Variation margins, fire sales, and information-constrained optimality*

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Abstract

In order to share risk, protection buyers trade derivatives with protection sellers. Protection sellers' actions affect the riskiness of their assets, which can create counterparty risk. Because these actions are unobservable, moral hazard limits risk sharing. To mitigate this problem, privately optimal derivative contracts involve variation margins. When margins are called, protection sellers must liquidate some assets, depressing asset prices. This tightens the incentive constraints of other protection sellers and reduces their ability to provide insurance. Despite this fire-sale externality, equilibrium is information-constrained efficient. Investors, who benefit from buying assets at fire-sale prices, optimally supply insurance against the risk of fire sales.

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1 Introduction

In the aftermath of the crisis regulators called for higher margins in derivative contracts, while cautioning that margins could lead to inefficient fire-sales and negative pecuniary externalities.[1] We offer a general equilibrium analysis of this issue, investigating whether privately optimal variation margins are socially optimal.[2]

Our model features three types of agents: protection buyers, protection sellers and investors. Protection sellers are, e.g., investment banks or specialised insurance companies. Protection buyers, e.g., commercial banks, share risk with protection sellers by trading derivative contracts. Protection sellers are endowed with risky assets, backing the contractual payments implied by their derivative positions. But they have limited liability, which creates counterparty risk, as protection sellers default when contractual payments exceed the value of their assets.[3] Protection sellers can take actions that reduce or increase the risk of their assets. The key friction in our model is that such actions are unobservable, i.e., there is moral hazard.[4] Finally, investors, e.g., sovereign or pension funds, can hold the risky assets of protection sellers, but are less efficient at doing so, e.g, because they are less able to manage or bear risk.

We also assume there is a publicly observable signal on the future value of protection buyers' risky assets. For example, if protection buyers are commercial banks hedging the risk on their real-estate loans, the signal can be provided by a retail-estate market index. The signal occurs after initial contracting, but before protection sellers take actions that reduce

[1]E.g., the Committee on the Global Financial System (2010) or the European Systemic Risk Board (2017).

[2]McDonald and Paulson (2015) offer an informative discussion of variation margins.

[3]Fleming and Sarkar (2014) discuss counterparty default in the Lehman bankruptcy.

[4]We consider two alternative specifications of the unobservable actions problem: one in which protection sellers must exert costly effort to reduce downside risk, as in Holmström and Tirole (1997), and one in which protection sellers can engage in risk-shifting, as in Jensen and Meckling (1976). In both cases, the undesirable action hurts protection buyers by increasing the probability of counterparty default. Both specifications yield the same economic insights.

or increase the risk of their assets.

Our first contribution is to characterize the information-constrained optimum, i.e., the second best. It is the set of consumptions and asset allocations, contingent on all publicly observable information, that maximize a weighted average of the three types of agents' expected utilities, subject to incentive, participation, and resource constraints.

The second best has two key characteristics. First, moral hazard prevents perfect risk sharing. Protection sellers' incentive constraints limit their ability to provide insurance, driving a wedge between their marginal rate of substitution and that of protection buyers. Second, following a bad signal (which in the example of real-estate loans could be a drop in the real-estate index), a transfer of assets from protection sellers to investors can be optimal. When the signal reveals bad news about the protection buyers' assets, it becomes more likely that some of the output of protection sellers' assets will be transferred to protection buyers. This increases protection sellers' incentives to take on risk. To alleviate this moral hazard problem and thus improve risk sharing, it is beneficial to reduce the amount of risky assets left under the control of protection sellers. But transferring assets to investors is costly, because they are less efficient at holding them. In the second best, the marginal cost of inefficient asset allocation is equal to the marginal benefit of better risk sharing.

Our second contribution is to analyze market equilibrium. Market participants can write and trade contracts contingent on all observable variables, so there is no *exogenous* market incompleteness. Yet, incentive constraints limit the amount of insurance that protection sellers can credibly promise, which generates *endogenous* market incompleteness. We show that privately optimal contracts between protection buyers and protection sellers involve derivatives (e.g., Credit Default Swaps insuring protection buyers against the default of their loans) and variation margins.[5] After a bad signal variation margins are called, requesting

[5]We build on the partial equilibrium analysis of risk-sharing under moral hazard in Biais, Heider and

protection sellers to deposit cash on their margin account. To do so, protection sellers must liquidate risky assets. Reducing the amount of risky assets under the control of protection sellers alleviates moral hazard, but implies selling risky assets to investors who are less efficient at holding them. Thus, variation margins trigger price drops, which can be interpreted as fire sales.[6]

Fire sales generate externalities, but do they create inefficiencies? One might be tempted to think so, because a protection seller's incentive constraint depend on market prices: What protection sellers can promise to pay without jeopardizing their incentives (cash on margin account, plus pledgeable income from risky assets under management) must exceed the liability from the derivative contract. When one protection seller liquidates assets, this contributes to depressing the price at which all protection sellers liquidate their assets, reducing cash proceeds deposited on margin accounts and tightening incentive constraints.[7]

Our third contribution, however, is to establish that market equilibrium, and the corresponding fire sales, are information-constrained efficient. The intuition is the following. Protection buyers are hurt by fire sales because they obtain less insurance from protection sellers, but investors benefit from the fire sale because they can buy underpriced assets.[8] Since protection buyers and investors have opposite exposures to fire sale risk, they benefit from insuring one another. In equilibrium they exploit this risk sharing opportunity until their marginal rates of substitution are equalised, just as in the second best. Still, and as in

Hoerova (2017). In a similar framework, Bolton and Oehmke (2015) analyse whether, under moral hazard, derivatives should be priviledged in bankruptcy.

[6]Shleifer and Vishny (1992) call a fire sale a forced sale of an asset at a dislocated price. Bian et al. (2018) document margin-induced fire sales triggering price drops in equity markets. Ellul et al. (2011) find that fire sales of downgraded corporate bonds by insurance companies trigger price declines. Merrill et al. (2012) document fire sales of residential mortgage-backed securities (RMBS). While in our model fire sales are due to moral hazard, in Dow and Han (2018) they reflect adverse selection.

[7]Chernenko and Sundaram (2018) find that mutual funds belonging to the same fund family try to mitigate fire-sale externalities on the other funds of the family by holding back on asset sales.

[8]Meier and Servaes (2018) show that firms buying distressed assets in fire sales earn excess returns.

the second best, the marginal rate of substitution of protection buyers differs from that of protection sellers because incentive constraints prevent perfect risk sharing.

Our paper is related to the literature on equilibrium inefficiency in incomplete markets, e.g., Stiglitz (1982), Greenwald and Stiglitz (1986), Geanakoplos and Polemarchakis (1986), Gromb and Vayanos (2002) and Lorenzoni (2008). In Gromb and Vayanos (2002) financially constrained arbitrageurs supply insurance to hedgers. When arbitrageurs suffer losses, their leverage constraints tighten, and they have to liquidate their positions. Markets are incomplete because hedgers cannot directly trade with one another. In Lorenzoni (2008) entrepreneurs borrow to fund investment projects. Financial constraints compel entrepreneurs to sell assets after negative shocks. Markets are incomplete because entrepreneurs cannot insure against these shocks. Both in Gromb and Vayanos (2002) and in Lorenzoni (2008), the combination of financial constraints and market incompleteness generates pecuniary externalities, leading to equilibrium constrained inefficiency.[9] In Davila and Korinek (2017), when markets are exogenously incomplete equilibrium is inefficient, but when there is no exogenous market incompleteness first-best risk sharing is achieved.[10] These results stand in contrast to ours. In contrast with Gromb and Vayanos (2002) and Lorenzoni (2008), we find that, in spite of financial constraints, equilibrium is second best. In contrast with Davila and Korinek (2017), we find that, even with a complete set of contingent contracts, equilibrium is not first best. These differences arise because in our model constraints do not reflect exogenous market incompleteness, but moral hazard, generating endogenous market incompleteness.

Our analysis is in line with Prescott and Townsend (1984) and Alvarez and Jermann (2000) who find that, with optimal contracting and complete markets, equilibrium is con-

[9] Analyses of pecuniary fire-sale externalities leading to inefficiencies in other settings are, for example, Caballero and Krishnamurthy (2003), Stein (2012), and He and Kondor (2016).

[10] See their application 1.

strained efficient.[11] The major difference between our setting and theirs concerns the timing of events. In our setting, unlike in theirs, after initial contracting but before agents' actions and resolution of uncertainty, a signal is observed and interim trades can take place. It is at this interim point that variation margins are called and fire sales can occur, affecting prices in the incentive constraint. There is no such mechanism, and in particular no price in the incentive constraint, in Prescott and Townsend (1984) and Alvarez and Jermann (2000). That equilibrium is constrained efficient although prices enter the incentive constraints is one of the distinguishing features of our analysis.

In contrast with our focus on risk sharing, Acharya and Viswanathan (2011), Brunnermeier and Pedersen (2009), and Kuong (2016) study lending. In Acharya and Viswanathan (2011), as in our paper, the incentive-compatibility constraint features pledgeable income that increases in the price of assets. Brunnermeier and Pedersen (2009) examine margin spirals resulting from fire sales. In Kuong (2016) the fire sale is self-fulfilling. When creditors expect low collateral liquidation value, they require high interest rate. In response, borrowers engage in risk taking. Eventually, this leads to a large number of defaults, triggering many collateral liquidations, leading to low collateral value. Unlike these papers, we conduct a normative analysis, characterising the second best and comparing it to market equilibrium.

Fostel and Geanakoplos (2015) offer an analysis of general equilibrium under collateral constraints. Margins have a different meaning in their analysis and ours. In their analysis, the agent buys one unit of a risky asset and deposits it as collateral, which, in itself entails no inefficiency. Against this collateral, the agent can borrow an amount equal to the minimum value of the risky asset at maturity.[12] The *initial* margin is the cash down-payment, equal to the difference between the price of the asset and the amount borrowed. The larger the

[11] See also Kehoe and Levine (1993), Kilenthong and Townsend (2014) and Kocherlakota (1998).

[12] Rampini, Sufi and Vishwanathan (2014) also consider collateral constraints. They show how these constraints affect risk management.

amount owed by the agent, the lower the *initial* margin.

In our analysis the agent is endowed with the risky asset and engages in a derivative transaction. If the derivative position moves against the agent, so that the agent is now in debt towards her counterparty, she must deposit a *variation* margin. To do so she sells some of the risky asset, which is inefficient because of fire sales. The larger the amount owed by the agent, the larger the *variation* margin.

Section 2 presents the model and its mapping to real markets and institutions. Section 3 analyses the first best, Section 4 the second best, and Section 5 market equilibrium. Section 6 analyses the case in which protection sellers can engage in risk shifting. Section 7 presents implications of our analysis.

2 Economic environment

2.1 Model

There are three dates: time 0, 1 and 2, one consumption good, and assets generating consumption good at time 2.

Agents and endowments: There is a unit-mass continuum of protection buyers, each with utility u, increasing and concave, and endowed at time 0 with one unit of a risky asset, paying $\tilde{\theta}$ units of the consumption good at time 2. There is also a unit-mass continuum of investors each with utility v, increasing and concave, and endowed at time 0 with e units of a safe asset, each paying 1 unit of consumption good at time 2. Finally, there is a unit-mass continuum of risk-neutral protection sellers, each endowed with one unit of a productive asset, paying $\tilde{R}$ units of consumption good at time 2.

Assets payoffs: The payoff of the protection buyers' asset at time 2, $\tilde{\theta}$, can be $\bar{\theta}$ with probability π, or $\underline{\theta}$ with probability $1-\pi$. While that payoff is exogenous, the payoff of a protection seller's asset depends on his action. The action is not observable, which coupled with limited liability, creates a moral-hazard problem.

The first specification of the moral-hazard problem we consider (in Sections 3, 4 and 5) is as in Holmström and Tirole (1997). Each unit of the protection sellers' asset yields R at time 2 for sure if protection sellers exert risk-management effort, at cost ψ per unit, at time 1. When consuming c_S units of the consumption good and exerting effort over a units of the asset, a protection seller obtains utility $c_S - a\psi$. If a protection seller does not exert risk-management effort, his asset's payoff is R with probability μ and 0 with probability $1-\mu$. We assume $R - \psi > \mu R$, so that protection seller's effort is efficient. Following Holmström and Tirole (1997), pledgeable income, i.e., the part of the physical return that can be promised without jeopardising incentives, is

$$\mathcal{P} \equiv R - \frac{\psi}{1-\mu} > 0. \tag{1}$$

To allow protection sellers that exert effort to fully insure protection buyers, we assume that

$$R > \pi(\bar{\theta} - \underline{\theta}). \tag{2}$$

For given effort decisions, $\tilde{\theta}$ and $\tilde{R}$ are independent. So there is no exogenous correlation between the valuations of the two assets. In spite of this simplifying assumption, we show below that moral hazard creates endogenous positive correlation between the two assets.

In Section 6, we consider an alternative specification of the unobservable action problem: risk-shifting, à la Jensen and Meckling (1976). In both specifications, the undesirable action (not exerting costly effort, or risk-shifting) hurts the protection buyer by increasing downside risk on the protection seller's assets and, correspondingly, counterparty risk for the protection buyer. We show that the same economic mechanisms are at play in the two specifications.

Signals: At time 1 an advanced signal $\tilde{s}$ about $\tilde{\theta}$ is publicly observed. When the final realisation of $\tilde{\theta}$ is $\bar{\theta}$, the signal is $\bar{s}$ with probability $\lambda > 1/2$ and $\underline{s}$ with probability $1-\lambda$. When the final realisation is $\underline{\theta}$, it is $\bar{s}$ with probability $1-\lambda$ and $\underline{s}$ with probability λ.

Asset transfers: Effort takes place at time 1, after the signal is publicly observed. Before effort is exerted (but after observing the public signal), a fraction α of the productive asset can be transferred from protection sellers to investors. This is costly because investors are less efficient than protection sellers at managing assets. Investors' per-unit cost of managing the asset is larger than that of protection sellers: $\psi_I(\alpha) > \psi, \forall\alpha$. When consuming c_I units of the consumption good and exerting effort over α units of asset, an investor obtains utility $v(c_I - \alpha\psi_I(\alpha))$. We assume $\psi_I' \geq 0$ and $\psi_I'' \geq 0$. Thus, investors' marginal cost, $\psi_I(\alpha) + \alpha\psi_I'(\alpha)$, is increasing. Yet, we assume it is efficient that investors exert effort even when holding all of the asset, i.e., $R - \psi_I(1) \geq \mu R$. We also maintain the following assumption:

$$\psi_I(1) + \psi_I'(1) > \frac{\psi}{1-\mu} > \psi_I(0). \tag{3}$$

As will be seen below, the right inequality in (3) allows for asset transfers, by making them not too inefficient when α is close to 0. The left inequality in (3) precludes full transfer of assets ($\alpha = 1$) because this would be too inefficient.[13]

Risk-sharing and moral hazard: Risk-averse protection buyers seek insurance against the risk $\tilde{\theta}$. They can turn to protection sellers or to investors, facing the following trade-

[13] In general, assets could also be transferred to protection buyers. For simplicity we rule this out by assuming protection buyers do not have the technology to manage the assets of protection sellers.

off. On the one hand, protection sellers are efficient providers of insurance, as they are risk-neutral, but they have a moral-hazard problem. On the other hand, investors are less efficient at managing the productive asset and at providing insurance since they are risk-averse.

When investor utility $v(0)$ is sufficiently low, there is no moral hazard problem with investors because threatening to give zero consumption after a low asset return is punishment enough to suppress the incentive to shirk. We hereafter assume this is the case, so we only need to impose incentive constraints for sellers, not for investors.

Sequence of events: Summarising, the sequence of events is as follows. At time 0, agents receive their endowments. At time 1, first the signal s is observed, then a fraction $\alpha(s)$ of the productive asset can be transferred from protection sellers to investors, and then holders of the productive asset decide whether to exert effort or not. At time 2, assets' payoffs are realised and publicly observed, and consumption takes place.

2.2 Mapping the model to real markets and institutions

Protection buyers can be commercial banks seeking to insure risk, while protection sellers can be investment banks or specialised firms providing this insurance. Prior to the 2007-09 crisis, banks frequently bought protection against credit-related losses on corporate loans and mortgages. Out of $533 billion (net notional amount) of credit default swaps sold by AIG at year-end 2007, 71% were categorized as such "Regulatory Capital" contracts (see Harrington, 2009).

Protection sellers take decisions that increase or reduce the riskiness of their assets. In the case of a loan portfolio, the effort reducing downside risk corresponds to the screening (due dilligence) and monitoring of loans. Lack of screening and monitoring leads to a higher risk of losses. For example, the report of the Financial Crisis Inquiry Commission (2011) states that "investors relied blindly on credit rating agencies as their arbiters of risk instead of doing their own due dilligence" and "... Merrill Lynch's top management realized that the company held \$55 billion in "super-senior" and supposedly "super-safe" mortgage-related securities that resulted in billions of dollars in losses".

The sellers' assets can also include financial securities and portfolio positions, whose risk is affected by the protection seller's management of collateral, liquidity, and exposures. For example, as part of its securities-lending activity, AIG received cash-collateral from its counterparties. Instead of holding this collateral in safe and liquid assets, such as Treasury bonds, AIG bought risky illiquid instruments, such as Residential Mortgages Backed Securities. As the value of these securities dropped, this resulted in approximately \$21 billion of losses for the company in 2008 (see McDonald and Paulson, 2015). More generally, before the crisis, several institutions failed to manage emerging risks, ignored warnings from their risk managers and increased their mortgage exposures even as the US housing market had begun to show signs of weakening (see Financial Crisis Inquiry Commission, 2011).

Moreover, consistent with our assumption that lack of proper risk-management effort increases downside risk, Ellul and Yerramilli (2013) document that banks with a weaker risk-management function at the onset of the financial crisis had higher tail risk and higher non-performing loans during the financial-crisis years.

3 First best

The first best obtains when protection-sellers' effort is observable. In that case, effort is always requested by the planner and exerted by protection sellers. Hence, the protection sellers' assets always yield R. The state variables, on which decisions and consumptions are contingent, are the realisations of the protection buyers' asset (θ) and the signal (s).

The social planner chooses the consumptions of protection buyers ($c_B(\theta, s)$), protection sellers ($c_S(\theta, s)$) and investors ($c_I(\theta, s)$), as well as the fraction of protection sellers' assets transferred to investors ($\alpha(s)$), to maximise the expected utility of protection buyers and investors (with respective Pareto weights ω_B and ω_I):

$$\omega_B E[u(c_B(\tilde{\theta}, \tilde{s}))] + \omega_I E[v(c_I(\tilde{\theta}, \tilde{s}) - \alpha(\tilde{s})\psi_I(\alpha))], \tag{4}$$

We assume that the social planner places no weight on protection sellers, i.e., $\omega_S = 0$. Correspondingly, when analysing the market equilibrium, we will assume zero bargaining power for the protection sellers.[14] The constraints are the participation constraint of protection buyers,

$$E[u(c_B(\tilde{\theta}, \tilde{s}))] \geq E[u(\tilde{\theta})], \tag{5}$$

the participation constraint of investors,

$$E[v(c_I(\tilde{\theta}, \tilde{s}) - \alpha(\tilde{s})\psi_I(\alpha))] \geq v(1), \tag{6}$$

the participation constraint of protection sellers,

$$E[c_S(\tilde{\theta}, \tilde{s}) - (1 - \alpha(\tilde{s}))\psi] \geq R - \psi, \tag{7}$$

[14] When effort is unobservable, protection sellers are agents, while protection buyers are principals. Our assumption that protection buyers have all the bargaining power is in line with the principal-agent literature, in which the principal makes a take-it-or-leave-it offer to the agent. Our assumption that $\omega_s = 0$ implies that, in Section 4, we only characterise a subset of the information-constrained Pareto frontier. This does not affect qualitatively our welfare analysis, as we show in Section 5 that the equilibrium implements a point on that subset of the information-constrained Pareto frontier.

the resource constraint in each state,

$$c_B(\theta, s) + c_I(\theta, s) + c_S(\theta, s) \leq \theta + 1 + R, \qquad \forall(\theta, s), \tag{8}$$

and the constraint that $\alpha(s)$ must be between 0 and 1. The participation constraints reflect the respective autarky payoffs of protection buyers ($E[u(\tilde{\theta})]$), investors ($v(1)$) and protection sellers ($R - \psi$). Our first proposition states the solution of the first-best problem.

Proposition 1 *In the first best, there is no transfer of the productive asset,* $\alpha(s) = 0, \forall s$, *and protection buyers and investors receive constant consumption,* $c_B(\theta, s) = c_B, c_I(\theta, s) = c_I$. *Their total consumption is*

$$c_B + c_I = E[\tilde{\theta}] + 1, \tag{9}$$

while protection sellers' consumption is

$$c_S(\theta, s) = \theta - E[\tilde{\theta}] + R, \quad \forall(\theta, s). \tag{10}$$

Note that assumption (2) ensures that protection seller's consumption is always positive in the first-best. Also, in the first best, the productive asset is held entirely by its most efficient holders, the protection sellers, i.e., $\alpha(s) = 0$. Moreover, the risk-neutral protection sellers fully insure the risk-averse agents, whose consumption is equal to the expected value of their endowment. Hence, marginal rates of substitution between consumption in different states are equal to one for all agents. Figure 1 illustrates the market implementation of the first best.

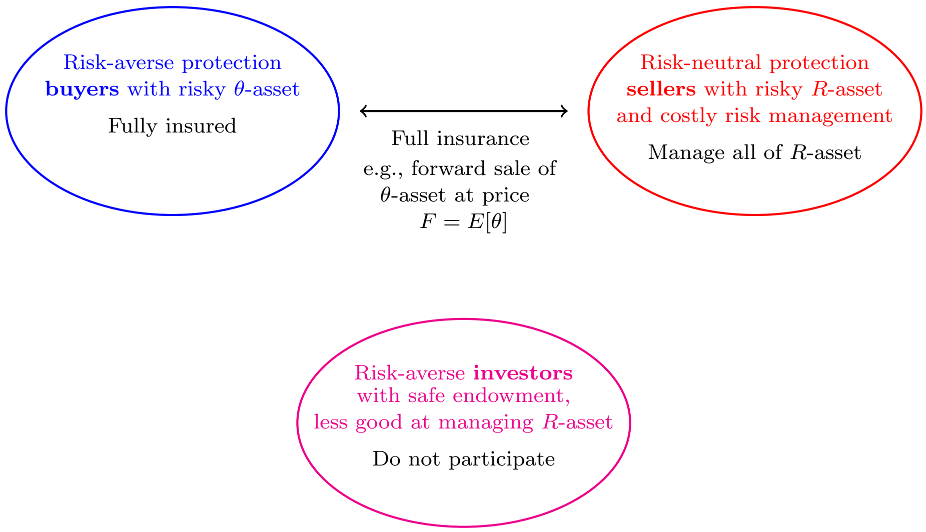

Figure 1: Market implementation of the first best

4 Second best

Now turn to the second best, when protection sellers' effort is unobservable. The social planner still chooses consumptions and asset transfers to maximise his objective function (4) under participation constraints (5), (6), and (7), and resource constraints (8). In addition, the planner is constrained by the protection sellers' incentive-compatibility condition. In what follows, we assume that the first-best allocation is not feasible in this more constrained, second-best problem:

$$\mathcal{P} < E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}]. \tag{11}$$

As will be clear below, if (11) did not hold, the pledgeable return would be sufficiently large for protection sellers to credibly promise full insurance in spite of moral-hazard.

4.1 Incentive compatibility

Protection sellers decide on effort after the realisation of the signal s. The incentive constraint is that they prefer effort to shirking:

$$E[c_S(\tilde{\theta}, s) - (1 - \alpha(s))\psi | s] \geq \mu E[c_S(\tilde{\theta}, s)|s] \quad \forall s.$$

The left-hand side is protection sellers' (on-equilibrium-path) expected consumption net of the cost of effort. The right-hand side is their (off-equilibrium-path) expected consumption when shirking. Under shirking, the asset yields R only with probability μ. In this case, protection sellers still receive the same expected consumption as under effort. But with probability $1 - \mu$, the asset returns zero. In order to relax the incentive-compatibility constraint, the planner optimally allocates zero consumption to limited-liability protection sellers in the (out-of-equilibrium) event of a zero asset return.

The incentive-compatibility constraint rewrites as

$$E[c_S(\tilde{\theta}, s)|s] \geq (1 - \alpha(s))\frac{\psi}{1 - \mu}. \tag{12}$$

The left-hand side of (12) is the expected consumption of protection sellers after observing the signal s. The right-hand side is the incentive-adjusted cost of managing the fraction of assets protection sellers still control after a possible transfer. Transferring assets from protection sellers to investors relaxes the incentive constraint.

4.2 Risk-sharing in the second best

Our first result is that protection buyers and investors are exposed only to the risk associated with the signal s. Correspondingly we write their respective consumptions as $(c_B(\bar{s}), c_B(\underline{s}))$

and $(c_I(\bar{s}), c_I(\underline{s}))$.

Lemma 1 *The consumption of protection buyers and investors depends only on the realisation of the signal s, but not on the realisation θ of protection buyers' assets.*

The incentive constraint (12) implies that only the expected consumption of protection sellers conditional on the signal matters for incentives. For a given $E[c_S(\tilde{\theta}, s)|s]$, the split between $c_S(\bar{\theta}, s)$ and $c_S(\underline{\theta}, s)$ does not affect the incentive constraint or the participation constraint of protection sellers. Hence, it is optimal to set $c_S(\bar{\theta}, s)$ and $c_S(\underline{\theta}, s)$ to fully insure protection buyers conditional on the realisation of s, by equalising their marginal utility in states $(\bar{\theta}, s)$ and $(\underline{\theta}, s)$. Similarly, it is optimal to equalise investors' marginal utility in these two states. The next lemmas further characterize the second-best outcome.

Lemma 2 *In the second best, the resource constraints as well as the participation constraint of protection sellers bind. Moreover, one, and only one, of the two incentive-compatibility conditions (after $\bar{s}$ or $\underline{s}$) binds.*

Lemma 3 *After a good signal, the incentive-compatibility constraint of protection sellers is slack and there is no asset transfer, $\alpha(\bar{s}) = 0$. After a bad signal, the incentive-compatibility constraint of protection sellers binds. Moreover, the consumption of protection buyers is larger after a good signal than after a bad signal, $c_B(\bar{s}) > c_B(\underline{s})$.*

After a good signal, protection sellers' expected consumption is large, which relaxes their incentive constraint. As a result, there is no need to transfer protection sellers' assets to less efficient investors ($\alpha(\bar{s}) = 0$).

After a bad signal, the opposite happens. Protection sellers' expected consumption is low, which tightens their incentive constraint. Because of the binding incentive constraint, protection buyers cannot be fully insured and remain exposed to signal risk ($c_B(\bar{s}) > c_B(\underline{s})$).

Combining Lemmas 1, 2, and 3, we obtain the next proposition, characterising the consumption of protection buyers and investors for a given level of asset transfers after a bad signal $\alpha(\underline{s})$.

Proposition 2 *After $\underline{s}$, the total consumption of protection buyers and investors is*

$$c_B(\underline{s}) + c_I(\underline{s}) = 1 + E[\tilde{\theta}|\underline{s}] + \alpha(\underline{s})R + (1 - \alpha(\underline{s}))\mathcal{P}, \tag{13}$$

while after $\bar{s}$ it is

$$c_B(\bar{s}) + c_I(\bar{s}) = 1 + E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}[\alpha(\underline{s})(R - \psi) + (1 - \alpha(\underline{s}))\mathcal{P}]. \tag{14}$$

Signal risk is perfectly shared between protection buyers and investors

$$\frac{v'(c_I(\underline{s}) - \alpha(\underline{s})\psi_I(\underline{s}))}{v'(c_I(\bar{s}))} = \frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}. \tag{15}$$

Consumption is split between protection buyers and investors according to their Pareto weights:

$$\frac{u'(c_B(s))}{v'(c_I(s) - \alpha(s)\psi_I(\alpha(s))} = \frac{\omega_I + \lambda_I}{\omega_B + \lambda_B}, \tag{16}$$

where λ_B and λ_I are the respective Lagrange multipliers of the participation constraint of protection buyers (5) *and investors* (6).

Since there is there is no incentive problem between protection buyers and investors, they share signal risk perfectly, as reflected in equation (15). While in the first best, the joint consumption of protection buyers and investors is given by the unconditional expectation of their joint endowment, reflecting full insurance, the second best involves the *conditional* expectation of their joint endowment, which is lower after bad news than after good news.

The joint consumption of protection buyers and investors after bad news (in equation (13)) also includes the protection sellers' pledgeable income, which without asset transfers is just $\mathcal{P}$, and with assets transfers is increased by $\alpha(\underline{s})(R-\mathcal{P})$.[15]

To complete the analysis of the second best, the next proposition characterizes asset transfers after bad news.

Proposition 3 *If*

$$\left.\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\right|_{\alpha(\underline{s})=0} - 1 > \frac{\psi_I(0)-\psi}{\frac{\psi}{1-\mu}-\psi_I(0)}, \tag{17}$$

then, in the second best, the asset transfer is interior, $\alpha(\underline{s}) \in (0,1)$, *and such that*

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))} - 1 = \frac{\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s}))-\psi}{\frac{\psi}{1-\mu}-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))}, \tag{18}$$

where $c_B(\underline{s})$ *and* $c_B(\bar{s})$ *are as given in Proposition 2. Otherwise,* $\alpha(\underline{s}) = 0$.

To interpret the left-hand sides of (17) and (18), recall that Lemma 3 implies there is signal risk: $c_B(\underline{s}) < c_B(\bar{s})$, which, in turn, implies the marginal rate of substitution, $\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}$, is larger than 1. The worse the insurance, the larger this marginal rate of substitution. Thus, the expression on the left-hand-sides of (17) and (18) reflects the extent to which protection buyers bear signal risk.

While the left-hand sides of (17) and (18) reflect the preferences of protection buyers, the right-hand sides of (17) and (18) reflect the technology and incentives of those who hold the productive asset. The denominator of the right-hand side of (17) and (18) is the wedge between the productive asset's marginal pledgeable income when it is held by investors $(R-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s}))))$ and its counterpart when it is held by protection sellers $(\mathcal{P})$.

[15]The joint consumption of protection buyers and investors after good news (in equation (14)) is then pinned down by the binding participation and incentive constraints of protection sellers.

Thus, it measures how much more income one can pledge by transferring the productive asset from protection sellers to investors. The numerator is the wedge between the investors' and the protection sellers' marginal cost of effort. Thus, the right-hand-sides of (17) and (18) can be interpreted as a marginal rate of transformation, reflecting the marginal cost of an increase in incentive-compatible insurance.

Condition (17) means that, at $\alpha(\underline{s}) = 0$, the marginal social benefit of a small asset transfer exceeds its marginal social cost. Since $\psi_I' \geq 0$ and $\psi_I'' \geq 0$, the marginal cost of effort for investors $\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s}))$ is increasing. So the right-hand-side of (18) is increasing, and takes its minimum value at $\alpha(\underline{s}) = 0$, as in the right-hand-side of (17). Furthermore, by (3), there exists threshold $\hat{\alpha} < 1$ at which the right-hand-side of (18) goes to infinity. Hence, under (17), there exists an interior value of $\alpha(\underline{s}) \in (0, \hat{\alpha})$ for which the marginal social benefit of additional insurance is equal to its marginal social cost. This pins down the optimal asset transfer after bad news in the second best.

Figure 2 illustrates the interaction of protection buyers, protections sellers, and investors in the second best when there are asset transfers after bad news.

5 Market equilibrium

We do not rule out trading of any contract based on the publicly observed variables (s, R, θ), but, as will be clear below, the following three markets are sufficient:

Market for insurance against the realization of $\tilde{\theta}$: Protection buyers and protection sellers participate in this market at time 0. In line with our simplifying assumption that the social planner places no weight on protection sellers, we assume protection buyers have all the bargaining power, so protection sellers are held to their reservation utility. Each pro-

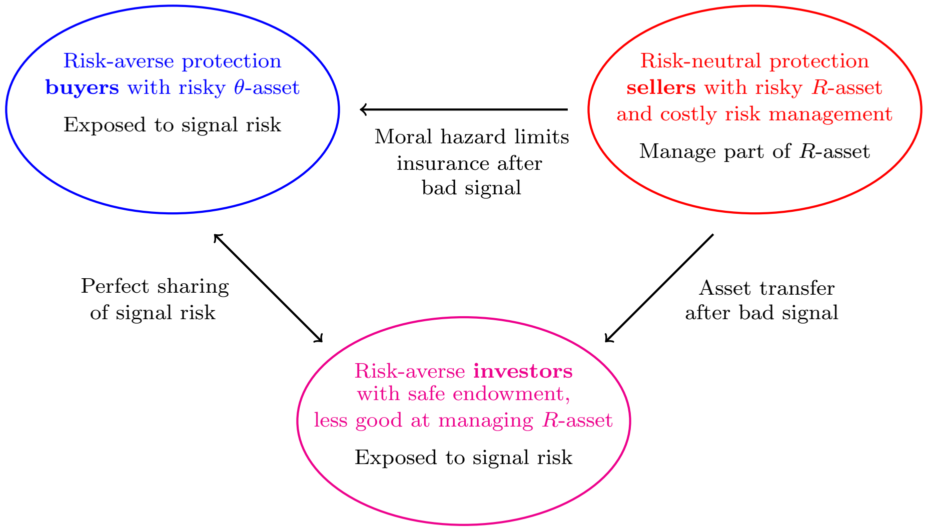

Figure 2: Second-best allocation with asset transfers after a bad signal

tection buyer is matched with one protection seller and makes an exclusive take-it-or-leave-it offer.[16] The offer includes time 2 transfers, $\tau(\theta, s, R)$ and variation margins (explained below). A positive transfer $\tau(\theta, s, R) > 0$ denotes a payment from the seller to the buyer and vice versa.

Market for protection sellers' assets: In the previous section we showed that the second best can involve asset transfers from protection sellers to investors after bad news. Therefore, we allow the protection buyer to request his counterparty to sell a fraction $\alpha_S \geq 0$ of his assets after a bad signal. The asset sale occurs at time 1, after the realisation of the signal, and before effort is exerted. The price is denoted by p. While supply α_S stems from protection sellers, demand α_I stems from investors. All market participants are competitive.

Requesting protection sellers to liquidate a fraction of their assets and deposit the proceeds on the margin account is a variation margin call. The procceeds $\alpha_S p$ still belong to protection

[16]For an analysis of issues arising with non-exclusive contracting, see Acharya and Bisin (2014).

sellers but are are ring-fenced from moral hazard and can be used to pay protection buyers at time 2.

Market for insurance against signal risk: Without moral hazard this market is not needed. Moral hazard, in contrast, limits the extent to which protection sellers can insure protection buyers, leaving the latter exposed to signal risk. This opens the scope for signal risk-sharing between protection buyers and investors. The corresponding market is held at time 0, and enables participants to exchange consumption after a bad signal against consumption after a good signal. Owners of one unit of the contract receive q units of consumption good after a bad signal and pay 1 unit of consumption good after a good signal. We denote protection buyers' demand by x_B and investors' supply by x_I.

Equilibrium: Equilibrium consists of transfers $\tau(\theta, s, R)$, prices (p, q), and trades (α_S, α_I) and (x_B, x_I), such that all participants behave optimally and markets clear: $\alpha_I = \alpha_S$ and $x_B = x_I$. To solve for equilibrium, we take the following steps. First, we derive incentive and participation constraints. Second, we characterize investors' trading decisions, α_I and x_I, for given prices. Third, we analyse contracting between protection buyers and protection sellers. Fourth, we impose market clearing.

5.1 Protection sellers' incentive and participation constraints

Incentive compatibility: As in the second best, the incentive-compatibility condition of protection sellers after a good signal is slack. After a bad signal ($\underline{s}$), the incentive-compatibility condition under which protection sellers exert effort is

$$
\begin{aligned}
(1-\alpha_S)(R-\psi)+\alpha_S p - E[\tau(\tilde{\theta},\tilde{s},R))|\underline{s}] \geq & \mu((1-\alpha_S)R+\alpha_S p - E[\tau(\tilde{\theta},\tilde{s},R))|\underline{s}]) \\
& + (1-\mu)E[\max[\alpha_S p - \tau(\tilde{\theta},\tilde{s},0),0]|\underline{s}]. \qquad (19)
\end{aligned}
$$

The left-hand side of (19) is the expected gain of protection sellers on the equilibrium path: They exert effort and obtain $R-\psi$ for each of the $1-\alpha_S$ units of the productive asset they keep. In addition, protection sellers own the proceeds from the asset sale, $\alpha_S p$, deposited in the margin account. Finally, the expected net payment by protection sellers to protection buyers is $E[\tau(\tilde{\theta},\tilde{s},R))|\underline{s}]$.

The right-hand side of (19) is the expected profit of protection sellers if they deviate and do not exert effort. In that case, with probability μ, protection sellers' productive assets still generate R, and their expected gain is the same as on the equilibrium path, except that the cost of effort, $(1-\alpha_S)\psi$, is not incurred. With probability $1-\mu$, the productive assets held by protection sellers generate no output. In that case, because of limited liability, protection sellers cannot pay more than $\alpha_S p$. Hence their gain is $\max[\alpha_S p - \tau(\theta,s,0),0]$.

It is optimal to set $\tau(\theta,s,0)=\alpha_S p$. This relaxes the incentive constraint by reducing the right-hand side of (19), and does not affect the rest of the analysis because transfers $\tau(\theta,s,0)$ only occur off the equilibrium path. Protection sellers' incentive constraint thus reduces to

$$
\alpha_S p + (1-\alpha_S)\mathcal{P} \geq E[\tau(\tilde{\theta},\tilde{s})|\underline{s}] \qquad (20)
$$

where we write $\tau(\tilde{\theta},\tilde{s},R)=\tau(\tilde{\theta},\tilde{s})$ to simplify the notation. The right-hand side of (20) is how much protection sellers expect to pay protection buyers, which can be interpreted as the implicit debt of protection sellers. The left-hand side of (20) is how much protection sellers can credibly pledge to pay, i.e., the sum of i) the pledgeable part ($\mathcal{P}$) of output (R) obtained on the $1 - \alpha_S$ units of productive assets kept by protection sellers, and ii) the cash proceeds from asset sales, deposited on margin account ($\alpha_S p$). Because of ii), prices enter the incentive constraint. As discussed below, this generates pecuniary externalities.

Participation constraint: A protection seller accepts the contract if it gives her equilibrium expected gains no smaller than her autarky payoff. This requires that the expected net payments to protections sellers be larger than their expected opportunity cost of asset sales, i.e.,

$$E[-\tau(\tilde{\theta}, \tilde{s})] \geq \Pr[\underline{s}]\alpha_S(R - \psi - p) \tag{21}$$

5.2 Investors' optimal trades

When selling x_I units of the insurance contract against signal risk and buying α_I units of protection sellers' assets, investors obtain time 2 consumption equal to $e + x_I$ after a good signal and $e - qx_I + \alpha_I(R - \psi_I(\alpha_I) - p)$ after a bad signal.[17] Their expected utility is

$$\Pr[\bar{s}]v(e + x_I) + \Pr[\underline{s}]v(e - qx_I + \alpha_I(R - \psi_I(\alpha_I) - p)). \tag{22}$$

Investors' supply of insurance against signal risk: At time 0, investors choose x_I to maximise (22). The first-order condition is[18]

$$\Pr[\bar{s}]v'(e + x_I) = \Pr[\underline{s}]qv'(e - qx_I + \alpha_I(R - \psi_I(\alpha_I) - p)), \tag{23}$$

[17] We assume the endowment e is large enough to settle all the transactions of an investor at $t = 1$.

[18] The second-order condition $\Pr[\bar{s}]v''(e + x_I) + q^2\Pr[\underline{s}]v''(e - qx_I + \alpha_I(R - \psi_I(\alpha_I) - p)) < 0$ holds by the concavity of the utility function.

which implies that x_I decreases in q.[19] Equation (23) rewrites as

$$q = \frac{\Pr[\bar{s}]}{\Pr[\underline{s}]} \frac{v'(e + x_I)}{v'(e - qx_I + \alpha_I(R - \psi_I(\alpha_I) - p))}, \tag{24}$$

which states that the price of insurance against signal risk is equal to the probability-weighted marginal rate of substitution between consumption after good and bad news.

Investors' demand for protection sellers' assets: At time 1, after a bad signal, investors choose α_I to maximise their utility $v(e - qx_I + \alpha_I(R - \psi_I(\alpha_I) - p))$. When $p \geq R - \psi_I(0)$, the price of the asset is so high that investors' demand is 0. Otherwise, their demand is pinned down by the first-order condition:

$$p = R - \left[\psi_I(\alpha_I) + \alpha_I \psi_I'(\alpha_I)\right], \tag{25}$$

which states that the price is equal to the marginal valuation of the investor for the asset. Because the marginal cost $\psi_I(\alpha_I) + \alpha_I \psi_I'(\alpha_I)$ is increasing, (25) implies that investors' demand for the asset is decreasing in p.[20]

5.3 Contracting between protection buyers and sellers

Protection buyers' privately-optimal contract specifies transfers $\tau(\theta, s)$ and an asset sale α_S. Moreover, protection buyers demand x_B units of the insurance against signal risk, receiving qx_B after bad news and paying x_B after good news. Correspondingly, the consumption of protection buyers at time 2 is $\theta + \tau(\theta, \bar{s}) - x_B$ after a good signal and $\theta + \tau(\theta, \underline{s}) + qx_B$ after

[19] The left-hand side of (23) is decreasing in x_I, while the right-hand side is increasing in x_I. Their intersection pins down the optimal supply of insurance by investors, x_I. Now, the right-hand side is increasing in q. Thus, an increase in q shifts up the right-hand side of (23), which leads to an intersection between the right- and the left-hand sides of (23) at a lower value of x_I.

[20] Increasing marginal cost also implies the second-order condition holds.

a bad signal. They choose x_B, $\tau(\theta, s)$ (for all $\theta \in \{\underline{\theta}, \bar{\theta}\}$ and $s \in \{\underline{s}, \bar{s}\}$), as well as $\alpha_S \in [0, 1]$ to maximise

$$Pr[\bar{s}]E[u(\tilde{\theta} + \tau(\tilde{\theta}, \tilde{s}) - x_B)|\bar{s}] + Pr[\underline{s}]E[u(\tilde{\theta} + \tau(\tilde{\theta}, \tilde{s}) + qx_B)|\underline{s}], \tag{26}$$

subject to the protection seller's incentive and participation constraints, (20) and (21). The next lemma states protection buyers' consumption as a function of α_S.

Lemma 4 *In equilibrium, in the privately-optimal contract between protection buyers and protection sellers, protection sellers' participation and incentive constraints bind. Moreover, protection buyers receive full insurance conditional on the signal, i.e., for a given realisation of the signal, their consumption does not depend on the realisation of* $\tilde{\theta}$*:*

$$c_B(\bar{\theta}, \bar{s}) = c_B(\underline{\theta}, \bar{s}) = E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}[\alpha_S(R - \psi) + (1 - \alpha_S)\mathcal{P}] - x_B, \tag{27}$$

$$c_B(\bar{\theta}, \underline{s}) = c_B(\underline{\theta}, \underline{s}) = E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1 - \alpha_S)\mathcal{P} + qx_B. \tag{28}$$

Lemma 4 is similar to Lemma 1. Both in the second best and in market equilibrium, protection buyers are fully insured conditional on the signal. The new element in the market equilibrium is that price p shows up in (28). Other things equal, raising the price relaxes the incentive constraint, which enables to provide more insurance to protection buyers.

The next lemma states what fraction of their assets protection sellers are required to sell after bad news.

Lemma 5 *When*

$$p \leq \mathcal{P} + (R - \psi - \mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}\mathcal{P} - x_B\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx_B)} \tag{29}$$

then $\alpha_S = 0$, otherwise α_S is strictly positive and such that

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + q x_B)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x_B\right)} = \frac{\lambda_1}{(p-\mathcal{P})Pr[\underline{s}]\lambda_S} + \frac{\frac{\psi}{1-\mu} - \psi}{p - \mathcal{P}}, \tag{30}$$

where λ_1 is the Lagrange multiplier of the constraint $\alpha_S \leq 1$.

5.4 Equilibrium

Equilibrium in the market for insurance against signal risk: Taking the first-order condition with respect to x_B in (26), we obtain

$$q = \frac{\Pr[\bar{s}]}{\Pr[\underline{s}]} \frac{u'(\theta + \tau(\theta, \bar{s}) - x_B)}{u'(\theta + \tau(\theta, \underline{s}) + q x_B)}. \tag{31}$$

Since the right-hand side of (31) is increasing in x_B, (31) implies x_B is increasing in q, while (24) implies that x_I decreases in q. At equilibrium, q is such that $x_B = x_I$. Combining (24) and (31), we obtain our next proposition:

Proposition 4 *Equilibrium in the market for insurance against signal risk involves price q^* and trading volume x^* such that*

$$q^* = \frac{Pr[\bar{s}]}{Pr[\underline{s}]} \frac{v'(e + x^*)}{v'(e - q^* x^* + \alpha_I(R - \psi_I(\alpha_I) - p))} = \frac{Pr[\bar{s}]}{Pr[\underline{s}]} \frac{u'(\theta + \tau(\theta, \bar{s}) - x^*)}{u'(\theta + \tau(\theta, \underline{s}) + q^* x^*)}. \tag{32}$$

Equation (32) states that in equilibrium, the marginal rates of substitution between consumption after a bad signal and after a good signal are equated among protection buyers and investors, i.e., they share signal risk optimally, as in the second best (see Proposition 2). Moreover, this marginal rate of substitution (weighted by the probabilities of a good and a bad signal) is equal to the price of insurance against signal risk.

As long as protection buyers are exposed to signal risk, we have

$$\frac{u'(\theta + \tau(\theta, \bar{s}) - x^*)}{u'(\theta + \tau(\theta, \underline{s}) + qx^*)} < 1,$$

which, combined with (32), implies that insurance against signal risk is not actuarially fair. Investors who supply protection buyers with insurance against a bad signal earn profits on average. This, in turn, means that investors' equilibrium supply is strictly positive. Thus, the market for insurance against signal risk is active, i.e., $x^* > 0$. Protection buyers (who cannot get full insurance from protection sellers because of moral hazard) demand a strictly positive amount of additional insurance from investors.

Equilibrium in the market for protection sellers' assets: Given equilibrium (q^*, x^*) in the market for insurance against signal risk, equilibrium in the market for protection sellers' assets is defined by a price p^* and a trading volume α^*, such that the market clears, i.e., $\alpha_S(p^*) = \alpha_I(p^*) = \alpha^*$. The next proposition characterises equilibrium in the market for protection sellers' assets.

Proposition 5 *If*

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^*)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}\mathcal{P} - x^*\right)} - 1 > \frac{\psi_I(0) - \psi}{\frac{\psi}{1-\mu} - \psi_I(0)}, \tag{33}$$

the equilibrium level of asset sales α^* is strictly positive and (α^*, p^*) is such that

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha^* p^* + (1-\alpha^*)\mathcal{P} + qx^*)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{Pr[\underline{s}]}{Pr[\bar{s}]}[\alpha^*(R-\psi) + (1-\alpha^*)\mathcal{P}] - x^*\right)} - 1 = \frac{\psi_I(\alpha^*) + \alpha^*\psi_I'(\alpha^*) - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha^*) + \alpha^*\psi_I'(\alpha^*))}, \tag{34}$$

and

$$p^* = R - [\psi_I(\alpha^*) + \alpha_I\psi_I'(\alpha^*)]. \tag{35}$$

Otherwise, if (33) does not hold, there are no asset sales in equilibrium, i.e., $\alpha^* = 0$.

Equation (34) is similar to (18). In both cases, the left-hand side captures the extent to which protection buyers bear signal risk and the right-hand side describes the trade-off when protection sellers sell their asset to investors. The numerator is the difference in the marginal cost of effort between investors and protection sellers. The denominator is the difference in pledgeable return between assets sold to investors (at price p^*, given by (35), with the proceeds deposited in the margin account) and assets kept under the management of protection sellers ($\mathcal{P}$).

5.5 Equilibrium constrained efficiency

Comparing Lemma 4 and Propositions 4 and 5 to Propositions 2 and 3, we obtain the following welfare theorem.

Proposition 6 *Market equilibrium is information-constrained Pareto efficient.*

It is striking that, in spite of moral hazard, the market equilibrium is second best, all the more so as the price in the incentive constraint (20) induces a pecuniary externality. This is because there are actually two countervailing pecuniary externalities. When one protection buyer demands larger margins, this depresses the price, which tightens the incentive constraint of all protection sellers. This negative pecuniary externality tends to increase the amount of signal risk protection buyers must bear. There is, however, a countervailing, stabilising effect. The decline in the price increases the profits of investors after a bad signal. Thus, while a negative signal is a negative shock for protection buyers, it is a positive shock for investors. This creates scope for risk-sharing gains from trade between investors and protection buyers. Investors and protection buyers fully exploit this risk-sharing opportunity until their marginal rates of substitution are equalised, exactly as in the second best.

In the Supplementary Appendix we offer complementary remarks on the above analysis. First, we show that if the market is exogenously incomplete, i.e., the market for insurance against signal risk is shut down ($x_B = x_I = 0$), equilibrium differs from the second best. Second, we argue that insurance against signal risk could be provided to protection sellers instead of protection buyers, without changing the equilibrium outcome.

6 Risk-shifting

So far, the protection seller had to exert unobservable risk-management effort to improve asset returns in the sense of first-order stochastic dominance as in Holmström and Tirole (1997). We now consider another type of unobservable action: "risk-shifting" à la Jensen and Meckling (1976) (we follow the set-up in Biais and Casamatta (1999)).

6.1 Model

We assume there is one risk-averse protection buyer seeking to hedge risk $\tilde{\theta}$ and one competitive protection seller, whose per-unit asset return $\tilde{R}$ can be high (H), medium ($M \in (0, H)$), or 0 (in which case the limited-liability protection seller defaults on any obligation). Without risk-shifting, the probability of the high return is $\kappa > 0$, and the probability of the medium return is $1 - \kappa$. In that case, the expected return of the asset is $E[\tilde{R}] = \kappa H + (1 - \kappa)M$. Risk-shifting increases the probability of 0 return to $b \in (0, 1)$ and the probability of H to $\kappa + a \in (\kappa, 1)$. Correspondingly, risk-shifting reduces the probability of M to $1 - (\kappa + a + b) \geq 0$.[21] We denote expectations when there is risk-shifting with $\hat{E}$. Risk-shifting generates second-order stochastic dominance: the expected return under

[21] If κ was equal to 0, the problem would be degenerate. Return H would only occur under risk-shifting. Thus, the optimal contract would give the protection seller zero pay-off after H. This would deter risk-shifting without introducing any distortion or agency cost.

risk-shifting $\hat{E}[\tilde{R}] = (\kappa + a)H + (1 - (\kappa + a + b))M$ is lower than $E[\tilde{R}]$, i.e., $(a + b)M > aH$.

As before, there is a continuum of risk-averse investors who can manage an amount α of the protection seller's risky asset at a cost $\psi_I(\alpha)$ per unit. Investors can mutualize the risk of the assets they hold so that the per-unit return each investor obtains is given by $E[\tilde{R}]$.[22] It will be useful to define

$$\hat{\mathcal{P}} \equiv \frac{(a + b)M - aH}{b} > 0, \tag{36}$$

which plays the same role as the pledgeable income $\mathcal{P}$ in the set-up with costly risk-management effort (1). It is the maximum return the protection seller can pledge to outsiders without engaging in risk-shifting. The pledgeable return is less than the expected return without risk-shifting, $\hat{\mathcal{P}} < E[\tilde{R}]$.

6.2 Second-best

As before, the social planner chooses the consumptions of the protection buyer, the protection seller, and investors, as well as the fraction of the protection seller's asset transferred to investors to maximize the expected utility of the protection buyers and investors (as before, we set the Pareto-weight of the protection seller to 0):

$$\omega_B E[u(c_B(\tilde{\theta}, \tilde{s}, \tilde{R}))] + \omega_I E[v(c_I(\tilde{\theta}, \tilde{s}, \tilde{R}) - \alpha(\tilde{s})\psi_I(\alpha))]$$

The incentive constraints are $E[c_S(\tilde{\theta}, s, \tilde{R})|s] \geq \hat{E}[c_S(\tilde{\theta}, s, \tilde{R})|s] \quad \forall s$. The protection seller avoids risk-shifting if, conditional on the signal, it gives her less consumption. Writing out

[22] They have no incentive to risk-shift because $E[\tilde{R}] > \hat{E}[\tilde{R}]$.

the expectations, the incentive constraints become

$$\frac{a+b}{a}\big(\Pr[\bar{\theta}|s]c_S(\bar{\theta},s,M)+\Pr[\underline{\theta}|s]c_S(\underline{\theta},s,M)\big)$$
$$\geq \big(\Pr[\bar{\theta}|s]c_S(\bar{\theta},s,H)+\Pr[\underline{\theta}|s]c_S(\underline{\theta},s,H)\big) \quad \forall s. \tag{37}$$

The protection seller must not expect to receive too much when the asset return is H as this would induce her to increase the probability of the H return by risk-shifting.

The participations constraints of the protection buyer, the protection seller, and the investors are, respectively:

$$E[u(c_B(\tilde{\theta},\tilde{s},\tilde{R}))] \geq E[u(\tilde{\theta})] \tag{38}$$

$$E[c_S(\tilde{\theta},\tilde{s},\tilde{R})] \geq E[\tilde{R}] \tag{39}$$

$$E[v(c_I(\tilde{\theta},\tilde{s},\tilde{R})-\alpha(\tilde{s})\psi_I(\alpha))] \geq v(1). \tag{40}$$

Finally, the resource constraints are

$$c_B(\theta,s,R)+c_I(\theta,s,R)+c_S(\theta,s,R) \leq \theta+1+(1-\alpha(s))R+\alpha(s)E[\tilde{R}] \quad \forall(\theta,s,R) \tag{41}$$

As in the case of costly effort, we assume the first-best allocation (i.e., full insurance, no asset transfer) is not feasible in the second-best problem:[23]

$$\hat{\mathcal{P}} < E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}], \tag{42}$$

[23] To see this, plug the first-best consumption of the protection seller, $c_S(\theta,s,R)=\theta-E[\tilde{\theta}]+R$ into the incentive constraint after the bad signal (we show below that the incentive constraint after the good signal never binds).

The following proposition characterizes the second-best allocation.[24]

Proposition 7 *All resource constraints* (41) *and the participation constraint of the protection seller* (39) *bind. The protection buyer's incentive constraint* (37) *is slack after a good signal, $\bar{s}$, and binds after a bad signal, $\underline{s}$. Conditional on the signal s and the protection seller's asset return R, the protection buyer and investors are not exposed to θ-risk, $c_B(\bar{\theta}, s, R) = c_B(\underline{\theta}, s, R)$ and $c_I(\bar{\theta}, s, R) = c_I(\underline{\theta}, s, R)$. The consumption levels of the protection buyer and investors can be ranked as follows:*

$$c_B(\underline{s}, M) < c_B(\bar{s}, M) = c_B(\bar{s}, H) < c_B(\underline{s}, H)$$

$$c_I(\underline{s}, M) < c_I(\bar{s}, M) = c_I(\bar{s}, H) < c_I(\underline{s}, H)$$

The protection buyer and investors perfectly share signal risk and R-risk:

$$\frac{u'(s, R)}{u'(s', R')} = \frac{v'(s, R)}{v'(s', R')} \quad \forall \quad s, s' \in \{\underline{s}, \bar{s}\} \text{ and } R, R' \in \{H, M\} \tag{43}$$

There is no asset transfer after a good signal, $\alpha(\bar{s}) = 0$. A necessary condition for an asset transfer after a bad signal is $E[\tilde{R}] - \hat{\mathcal{P}} \geq \psi_I(0)$. A sufficient condition for an asset transfer after a bad signal, $\alpha(\underline{s}) > 0$, is:

$$\left.\frac{u'(c_B(\underline{s}, M))}{u'(c_B(\bar{s}, M))}\right|_{\alpha(\underline{s})=0} - 1 > \frac{a+b}{b(1-\kappa)} \frac{\psi_I(0)}{E[\tilde{R}] - \psi_I(0) - \hat{\mathcal{P}}}$$

An interior asset transfer after a bad signal $\alpha(\underline{s}) \in (0, 1)$ is characterized by

$$\frac{u'(c_B(\underline{s}, M))}{u'(c_B(\bar{s}, M))} - 1 = \frac{a+b}{b(1-\kappa)} \frac{\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s}))}{E[\tilde{R}] - (\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s}))) - \hat{\mathcal{P}}}. \tag{44}$$

[24] See the Supplementary Appendix for the proof.

The results in Proposition 7 mirror those in Section 4. The protection buyer and investors are exposed to signal risk and perfectly share it (equation (43)). The optimal asset transfer is such that marginal social benefit equals the marginal social cost (equation (44)). The marginal social benefit of $\alpha(\underline{s})$ is the extent of signal-risk sharing the asset transfer achieves for the protection buyer. The extent of risk-sharing is given by the wedge between the marginal rate of substitution, $\frac{u'(c_B(\underline{s},M))}{u'(c_B(\bar{s},M))}$, and one (what the marginal rate would be equal to under perfect risk-sharing). The marginal social cost of $\alpha(\underline{s})$ is given by the marginal cost of investors managing the seller's asset, $\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s}))$, relative to the increase in the pledgeable return, $E[\tilde{R}] - (\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s}))) - \hat{\mathcal{P}}$, when the asset is transferred from the seller to investors.

A difference between risk-shifting and costly risk-management effort is that with risk-shifting, the risk of the protection seller's asset is present on the equilibrium path. After bad news, in order to relax incentive constraints, the planner allocates more (resp. less) consumption to the protection buyer (resp. seller) when the asset return is H than when it is M. The corresponding risk is perfectly shared by protection buyer and investors.[25]

6.3 Market equilibrium

The structure of the market equilibrium problem with risk-shifting is similar to that with costly risk-management effort. There is a market for insurance against $\tilde{\theta}$-risk, a market for the protection seller's asset, and a market for insurance against signal risk. Because the risk of the seller's asset is present on the equilibrium path in the risk-shifting set-up, we also have a market for contracts to share $\tilde{R}$-risk when the bad signal occurs (only then does the protection seller's incentive constraint bind and this risk occurs). The owner of one unit of

[25]The extra term $\frac{a+b}{b(1-\kappa)}$ appears because of this risk. It matters whether we write (44) in terms of $c_B(s,M)$ or $c_B(s,H)$ and the extra term makes the necessary adjustment.

this contract receives z units of consumption if the asset return is H and pays one unit of consumption if the asset return is M. We denote the protection buyer's demand by y_B and the investors' supply by y_I.

Protection seller's incentive and participation constraints: As before, we can just focus on the protection seller's incentive constraint after a bad signal. Also as before, it is optimal not to leave the protection seller with a positive pay-off when she defaults (which happens with probability b): $\tau(\theta, \underline{s}, 0) = \alpha_S p$ for all θ. The incentive constraint then is

$$\begin{aligned}
&\alpha_S p + (1-\alpha_S)E[\tilde{R}] - \kappa E[\tau(\tilde{\theta}, \underline{s}, H)|\underline{s}] - (1-\kappa)E[\tau(\tilde{\theta}, \underline{s}, M)|\underline{s}] \\
&\geq (1-b)\alpha_S p + (1-\alpha_S)\hat{E}[\tilde{R}] - (\kappa+a)E[\tau(\tilde{\theta}, \underline{s}, H)|\underline{s}] - (1-(\kappa+a+b))E[\tau(\tilde{\theta}, \underline{s}, M)|\underline{s}],
\end{aligned}$$

which rewrites as

$$\alpha_S p + (1-\alpha_S)\hat{\mathcal{P}} \geq \left(\frac{a+b}{b}\right) E[\tau(\tilde{\theta}, \underline{s}, M)|\underline{s}] - \frac{a}{b} E[\tau(\tilde{\theta}, \underline{s}, H)|\underline{s}], \tag{45}$$

This incentive constraint (45) mirrors the incentive constraint in the costly-effort case (20). The left-hand side is how much the protection seller can pledge to pay, i.e., the sum of the proceeds from the asset sale and the pledgeable return of the proportion of her asset she still manages. The right-hand side is the expected transfer to the protection buyer. Note that a higher transfer from the protection seller to the protection buyer when the asset return is H actually relaxes the incentive constraint. This transfer makes risk-shifting, which increases the probability of the high asset return, less attractive.

The protection seller's participation constraint is

$$Pr[\bar{s}]\left(E[\tilde{R}] - E[\tau(\theta, \bar{s}, \tilde{R})\right) + Pr[\underline{s}]\left(\alpha_s p + (1-\alpha_S)E[\tilde{R}] - E[\tau(\theta, \underline{s}, \tilde{R})]\right) \geq E[\tilde{R}],$$

which rewrites as,

$$E[-\tau(\tilde{\theta}, \underline{s}, \tilde{R})] \geq \Pr[\underline{s}]\alpha_S(E[\tilde{R}] - p). \tag{46}$$

The participation constraint (46) mirrors the participation constraint in the costly-effort case (21). Expected net payments to the protection seller must be larger than the expected opportunity cost of selling a fraction α_S of her assets at price p after a bad signal.

Investors' optimal trades: At time $t = 0$ investors trade contracts with the protection buyer:

$$\begin{aligned}\max_{x_I, y_I} \ \Pr[\bar{s}]v(e + x_I) + \Pr[\underline{s}]\Big[&\kappa v(e - qx_I + y_I + \alpha_I(E[\tilde{R}] - \psi_I(\alpha) - p)) \\ &+(1-\kappa)v(e - qx_I - zy_I + \alpha_I(E[\tilde{R}] - \psi_I(\alpha) - p))\Big].\end{aligned} \tag{47}$$

This is similar to equation (22) except that there is $\tilde{R}$-risk when the signal is bad, and we have the terms y_I and z for the contract to share this risk.

After a bad signal at $t = 1$, investors choose α_I to maximise the expression in square brackets in (47). The derivative of this expression with respect to α_I yields the following demand when $\alpha_I > 0$:[26]

$$p = E[\tilde{R}] - [\psi_I(\alpha_I) + \alpha_I \psi_I'(\alpha_I)], \tag{48}$$

which is downward sloping and mirrors the demand function (25) in the costly-effort set-up.

Contracting between protection buyer and seller: The protection buyer contract with the protection seller specifies transfers $\tau(\theta, s, R)$ and an asset sale after bad news α_S to maximise her expected utility $E[u(\tilde{\theta}, \tilde{s}, \tilde{R})]$ subject to the incentive (45) and participation (46) constraints of the protection seller. In the Supplementary Appendix we prove the following

[26] If $p > E[\tilde{R}] - \psi_I(0)$ then the demand is zero.

results:

Proposition 8 *The participation and incentive constraints of the protection seller bind. Conditional on the signal and the return of the protection seller's asset, the transfers fully insure the protection buyer against the risk $\tilde{\theta}$ of his asset. After bad news, the protection buyer receives a larger transfer for each realization of θ if the protection seller's asset return is H than when it is M. After good news, these transfers are equal. If*

$$p \leq \frac{E[\tilde{R}] + \hat{\mathcal{P}}\left(\frac{u'(\underline{s},M)}{u'(\bar{s},M)} - 1\right)\frac{b(1-\kappa)}{a+b}}{1 + \left(\frac{u'(\underline{s},M)}{u'(\bar{s},M)} - 1\right)\frac{b(1-\kappa)}{a+b}},$$

then $\alpha_S = 0$. Otherwise α_S is strictly positive, increasing in p, and satisfies[27]

$$\frac{u'(\underline{s},M)}{u'(\bar{s},M)} - 1 = \frac{a+b}{b(1-\kappa)}\frac{E[\tilde{R}] - p}{p - \hat{\mathcal{P}}}. \tag{49}$$

These results mirror those obtained for costly unobservable effort (Lemmas 4 and 5). Given the realisation of the signal s and the protection seller's asset return R, the transfers insure the protection buyer against the remaining risk of his own asset. The new element is the presence of the risk of the protection seller's asset, i.e., whether the return is H or M. To counter the protection seller's incentive to shift risk, the protection buyer gets a higher transfer after return H than after M.

We can therefore write the protection buyer's expected utility as

$$\max_{x_I, y_I} \Pr[\bar{s}]u(\theta + \tau(\theta, \bar{s}, R) - x_B) + \Pr[\underline{s}]\Big[\kappa u(\theta + \tau(\theta, \underline{s}, H) + qx_B - y_B) \tag{50}$$
$$+(1-\kappa)u(\theta + \tau(\theta, \underline{s}, M) + qx_B + xy_B)\Big].$$

[27]Recall that $u'(\underline{s}, M)$ and $u'(\bar{s}, M)$ depend on α_S via the transfers (see the Supplementary Appendix for details).

The expression pinning down the margin call, (49), and therefore the supply of the protection seller's asset, is similar to (30). The left-hand side describes the departure from full insurance again signal risk. The right-hand side describes the trade-off per unit sold. The numerator of the second fraction is the cost of losing the expected return $E[\tilde{R}]$ in return for the price p. The denominator is the gain in pledgeable return: cash p instead of $\hat{\mathcal{P}}$.

Equilibrium: Market clearing for the contract to share signal risk, $x_I = x_B$ and the contract to share $\tilde{R}$-risk after a bad signal, $y_I = y_B$, leads to equilibrium prices (q^*, z^*) that result in perfect risk-sharing between the protection buyer and investors (for details see the Supplementary Appendix):

$$\frac{u'(s,R)}{u'(s',R')} = \frac{v'(s,R)}{v'(s',R')} \quad \forall \quad s,s' \in \{\underline{s}, \bar{s}\} \text{ and } R,R' \in \{H,M\} \tag{51}$$

Market clearing in the market for the protection seller's asset, $\alpha_S = \alpha_I = \alpha^*$, where demand and supply are given by (48) and (49), respectively, yields (for $\alpha^* > 0$):

$$\frac{u'(\underline{s},M)}{u'(\bar{s},M)} - 1 = \frac{a+b}{b(1-\kappa)} \frac{\psi_I(\alpha^*) + \alpha^* \psi_I'(\alpha^*)}{E[\tilde{R}] - [\psi_I(\alpha^*) + \alpha^* \psi_I'(\alpha^*)] - \hat{\mathcal{P}}}. \tag{52}$$

We are now ready to check whether the equilibrium allocation satisfies the optimality conditions of the second-best. The two key optimality conditions are perfect risk-sharing between the protection buyer and investors, (43), and the characterization of the optimal asset transfer, (44).

The market achieves optimal risk-sharing, (51), trough transfers $\tau(\theta, s, R)$, the asset sale α^*, and the contracts sharing $\tilde{R}$-risk and signal risk. The equilibrium asset transfer, (52), is determined by the same marginal benefit and marginal cost as the second-best transfer. The benefit, on the left-hand side, is given by the extent of signal risk the protection buyer has to bear. The cost, on the right-hand side, is given by the marginal cost of investors relative to the gain in the pledgeable return. Following the same steps as in the proof of Proposition 6, we therefore obtain:

Proposition 9 *Market equilibrium is information-constrained Pareto efficient in the risk-shifting set-up.*

7 Implications

Economic balance sheet: The variation-margin call requests that protection sellers deposit safe assets in a margin account. These safe assets, however, are still owned by protection sellers. Therefore the margin deposit is on the asset side of their balance sheet, in line with the remark of McDonald and Paulson (2015, page 92) that the "transfer of funds based on a market value change is classified as a change in collateral and not as payment."

Figure 3 shows the balance sheet of protection sellers at time 1. This is not an accounting balance sheet, but an economic one, showing the value of the assets and liabilities of protection sellers, including those corresponding to derivative positions, implied by our theoretical analysis. After good news, the derivative position of the protection seller is expected to make a positive profit. This "marked-to-market" position is on the asset side of the balance sheet of protection sellers along with the portfolio of loans under management (which will, on the equilibrium path, return R at time 2 since the protection seller will exert effort). On the other side of the balance sheet, there is just the equity of protection sellers. After bad news, the derivative position of protection sellers is expected to be loss-making, which triggers the variation-margin call. Correspondingly, on the asset side of the balance sheet, there is the margin deposit (αp) and the downsized portfolio of loans, $(1-\alpha)R$, while on the other side of the balance sheet, there is the liability corresponding to the net value of the derivative position $\alpha p + (1-\alpha)\mathcal{P}$ and the lower equity of protection sellers.

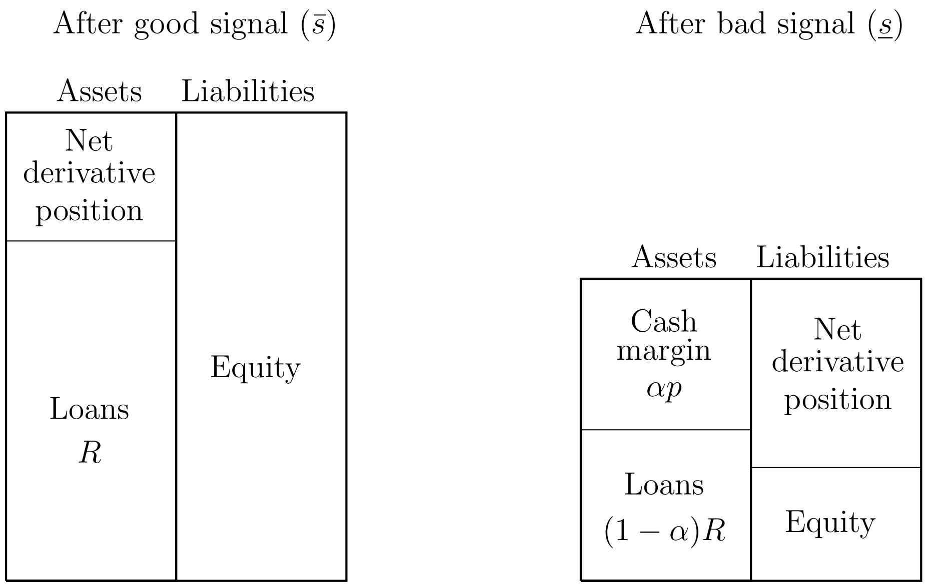

Figure 3: Balance sheet of protection sellers at $t = 1$

Variation margins vs solvency regulation: In line with stylised facts, variation margins are called when equity capital drops due to derivative losses. The ensuing asset sales could be compared to those triggered by solvency regulation. However, while solvency regulation aims at *reducing* leverage, variation margin calls allow an *increase* leverage: After bad news, the total value of assets (margin deposits plus risky assets under management) is $R - \alpha(R-p)$. This is decreasing in α, reflecting that assets under management are more profitable than cash in the margin account. The value of liabilities, i.e., the expected loss on the derivative position, is $\mathcal{P} + \alpha(p - \mathcal{P})$. This is increasing in α, reflecting that larger variation margins enable to promise more insurance ex ante, generating larger liabilities after bad news. Thus equity (assets minus liabilities) decreases in α. This striking difference between the consequences of variation margins and those of solvency regulation underscores the difference between our asset-side view of variation margins, which change the structure of assets to enhance pledgeability, and the liability-side approach of solvency regulation, which changes the structure of liabilities to reduce leverage.

Contagion between asset classes: Bad news lower the conditional expectation of the final value of protection buyers' assets. In the first best, there is no simultaneous change in the valuation of protection sellers' assets. In contrast, under moral hazard, after bad news variation margin calls lead to asset sales lowering the price of protection sellers' assets. Thus, moral hazard generates endogenous correlation between protection buyers' and protection sellers' assets. This can be interpreted as contagion after bad news, and is in line with the empirically observed increase in correlation across asset classes during bear markets (see, e.g., Ang and Chen, 2002).

Price of protection: An increase in variation-margins α generates productive inefficiencies by reducing the return on protection sellers' assets. The break-even constraint of protection sellers then requires them to make lower average transfers to protection buyers. Empirically, this means that the price of protection, proxied for example by CDS spreads, should increase when protection sellers expect larger margin calls, i.e., larger negative shock to their balance sheets. This matches the recent finding by Siriwardane (2018) that negative shocks to the capital of protection sellers in the CDS market increase the cost of insurance they provide. Our model further predicts that the effect Siriwardane (2018) documents should be more pronounced when the asset-price impact of margin calls is large.

Policy: Regulators are concerned by margin-induced fire sales (see, e.g., European Systemic Risk Board, 2017, p.5). As noted by Meier and Servaes (2018), however, buyers of underpriced assets benefit from fire sales. Meier and Servaes (2018) argue that a welfare analysis should weigh these benefits against the losses of the sellers. Our comparison of market equilibrium and second best considers the ex-ante as well as the ex-post consequences of fire sales. Ex post, during the fire sale, the profits of asset buyers are the mirror image of the losses of asset sellers. Ex ante, before the fire sale, what matters for welfare is the way in which those profits and losses are taken into account. When all market participants rationally anticipate the risk of fire sales, efficient insurance against that risk is supplied and equilibrium is constrained efficient. This points to a form of complementarity between markets. To efficiently share risk, we need both i) insurance markets against the final value of protection buyers' assets, and ii) insurance markets against the interim risk of fire sales. In practice, margin calls are often triggered by drops in the market valuation of the insured asset. In that case, the above mentioned final and interim risks correspond to different maturities of derivatives with the same underlying asset. When overseeing the development of derivatives markets, regulators and market organisers should therefore ensure that the set of maturities and risks traded are comprehensive enough. They should also make sure all market participants are aware of the risk of fire sales and can sell or buy insurance against that risk.

8 Conclusion

When protection sellers take large positions, likely to generate large losses, moral hazard creates counterparty risk. To mitigate this problem, positions must be limited (preventing perfect insurance) and variation margins must be called after bad news. To deposit cash on their margin account, protection sellers need to liquidate risky assets triggering fire sales. From a positive point of view, we show that moral hazard generates endogenous correlation between otherwise independent asset classes. From a normative point of view, we show that with optimal contracts equilibrium is information-constrained efficient, in spite of the presence of prices in incentive constraints.

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Appendix

A Proofs

Proof of Proposition 1

The Lagrangian is:

$$\begin{aligned} L_{FB}(c_B(\theta,s),c_S(\theta,s),c_I(\theta,s),\alpha(s)) = {} & \omega_B E[u(c_B(\theta,s))] + \omega_I E[v(c_I(\theta,s)-\alpha(s)\psi_I(\alpha))] \\ & +\lambda_B[E[u(c_B(\theta,s))]-E[u(\theta)]] \\ & +\lambda_I[E[v(c_I(\theta,s)-\alpha(s)\psi_I(\alpha))]-v(1)] \\ & +\lambda_S[E[c_S(\theta,s)-(1-\alpha(s))\psi]-(R-\psi)] \\ & +\sum_{\theta,s}\lambda(\theta,s)[\theta+1+R-(c_B(\theta,s)+c_S(\theta,s)+c_I(\theta,s))] \\ & +\sum_{s}(\lambda_1(s)[1-\alpha(s)]+\lambda_0(s)[\alpha(s)]) \end{aligned}$$

First-order conditions with respect to $c_B(\theta,s), c_I(\theta,s), c_S(\theta,s)$ and $\alpha(s)$ are

$$(\omega_B+\lambda_B)\Pr[\theta,s]u'(\theta,s)=\lambda(\theta,s), \tag{A.1}$$

$$(\omega_I+\lambda_I)\Pr[\theta,s]v'(\theta,s)=\lambda(\theta,s), \tag{A.2}$$

$$\lambda_S\Pr[\theta,s]=\lambda(\theta,s), \tag{A.3}$$

and

$$-(\omega_I+\lambda_I)\Pr[s]E[v'(c_I(\theta,s)-\alpha(s)\psi_I(\alpha))(\psi_I+\alpha(s)\psi_I')|s]+\lambda_S\Pr[s]\psi=\lambda_1(s)-\lambda_0(s), \tag{A.4}$$

respectively (where, in (A.4), we have used $\Pr[\theta,s]=\Pr[\theta|s]\Pr[s]$). The second-order conditions with respect to $c_B(\theta,s), c_I(\theta,s)$ and $c_S(\theta,s)$ hold because of decreasing marginal utilities. The second-order condition with respect to α is:

$$\begin{aligned} -(\omega_I+\lambda_I)\Pr[s]E[-v''(c_I(\theta,s)-\alpha(s)\psi_I(\alpha))(\psi_I+\alpha(s)\psi_I')^2 \\ +v'(c_I(\theta,s)-\alpha(s)\psi_I(\alpha))(2\psi_I'+\alpha(s)\psi_I'')|s]\le 0, \end{aligned} \tag{A.5}$$

which holds since $\psi_I''\ge 0$ and $v''<0$.

Equations (A.1), (A.2) and (A.3) imply

$$(\omega_B+\lambda_B)u'(\theta,s)=(\omega_I+\lambda_I)v'(\theta,s)=\lambda_S \qquad \forall(\theta,s). \tag{A.6}$$

Because neither $\omega_B,\omega_I,\lambda_B,\lambda_I$, nor λ_S depend on the state (θ,s), equation (A.6) implies that the marginal utilities of buyers and investors are constant across states. Hence, $c_B(\theta, s) = c_B$ and $c_I(\theta, s) = c_I$.

The resource constraints bind, $\lambda(\theta, s) > 0$. Suppose not. Because $v', u' > 0$, $\Pr[\theta, s] > 0$, this implies $\omega_B + \lambda_B = 0$ and $\omega_I + \lambda_I = 0$, and hence, $\omega_B = \omega_I = 0$. But because we also have $\omega_S = 0$ (by assumption), the planner's objective would then become zero.

The participation constraint of the sophisticated investors binds, $\lambda_S > 0$. Because $\Pr[\theta, s] > 0$, this is immediate once $\lambda(\theta, s) > 0$.

There is no asset transfer in any state, $\alpha(s) = 0$. Suppose there were positive asset transfers, i.e., $\alpha(s) > 0$. Using the second equality in (A.6), dividing by $\lambda_S \Pr[s]$, and rearranging, the first-order condition with respect to $\alpha(s)$ becomes

$$\psi - \psi_I(\alpha(s)) = \frac{\lambda_1}{\lambda_S \Pr[s]} + \alpha(s)\psi_I'.$$

Given that $\lambda_S > 0$, $\psi_I' \geq 0$ and $\psi < \psi_I(\alpha(s))$ when $\alpha(s) > 0$, this is a contradiction: the left-hand side is negative while the right-hand side is weakly positive.

Given constant consumption for buyers and investors, and the binding resource constraints, we have $c_B + c_I + c_S(\theta, s) = \theta + 1 + R \quad \forall(\theta, s)$. Using this to substitute for $c_S(\theta, s)$ in the binding participation constraint of investors, together with $\alpha(s) = 0$, we have $c_B + c_I = E[\tilde{\theta}] + 1$.

Proof of Lemma 1

The Lagrangian of the second-best maximisation problem is

$$\begin{aligned} L_{SB}(c_B(\theta,s), c_S(\theta,s), c_I(\theta,s), \alpha(s)) = {} & \omega_B E[u(c_B(\theta,s))] + \omega_I E[v(c_I(\theta,s) - \alpha(s)\psi_I(\alpha))] \\ & + \sum_s \lambda_{IC(s)} \left[E[c_S(\theta,s)|s] - \frac{(1-\alpha(s))\psi}{1-\mu} \right] \\ & + \lambda_B[E[u(c_B(\theta,s))] - E[u(\theta)]] \\ & + \lambda_I[E[v(c_I(\theta,s) - \alpha(s)\psi_I(\alpha))] - v(1)] \\ & + \lambda_S[E[c_S(\theta,s) - (1-\alpha(s))\psi] - (R-\psi)] \\ & + \sum_{\theta,s} \lambda(\theta,s)[\theta + 1 + R - (c_B(\theta,s) + c_S(\theta,s) + c_I(\theta,s))] \\ & + \sum_s \left(\lambda_1(s)[1-\alpha(s)] + \lambda_0(s)[\alpha(s)]\right). \end{aligned}$$

First-order conditions with respect to $c_B(\theta, s)$ and $c_I(\theta, s)$ are the same as in the first best, (A.1) and (A.2), respectively. The first-order conditions with respect to $c_S(\theta, s)$ and

$\alpha(s)$ are altered, to take into acount the incentive constraint, and write

$$\lambda_{IC(s)}\Pr[\theta|s] + \lambda_S \Pr[\theta, s] = \lambda(\theta, s) \tag{A.7}$$

and

$$-(\omega_I+\lambda_I)\Pr[s]E[v'(\theta,s)|s](\psi_I(\alpha)+\alpha(s)\psi_I')+\lambda_{IC(s)}\frac{\psi}{1-\mu}+\lambda_S\Pr[s]\psi = \lambda_1(s)-\lambda_0(s), \tag{A.8}$$

respectively. The second-order conditions are as in the first best.

The first-order conditions with respect to $c_B(\theta, s)$ and $c_S(\theta, s)$, (A.1) and (A.7), respectively imply

$$u'(\theta, s) = \frac{1}{\omega_B + \lambda_B}\left(\lambda_{IC(s)}\frac{1}{\Pr[s]} + \lambda_S\right) \tag{A.9}$$

while the first-order conditions with respect to $c_I(\theta, s)$ and $c_S(\theta, s)$, (A.2) and (A.7), respectively imply

$$v'(\theta, s) = \frac{1}{\omega_I + \lambda_I}\left(\lambda_{IC(s)}\frac{1}{\Pr[s]} + \lambda_S\right). \tag{A.10}$$

Because their right-hand sides are independent of θ, (A.9) and (A.10) imply that, for a given realisation of the signal s, the marginal utility of consumption of the protection buyers and investors is the same in $(\bar{\theta}, s)$ and $(\underline{\theta}, s)$.

Proof of Lemma 2

First, we prove that the resource constraints bind, $\lambda(\theta, s) > 0$. Suppose not. Because $v', u' > 0$, $\Pr[\theta, s] > 0$, by (A.1) and (A.2), this implies $\omega_B + \lambda_B = 0$ and $\omega_I + \lambda_I = 0$, and hence, $\omega_B = \omega_I = 0$, a contradiction.

Second, we prove that the participation constraint of the protection seller binds. Suppose not, $\lambda_S = 0$. Then, using $\lambda(\theta, s) > 0$ in (A.7) yields $\lambda_{IC(s)} > 0$ for all (θ, s), i.e., both incentive constraints bind. From the binding incentive constraints, we have $E[c_S(\theta, s)|s] = \frac{1-\alpha(s)}{1-\mu}\psi$ and hence,

$$E[c_S(\theta, s)] = \Pr[\bar{s}]\frac{1-\alpha(\bar{s})}{1-\mu}\psi + \Pr[\underline{s}]\frac{1-\alpha(\underline{s})}{1-\mu}\psi = (1 - E[\alpha(s)])\frac{\psi}{1-\mu}. \tag{A.11}$$

Substituting this into the slack participation constraint of the protection seller yields

$$(1 - E[\alpha(s)])\frac{\psi}{1-\mu} - (1 - E[\alpha(s)])\psi > R - \psi$$

and, after some rearranging,

$$-E[\alpha(s)]\psi\frac{\mu}{1-\mu} > R - \frac{\psi}{1-\mu},$$

which contradicts the assumption that $\mathcal{P} > 0$.

Third, we prove that one of the two incentive constraints (or both) must bind. If not, then the first-best allocation would solve the second best problem. Now, with the seller's first-best consumption (10) and $\alpha(s) = 0$, the incentive constraint after a bad signal becomes

$$\Pr(\bar{\theta}|\underline{s})(\bar{\theta} - E[\tilde{\theta}] + R) + \Pr(\underline{\theta}|\underline{s})(\underline{\theta} - E[\tilde{\theta}] + R) \geq \frac{\psi}{1-\mu},$$

i.e., $E[\tilde{\theta}|\underline{s}] - E[\tilde{\theta}] + R \geq \frac{\psi}{1-\mu}$, which violates assumption (11).

Fourth, we prove that both ICs cannot bind at the same time. Suppose they do. Then, we have again (A.11), which after substituting the binding participation constraint of the protection seller and rearranging yields $-E[\alpha(s)]\psi\frac{\mu}{1-\mu} = R - \frac{\psi}{1-\mu}$, which contradicts the assumption that $\mathcal{P} > 0$.

Proof of Lemma 3

First, we prove that when the incentive-compatibility condition in state s is slack, then $\alpha(s) = 0$. Suppose not, i.e., $\alpha(s) > 0$ and $\lambda_{IC(s)} = 0$. Then, using (A.2) and (A.7), (A.8) becomes

$$-\lambda_S \Pr[s](\psi_I(\alpha(s)) + \alpha(s)\psi_I'(\alpha(s))) + \lambda_S \Pr[s]\psi = \lambda_1(s).$$

Dividing by $\lambda_S \Pr[s] > 0$ and rearranging yields

$$\psi - \psi_I(\alpha(s)) = \frac{\lambda_1(s)}{\lambda_S(s)\Pr[s]} + \alpha(s)\psi_I'(\alpha(s)).$$

Given that $\psi_I' \geq 0$ and $\psi < \psi_I$ when $\alpha(s) > 0$, we obtain the desired contradiction. The left-hand side is negative while the right-hand side is weakly positive.

Second, we prove that the incentive-compatibility condition after a bad signal binds. Suppose not, $\lambda_{IC((\underline{s}))} = 0$, and only the incentive constraint after the good signal binds. Now, given that the incentive constraint after a bad signal is slack, so that $\alpha(\underline{s}) = 0$, and the incentive constraint after a good signal binds, we have

$$E[c_S(\theta, s)|\bar{s}] = \frac{(1-\alpha(\bar{s}))\psi}{1-\mu} = \frac{\psi}{1-\mu} - \frac{\alpha(\bar{s})\psi}{1-\mu}$$
$$E[c_S(\theta, s)|\underline{s}] > \frac{\psi}{1-\mu},$$

which implies that

$$E[c_S(\theta, s)|\underline{s}] - E[c_S(\theta, s)|\bar{s}] > 0. \tag{A.12}$$

Next, from the binding resource constraints and full risk-sharing conditional on the signal, we have

$$c_S(\theta, s) = \theta + 1 + R - (c_B(s) + c_I(s)) \tag{A.13}$$

and hence

$$E[c_S(\theta, s)|\bar{s}] = E[\tilde{\theta}|\bar{s}] + 1 + R - (c_B(\bar{s}) + c_I(\bar{s})) \tag{A.14}$$

$$E[c_S(\theta, s)|\underline{s}] = E[\tilde{\theta}|\underline{s}] + 1 + R - (c_B(\underline{s}) + c_I(\underline{s})) \tag{A.15}$$

so that

$$E[c_S(\theta, s)|\underline{s}] - E[c_S(\theta, s)|\bar{s}] = E[\tilde{\theta}|\underline{s}] - E[\tilde{\theta}|\bar{s}] - [c_B(\underline{s}) - c_B(\bar{s}) + c_I(\underline{s}) - c_I(\bar{s})]. \tag{A.16}$$

To obtain that expression in (A.16) is weakly negative, so that we have the contradiction to (A.12), the term in squared brackets with the consumptions must be weakly positive (because the signal is (weakly) informative, we have $E[\tilde{\theta}|\underline{s}] - E[\tilde{\theta}|\bar{s}] \leq 0$). From (A.1), (A.7) and the slack incentive constraint after a bad signal, we have

$$(\omega_B + \lambda_B)u'(\theta, \bar{s}) = \lambda_S + \frac{\lambda_{IC(\bar{s})}}{\Pr[\bar{s}]}$$
$$(\omega_B + \lambda_B)u'(\theta, \underline{s}) = \lambda_S$$

Together with full risk-sharing conditional on the signal, this implies that $c_B(\underline{s}) \geq c_B(\bar{s})$.The same type of argument also establishes that $c_I(\underline{s}) \geq c_I(\bar{s})$.Hence, the term in squared brackets in (A.16) is (weakly) positive, which yields the desired contradiction.

Third, we analyse the ranking of the consumptions of the protection buyers after bad and good signals. Combining (A.2) with (A.7), and using the fact that there is full risk-sharing conditional on the signal and that only the incentive constraint after the bad signal binds, we obtain:

$$(\omega_B + \lambda_B)\Pr[\theta, \bar{s}]u'(c_B(\bar{s})) = \lambda_S \Pr[\theta, \bar{s}]$$
$$(\omega_B + \lambda_B)\Pr[\theta, \underline{s}]u'(c_B(\underline{s})) = \lambda_{IC(\underline{s})}\Pr[\theta|\underline{s}] + \lambda_S \Pr[\theta, \underline{s}]$$

so that

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))} = 1 + \frac{\lambda_{IC(\underline{s})}}{\Pr[\underline{s}]\lambda_S}. \tag{A.17}$$

Because $\lambda_{IC(\underline{s})} > 0$ and $\lambda_S > 0$, we have imperfect risk-sharing across signals with $c_B(\underline{s}) < c_B(\bar{s})$.

Proof of Proposition 3

First, we write down more precisely the first-order optimality condition with respect to $\alpha(\underline{s})$. Using (A.2) and Lemma 1, the derivative of the Lagrangian with respect to $\alpha(\underline{s})$ is

$$\frac{\partial L_{SB}}{\partial \alpha(\underline{s})} = -\lambda(\theta, s)\Pr[\underline{s}]\frac{1}{\Pr[\theta, \underline{s}]}(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))+\lambda_{IC(\underline{s})}\frac{\psi}{1-\mu}+\lambda_S\Pr[\underline{s}]\psi-(\lambda_1(\underline{s})-\lambda_0(\underline{s}))\,.$$

Using (A.7), this rewrites as

$$\frac{\partial L_{SB}}{\partial \alpha(\underline{s})} = -(\lambda_S\Pr[\underline{s}]+\lambda_{IC(\underline{s})})(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))+\lambda_{IC(\underline{s})}\frac{\psi}{1-\mu}+\lambda_S\Pr[\underline{s}]\psi-(\lambda_1(\underline{s})-\lambda_0(\underline{s}))\,.$$

Collecting terms,

$$\begin{aligned}\frac{\partial L_{SB}}{\partial \alpha(\underline{s})} =& \lambda_{IC(\underline{s})}\left[\frac{\psi}{1-\mu}-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))\right] \\ &+\lambda_S\Pr[\underline{s}]\big[\psi-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))\big]-(\lambda_1(\underline{s})-\lambda_0(\underline{s}))\,. \end{aligned} \tag{A.18}$$

Second, we show that under (17) there must be some asset transfer, i.e., $\alpha(\underline{s}) > 0$. Suppose not, i.e., suppose we have $\alpha(\underline{s}) = 0$. Then, $\lambda_1(\underline{s}) = 0$ and, by (A.18), the optimality condition such that $\alpha(\underline{s}) = 0$, $\frac{\partial L_{SB}}{\partial \alpha(\underline{s})} \leq 0$, writes as

$$\frac{\lambda_{IC(\underline{s})}}{\lambda_S\Pr[\underline{s}]}\left[\frac{\psi}{1-\mu}-\psi_I(0)\right]+\big[\psi-\psi_I(0)\big] \leq -\frac{\lambda_0(\underline{s})}{\lambda_S\Pr[\underline{s}]}. \tag{A.19}$$

Now, (A.17) yields

$$\frac{\lambda_{IC(\underline{s})}}{\Pr[\underline{s}]\lambda_S} = \left.\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\right|_{\alpha(\underline{s})=0} - 1. \tag{A.20}$$

Substituting into (A.19) yields

$$\left.\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\right|_{\alpha(\underline{s})=0} - \frac{\frac{\psi}{1-\mu}-\psi}{\frac{\psi}{1-\mu}-\psi_I(0)} \leq -\frac{\lambda_0(\underline{s})}{\lambda_S\Pr[\underline{s}]\left[\frac{\psi}{1-\mu}-\psi_I(0)\right]}. \tag{A.21}$$

This contradicts (17), which is equivalent to

$$\left.\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\right|_{\alpha(\underline{s})=0} > \frac{\frac{\psi}{1-\mu}-\psi}{\frac{\psi}{1-\mu}-\psi_I(0)}.$$

Third, we characterise asset transfers when they are interior, i.e., when $\alpha(\underline{s}) \in (0,1)$. In that case, (A.18) and (A.17) imply

$$\left[\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}-1\right]+\frac{\psi-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))}{\frac{\psi}{1-\mu}-(\psi_I(\alpha(\underline{s}))+\alpha(\underline{s})\psi_I'(\alpha(\underline{s})))}=0,$$

where $c_B(\underline{s})$ and $c_B(\bar{s})$ are as given in Proposition 2.

Proof of Lemma 4

First, we write down the Lagrangian of the protection buyer and use it to show that the participation constraint of protection sellers bind. The Lagrangian is:

$$\begin{aligned} L(\tau(\theta,s),\alpha_S,x_B) = {} & \Pr[\bar{s}]E[u(\theta+\tau(\theta,s)-x_B)|\bar{s}]+\Pr[\underline{s}]E[u(\theta+\tau(\theta,s)+qx_B)|\underline{s}] \\ & +\lambda_{IC}\left[\alpha_S p+(1-\alpha_S)\mathcal{P}-E[\tau(\theta,s)|\underline{s}]\right] \\ & +\lambda_S\left[\Pr[\bar{s}](R-\psi)+\Pr[\underline{s}]((1-\alpha_S)(R-\psi)+\alpha_S p)-E[\tau(\theta,s)]-(R-\psi)\right] \\ & +\lambda_1[1-\alpha_S]-\lambda_0\alpha_S. \end{aligned} \tag{A.22}$$

The first-order conditions of (A.22) with respect to $\tau(\theta,\bar{s})$ and $\tau(\theta,\underline{s})$ are:

$$\begin{aligned} \Pr[\bar{s}]\Pr[\theta|\bar{s}]u'(\theta,\bar{s}) &= \lambda_S\Pr[\theta,\bar{s}] && \forall\theta \\ \Pr[\underline{s}]\Pr[\theta|\underline{s}]u'(\theta,\underline{s}) &= \lambda_S\Pr[\theta,\underline{s}]+\lambda_{IC}\Pr[\theta|\underline{s}] && \forall\theta \end{aligned}$$

which simplify to

$$u'(\theta,\bar{s}) = \lambda_S \qquad \forall\theta \tag{A.23}$$

$$u'(\theta,\underline{s}) = \lambda_S+\frac{\lambda_{IC}}{\Pr[\underline{s}]} \qquad \forall\theta. \tag{A.24}$$

(A.23) implies that $\lambda_S > 0$, i.e., the participation constraint of protection sellers binds.

Second, we use the first-order conditions with respect to $\tau(\theta,\bar{s})$ and $\tau(\theta,\underline{s})$ to show that the protection buyer is fully insured conditional on the signal. Because the right-hand sides of (A.23) and (A.24) do not depend on θ, we have:

$$\bar{\theta}+\tau(\bar{\theta},\bar{s}) = \underline{\theta}+\tau(\underline{\theta},\bar{s}) \tag{A.25}$$

$$\bar{\theta}+\tau(\bar{\theta},\underline{s}) = \underline{\theta}+\tau(\underline{\theta},\underline{s}). \tag{A.26}$$

Thus, conditional on the realisation of the signal s,the protection buyer is fully insured against remaining θ-risk.

Third, we prove by contradiction that the incentive-compatibility condition of the protection seller binds. To do so, we proceed in two steps.

The first step is to prove that, if the incentive-compatibility condition of the protection seller was slack, there would be no asset sale in equilibrium. This first step proceeds by contradiction. Suppose $\lambda_{IC} = 0$ and $\alpha_S = \alpha_I > 0$. Consider the first-order condition of the Lagrangian (A.22) with respect to α_S, when $\alpha_S > 0$ (and hence, $\lambda_0 = 0$) and $\lambda_{IC} = 0$:

$$-\lambda_s \Pr[\underline{s}](R - \psi - p^*) = \lambda_1, \tag{A.27}$$

where p^* is the equilibrium price in the asset market. From the investors' demand for the productive asset, we know that $\alpha_I > 0$ requires $p^* < R - \psi_I(0)$. Because $\psi_I(0) \geq \psi$ by assumption, the left-hand side of (A.27) is strictly negative, which contradicts the fact that the right-hand side is weakly positive.

The second step is to prove that slack protection seller's incentive constraint would contradict our assumption that $\mathcal{P} < E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}]$. Suppose $\lambda_{IC} = 0$. Equations (A.23) and (A.24) imply full insurance, $\tau(\theta, \bar{s}) = \tau(\theta, \underline{s}) \equiv \tau(\theta)$ for all θ, and $\bar{\theta} + \tau(\bar{\theta}) = \underline{\theta} + \tau(\underline{\theta})$. Using that $\alpha_S = 0$ and there is full insurance when $\lambda_{IC} = 0$, and substituting the binding participation constraint, we obtain $\tau(\bar{\theta}) = -(1-\pi)(\bar{\theta} - \underline{\theta})$ and $\tau(\underline{\theta}) = \pi(\bar{\theta} - \underline{\theta})$. Using this in the slack incentive constraint yields

$$\begin{aligned}\mathcal{P} >& \Pr[\bar{\theta}|\underline{s}](-1)(1-\pi)(\bar{\theta} - \underline{\theta}) + (1 - \Pr[\bar{\theta}|\underline{s}])\pi(\bar{\theta} - \underline{\theta})\\ =& (\pi - \Pr[\bar{\theta}|\underline{s}])(\bar{\theta} - \underline{\theta})\\ =& E[\tilde{\theta}] - E[\tilde{\theta}|\underline{s}],\end{aligned}$$

a contradiction.

Fourth, we compute the transfers. The binding incentive and participation constraints imply

$$E[\tau(\theta, s)|\bar{s}] = -\frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\big[\alpha_S(R - \psi) + (1 - \alpha_S)\mathcal{P}\big], \tag{A.28}$$

$$E[\tau(\theta, s)|\underline{s}] = \alpha_S p + (1 - \alpha_S)\mathcal{P}. \tag{A.29}$$

Equations (A.28) and (A.29), together with full insurance conditional on the signal, (A.25) and (A.26), yield the set of transfers, for a given α_S:

$$\begin{aligned}\tau^*(\bar{\theta}, \underline{s}) &= -\Pr[\underline{\theta}|\underline{s}](\bar{\theta} - \underline{\theta}) + \alpha_S p + (1 - \alpha_S)\mathcal{P},\\ \tau^*(\underline{\theta}, \underline{s}) &= \Pr[\bar{\theta}|\underline{s}](\bar{\theta} - \underline{\theta}) + \alpha_S p + (1 - \alpha_S)\mathcal{P},\\ \tau^*(\bar{\theta}, \bar{s}) &= -\Pr[\underline{\theta}|\underline{s}](\bar{\theta} - \underline{\theta}) - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\big[\alpha_S(R - \psi) + (1 - \alpha_S)\mathcal{P}\big],\\ \tau^*(\underline{\theta}, \bar{s}) &= \Pr[\bar{\theta}|\underline{s}](\bar{\theta} - \underline{\theta}) - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\big[\alpha_S(R - \psi) + (1 - \alpha_S)\mathcal{P}\big].\end{aligned}$$

Proof of Lemma 5

The first-order condition of the Lagrangian (A.22) with respect to α_S is

$$\lambda_{IC}(p-\mathcal{P}) - \lambda_s \Pr[\underline{s}](R-\psi-p) = \lambda_1 - \lambda_0. \tag{A.30}$$

From (A.23) and (A.24) we have

$$\frac{u'(\theta,\underline{s})}{u'(\theta,\bar{s})} = 1 + \frac{\lambda_{IC}}{\Pr[\underline{s}]\lambda_S} > 1, \tag{A.31}$$

where the inequality follows from the binding incentive constraint stated in Lemma 4.

Combining (A.30) and (A.31), and using the consumptions in Lemma 4, we obtain

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d\right)} = \frac{\lambda_1 - \lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S} + \frac{R-\psi-\mathcal{P}}{p-\mathcal{P}}. \tag{A.32}$$

Next, we show that when $p > \mathcal{P} + (R-\psi-\mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$ then $\alpha_S > 0$. In that case, (A.32) with $\lambda_0 = 0$ yields (30). Suppose not, $\alpha_S = 0$, so that $\lambda_0 > 0$ and $\lambda_1 = 0$. Then solving (A.32) with $\alpha_S = 0$ for p yields

$$\left[\frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)} + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\right](p-\mathcal{P}) = R-\psi-\mathcal{P}$$

$$p = \mathcal{P} + \frac{R-\psi-\mathcal{P}}{\left[\frac{u'(E[\tilde{\theta}|\underline{s}]+\mathcal{P}+qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)} + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\right]}.$$

This contradicts the assumption that $p > \mathcal{P} + (R-\psi-\mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}]+\mathcal{P}+qx^d)}$, because

$$\frac{R-\psi-\mathcal{P}}{\left[\frac{u'(E[\tilde{\theta}|\underline{s}]+\mathcal{P}+qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)} + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\right]} < (R-\psi-\mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)},$$

as

$$1 < \frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}\left[\frac{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)} + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\right],$$

due to

$$1 < 1 + \frac{\lambda_0}{(p-\mathcal{P})\Pr[\underline{s}]\lambda_S}\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}.$$

Finally, we show that when $p \leq \mathcal{P} + (R - \psi - \mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$, then $\alpha_S = 0$. To do so, we proceed in three steps, corresponding to different values of p.

First, when $p < \mathcal{P}$, $\alpha_S = 0$. Suppose not, $\alpha_S > 0$ and hence $\lambda_0 = 0$. Then, the first term on the right-hand side of (A.32) is weakly negative and the second term is strictly negative. Hence, the right-hand side is strictly negative while the left-hand side is strictly positive.

Second, when $\mathcal{P} < p \leq \mathcal{P} + (R - \psi - \mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$, then $\alpha_S = 0$. Suppose not, $\alpha_S > 0$ and hence, $\lambda_0 = 0$. Then, solving (A.32) for p yields

$$\begin{aligned} p \;=\; & \frac{\lambda_1}{\Pr[\underline{s}]\lambda_S}\frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)} \\ & + \mathcal{P} + (R - \psi - \mathcal{P})\frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)}. \end{aligned}$$

This price decreases when α_S decreases (since the ratio of marginal utilities is strictly increasing in α_S). Yet, with $\alpha_S > 0$, the price will always be larger than the largest price allowed in the starting condition

$$p = \mathcal{P} + (R - \psi - \mathcal{P})\frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$$

because $\lambda_1 \geq 0$ and

$$\frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)} > \frac{u'(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x^d)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx^d)}$$

when $\alpha_S > 0$.

Third, when $p = \mathcal{P}$, then $\alpha_S = 0$. Suppose not, $\alpha_S > 0$ and hence, $\lambda_0 = 0$. As $p \to \mathcal{P}$, the right-hand side of (A.32) goes to infinity, contradiction since the left-hand side is finite.

Proof of Proposition 5

Lemma 5 states that if

$$p \leq \mathcal{P} + (R - \psi - \mathcal{P})\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x_B\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx_B)},$$

then $\alpha_S = 0$, otherwise $\alpha_S > 0$, where α_S is given by (30).

Moreover, the above analysis of investors trades showed that if $p < R - \psi_I(0)$ then $\alpha_I > 0$, while otherwise $\alpha_I = 0$. So two cases must be distinguished.

If

$$\frac{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}\mathcal{P} - x_B\right)}{u'(E[\tilde{\theta}|\underline{s}] + \mathcal{P} + qx_B)} > \frac{\frac{\psi}{1-\mu} - \psi}{R - \psi - \mathcal{P}},$$

then $\alpha^* = 0$ and p^* is any price in $[R - \psi_I(0), \hat{p}(x^d, q)]$.

Otherwise, there exists (p^*, α^*) such that $\alpha_S(p^*) = \alpha_I(p^*) = \alpha^* > 0$. A sufficient condition for $\alpha^* < 1$ is provided by (3), which implies $\psi_I(1) + \psi_I' > \frac{\psi}{1-\mu}$. To see this, proceed by contradiction and suppose $\alpha^* = 1$. Then, (25) implies the price is $p^* = R - (\psi_I(1) + \psi_I')$. Substituting into (30)

$$\frac{u'(E[\tilde{\theta}|\underline{s}] + \alpha_S p + (1-\alpha_S)\mathcal{P} + qx^d)}{u'\left(E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha_S(R-\psi) + (1-\alpha_S)\mathcal{P}] - x^d\right)} = \frac{\lambda_1}{\left(\frac{\psi}{1-\mu} - (\psi_I(1) + \psi_I')\right)\lambda_S \Pr[\underline{s}]} + \frac{R - \psi - \mathcal{P}}{\frac{\psi}{1-\mu} - (\psi_I(1) + \psi_I')}$$

The left-hand side is strictly positive but if $\psi_I(1) + \psi_I' > \frac{\psi}{1-\mu}$, the right-hand side is strictly negative, so we have a contradiction.

In the second case, the price p^* is obtained by applying (25). Substituting this price into (30) while setting $\lambda_1 = 0$ yields (34).

Proof of Proposition 6

To prove Proposition 6, we first recall the equilibrium conditions, then we recall the second-best conditions, and finally we show that for any allocation that satisfies the equilbrium conditions there exists a set of Pareto weights such that this allocation satisfies the conditions for second-best optimality.

Equilibrium allocation: Substituting equilibrium prices and trades α^*, p^*, x^*, and q^* into (27) and (28), equilibrium protection buyers' consumption is

$$c_B(\bar{\theta}, \bar{s}) = c_B(\underline{\theta}, \bar{s}) = E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha^*(R-\psi) + (1-\alpha^*)\mathcal{P}] - x^*, \tag{A.33}$$

$$c_B(\bar{\theta}, \underline{s}) = c_B(\underline{\theta}, \underline{s}) = E[\tilde{\theta}|\underline{s}] + \alpha^* p^* + (1-\alpha^*)\mathcal{P} + q^* x^*. \tag{A.34}$$

Similarly, substituting α^*, p^*, x^*, and q^* into investors' consumptions

$$c_I(\bar{\theta}, \bar{s}) = c_I(\underline{\theta}, \bar{s}) = 1 + x^*, \tag{A.35}$$

$$c_I(\bar{\theta}, \underline{s}) = c_I(\underline{\theta}, \underline{s}) = 1 - q^* x^* + \alpha^*(R - p^*). \tag{A.36}$$

Substituting α^*, p^*, x^*, q^*, (A.33) and (A.34) into (32), marginal rates of substitution between consumption after good news and after bad news are equalised for protection buyers and investors.

$$\frac{v'(c_I(\theta, \underline{s}) - \alpha^* \psi_I(\alpha^*))}{v'(c_I(\theta, \bar{s}))} = \frac{u'(c_B(\theta, \underline{s}))}{u'(c_B(\theta, \bar{s}))}. \tag{A.37}$$

Substituting (A.33) and (A.34) into condition (33), the condition writes as

$$\left.\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\right|_{\alpha=0} > \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - \psi_I(0)}. \tag{A.38}$$

When that condition does not hold, $\alpha^* = 0$. When it holds, substituting α^*, p^*, x^*, q^*, into (34), the marginal rate of substitution between consumption after bad news and consumption after good news is equal to what we intepreted, in the discussion of equation (18) in Proposition 3, as the marginal cost of insurance:

$$\frac{u'(c_B(\theta, \underline{s})}{u'\left(c_B(\theta, \bar{s})\right)} = \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha^*) + \alpha^* \psi_I'(\alpha^*))}. \tag{A.39}$$

Second best allocation: Equations (13) and (14) state the total consumption of protection buyers and investors, after bad news and after good news, in the second best:

$$c_B(\underline{s}) + c_I(\underline{s}) = 1 + E[\tilde{\theta}|\underline{s}] + \alpha(\underline{s})R + (1 - \alpha(\underline{s}))\mathcal{P}, \tag{A.40}$$

$$c_B(\bar{s}) + c_I(\bar{s}) = 1 + E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha(\underline{s})(R - \psi) + (1 - \alpha(\underline{s}))\mathcal{P}]. \tag{A.41}$$

Equation (15) states that in the second best marginal rates of substitution are equalised between protection buyers and investors:

$$\frac{v'(c_I(\underline{s}) - \alpha(\underline{s})\psi_I(\underline{s}))}{v'(c_I(\bar{s}) - \alpha(\bar{s})\psi_I(\bar{s}))} = \frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}. \tag{A.42}$$

Inequality (17) states the condition under which asset transfers are strictly positive in the second best:

$$\left.\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))}\right|_{\alpha(\underline{s})=0} > \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - \psi_I(0)}; \tag{A.43}$$

if that condition does not hold, then there are no asset transfers in the second best.

Equation (18) gives the interior asset transfer:

$$\frac{u'(c_B(\underline{s}))}{u'(c_B(\bar{s}))} = \frac{\frac{\psi}{1-\mu} - \psi}{\frac{\psi}{1-\mu} - (\psi_I(\alpha(\underline{s})) + \alpha(\underline{s})\psi_I'(\alpha(\underline{s})))}. \tag{A.44}$$

Finally, equation (16) states how total consumption is split between protection buyers and investors as a function of their Pareto weights:

$$\frac{u'(c_B(s))}{v'(c_I(s) - \alpha(s)\psi_I(\alpha(s))} = \frac{\omega_I + \lambda_I}{\omega_B + \lambda_B}. \tag{A.45}$$

Investors' and protection buyers' consumptions and asset transfers such that (A.40), (A.41), (A.42), (A.43), (A.44) and (A.45) hold are second best.

Comparing second best and equilibrium allocations: Consider an equilibrium allocation $\mathcal{E} = \{c_I(\theta, s), c_B(\theta, s), \alpha^*\}$. It is such that i) (A.33) to (A.36) hold and ii) if (A.38) holds, then (A.39) holds.

Equilibrium is information-constrained Pareto efficient if $\mathcal{E}$ satisfies the second-best optimality conditions, (A.40) to (A.45). Out of these six conditions, 5 are obviously satisfied:

Adding (A.33) to (A.35), and (A.34) to (A.36), in equilibrium the total consumption of protection buyers and investors is

$$c_B(\theta, \bar{s}) + c_I(\theta, \bar{s}) = 1 + E[\tilde{\theta}|\bar{s}] - \frac{\Pr[\underline{s}]}{\Pr[\bar{s}]}[\alpha^*(R - \psi) + (1 - \alpha^*)\mathcal{P}], \forall\theta, \tag{A.46}$$

after good news and

$$c_B(\theta, \underline{s}) + c_I(\theta, \underline{s}) = 1 + E[\tilde{\theta}|\underline{s}] + \alpha^* R + (1 - \alpha^*)\mathcal{P}, \forall\theta, \tag{A.47}$$

after bad news. (A.47) is equivalent to (A.40), while (A.46) is equivalent to (A.41).

Equation (A.37) shows that in equilibrium the MRS of protection buyers and investors are equalised, exactly as requested in the second best, in (A.42).

Third, (A.38) is equivalent to (A.43), and (A.39) is equivalent to (A.44).

So, it only remains to check that $\mathcal{E}$ satisfies (A.45). To do so, we need to show that there are Pareto weights ω_I and ω_B such that (A.45) holds for the consumptions in $\mathcal{E}$. Now, investors are strictly better off when participating in the market equilibrium than in autarky, since they strictly prefer to trade in the market for insurance against signal risk. Protection buyers also are strictly better off since they can, at least, extract all the surplus from contracting with protection sellers with $\alpha = 0$. Consequently, the participation constraints of protection buyers and investors are slack, implying $\lambda_I = \lambda_B = 0$. Hence, (A.45) holds for the consumptions in $\mathcal{E}$ if and only if there exist Pareto weights ω_I and ω_B such that

$$\frac{u'(c_B(s))}{v'(c_I(s)-\alpha(s)\psi_I(\alpha(s)))}=\frac{\omega_I}{\omega_B}.$$

This is always the case. To see this, pick an arbitrary ω_B, then set

$$\omega_I=\omega_B\frac{u'(c_B(s))}{v'(c_I(s)-\alpha(s)\psi_I(\alpha(s)))}.$$