases aggregate consumption in the opposite case. (This is parallel to the classic Phelps, 1961, result.) The second we call the generational effect: the introduction of government debt improves the allocation of aggregate consumption between the young and the old. This effect is similar to the Pareto improvement that can result from pay-as-you-go social security when the growth rate exceeds the interest rate (Samuelson, 1958; Weil, 2008). In a competitive economy, the aggregate and generational effects of debt work in the same direction. In an economy with market power, when the economy’s growth rate is greater than the real interest rate but less than the net marginal product of capital, the two effects conflict. These findings show that, in the presence of market power, the real interest rate by itself is a poor guide for judging the welfare effect of increased government indebtedness.

To be sure, while this paper studies the role of market power, the real interest rate and the marginal product of capital can differ for other reasons. Capital returns are risky, and other things being equal, the risk premium depresses the risk-free rate. (See, for example, Abel et al., 1989, and more recently Barro, 2020.) This phenomenon does not arise here because we consider models with certainty. Of course, in the world, both uncertainty and market power influence the real interest rate. But we believe we can best clarify matters by considering only one of these forces. Just as much previous work has focused on the case of uncertainty without market power, here we focus on the case of market power without uncertainty.
Our analysis is related to several strands of the literature. Farhi and Gourio (2018) and Basu (2019) note that market power can drive a wedge between the marginal product of capital and the cost of capital, but they do not explore the implications for optimal capital accumulation or fiscal policy. Judd (1997) suggests that imperfect competition can imply that the optimal tax on capital is negative. He aims to use tax policy to correct the distortion that we take as given. Reis (2021) examines debt policy in an economy where credit-market imperfections cause the interest rate to fall below the marginal product of capital. Though the wedge in his model arises from a very different source, his analysis shares with ours the importance of distinguishing between the social return on physical capital and the private return on financial assets. Finally, much of the endogenous growth literature, such as Aghion and Howitt (1992), emphasizes the role of market power as a driver of technological innovation. By contrast, we follow in the neoclassical tradition of taking technological change as given to explore the implications of market power for optimal capital accumulation and fiscal policy.

1. THE SOLOW MODEL WITH MARKET POWER

This section generalizes the standard continuous-time Solow model by incorporating firms with market power. We proceed in four steps. First, we examine the behavior of individual firms. Second, we imbed these firms into the standard Solow framework. Third, we reconsider Phelps’s conclusions about optimal capital accumulation. Fourth, we provide a numerical calibration to roughly match the U.S. economy.

1.1 The behavior of individual firms

The economy consists of \( N \) identical firms, each of which sells a differentiated product. Each firm produces its good using capital, labor, and intermediate goods purchased from other firms. The firm’s output can be used as consumption, investment, or an intermediate good for other firms.

Following Rotemberg and Woodford (1995), we assume that producing one unit of the firm’s output requires one unit of a composite of capital and labor and \( \lambda \) units of intermediate goods. The composite is produced by the function

\[
Q = F[K, A(L - X)]
\]

where \( Q \) is the composite (which in turn equals the quantity of the firm’s output), \( K \) is capital, \( L \) is labor, \( A \) is a measure of the state of labor-augmenting technology, \( X \) is a constant that represents fixed overhead labor \( (0 \leq X < 1) \), and \( F(\bullet) \) is homogenous of degree one. If \( X = 0 \), firms have constant returns to scale. If \( X > 0 \), then because of this fixed cost, firms exhibit increasing returns.
Increasing returns to scale are not required for our analysis, but it is important to allow for this possibility. A sizable markup with constant returns would imply implausibly high pure profits as a share of national income. Similarly, intermediate goods are not crucial for the theory but are important when calibrating the model to actual economies.

Each firm has market power and charges a markup $\gamma$ over marginal cost:

$$P = \gamma MC$$

where $P$ is the price of the firm’s output. In this part of the paper, we take the markup as exogenously given. The markup is presumably determined by the elasticity of demand and perhaps other aspects of market structure, which for now we do not need to specify. To ensure the solution to the model is well-behaved, we assume $\gamma \lambda < 1$.

Each firm hires labor at rate $W$ and rents capital at rate $R$ in competitive factor markets. It also buys goods from other firms, which it uses as its intermediate goods, at price $P_i$. An extra unit of output can be produced with either additional capital or additional labor, plus the necessary intermediate goods. Therefore, cost minimization implies that

$$MC = \frac{R}{F_1} + \lambda P_i = \frac{W}{AF_2} + \lambda P_i$$

where $F_1$ and $F_2$ are the partial derivatives of $F(\cdot)$ with respect to the first and second arguments.

Because of the symmetry among the many identical firms, all firms’ prices are the same, so

$$P_i = P.$$ 

As a result, the share of revenue each firm spends on intermediate goods equals the parameter $\lambda$. Combining the last three equations, the price of output can be written as

$$P = \left( \frac{\gamma}{1 - \gamma \lambda} \right) \frac{R}{F_1} = \left( \frac{\gamma}{1 - \gamma \lambda} \right) \frac{W}{AF_2}.$$ 

The price of the firm’s output depends on its markup, the cost of capital and labor, its production function, and the intermediate-goods share.

To relate these firm-level variables to measures in the national income accounts, note that the firm’s real value added is its gross output $Q$ minus the value of intermediate goods $\lambda Q$:

$$Y = (1 - \lambda)Q = (1 - \lambda)F[K, A(L - X)].$$

---

$^1$ We use this formulation of fixed costs as overhead labor because it prevents fixed costs from vanishing in importance as the economy grows.
The marginal products of capital and labor in terms of value added are

\[ \frac{\partial Y}{\partial K} = (1 - \lambda)F_1 \]

\[ \frac{\partial Y}{\partial L} = (1 - \lambda)AF_2. \]

From these equations and the preceding equation for \( P \), we obtain

\[ P = \mu \frac{R}{\partial Y / \partial K} = \mu \frac{W}{\partial Y / \partial L} \]

where

\[ \mu = \frac{(1 - \lambda)\gamma}{1 - \lambda\gamma}. \]

Equivalently,

\[ \frac{R}{P} = \frac{\partial Y / \partial K}{\mu} \]

\[ \frac{W}{P} = \frac{\partial Y / \partial L}{\mu}. \]

This equation shows the wedge \( \mu \) between the marginal products of capital and labor and their real factor prices. This wedge plays a crucial role in what follows.

The variable \( \mu \) can be interpreted as the economy-wide markup: the ratio of the price to the social cost of producing a marginal unit of output. As Rotemberg and Woodford (1995) and Basu (2019) emphasize, when expenditure on intermediate goods is a substantial share of firm revenue, the economy-wide markup is much larger than the firm-level markup \( \gamma \). This difference arises because of double marginalization: the cost of producing intermediate goods is marked up repeatedly along the chain of production. Hereafter, we work mostly in terms of the economy-wide markup. The firm-level markup and the intermediate-goods share will return, however, when we calibrate the model.

The economy-wide markup determines the pure profits that accrue to firms. Each firm’s share of profits in its value added is:

\[ \Pi = \frac{PY - RK - WL}{PY}. \]

Using the above relationships and Euler’s theorem, we can express the profit share as
\[ \Pi = 1 - \frac{1}{\mu} - \frac{X}{L} \]

where \( \varphi = WL/(PY) \) is the labor share of value added. A firm’s profit share depends on the economy-wide markup \( \mu \) and the size of the fixed labor cost \( X \) relative to its labor input \( L \). Though there may be sizable pure profits in this economy, that need not be the case, depending on the magnitude of the fixed cost.

In this classical model, only relative prices matter. Therefore, without loss of generality, we can hereafter normalize \( P \), the price of output for these identical firms, to equal one.

### 1.2 The aggregate economy

Now let’s embed these firms with market power in an otherwise standard Solow growth model as presented, for example, in the first chapter of Romer (2019). The economy’s final output is the sum of the valued added of the \( N \) firms. The economy’s aggregate production function is

\[ Y = (1 - \lambda)F[K, A(L - NX)] \]

where \( Y, K, \) and \( L \) now represent aggregate output, capital, and labor. Because the firms are identical and the function \( F(\bullet) \) is homogenous of degree one, this aggregate production function follows from the individual firms’ production functions for value added.

Final output can be used for consumption or investment. Labor is inelastically supplied and grows at rate \( n \). Technological progress increases \( A \) at rate \( g \). The economy saves an exogenous fraction \( s \) of output, and capital depreciates at rate \( \delta \). To ensure balanced growth, we assume the number of firms grows with the population:

\[ N = \theta L \]

where \( \theta \) is a constant. Its reciprocal is the number of workers in each firm. As a result, the aggregate production function becomes

\[ Y = (1 - \lambda)F[K, AL(1 - \theta X)]. \]

As is standard, we define \( AL \) to be the effective labor force, which grows at rate \( n + g \). The economy’s production function per effective worker is

\[ y = f(k) \]

where \( y = Y/(AL) \), \( k = K/(AL) \), and \( f(k) = (1 - \lambda)F(k, 1 - \theta X) \). Throughout the analysis, we hold \( \lambda, \theta, \) and \( X \) constant, so these arguments in the production function can be suppressed. Except for the fixed cost and the correction for intermediate goods, the production function is standard. We assume that \( f'(k) > 0 \) and \( f''(k) < 0 \).
The equation for capital accumulation is also standard:

\[
\frac{dk}{dt} = sf(k) - (n + g + \delta)k.
\]

Therefore, the steady-state capital stock \( k^* \), defined by \( \frac{dk}{dt} = 0 \), is determined by

\[
sf(k^*) = (n + g + \delta)k^*.
\]

As is usual, output per effective worker is constant in steady state, and total output grows at rate \( n + g \). Notice that the markup \( \mu \) does not affect the steady-state capital stock and output. This result follows from the Solow model's assumption of an exogenously fixed saving rate. The markup does, however, affect the distribution of national income among capital, labor, and profit.

The markup also affects the real interest rate \( r \). In particular, recall that each unit of capital is paid the following:

\[
R = \frac{f'(k^*)}{\mu}
\]

where \( \mu \) is the economy-wide markup. The return on holding capital is \( R - \delta \). Arbitrage requires that this return equals the return on holding a financial asset \( r \). In other words, as is standard, the rental price of capital \( R \) equals the cost of capital \( r + \delta \). Therefore,

\[
r + \delta = \frac{f'(k^*)}{\mu}
\]

or

\[
r = \frac{f'(k^*)}{\mu} - \delta.
\]

This equation shows that, other things being equal, increased market power reduces the real interest rate.

In essence, market power reduces firms' demand for capital. Because the Solow model assumes a fixed saving rate, the supply of capital is inelastic, so the reduced demand is reflected in a lower cost of capital. Hence, the economy has a lower real interest rate. For example, consistent with our calibration below, suppose that the gross marginal product of capital \( f' \) is 13.3\% and the intermediate goods share \( \lambda \) is 0.45. Then an increase in the firm-level markup \( \gamma \) from 1.0 to 1.2 increases the economy-wide markup \( \mu \) from 1.0 to 1.43 and reduces the real interest rate by 4.0 percentage points.
1.3 Dynamic efficiency

Let’s now examine the optimal accumulation of capital. In this economy, steady-state consumption per person is

\[ c^* = f(k^*) - (n + g + \delta)k^*. \]

As a result, the consumption-maximizing steady-state capital stock, which Phelps (1961) dubbed the Golden Rule capital stock, is determined by:

\[ f'(k_{GR}^*) = n + g + \delta. \]

At the Golden Rule capital stock, the net marginal product of capital \( f' - \delta \) equals the growth rate of the economy \( n + g \). If the saving rate is larger than necessary to yield this capital stock, the economy is said to be dynamically inefficient because the economy can reduce saving and increase consumption at all points of time. The existence of market power does not alter these familiar results.

How can one judge whether an economy is dynamically efficient? The key is to compare the net marginal product of capital \( f' - \delta \) with the economy’s growth rate \( n + g \). Because of diminishing marginal product, if \( f' - \delta < n + g \), the economy has more capital than at the Golden Rule steady state, and if \( f' - \delta > n + g \), it has less.

The real interest rate might seem useful here. Under competition, the markup \( \mu \) equals one, and the real interest rate equals the net marginal product of capital \( f' - \delta \). That is one reason why comparisons of the real interest rate with the growth rate appear so often in discussions of economic growth. This approach is not robust, however. With market power, the real interest rate equals \( f'/\mu - \delta \). If \( f'/\mu > n + g > f'/(\mu - \delta) \), an observer who assumed the economy is competitive would measure the net marginal product of capital with the interest rate and erroneously conclude that the economy is dynamically inefficient.

Another approach to judging dynamic efficiency, rather than relying on the real interest rate, is to measure the marginal product of capital with capital income per unit of capital. (For example, see Abel et al., 1989, and Reis, 2021.\(^2\)) But this approach does not solve the problems introduced by market power. If capital income reported in the national accounts combines the net payments to capital \( RK - \delta K \) and pure profits \( \Pi Y \), then the measured net return per unit of capital, which we denote as \( m \), is

\[^2\text{In particular, Abel et al. (1989) recommend comparing the gross cash flow generated by capital [which in our notation equals } (m + \delta)K\text{] with gross investment [which equals } (n + g + \delta)K\text{ in the steady state. In our framework, this is equivalent to comparing net capital income per unit of capital } m \text{ with the growth rate } n + g \text{. Abel et al. show that this condition works under uncertainty and argue, therefore, that it is better than drawing inferences from the risk-free real interest rate. Throughout their analysis, they assume constant returns to scale and competitive markets.}]}\]
\[ m = \frac{RK - \delta K + \Pi Y}{K}. \]

This can be written as

\[ m = \frac{f'}{\mu} - \delta + \frac{\Pi y}{k}. \]

As a gauge of the true marginal value of capital \( f' - \delta \), this measure suffers from two problems. First, the markup reduces the payments to capital below its marginal product, which tends to decrease the measured return on capital. Second, pure profits appear to accrue to capital, which tends to increase the measured return on capital. These two biases push in opposite directions. Depending on the size of the markup and the size of the fixed costs, either bias could be larger.

We can, however, correct the measured return \( m \) for these biases. To do so, it is useful to define steady-state Tobin’s \( q \): the ratio of the market value of existing firms to the replacement cost of capital. If firms are infinitely lived, we can use the well-known Gordon growth formula to evaluate their market value:

\[ M = \frac{RK - \delta K - gK + \Pi Y}{(r - g)}. \]

The numerator is the firms’ dividends: their income \( RK + \Pi Y \) minus their expenditure on capital investment \( \delta K + gK \). (To maintain growth at rate \( g \), firms must both replace depreciating capital and increase their capital stocks at rate \( g \).) Dividends are discounted at \( r - g \) because they grow at rate \( g \) in the steady state. Tobin’s \( q \) is

\[ q = \frac{M}{K}. \]

Using these expressions and the fact that \( R = r + \delta \), Tobin’s \( q \) can be written as

\[ q = 1 + \frac{\Pi y}{(r - g)k}. \]

The excess of \( q \) over 1 equals the present value of pure profits from existing firms per unit of capital.

We can now derive an expression for the marginal product of capital as a function of variables that can be observed or estimated:

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3 If firms are not infinitely lived but instead die with a hazard rate of \( d \) (and are replaced by new firms to maintain the average firm size, \( 1/\theta \)), then the value of existing firms is determined by a discount rate of \( r + d - g \). All the expressions that follow remain the same, except \( g \) is replaced with \( g - d \). Firm death would also allow for the possibility of \( r < g \) without sending the market value of firms to infinity.
\[ f' = \mu \left( \frac{m - g}{q} + g + \delta \right). \]

This equation follows from the equations for \( q, m, \) and \( r \). In the competitive case in which \( \mu = 1 \) and \( q = 1 \), it simplifies to the familiar \( f' = m + \delta \). More generally, we can infer the true marginal product of capital from the measured net return \( m \) with two corrections. The multiplication by \( \mu \) accounts for the underpayment of capital that arises from market power. The division by \( q \) accounts for the pure profits that firms receive.\(^4\)

In short, evaluating dynamic efficiency in an economy with market power is tricky. An observer requires an estimate of the marginal product of capital. The real interest rate is an unreliable gauge because it is depressed by market power. Measuring the return on capital as firm income per unit of capital yields an estimate that can be either above or below the marginal product. Fortunately, our last equation provides a way to estimate the marginal product of capital, which we will apply shortly.

1.4 An approximate calibration to the U.S. economy

Applying a simple model like this one to actual economies with all their complexities and ambiguities is inherently perilous. For example, the concept of investment is clear in theory, but in practice it raises a variety of hard questions, such as whether the data capture the many types of intangible investment that are so important in the modern economy. Nonetheless, to get a sense of whether the effects of market power are sizable, we calibrate the model with some numbers that roughly describe the U.S. economy today.

We use the following figures as our baseline:

- The capital stock is three times annual output. That is, \( K/Y = 3 \).
- Depreciation is 15% of output. That is, \( \delta K = .15Y \).
- Measured gross capital income is 33% of gross output. That is, \( 1 - \varphi = 0.33 \).
- Income per person grows at 2% per year. That is, \( g = .02 \).
- The share of intermediate goods in firm revenue is 45%. That is, \( \lambda = 0.45 \).
- Firms’ market value exceeds the replacement cost of their capital by 75%. That is, \( q = 1.75 \).
- Firms mark up prices 20% over their marginal cost. That is, \( \gamma = 1.2 \).

\(^4\) Abel et al. (1989) note that \( q \) was averaging around 1 when they were writing, and they use that fact to justify their assumption of constant returns and competition. We can now see the flaw in that inference. A value of \( q \) equal to 1 could arise because fixed costs are large enough to eliminate pure profits, but that fact by itself does not render the measured capital return \( m \) a useful gauge for the net marginal product of capital \( f' - \delta \). If \( q \) equals 1, the measured capital return understates the net marginal product of capital if \( \mu > 1 \).
The first five numbers are based on the national income accounts and related BEA data. The sixth is based on estimates of Tobin’s $q$ as we write this paper in the year 2021. This number is hard to pin down because stock market valuations vary substantially over time, so we will examine the sensitivity of our conclusions to plausible variations in it.\(^5\)

The most challenging number in the model to calibrate is the markup $\gamma$. Basu (2019) offers a good overview of the literature on estimating markups, which has not reached a consensus. Many studies find that markups have increased over the last several decades, though there is also not a consensus whether that is the case and, if so, by how much. We use an estimate of 1.2 as our baseline value of $\gamma$, but our sensitivity analysis allows $\gamma$ to range from 1.05 to 1.4. Most of the markup estimates cited by Basu lie within this range.

Taking these seven numbers as our baseline and applying the above equations, we can infer the other key variables in the model. From the firm-level markup $\gamma$ and the intermediate-goods share $\lambda$, the economy-wide markup $\mu$ is computed using the equation derived earlier:

$$\mu = \frac{(1 - \lambda)\gamma}{1 - \lambda \gamma}.$$

We find that $\mu$ equals 1.43.

Our numbers for the depreciation share and the capital–output ratio imply that the depreciation rate $\delta$ is 5%. Using the definition of the labor share $\varphi$, we can then compute the measured capital return $m$ as follows:

$$m = \frac{(1 - \varphi)k}{y} - \delta.$$

From $m$, we then obtain an estimate of the marginal product of capital:

$$f' = \mu \left(\frac{m - g}{q} + g + \delta\right).$$

Next, we compute the implied real interest rate:

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\(^5\) Most of the data used for this calibration are available in FRED. For reference, here are their sometimes-awkward official names: For $K$, Capital Stock at Constant National Prices for United States. For $Y$, Real Gross Domestic Product. For $\delta K$, Consumption of Fixed Capital. For $Q$, Gross Output by Industry: All Industries. (Recall that $Y/Q = 1 - \lambda$.) For $q$, (Nonfinancial Corporate Business; Corporate Equities; Liability, Level/1000)/Nonfinancial Corporate Business; Net Worth, Level. For $\varphi$, see BEA Table 1.12. We calculated $\varphi$ as compensation of employees divided by total income (the sum of compensation of employees, corporate profits before tax, net interest, rental income of persons, and capital consumption adjustment); proprietors’ income is excluded because it combines labor income and capital income for those businesses.
\[ r = \frac{f'}{\mu} - \delta. \]

In the end, we find a real interest rate \( r \) of 4.3\%, a measured net return on capital \( m \) of 6.0\%, and a social return on capital \( f' - \delta \) of 8.3\%. This estimate of the social return on capital is well above the economy’s average growth rate of about 3\%, indicating that the U.S. economy is dynamically efficient.

The share of pure profits in gross national income is calculated as follows:

\[ \Pi = (q - 1)(r - g)(k/y). \]

With the above numbers, pure profits are 5.1\% of income. Given the intermediate-goods share of 0.45, pure profits are 2.8\% of firms’ gross revenue.

Finally, we can also infer the degree of increasing returns, as measured here by the percentage of workers in the economy that represent a fixed cost:

\[ \frac{NX}{L} = \frac{1 - \frac{1}{\mu} - \Pi}{\varphi}. \]

This equation follows from the earlier equation for an individual firm’s profit share \( \Pi \) and the fact that there are \( N \) firms. For our calibration, \( NX/L \) is 38\%. Such a large number is required to explain why a substantial economy-wide markup of 43\% generates relatively small pure profits of only 5.1\% of income. Given that the intermediate-goods share is 0.45 and that labor represents 67\% of value added, the cost of this overhead labor is 14\% of firms’ gross revenue.\(^6\)

It is worth noting the real interest rate in our calibration of 4.3\% is higher than the return on risk-free assets like Treasury bills, which in recent years have returned close to zero (and sometimes less) in real terms. No doubt, this discrepancy occurs because our model excludes uncertainty. As many authors have noted, other things being equal, uncertainty together with risk aversion reduces the risk-free rate. It might be best to interpret the real interest rate in our model not as the risk-free rate but as the average return on the financial assets that fund capital accumulation. From this perspective, the calibrated estimate of 4.3\% seems reasonable.

\(^6\) The results change only slightly if firms are not infinitely lived (see footnote 3). Suppose that firms live on average for 50 years, so \( \delta \) is 2\%. Then under our baseline, the real interest rate \( r \) is 3.4\%, the measured net return on capital \( m \) is 6.0\%, and the social return on capital \( f' - \delta \) is 7.1\%. Thus, in this case, market power depresses the real interest rate below the social return on capital by 3.7 percentage points. The profit share \( \Pi \) is 7.7\%, and the fixed cost \( NX/L \) is 34\%. (Note that it is not clear how best to calibrate the death rate of firms. One might be tempted to look at the rate at which corporate entities disappear. But the end of an entity resulting from a merger or acquisition would not count as a firm death because the profits of that entity still accrue to the existing base of shareholders.)
Table 1 shows the sensitivity of our results to plausible changes in μ and q. For comparison, the top line shows the competitive benchmark (γ = 1, q = 1). In this case, all three measures of the rate of return (r, m, and f′ − δ) give an estimate of 6%, and there are no pure profits. The second panel shows the effect of varying γ around our baseline calibration (γ = 1.2, q = 1.75), and the third panel shows the effect of varying q. Except in the case when the economy is close to competitive (γ = 1.05), the real interest rate r is well below the social return on capital f′ − δ. The size of this wedge is very sensitive to the markup γ but not very sensitive to Tobin’s q. The social return on capital always exceeds the economy’s average growth rate of about 3%, often by a large margin, indicating dynamic efficiency.

Table 2 shows what happens as Tobin’s q and the markup γ vary together. The first panel shows the social return on capital f′ − δ, and the second panel shows the wedge between the social return and the real interest rate. We exclude entries that would imply negative fixed costs (which occurs when γ = 1 and q > 1). Note that for every plausible combination of these parameters, the estimated social return is substantial. And the wedge is large for all economies with significant market power.

The bottom line is that, when firms have market power, the social return on capital necessary to gauge dynamic efficiency can be much larger than the return earned by savers in the economy. In our main calibration, the gap is about 4 percentage points. The next section explores what this conclusion implies for fiscal policy.
TABLE 1

*Sensitivity check*

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<th>$r$</th>
<th>$m$</th>
<th>$f' - \delta$</th>
<th>$\Pi$</th>
<th>$NX/L$</th>
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Note: All entries are in percentage points.
### TABLE 2

*Sensitivity check, continued*

The social return on capital: \( f' - \delta \).

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The wedge between the social return on capital and the real interest rate: \( f' - \delta - r \).

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<td>9.5</td>
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</table>

Note: All entries are in percentage points.
2. THE SUSTAINABILITY AND WELFARE EFFECTS OF GOVERNMENT DEBT

We now consider the effects of government debt in an economy with market power. For this purpose, we move from the Solow growth model to the Diamond (1965) overlapping-generations (OLG) model, a fully specified general equilibrium model with optimizing agents. The Diamond model is a classic tool for analyzing fiscal policy.

We focus on the questions of whether the government can run a Ponzi scheme and, if so, to what effect. In other words, can the government run up a debt and then roll over the principal and the accumulating interest forever? And if it can avoid future taxes to service or repay the debt, how does such a policy affect welfare? This longstanding issue has received renewed interest recently, with prominent economists suggesting that low interest rates make such a policy feasible and perhaps desirable. (See, for example, Blanchard, 2019.) As we will see, the answers to these questions are markedly different in economies with market power than they are under competition.

In competitive OLG models with certainty, the effects of fiscal policy are closely linked to whether the economy is dynamically efficient. In that setting, dynamic efficiency can be determined by comparing the growth rate of the economy with either the net marginal product of capital or the real interest rate (because the two are equal). Under dynamic efficiency, a Ponzi scheme is infeasible in the sense that the government must eventually raise future taxes to service any debt it issues. Moreover, because government debt crowds out capital, it reduces welfare for future generations. On the other hand, if the economy is dynamically inefficient, Ponzi schemes are feasible, and issuing debt can yield a Pareto improvement by reducing the overaccumulation of capital.

As in the Solow model, the key effect of market power is to introduce a wedge between the marginal product of capital and the real interest rate. This wedge raises the possibility that the net marginal product of capital exceeds the economy’s growth rate while the real interest rate is less than the growth rate. In that case, the low interest rate allows the government to run a perpetual Ponzi scheme by making debt-financed transfers to every generation. The scheme is feasible because the debt–income ratio, rather than exploding, converges to a constant level. In contrast to the competitive case, however, the feasibility of this Ponzi scheme does not imply that it increases welfare. With the marginal product of capital above the growth rate, the crowding out of capital reduces aggregate consumption, and the welfare loss from this effect can exceed the welfare gains that people get from the transfers.

Reis (2021) focuses on this scenario as well, though the wedge in his model comes from a credit-market imperfection rather than from market power.
2.1 Assumptions

Aside from the introduction of market power, our essential assumptions follow the Diamond model. (See, for example, the second chapter of Romer, 2019.) A cohort of individuals is born each period and lives for two periods. For simplicity, we assume that the cohort size is constant over time and normalize it to one. When young, an individual works, earns income, and receives a transfer from the government (which can be positive or negative). She divides her after-transfer income between consumption and saving, which is allocated to capital and bonds issued by the government. When old, the individual consumes the gross return on her savings.\(^8\)

To introduce market power, we assume that each young person is a yeoman farmer who produces and sells a differentiated good. A farmer produces her good using an endowment of one unit of her own labor along with capital that she rents from the old in a competitive factor market. For simplicity, we consider a special case of the production function introduced in the Solow model in which there are no intermediate goods and output is a Cobb-Douglas function of capital and effective labor \(A(L-X)\). With one unit of labor \(L\), the farmer produces

\[
Y = F[K,A(1-X)] = K^a[A(1-X)]^{1-a}, \quad 0 < a < 1,
\]

where \(Y\) is output, \(K\) is the capital rented by the young farmer from the old, and the technology parameter \(A\) grows at the rate \(g\) each period. We again use the notation of \(y = Y/A\) for output per unit of effective labor, \(k = K/A\) for capital per unit of effective labor, and \(y = f(k) = k^a(1-X)^{1-a}\) for the production function in terms of \(y\) and \(k\).

Everyone consumes the goods produced by all the farmers. We let \(C_i\) denote an individual’s consumption of good \(i\), with \(i\) distributed uniformly on \([0,1]\). Following Dixit and Stiglitz (1977), we assume that utility depends on a consumption aggregate \(C\):

\[
C = \left( \int_0^1 C_i \epsilon^{\epsilon-1} \, di \right)^{\epsilon/(\epsilon-1)}, \quad \epsilon > 1,
\]

where \(\epsilon\) is the elasticity of substitution between different goods. We further assume that an individual’s utility in each period is a logarithmic function of consumption \(C\). The lifetime welfare of a person born in period \(t\) is

\[
V_t = \ln(C_{y,t}) + \beta \ln(C_{o,t+1}), \quad \beta > 0,
\]

where \(C_y\) and \(C_o\) are the levels of consumption \(C\) when young and old.

A young individual can produce a unit of capital \(K\) from a unit of the good that she produces. Therefore, as in the original Diamond model, one unit of a consumption good is freely

---

\(^8\)If we instead assume that the transfer is received when an individual is old, the model is a bit more complex but the main results do not change.
tradable for one unit of capital. Capital produced in period $t$ is rented out and used in production in period $t+1$ and then fully depreciates.

2.2 Equilibrium in the goods and capital markets

In each period, a young farmer chooses a price for her good to maximize profit, which is the difference between revenue from selling the good and the cost of renting capital to produce the good. We can think of this profit as a mixture of wages for the farmer’s labor and economic profits derived from her market power.

As is usual in the Dixit-Stiglitz model, the elasticity of demand for each farmer’s product equals the substitution parameter $\epsilon$, and the profit-maximizing price equals the farmer’s marginal cost times a markup $\mu = \epsilon/(\epsilon - 1)$. (Here, because there are no intermediate goods, the economy-wide markup and the firm-level markup are the same.) In equilibrium, every farmer charges the same price, produces the same level of output, and rents the same amount of capital. We normalize the price of each farmer’s good to one.

Each person divides her consumption expenditure equally among all the differentiated goods: $C_i$ is the same for every good $i$. This fact and the definition of the consumption aggregate $C$ imply that $C$ equals the common $C_i$, and that the price of a unit of $C$ equals one, the price of an individual good.

With the prices of all goods equal to one, the condition for profit-maximization becomes $1 = \mu(MC)$. Marginal cost $MC$ is $R/MPK$, where $R$ is the rental price of capital and $MPK$ is the marginal product of capital $f'(k)$. Therefore, as in the extended Solow model, $R = MPK/\mu$. This condition defines a farmer’s demand for capital. The supply of capital is determined by the saving decision made by the currently old generation in the previous period, as described below. The supply and demand for capital determine the equilibrium value of $R$.

2.3 Intertemporal optimization

Given the period-by-period equilibrium in the goods and capital markets, an individual who is young in period $t$ chooses a level of saving $S_t$ to maximize welfare $V_t$. Saving is divided between capital accumulation and purchases of government debt. The gross return on capital equals the rental price of capital in the next period, $R_{t+1}$. In our non-stochastic model, the gross return on government debt must also equal $R_{t+1}$ because capital and debt are perfect substitutes for savers.

The individual’s budget constraint is

$$C_{y,t} = F[K_t, A_t(1-X)] - R_t K_t + \Omega_t - S_t$$

(1)
\[ C_{o,t+1} = R_{t+1}S_t \]

where \( \Omega \) is the transfer from the government to the young (or a tax if \( \Omega < 0 \)). The first equation says that consumption when young equals earnings (the farmer’s output minus the cost of renting capital) plus the transfer minus saving. The second equation says that consumption when old equals the return on saving.

Given the utility function \( V_t \), the first-order condition for a person’s optimal consumption path is

\[ \frac{C_{o,t+1}}{C_{y,t}} = \beta R_{t+1}. \]  \hspace{1cm} (2)

That is, the ratio of a person’s consumption when old and young equals the discount factor times the interest rate when old. This equation and the budget constraint define \( C_{y,t}, S_t \), and \( C_{o,t+1} \) in terms of the interest rate in periods \( t \) and \( t + 1 \), the government transfer, and the level of \( K_t \), which was determined in period \( t - 1 \).

### 2.4 Baseline fiscal policy

We assume that the economy initially has a level of debt \( B_t \) that grows at the same rate as the productivity parameter \( A_t \), and we let \( b \) equal the constant ratio \( B_t/A_t \). In steady state, this policy implies a constant ratio of government debt to output. Each period, the government sets the transfer \( \Omega_t \) at the level needed to keep \( b \) constant.\(^9\)

The transfers or taxes implied by this policy are determined as follows. The level of debt evolves according to

\[ B_t = R_t B_{t-1} + \Omega_t \]

where the first term on the right is the interest and principal on the debt rolled over from the previous period and the second is the current transfer. The government’s policy of holding \( b \) constant implies \( B_t = A_t b \) and \( B_{t-1} = A_{t-1} b = A_t b/(1 + g) \), which lead to

\[ \Omega_t = A_t \left( 1 - \frac{R_t}{1 + g} \right) b. \]

In a steady state with a constant \( R \), there is a constant level of transfers per unit of effective labor,

---

\(^9\) This policy is feasible as long as the government chooses a level of debt that is less than the total savings of the young, who must buy the debt in equilibrium. See the Appendix.
\[ \omega = \Omega / A : \]

\[ \omega = \left( \frac{g - r}{1 + g} \right) b \]

where \( r = R - 1 \) is the net return on capital and the interest rate on debt.

Notice that the sign of the steady-state \( \omega \) depends on \( g - r \). If \( r > g \), then \( \omega < 0 \): in steady state, the government levies a perpetual tax to service the debt. If \( r < g \), however, \( \omega > 0 \): the government provides a perpetual transfer that is never financed with taxes. In this case, the government runs a Ponzi scheme. This policy is feasible because the debt–income ratio tends to fall when the economy’s growth rate exceeds the interest rate.

The feasibility of a Ponzi scheme when \( r < g \) is a well-known result in OLG models. This economy, however, differs from those in previous work that assumes competitive markets. Because market power places a wedge between the marginal product of capital and the interest rate, a Ponzi scheme may be feasible even if the economy is dynamically efficient. As we will see, this difference has important implications for the welfare effects of government debt.

### 2.5 Debt and steady-state capital

Let’s now consider the evolution of the capital stock under the assumed fiscal policy. In each period, the saving of a young individual is divided between government bonds and investment in capital, which will be productive in the following period. This implies that \( K_{t+1} = S_t - B_t \). As described above, \( S_t \) is determined by an individual’s optimal saving problem. The levels of \( R_t \) and \( R_{t+1} \), which appear in the individual’s optimization problem, are determined in equilibrium by the condition \( R = f'(k)/\mu \), and the transfer \( \Omega_t \) is determined by the condition that the government maintains a constant level of \( b \).

In the Appendix, we use these results to derive an equation that defines \( K_{t+1} \) as a function of \( K_t \) and \( B_t \). We show that the economy has a unique stable steady state. Let \( k^* \) be the level of capital per unit of effective labor in this steady state. (There is also an unstable steady state with a lower level of capital.) The steady-state \( k^* \) depends on \( b \), the level of government debt per unit of effective labor: \( k^* = k^*(b) \).

While the condition that determines \( k^* \) is complex, our main results follow from a simple property of \( k^*(b) \):

\[ \frac{dk^*}{db} < 0. \]

That is, an increase in government debt reduces steady-state capital. This result is proved in the Appendix. It is a conventional crowding-out effect: an increase in debt diverts part of saving from
capital accumulation to purchases of government bonds. We will see that all the steady-state effects of debt on welfare, whether positive or negative, flow from this crowding out.

2.6 Steady-state welfare

To understand the effects of debt, it is useful to analyze the basic determinants of welfare as follows. Consider the lifetime welfare of a person born in period $t$:

$$V_t = \ln(C_{y,t}) + \beta \ln(C_{o,t+1}).$$

In steady state, $C_{y,t} = A_t c_y$ and $C_{o,t+1} = A_{t+1} c_o$, where $c_y$ and $c_o$ are the consumption of the young and old per unit of effective labor. We can rewrite $C_{o,t+1}$ as $A_t (1 + g) c_o$. Aggregate consumption per unit of effective labor is $c = c_y + c_o$. Letting $z$ equal $c_o/c_y$, the ratio of consumption of the old and young in a given period, we can write $c_y = c/(1 + z)$ and $c_o = cz/(1 + z)$. Combining these expressions, the steady-state welfare of a person born at $t$ is

$$V_t^* = \ln \left( \frac{cA_t}{1 + z} \right) + \beta \ln \left[ \frac{cA_t (1 + g) z}{1 + z} \right]. \quad (3)$$

The last equation shows that steady-state welfare can be expressed in terms of two variables: $c$, aggregate consumption per unit of effective labor, and $z$, the old-to-young consumption ratio. This result will prove to be helpful because both $c$ and $z$ are determined in simple ways by $k$, the steady-state level of capital per unit of effective labor.

Specifically, the steady-state level of $c$ is determined by:

$$c = y - i$$

$$= f(k) - (1 + g) k$$

where $y$ and $i$ are output and investment per unit of effective labor. The second line uses the fact that investment equals capital in the following period, which is $(1 + g)$ times capital in the current period. This equation is the same as the equation for consumption per unit of effective labor in the Solow model with the assumptions here that the depreciation rate is one and the population growth rate is zero.

An expression for $z = c_o/c_y$ follows from the first-order condition for an individual’s optimization, which in steady state is:

$$\frac{C_{o,t+1}}{C_{y,t}} = \beta R.$$ 

Using the facts that $C_{y,t} = A_t c_y$, $C_{o,t+1} = A_t (1 + g) c_o$, and $R = f'(k)/\mu$, we obtain:
\[ z = \frac{\beta f'(k)}{\mu(1 + g)}. \]  

The ratio \( z \) depends on the marginal product of capital \( f'(k) \) because the marginal product influences the real interest rate and hence an individual’s allocation of resources over time.

We now have a compact way to express steady-state welfare. Welfare is a function of aggregate consumption per unit of effective labor \( c \) and the ratio \( z = c_0/c_y \), as shown by equation (3). Both \( c \) and \( z \) are determined by capital per unit of effective labor \( k \), as shown by equations (4) and (5).

2.7 Welfare effects of an increase in debt

We study the welfare effects of the following shift in fiscal policy. The economy starts in the stable steady state described above. Then, in some period \( t_0 \), the government issues a small, one-time, positive transfer to the young generation. This transfer is in addition to the usual transfer \( \omega \) determined above (so the young receive a larger than usual transfer or pay a smaller than usual tax). The one-time transfer is financed through a small increase in the debt level \( b \). After period \( t_0 \), the government returns to the policy of maintaining a fixed level of \( b \), but now at the higher level established at \( t_0 \), which implies a fall in steady-state capital \( k^* \). We ask how this policy affects the welfare of different generations.

At least one generation benefits from the increase in \( b \): the generation that is young in period \( t_0 \) and therefore receives the one-time transfer. This transfer allows the young to increase both their current consumption and their saving for future consumption. There is also a more subtle benefit: because the increase in government debt that finances the transfer crowds out capital, the lucky generation receives a higher interest rate on their saving. There is no adverse effect on the generation; in particular, no crowding out of capital has yet occurred when they are young, so there is no loss of earnings when young.10

What are the effects of higher debt on the welfare of future generations? We address this question by examining the steady-state effects of an increase in \( b \). For the case in which a higher \( b \) increases steady-state welfare as well as benefiting the first generation, the policy yields a Pareto improvement. (The Appendix establishes that welfare increases throughout the transition to the new steady state.) On the other hand, if the policy reduces steady-state welfare, it obviously does

\[ V_t(K_t, R_t, \Omega_t, R_{t+1}). \]

For the generation born at \( t_0 \), the policy intervention in that period does not affect \( K_t \) or \( R_t \), because these variables were determined in the previous period. The intervention increases the transfer \( \Omega_t \) that the generation receives, and it increases \( R_{t+1} \) because it reduces \( K_{t+1} \). Both the higher \( \Omega_t \) and the higher \( R_{t+1} \) raise the individual’s welfare.

---

10 Formally, the welfare of an individual born in period \( t \) can be written as a function of the aggregate variables that determine her budget constraint: \( V_t(K_t, R_t, \Omega_t, R_{t+1}) \). For the generation born at \( t_0 \), the policy intervention in that period does not affect \( K_t \) or \( R_t \), because these variables were determined in the previous period. The intervention increases the transfer \( \Omega \) that the generation receives, and it increases \( R_{t+1} \) because it reduces \( K_{t+1} \). Both the higher \( \Omega \) and the higher \( R_{t+1} \) raise the individual’s welfare.
not yield a Pareto improvement. In this case, a higher debt level increases welfare in the short run but imposes a long-run burden.

Recall that steady-state welfare is determined by the variables $c$ and $z = c_0/c_y$, and that both these variables are determined by $k$, the steady-state level of capital per unit of effective labor. Therefore, the effect of the debt level $b$ on steady-state welfare operates through its effect on $k$. Differentiating the expression for welfare (3) yields:

$$\frac{dV_t^*}{db} = \frac{dV_t^*(c,z)}{dk} \frac{dk}{db}.$$ 

This expression can in turn be expanded:

$$\frac{dV_t^*}{db} = \left( \frac{dV_t^*}{dc} \frac{dc}{dk} + \frac{dV_t^*}{dz} \frac{dz}{dk} \right) \frac{dk}{db}.$$

$\begin{align*}
(+)(?) & \quad (?)
\end{align*}$

Aggregate effect Generational effect

Under this equation, we have written the signs of the various derivatives that determine $dV_t^*/db$ and labels to identify the two different effects of debt on steady-state welfare. To interpret this equation, recall first that $dk/db$ is negative: debt crowds out capital. This fact implies that the effect of debt on welfare has the opposite sign of the effect of greater capital accumulation on welfare. The effect of capital on welfare consists of two effects operating through aggregate consumption $c$ and the ratio $z = c_0/c_y$. In general, both these effects have ambiguous signs.

We call the effect operating through $c$ the **aggregate effect**. It is clear from an individual’s utility function that $\partial V_t^*/\partial c > 0$: holding constant the distribution of consumption between the old and young, higher aggregate consumption raises welfare. The sign of the aggregate effect is determined therefore by the sign of $dc/dk$. Differentiating equation (4) yields a familiar condition:

$$\frac{dc}{dk} > 0 \text{ iff } f'(k) - 1 > g$$

and

$$\frac{dc}{dk} < 0 \text{ iff } f'(k) - 1 < g.$$ 

This condition is the usual one for dynamic efficiency: more capital increases consumption if the net marginal product of capital [$f'(k)$ minus the depreciation rate of 1] exceeds the economy’s
growth rate. When this condition holds, the aggregate effect of capital on welfare is positive; because debt crowds out capital, the welfare effect of debt through this channel is negative. When the condition does not hold, debt raises welfare by reducing the overaccumulation of capital.

We call the effect operating through $z$, the ratio of consumption when old and young, the *generational effect*. It is easy to show using equation (5) that $dz/dk < 0$: more capital shifts consumption away from the old. The reason is that more capital reduces the real interest rate and thereby causes individuals to reduce consumption when old relative to consumption when young.

We can derive $\partial V^*_t/\partial z$ using the expression for welfare, equation (3). Taking the partial derivative with respect to $z$ leads after some algebra (see Appendix) to

$$\frac{\partial V^*_t}{\partial z} > 0 \quad \text{iff} \quad g > r$$

and

$$\frac{\partial V^*_t}{\partial z} < 0 \quad \text{iff} \quad g < r.$$ 

Recall that $dz/dk < 0$ and $dk/db < 0$, which together mean that higher debt increases $z$. (The mechanism is that debt crowds out capital, which raises the interest rate, which in turn causes people to shift consumption toward old age). Combining these results, we find that an increase in debt raises welfare through the generational channel if $g > r$ and reduces welfare through this channel if $g < r$.

What explains this result? An increase in $z$ for a given $c$ means that more aggregate consumption is shifted from the young to the old within each period. Over a person’s life cycle, she loses consumption when young and gains consumption when old. Because $c$ grows at rate $g$ in steady state, the extra consumption that a person receives when old is $1 + g$ times the consumption she loses when young. From the first-order condition for optimal saving, the marginal utility of consumption when old equals $1/(1 + r)$ times the marginal utility of consumption when young. Combining these facts, the ratio of the utility gain when old to the loss when young is $(1 + g)/(1 + r)$. The net effect on welfare is positive if this ratio exceeds one, that is, if $g > r$.

We can build intuition for these results by recognizing that in this model, as in many OLG models, the steady-state effects of government debt are the same as those of a pay-as-you-go social security system. Specifically, in our model, the steady state with an amount $b$ of debt per unit of effective labor is equivalent to a social security system that transfers an amount $\Psi_t = bA_t[(1 + r)/(1 + g)]$ from the young to the old in period $t$. We can see this from the cash flows between individuals and the government. In the steady state with debt, a young individual in period $t$ receives a transfer $\Omega_t = A_t b(g - r)/(1 + g)$ and gives the government $bA_t$ to purchase its debt. Adding these two terms yields a net receipt of $-\Psi_t$. An old individual at $t$ purchased $bA_{t-1} = bA_t/(1 + g)$ of debt in the previous period and receives a current return of
At \[(1 + r)/(1 + g)\] = Ψ. Thus, in the steady state, all the cash flows from debt amount to a transfer of Ψ from young to old.

To focus on the generational effect of government debt or social security, it is useful to consider an endowment economy (Samuelson, 1958; Weil, 2008). In this setting, aggregate consumption is fixed, and social security shifts consumption toward the old: the policy affects welfare only through z. In an endowment economy, social security raises welfare if \(g > r\), where \(r\) is the interest rate on consumption loans (which in equilibrium are in zero net supply). As in our model, \(g > r\) implies that the utility gain from shifting consumption toward the old exceeds the loss when young. An endowment economy, however, does not have the aggregate effect because fiscal policy cannot alter total consumption \(c\).

To summarize: We can now sign the previously ambiguous terms in the welfare decomposition:

\[
\frac{dV^*_t}{db} = \left( \frac{\partial V^*_t}{\partial c} \times \frac{dc}{dk} \right) + \left( \frac{\partial V^*_t}{\partial z} \times \frac{dz}{dk} \right) \left( \frac{dk}{db} \right).
\]

\[
\begin{array}{c|c|c|c}
\text{Aggregate effect} & \text{Generational effect} \\
(+) & (sign(f' - 1 - g)) & (sign(g - r)) & (-) \quad (-)
\end{array}
\]

According to this equation, debt raises steady-state welfare through the aggregate effect if the economy’s growth rate \(g\) exceeds the net marginal product of capital \(f'(k) - 1\), and it raises welfare through the generational effect if \(g\) exceeds the interest rate \(r\). Because of market power, we know that \(r < f'(k) - 1\), so our results imply the following about the total effect of debt:

1. If \(g \leq r\), then \(\frac{dV^*_t}{db} < 0\).
2. If \(r < g < f'(k) - 1\), then \(\frac{dV^*_t}{db}\) is ambiguous.
3. If \(f'(k) - 1 \leq g\), then \(\frac{dV^*_t}{db} > 0\).

These results arise because the two effects of debt have the same sign in the first and third cases and opposite signs in the second case.

Note that the competitive benchmark of \(\mu = 1\) implies \(r = f'(k) - 1\). In this circumstance, the middle case of opposing aggregate and generational effects disappears, and the two effects always have the same sign. This result explains why previous research on debt in OLG models has not emphasized the difference between these effects: this work has studied competitive models in
which the two effects always push in the same direction.

2.8 A model with free entry of producers

So far, we have assumed that each young person produces a unique type of output with her labor, so the population size fixes the number of producers. An alternative approach is to assume that there are many firms, each of which produces a type of output, and that, with free entry, a zero-profit condition pins down the number of firms. Here we consider the robustness of our results by sketching out a model with free entry.

As in our main model, a cohort of individuals of size one is born each period and has one unit of labor when young. Instead of assuming a young person produces a good using her own labor, we now assume that firms produce output with capital they rent from the old and labor they rent from the young in competitive factor markets. Let $N$ be the number of firms that produce differentiated goods, with $N$ determined endogenously: $N$ adjusts to drive profits to zero.

Each firm’s output is given by a Cobb-Douglas production function:

$$ Y = K^\alpha [A(L - X)]^{1-\alpha} $$

where $K$ and $L$ are the firm’s capital and labor inputs. It does not matter who owns the firms because their profits are zero in equilibrium. The aggregate production function is

$$ Y = F[K, A(L - NX)] = K^\alpha [A(L - NX)]^{1-\alpha} $$

where $Y$, $K$, and $L$ represent the aggregate versions of these variables.

An individual supplies one unit of labor when young, and she has the same utility function as before except that the consumption aggregate $C$ is given by

$$ C = \left( \int_0^N C_i^{\frac{\varepsilon-1}{\varepsilon-1}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1, $$

As in Dixit and Stiglitz (1977), an increase in $N$ creates greater product variety, which increases $C$ for a given level of expenditure on consumption goods.

The supply of capital is again determined by the saving of the young in the previous period net of their purchases of government bonds. Profit-maximization again implies that firms’ prices are a markup $\mu = \varepsilon/(\varepsilon - 1)$ times marginal cost and, therefore, that factor prices satisfy $R = MPK/\mu$ and $W = MPL/\mu$. 

Because profits are zero, total labor payments equal output minus payments to capital. Using the fact that the number of workers is normalized one, we can write each worker’s earnings as

\[ W = F[K, A(1 - NX)] - RK. \]

This expression is the same as the pre-transfer income of a young person in our basic model (except that \( X \) is replaced by \( NX \) in the production function, since \( N \) is no longer normalized to one).

Consider now the equilibrium number of firms \( N \). As in our extended Solow model, because \( F(\bullet) \) is homogenous of degree one, the share of profits in a firm’s output can be written as

\[ \Pi = 1 - \frac{1}{\mu} - \frac{\varphi X}{L} \]

where again \( \varphi \) is labor’s share of income and \( L \) is the firm’s employment. The aggregate quantity of labor is fixed at 1 (the size of the young cohort of workers), so each firm’s employment is \( L = 1/N \). This implies that the profit share is decreasing in \( N \): a higher \( N \) reduces employment per firm and therefore makes the overhead labor term \( X \) larger relative to employment. With free entry, \( N \) rises until the profit share \( \Pi \) is driven to zero.

Setting \( \Pi = 0 \) and \( L = 1/N \) leads to an expression for the number of firms in the free-entry equilibrium:

\[ N = \frac{\mu - 1}{\mu \varphi X}. \]

In addition, the Cobb-Douglas production function implies that labor’s share is

\[ \varphi = \frac{1 - \alpha}{\mu(1 - NX)}. \]

These last two equations implicitly determine equilibrium \( N \) in terms of the parameters \( \mu, X, \) and \( \alpha \). Because these parameters are all fixed, \( N \) is independent of the levels of capital and debt. Thus, in this model, an increase in the debt level \( b \) does not affect the number of firms \( N \).

This modification of the model does not alter our conclusions about the welfare effects of debt. Because the crowding out of capital by debt does not change the equilibrium value of \( N \), the dynamics of the capital stock are the same here as they are in our baseline model. As a result, the welfare effects of debt are simply the aggregate and generational effects that we have already derived for a fixed number of producers.
2.9 Results for more general preferences and technology

The overlapping-generations economy we have been analyzing includes Cobb-Douglas production and logarithmic utility. Here we briefly summarize results from our working paper (Ball and Mankiw, 2021) for more general production and utility functions. This generalization shows how the properties of tastes and technology affect the strength of the aggregate and generational effects of government debt and, hence, the overall welfare effect when these two effects conflict.

The key parameters are the elasticity of substitution between capital and labor in production ($\nu$) and the intertemporal elasticity of substitution in consumption ($\sigma$). With Cobb-Douglas production and log utility, both parameters equal one. In the general case, $\nu$ and $\sigma$ matter because they affect the size of the generational effect. In limiting cases in which $\nu$ becomes large or $\sigma$ becomes small, the effect of capital $k$ on $z$, the ratio of consumption when old and young, approaches zero, which means the generational effect disappears. This implies that only the aggregate effect is operative, so debt reduces welfare as long as $f'(k) - 1 > 0$ (even if $r < g$). In the opposite limiting cases in which $\nu$ becomes small or $\sigma$ becomes large, the opposite result holds: the generational effect dominates the aggregate effect, so debt raises welfare as long as $r < g$ (even if $f'(k) - 1 > 0$).

To understand these results, consider the cases of a large $\nu$ or a small $\sigma$, either of which eliminates the generational effect. Recall that $k$ affects $z$ because an increase in $k$ reduces $r$, which in turn reduces $z$ due to intertemporal substitution:

$$\frac{dz}{dk} = \frac{dz}{dr} \frac{dr}{dk}$$

As $\nu$, the substitutability of capital and labor, becomes large, $dz/dk$ approaches zero because $dr/dk$ approaches zero. In this case, the production function approaches linearity, which implies a constant $f'(k)$ and hence a constant $r$. The logic is different, however, when $\sigma$ approaches zero. In this case, consumption when old and young are not substitutable, and $dz/dk$ approaches zero because $dz/dr$ approaches zero. Intertemporal preferences over consumption approach Leontief, so the ratio $z$ does not respond to the return on saving. In both limiting cases, the result that $dz/dk$ approaches zero eliminates the generational effect of debt.

In our working paper, as in the main model here, the number of producers is assumed to be fixed. A potential extension would be a model with both general tastes and technology and entry guided by a zero-profit condition. It appears that, in contrast to our analysis above, capital accumulation could influence the number of producers and hence product variety, introducing a new channel from debt to welfare. That is possible because, with a general production function, capital accumulation can affect labor’s share of income $\varphi$, which enters the condition determining the equilibrium number of producers. The result that labor’s share is fixed regardless of capital accumulation and government debt is special to the Cobb-Douglas production function. With other production functions, debt might either increase or decrease the number of producers and product...
variety.

3. CONCLUSION

This paper has addressed a classic topic: the optimal accumulation of capital and the welfare effects of government debt in neoclassical growth models. Over the past several decades, the thinking of many economists has been shaped by the Solow (1956) growth model and the Diamond (1965) overlapping-generations model, which assume certainty and competitive markets. In these models, the real interest rate reflects the marginal product of capital. As a result, the low real interest rates experienced in recent years have tempted some economists to conclude that capital accumulation now has only small social value at the margin and, therefore, that the crowding out of capital by government debt is of little concern.

The innovation in this paper is to generalize these two canonical models to include firms with market power. In this environment, the real interest rate earned by savers is below the net marginal product of capital. For a plausible calibration of the extended Solow model, the wedge between them is about 4 percentage points. The calibration also suggests that the U.S. economy is dynamically efficient. When market power is introduced into the Diamond model, government Ponzi schemes can have different implications for welfare than they do under competition. Even if a perpetual rollover of government debt and accumulating interest is feasible, the crowding out of capital may still reduce steady-state welfare.

Previous work has established a close connection between the feasibility of government Ponzi schemes and the possibility of rational speculative bubbles. (See, for example, Tirole, 1985, O'Connell and Zeldes, 1988, and Martin and Ventura, 2018.) This equivalence suggests that in an overlapping-generations model with market power, rational bubbles are possible whenever government Ponzi schemes are. But unlike under competition, these bubbles may reduce welfare because they could occur even in dynamically efficient economies. By diverting saving away from capital accumulation, bubbles may depress aggregate consumption. This topic could be addressed in future research.

Much previous work on dynamic efficiency and Ponzi schemes has stressed the role of uncertainty about capital returns. Under uncertainty, the risk-free real interest rate is below the expected marginal product of capital, and attempted Ponzi schemes by the government are risky (Abel et al., 1989; Ball, Elmendorf, and Mankiw, 1998; Blanchard, 2019). This paper abstracts from these issues by considering models with certainty. A more realistic analysis of the U.S. economy would include both uncertainty and market power. We also leave that topic for future work.


APPENDIX

Determination of the Steady-State Capital Stock

Combining an individual’s first-order condition for optimal saving, equation (2), and the budget constraint, equation (1), leads to an expression for saving:

\[ S_t = \frac{\beta}{1 + \beta} (F[K_t, A_t(1 - X)] - R_tK_t + \Omega_t). \]

To study the dynamics of the capital stock, we substitute \( S_t = K_{t+1} + B_t \) into this equation. We also use \( R = f'(k)/\mu \) and divide by \( A_t \) to derive the relationship between \( k_{t+1} \) and \( k_t \):

\[ (1 + g)k_{t+1} + b = \frac{\beta}{1 + \beta} \left[ f(k_t) - \frac{f'(k_t)k_t}{\mu} + \left( 1 - \frac{f'(k_t)}{\mu(1 + g)} \right) b \right]. \]

In our case, \( f(k) = k^\alpha (1 - X)^{1-\alpha} \), so the solution for \( k_{t+1} \) is

\[
k_{t+1} = \frac{1}{1 + g} \left\{ \left( \frac{\beta}{1 + \beta} \right) \left( 1 - \frac{\alpha}{\mu} \right) k_t^\alpha (1 - X)^{1-\alpha}
+ b \left[ \left( \frac{\beta}{1 + \beta} \right) \left( 1 - \frac{\alpha k_t^{\alpha-1} (1 - X)^{1-\alpha}}{\mu(1 + g)} \right) - 1 \right] \right\}.
\]

We can write this relationship as \( k_{t+1} = H(k_t, b) \), where \( H(*) \) is the right side of the last equation. This function has some useful properties: for a given \( b \), \( H(*) \) is strictly concave in \( k_t \) and satisfies the Inada conditions; \( \partial H/\partial b < 0 \) (higher debt reduces \( k_{t+1} \) for a given \( k_t \)); and \( H(k_t, 0) \to 0 \) as \( k_t \to 0 \).

A steady-state level of capital \( k^* \) is defined by setting \( k_{t+1} = k_t = k^* \). That is, \( k^* = H(k^*, b) \).

Because of the strict concavity and other properties of \( H(*) \), for any level of \( b \) below some bound \( \hat{b} \), there are two steady-state \( k^* \)'s. (That is, as \( k_{t+1} \) increases with \( k_t \), it crosses the 45-degree line at two points.) At the lower of the two \( k^* \)'s, \( \partial H/\partial k_t > 1 \) (\( H \) crosses the 45-degree line from below), so the steady state is unstable. At the higher \( k^* \), \( 0 < \partial H/\partial k_{t+1} < 1 \), so that steady state is stable. There is no steady state for \( b > \hat{b} \), with \( \hat{b} \) defined by

\[
\max_k [H(k, \hat{b}) - k] = 0.
\]

A level of \( b \) greater than \( \hat{b} \) precludes the possibility of a steady state because the level of debt exceeds the total saving of the young, so the bond market cannot clear.
The Effect of Debt on Steady-State Capital

For \( b < \bar{b} \), we consider how a small increase in \( b \) affects the stable \( k^* \). Differentiating the equation that defines \( k^* \), we obtain

\[
\frac{dk^*}{db} = \frac{\partial H}{\partial b} \frac{1}{1 - \frac{\partial H}{\partial k}}.
\]

The numerator of this expression is negative, and the denominator is positive because \( \frac{\partial H}{\partial k} < 1 \) at the stable steady state. This implies that

\[
\frac{dk^*}{db} < 0.
\]

A higher level of debt reduces the steady-state capital stock.

Welfare During the Transition Between Steady States

The text shows that a small increase in debt in period \( t_0 \) increases the welfare of the generation that is young in \( t_0 \) and has an ambiguous effect on steady-state welfare. Here we show that, if steady-state welfare rises, then welfare also rises for generations born during the transition to the new steady state. In this case, the increase in debt yields a Pareto improvement.

Here we let \( b_0 \) denote the level of debt per unit of effective labor before period \( t_0 \), and \( \Delta > 0 \) denote the small increase in debt per unit of effective labor at \( t_0 \). Let \( k^* \) be the steady-state level of \( k \) before \( t_0 \) and \( k^{**} \) be the steady-state \( k \) after \( t_0 \), when debt is higher. As already established, \( k^{**} < k^* \). The level of \( k \) in period \( t_0 \) is \( k^* \) (the one-time transfer does not affect \( k \) in the period when it occurs), and \( k \) in period \( t_0 + 1 \) is determined by the optimal saving problem of the generation born at \( t_0 \) (who receive the one-time transfer). Subsequently, the evolution of \( k \) is determined by

\[
k_{t+1} = H(k_t, b_0 + \Delta).
\]

Using these facts, one can show that \( k_{t+1} < k_t \) for \( t \geq t_0 \) and \( k_t \to k^{**} \) as \( t \to \infty \). That is, \( k_t \) falls monotonically from \( k^* \) and converges to \( k^{**} \).

Consider the welfare of an individual born in some period \( t > t_0 \), during the transition to the new steady state. We write the individual’s utility as a function of the aggregate variables that determine her budget constraint: \( W(k_y, b, R_o) \), where \( k_y \) is the level of capital when the individual is young and \( R_o \) is the return on her saving when she is old. (Recall that \( k_y \) and \( b \) determine the individual’s earnings and transfer when young.) For an individual born at \( t > t_0 \), the effect of the debt increase on welfare is

\[
\Theta = W(k_t, b_0 + \Delta, R_{t+1}) - W(k^*, b_0, R^*),
\]
where $R^* = f'(k^*)/\mu$. The first $W(\bullet)$ is the individual’s actual welfare given the debt increase at $t$, and the second is what her welfare would have been if the increase had not occurred and the economy remained in its initial steady state.

Define $\bar{\Delta}$ as the solution to $k_t = H(k_t, b_0 + \bar{\Delta})$. If the debt were equal to $b_0 + \bar{\Delta}$, then $k_t$ would be the steady-state level of capital. Since $k^* > k_t > k^{**}$ during the transition and the steady-state $k$ is decreasing in $b$, we know that $0 < \bar{\Delta} < \Delta$. Using the concept of $\bar{\Delta}$, we can write the welfare effect of the debt increase as the sum of two terms:

$$\Theta = \Theta_1 + \Theta_2$$

where

$$\Theta_1 = W(k_t, b_0 + \bar{\Delta}, R_t) - W(k^*, b_0, R^*)$$

$$\Theta_2 = W(k_t, b_0 + \Delta, R_{t+1}) - W(k_t, b_0 + \bar{\Delta}, R_t).$$

We show that both $\Theta_1$ and $\Theta_2$ are positive for $t > t_0$, implying that the debt increase raises welfare during the transition, if the debt increase raises steady-state welfare.

The expression $\Theta_1$ is the effect on steady-state welfare of increasing the debt by $\bar{\Delta}$, which would imply that $k_t$ is the steady-state $k$ and $R_t$ is the steady-state $R$. Since we assume a debt increase of $\Delta$ raises welfare and $\Delta$ is small, an increase of $\bar{\Delta} < \Delta$ also increases welfare. Therefore, $\Theta_1 > 0$.

Turning to $\Theta_2$, note that $R_{t+1} > R_t$ because $k_{t+1} < k_t$ and $R$ is decreasing in $k$, and that $W$ is increasing in $R_o$. In addition, $b_0 + \Delta > b_0 + \bar{\Delta}$, and $W$ is increasing in $b$ (this follows from the fact that a higher $b$ raises the transfer $\Omega$ if $r < g$, which is necessary for a debt increase to raise steady-state welfare). These facts imply that the first $W(\bullet)$ term in the expression for $\Theta_2$ exceeds the second because of both a higher $b$ and a higher $R_o$. Therefore, $\Theta_2 > 0$.

**The Sign of the Generational Effect**

We show above that the sign of the generational effect of debt on welfare is the same as the sign of $\partial V^*_t/\partial z$, where $V^*_t(c, z)$ is given by equation (3). Differentiating equation (3) and simplifying leads to

$$\frac{\partial V^*_t}{\partial z} = \frac{\beta - z}{z(1 + z)}.$$
From the expression for $z$ in equation (5) and the fact that $R = f'(k)/\mu$, we have $z = \beta R/(1 + g)$. Substituting this fact for the $z$ in numerator, and recalling that $r = R - 1$, leads to

$$\frac{\partial V^*_t}{\partial z} = \frac{\beta (g - r)}{z(1 + z)(1 + g)}.$$ 

As claimed in the text, $\partial V^*_t/\partial z$ has the same sign as $g - r$.

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