Abstract
This paper studies the optimal determination of bankruptcy exemptions for risk averse borrowers who use unsecured contracts but have the possibility of defaulting. In a large class of economies, knowledge of four variables is sufficient to determine whether a bankruptcy exemption level is optimal or should be increased or decreased. These variables are i) the composition of households’ liabilities, ii) the sensitivity of the credit supply schedule to exemption changes, iii) the probability of filing for bankruptcy with non-exempt assets, and iv) the value given by households to a marginal dollar in different states, which can be mapped to changes in households’ consumption. I recover empirical estimates of the sufficient statistics using U.S. data over the period 2008-2016 and find that increasing exemption levels improves overall welfare, although there is substantial variation in estimated welfare gains across U.S. states and income quintiles.

JEL codes: D52, K35, D14

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1 Introduction

The number of personal bankruptcy filings in the U.S. is on the order of 1 million per year, affecting roughly 1% of American households. A recurring policy question is whether bankruptcy procedures should be harsher or more lenient with bankrupt borrowers. The level of bankruptcy exemptions, which determines the amount of resources that a borrower is allowed to keep in bankruptcy, is a key determinant of the leniency of the personal bankruptcy system.\(^1\) Although it is widely believed that some form of credit relief is desirable, existing research has not identified how to determine the optimal exemption.

The lack of a simple but general framework to assess the welfare impact of exemption changes likely explains why comparable countries and regions have substantially different exemption levels. The wide dispersion in exemption levels across U.S. states is an example of this phenomenon. For instance, as of 2016, the homestead exemption for a bankrupt household in Massachusetts is $500,000, while the homestead exemption for an identical household in Alabama is $30,000.

In this paper, I study the question of how to set bankruptcy exemptions optimally. Using a canonical equilibrium model of unsecured credit, I show that, in a large class of economies, a few observable or potentially recoverable variables are sufficient to determine whether a bankruptcy exemption level is optimal, or should be increased or decreased. I then measure such observables to provide empirical estimates of the marginal welfare gains or losses associated with changing exemption levels for households in different income quintiles across U.S. states. These empirical estimates suggest that increasing current exemption levels is associated with welfare gains of varying degrees across states and income quintiles.

I first illustrate the main results of the paper in a baseline two-period model in which identical risk averse households borrow from competitive lenders using a debt contract with an option to declare bankruptcy. In this environment, I show that the marginal welfare change induced by a change in the exemption level \(m\) – measured in date 0 dollars – corresponds to the following expression:

\[
\frac{dW}{dm} = \frac{\partial q_0}{\partial m} b_1 + \pi_m E_m \left[ \beta u' \left( c^D_1 \right) \right]. \tag{1}
\]

This characterization shows that an elementary tradeoff determines whether it is optimal to increase or decrease the exemption level. On the one hand, a one-dollar increase in the exemption level \(m\) changes the price of debt issued by households by \(\frac{\partial q_0}{\partial m}\) (always negative), which, when multiplied by the amount of debt issued, \(b_1\), exactly captures the marginal cost of a more lenient bankruptcy procedure. This formalizes the notion that high exemptions hurt intertemporal smoothing by restricting credit supply.

\(^1\) I describe the institutional environment surrounding consumer bankruptcy in the U.S. in the Appendix. Figure 6 documents the evolution of bankruptcy filings in recent years. The Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005 is the last significant legislative change related to bankruptcy.
On the other hand, a one-dollar increase in the exemption level effectively increases the resources in the hands of bankrupt borrowers in those states in which they file for bankruptcy with non-exempt resources. Multiplying the probability of filing for bankruptcy with non-exempt resources, \( \pi_m \), by the valuation of a marginal dollar over those states, \( \mathbb{E}_m \left[ \frac{\beta u'(c_D^m)}{u'(c_0)} \right] \), exactly accounts for the marginal benefit of increasing exemptions. This formalizes the notion that high exemptions improve households' ability to insure. Importantly, all elements of Equation (1) have empirical counterparts, which opens the door to finding empirical estimates of marginal welfare changes.

It may come as a surprise that the characterization of the marginal welfare change, \( \frac{dW}{dmu'(c_0)} \), does not explicitly incorporate households' behavioral responses regarding how much to borrow and when to default. This occurs because households borrow and default optimally. The same logic applies to other optimally chosen variables, including labor supply or education decisions. This is a central insight of this paper that has thus far been overlooked in the context of bankruptcy design.

Because all right hand side variables in Equation (1) are endogenous to the exemption level, I explore the mapping between primitives and welfare through the identified sufficient statistics. In particular, I numerically illustrate how higher levels of default deadweight losses make bankruptcy exemptions less attractive by increasing the marginal cost of increasing exemptions. Intuitively, high levels of deadweight losses make credit supply schedules more sensitive to changes in the exemption level, which makes high exemptions less desirable. I also illustrate how increasing households' income uncertainty makes bankruptcy exemptions more desirable by increasing the marginal benefit of increasing exemptions. Intuitively, higher uncertainty over future income realizations increases the probability of filing for bankruptcy, which increases the insurance value of higher exemption levels.\(^2\)

Subsequently, I extend the characterization of the marginal welfare change to a dynamic model with general preferences, richer investment and borrowing opportunities, flexible income processes, and that features household heterogeneity. The insights from the baseline model extend to this more general environment after appropriately weighting a sequence of the same marginal cost and benefit terms identified in Equation (1). In particular, I show that the identified sufficient statistics accommodate rich specifications of households' income processes that may include permanent and transitory shocks, health or family shocks, and life cycle considerations, which previous literature has shown to be important determinants of households' behavior. I also show that the precise determination of households' bankruptcy decisions, often challenging to characterize, only affects welfare through the identified sufficient statistics. In the context of this general model, I discuss how to account for multiple relevant institutional features, including the possibility of filing for bankruptcy under

\(^2\)This result does not necessarily hold in a fully dynamic model, especially if the income uncertainty is transitory. This idea is emphasized in Livshits et al. (2007).
Chapter 7 or 13, the role of informal bankruptcy, and the existence of filing fees, among others.

This paper builds on existing structural work, since normative statements require a well-defined microfounded model. However, the new approach developed in this paper is most powerful when used to conduct a direct measurement exercise. In Section 4, building on the general model, I combine data from multiple sources to construct credible empirical counterparts of the identified sufficient statistics for income quintiles across U.S. states in the period 2008-2016. In order to measure the marginal cost of increasing exemptions, I first estimate sensitivities of credit supply schedules to exemption levels by using variation in exemption levels over time for different states. I then combine the estimated sensitivities with measures of the level of unsecured personal debt and auto loans per income quintile. In order to measure the marginal benefit of increasing exemptions, I first estimate transition probabilities into bankruptcy (with non-exempt resources) for households in different quintiles. I then combine them with measures of households’ relative valuations across states, calculated from consumption expenditure changes across states. Using a recursive approach, I combine the directly measured estimates of two-period flow gains/losses with estimates of the transition dynamics of households across different income quintiles to be able to compute marginal welfare gains in net present value terms that account for households’ income dynamics.

The baseline empirical estimates yield three main conclusions. First, increasing the level of exemptions is associated with welfare gains across all quintiles over all the states considered. This is due to the low estimated sensitivity of credit supply schedules to exemption changes, which makes marginal costs an order of magnitude smaller than benefits. Second, the marginal welfare gains are higher in southern states, which experience a large incidence of bankruptcies, feature a large share of bankrupt borrowers with non-exempt resources, and whose households’ perceive transition towards low income states as costlier in terms of consumption. Third, the marginal welfare gains are higher in willingness-to-pay terms for households in middle to higher income quintiles. This occurs because these households value the insurance offered by exemptions in bad states more than households that do not have much to insure.

On average, households value a marginal increase in the exemption level of one-dollar at 0.48 basis points. This number should be interpreted as households’ marginal willingness-to-pay for an exemption increase. That is, the net present value of the benefits associated with an increase in exemptions of $50,000 is roughly $240 for an average household, although there is substantial variation across states and income quintiles. In some states, this number can be close to $2,000, while for others it is close to zero. Extrapolating to the U.S. as a whole, a general $50,000 increase in exemptions is valued at $30bn. I discuss in detail the sensitivity of the results to estimate choices and preference parameters.

Lastly, I show how to account for three additional channels that modify the characterization of marginal welfare changes. First, I consider households who make borrowing or default decisions under distorted beliefs or preferences. In this case, the welfare impact of an exemption change must account for the impact
on households’ borrowing or default decisions. Second, I show that, provided that lenders’ profits are invariant to the exemption level, the sufficient statistics identified remain valid, regardless of whether lenders have market power. Otherwise, computations of welfare changes must account for the impact of exemption changes on lenders’ profits. Third, I show that the general equilibrium impact of exemption changes on prices (pecuniary effects) and aggregate demand also have a first-order effect on welfare. Even though each extension introduces a new channel that needs to be accounted for when calculating marginal welfare changes, the main forces highlighted in this paper do not vanish and still provide a natural starting point to assess the desirability of varying exemptions.

In the Appendix, I describe examples of the general environment analyzed in the paper and study additional extensions, including long-term debt and pooling and exclusion of borrowers, among others.

Related Literature This paper contributes to several strands of the literature on bankruptcy. The theoretical results in this paper contribute to the literature on applications of general equilibrium with incomplete markets, which has studied the possibility of default in general environments. Zame (1993) and Dubey et al. (2005) are the first to formalize the core tradeoffs behind the design of bankruptcy systems. They show that introducing a default option may be welfare improving in a model with incomplete markets since it creates new insurance opportunities by introducing contingencies into existing contracts. While these papers take exemptions/default penalties as predetermined, I focus on the optimal determination of exemptions. Interestingly, despite acknowledging in Section 7 of their paper that the optimal penalty associated with default is neither zero nor infinite, Dubey et al. (2005) do not pursue its optimal determination. It is well known that default is only beneficial when markets are initially incomplete. Allowing for default when agents can write fully state contingent contracts, as in Kehoe and Levine (1993), Kocherlakota (1996), Alvarez and Jermann (2000), or Chien and Lustig (2010), only restricts the contracting space, reducing welfare unequivocally.

Many recent contributions to the question of consumer bankruptcy have been quantitative. For instance, the papers by Chatterjee et al. (2007) and Livshits et al. (2007) – by now standard references – provide a careful quantitative structural analysis of unsecured credit and bankruptcy from a macroeconomic perspective. See Livshits (2015) for a recent survey. Because of their rich equilibrium features, these papers have evaluated numerically the welfare implications of alternative bankruptcy procedures. I take a complementary but different approach. As in this literature, I use a microfounded model to be able to make normative statements. However, I do not calibrate the primitives of a specific model. Instead, I seek to provide analytical insights and identify a small set of observables that must be measured to determine optimal exemptions.

within a large class of models.

Methodologically, this paper develops a sufficient statistic approach for the problem of bankruptcy and security design. It derives formulas for the welfare consequences of policies in terms of observables rather than primitives – see Chetty (2009) and Weyl (2015). An advantage of this approach is that it applies to a large class of models. Applications of this approach include Diamond (1998) and Saez (2001) – hence the title analogy – on income taxation, Chetty (2006) and Shimer and Werning (2007) on social and unemployment insurance, Arkolakis et al. (2012) on the welfare gains from trade, Berger et al. (2017) on the impact of house prices changes on consumption, and Alvarez et al. (2016) and Auclert (2016) on monetary policy. In the context of financial markets, most closely related is the work of Matvos (2013), who uses sufficient statistics to estimate the benefits of contractual completeness. See also Dávila and Goldstein (2018) on deposit insurance.

The empirical findings of this paper contribute to the growing empirical literature that studies the impact on bankruptcy with unsecured credit. Earlier work providing evidence on consumer bankruptcies includes Gropp et al. (1997), Sullivan et al. (2001), and Fay et al. (2002). More recently, Mahoney (2015), Dobbie and Song (2015), Dobbie and Goldsmith-Pinkham (2014), Cerqueiro and Penas (2016), Albanesi and Nosal (2015), and Herkenhoff et al. (2016) shed light on the impact of bankruptcy procedure on household decisions, including health insurance, consumption, labor supply, and borrowing choices. The interest rate regressions that recover the sensitivity of interest rates schedules to exemption levels are most closely related to the work of Severino and Brown (2017), who empirically identify the impact of exemption changes on the quantity of credit and interest rates. See White (2007, 2011) for recent surveys of this body of work.

Finally, since varying exemptions affects the ability to enforce contracts, this paper relates to the theoretical literature on optimal contract enforcement. Krasa and Villamil (2000) study optimal contracting in a costly state verification environment when enforcement is endogenous but costly. Koeppl (2007) studies an optimal dynamic risk sharing problem along the lines of Kehoe and Levine (1993), in which the degree of commitment to repaying loans is endogenously determined. Kovrijnykh (2013) studies the impact of partial commitment on allocations and welfare in a related environment. Koeppl et al. (2014) jointly characterize the efficient choice of contract enforcement and the efficient allocation of capital in a production economy.

Outline Section 2 introduces the main results of the paper within the baseline model. Section 3 extends the main results to a general dynamic environment. Section 4 presents the empirical application, and Section 5 studies additional channels relevant to the design of optimal exemptions. Section 6 concludes. All proofs and derivations are in the Appendix.
2 Baseline Environment

This section introduces the main results of the paper in the simplest possible model: a two-period environment in which risk averse households borrow from competitive lenders using a debt contract with an option to declare bankruptcy. To simplify the exposition, I proceed as if the model features an interior solution with positive borrowing and describe the pertinent regularity conditions in the Appendix.

2.1 Environment

There are two dates, denoted by $t = \{0, 1\}$. At date 1, there are a continuum of possible states $s \in [\underline{s}, \bar{s}]$ with cdf $F(s)$. There is a single consumption good (dollar) in this economy, which serves as numeraire. There is a unit measure of ex-ante identical households who borrow from a unit measure of lenders.\footnote{Alternatively, the problem studied in this section can be interpreted as the problem of a single household who borrows from competitive lenders.}

Households

Households, who are risk averse, have well-behaved preferences represented by the discounted sum of expected utilities of consumption, with a discount factor $\beta \in (0, 1)$. Formally, households’ objective function corresponds to

$$u(c_0) + \beta \mathbb{E}[u(c_1(s))],$$  \hspace{1cm} (2)

where $u(c_0)$ denotes the flow utility of date 0 consumption and $u(c_1(s))$ denotes the flow utility of date 1 consumption in state $s$.

At date 0, households decide how much to consume and how much to borrow. Households consume their initial endowment of $n_0$ dollars in addition to the amount they borrow from lenders using a defaultable non-contingent contract (a debt contract). Formally, households face the following date 0 budget constraint

$$c_0 = n_0 + Q_0(b_1, m),$$  \hspace{1cm} (3)

where $n_0$ denotes households’ initial endowment and $Q_0(b_1, m)$ denotes the total amount raised from lenders by a household who takes on debt with face value $b_1$ when the bankruptcy exemption level (described below) is $m$ dollars. The schedule $Q_0(b_1, m)$, determined by lenders’ as described below, can be written as the product of the face value of debt, $b_1$, and the price of debt, $q_0(b_1, m)$, as follows $Q_0(b_1, m) = q_0(b_1, m)b_1$. Note that the inverse of the price of debt, $\frac{1}{q_0(b_1, m)}$, represents an interest rate schedule. When choosing $b_1$, households understand that $Q_0(b_1, m)$ or, equivalently, $q_0(b_1, m)$, are functions of the promised repayment $b_1$.

At date 1, households receive a stochastic endowment $n(s) > 0$ that depends on the realization of the state $s$. After $s$ is realized, households decide whether to satisfy their promised repayment $b_1$ or to default.\footnote{The terms bankruptcy and default are synonyms in this paper. See Section 3 for an explicit distinction between formal bankruptcy and informal bankruptcy (default) and for how to interpret the model to account for Chapter 7 and Chapter 13 bankruptcies.} If households repay, they
simply consume their endowment net of the repayment $b_1$. If households default, they keep their endowment up to the bankruptcy exemption of $m$ dollars, while any non-exempt resources, $\max \{ n(s) - m, 0 \}$, are transferred to lenders. Formally, households’ consumption after repayment ($N$) and after default ($D$), corresponds to

\[ c_1^N (s) = n(s) - b_1 \]  
\[ c_1^D (s) = \min \{ n(s), m \}. \]  

Therefore, we can recursively express the problem faced by households at date 0 and their indirect utility for a given exemption $m$, $W(m)$, as follows

\[ W(m) = \max_{b_1} u(c_0) + \beta \tilde{V}(b_1, m), \]  
\[ \text{s.t. Eq. (3)}, \]  

where $\tilde{V}(b_1, m) = \mathbb{E}[V(b_1, s, m)]$ and where $V(b_1, s, m)$ denotes the date 1 continuation value for a household with debt $b_1$ in state $s$ for a given exemption $m$, defined as

\[ V(b_1, s, m) = \max \{ u(c_1^D(s)), u(c_1^N(s)) \}, \]  
\[ \text{s.t. Eq. (4) and (5)}. \]  

**Lenders** The supply of credit is determined by the schedule $Q_0(b_1, m)$, which defines the total dollar amount that households receive from lenders at date 0 when they take a loan with face value $b_1$. Provided that $Q_0(b_1, m)$ is well behaved, the main results of the paper can be derived without further assumptions on lenders’ behavior. However, to study a fully specified environment, I assume that lenders are risk neutral, perfectly competitive, and that they require a given rate of return $1 + r$. Under these assumptions, $Q_0(b_1, m)$ corresponds to

\[ Q_0(b_1, m) = \frac{\delta \int \max \{ n(s) - m, 0 \} dF(s) + b_1 \int c_1 N dF(s)}{1 + r}, \]  

where $1 - \delta$ denotes the deadweight loss associated with default and $D$ and $N$ respectively denote the default and no default regions, determined in equilibrium.

**Equilibrium definition** Given an exemption level $m$, an equilibrium is defined as a set of consumption and debt allocations $c_0$, $c_1(s)$, and $b_1$, default decisions, and a credit supply schedule $Q_0(b_1, m)$ such that i) households borrow and default optimally, internalizing the credit supply schedule, and ii) lenders’ credit supply schedule satisfies a zero-profit condition.

### 2.2 Equilibrium Characterization

I sequentially characterize households’ default decision, lenders’ credit supply schedule, and households’ borrowing choice. At date 1, households default whenever their consumption defaulting is higher than their consumption repaying.
Formally, households default decision can be expressed as

\[
\begin{align*}
\text{if } n(s) - b_1 > m, & \quad \text{Default} \\
\text{if } n(s) - b_1 \leq m, & \quad \text{No Default.}
\end{align*}
\]

As Figure 1 illustrates, households find optimal to default for low values of \(n(s)\). This model incorporates both forced/involuntary default and strategic/voluntary default. Forced default happens when a household does not have enough resources to fully pay back its debt, which occurs when \(n(s) < b_1\). Strategic default happens when households have enough resources to repay their debt but decide not to do so, which occurs when \(b_1 < n(s) < m + b_1\).

Given borrowers’ default decision, lenders offer the following credit supply schedule in equilibrium

\[
Q_0(b_1, m) = \delta \int_{\hat{s}}^{\hat{s}} \max \{n(s) - m, 0\} dF(s) + b_1 \int_{\hat{s}}^{\hat{s}} dF(s) + \frac{b_1 (\hat{s} - b_1)}{1 + r^f},
\]

where \(\hat{s} = n^{-1}(m + b_1)\) denotes the threshold state that separates the default region from the no default region and \(\hat{s} = n^{-1}(m)\) denotes the threshold state that separates the partial recovery default region from the zero recovery default region. Figure 2 illustrates the repayment received by lenders. As shown in the Appendix, the credit supply schedule decreases with the exemption \(m\), that is, \(\frac{\partial Q_0(b_1, m)}{\partial m} = \frac{\partial \hat{s}_0(b_1, m)}{\partial m} \leq 0\), \(\forall b_1\). This occurs for two reasons. First, an increase in \(m\) reduces lenders’ recovery rates in default states. Second, an increase in \(m\) expands the default region, which increases deadweight losses associated with default. Both effects are passed to households through a lower \(Q_0(b_1, m)\). I show
that $Q_0(b_1, m)$ is increasing in $b_1$ when $b_1 = 0$, that is, $\lim_{b_1 \to 0} \frac{\partial Q_0(b_1, m)}{\partial b_1} > 0$, and single peaked, provided that the deadweight loss of default is non-zero.

At date 0, households optimally choose the equilibrium level of borrowing by solving
\[
\max_{b_1} u(c_0) + \beta \tilde{V}(b_1, m),
\]
where $c_0 = n_0 + Q_0(b_1, m)$ and
\[
\tilde{V}(b_1, m) = \int^{\hat{s}}_s u(c^D_1(s)) \, dF(s) + \int^{\hat{s}}_{\hat{s}} u(c^N_1(s)) \, dF(s),
\]
where $\hat{s} = n^{-1}(m + b_1)$. At the optimum, households' choose how much to borrow according to
\[
u'(c_0) \frac{\partial Q_0}{\partial \theta_1} = \beta \int^{\hat{s}}_{\hat{s}} u'(c^N_1(s)) \, dF(s). \tag{11}
\]
The left hand side of Equation (11) captures the marginal benefit of increasing the promised repayment $b_1$. It corresponds to the increase in resources available at date 0, $\frac{\partial Q_0}{\partial \theta_1} = q_0 + \frac{\partial q_0}{\partial \theta_1} b_1$, valued at households’ date 0 marginal utility, $u'(c_0)$. The right hand side of Equation (11) captures the marginal cost of increasing the promised repayment. It corresponds to the discounted marginal utility of a dollar aggregated over the states in which households actually repay their debt.

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To simplify the exposition, I proceed in this section as if $\underline{s} < \hat{s} < \bar{s}$. The Appendix includes a full characterization.
Differentiating Equation (11) with respect to $m$ determines how households’ equilibrium borrowing reacts to changes in the exemption level. I show in the Appendix that the sign of $\frac{db_1}{dm}$, which is ambiguous, is determined by a combination of income, substitution, and direct effects. However, because households borrow optimally, the value of $\frac{db_1}{dm}$ is not necessary to compute the welfare impact of varying the exemption level, as shown next.

2.3 Main Results

Given that Equations (9), (10), and (11) fully characterize the equilibrium for a given exemption level, it is now possible to understand how changes in the exemption level $m$ affect social welfare. Because lenders make zero profit in equilibrium, maximizing households’ date 0 indirect utility $W(m)$, defined in Equation (6), maximizes social welfare in this economy. Proposition 1, which presents the central result of this paper, provides a test for whether to optimally increase or decrease the exemption level.

Proposition 1. (Directional test for a change in the exemption level $m$)

The normalized welfare change induced by a marginal change in the bankruptcy exemption $m$, expressed as a money-metric in date 0 dollars, is given by:

$$\frac{dW}{dm} = \frac{\partial q_0}{\partial m} b_1 + \pi_m E_m \left[ \frac{\beta u'(c^D_1)}{u'(c_0)} \right],$$

where $\pi_m = \int_{\hat{s}}^{\tilde{s}} dF(s)$ denotes the probability of filing for bankruptcy while claiming the full exemption, $\hat{s} = n^{-1}(m + b_1)$ denotes the threshold that separates the bankruptcy region from the no-bankruptcy region, $\tilde{s} = n^{-1}(m)$ denotes the threshold that separates the region in which households claim the full exemption from the region in which they do not, and $E_m \left[ \frac{\beta u'(c^D_1)}{u'(c_0)} \right] = \beta$

$$\frac{\pi_m}{\int_{\hat{s}}^{\tilde{s}} dF(s)} \int_{\hat{s}}^{\tilde{s}} \frac{u'(c^D_1(s))}{u'(c_0)} dF(s)$$

denotes the conditional expected value as of date 0 of a claim that pays a dollar in states in which bankrupt households claim the full exemption.

Proposition 1 formalizes the tradeoff between intertemporal smoothing and insurance behind the optimal design of bankruptcy procedures. On the one hand, an increase in the exemption level $m$ makes households’ borrowing more expensive through a reduction in the price of the debt issued, $\frac{\partial q_0}{\partial m} < 0$ (equivalently, through an increase in the interest rate charged by lenders). This increase in borrowing costs captures the marginal cost of a more lenient bankruptcy procedure and formalizes the notion that high exemptions hurt intertemporal smoothing by restricting credit supply through the response of lenders.

On the other hand, an increase in the exemption level $m$ increases the resources that bankrupt households are allowed to keep when claiming the full exemption, that is, in states in which they file for bankruptcy with non-exempt
resources. From an ex-ante perspective, households value these additional resources by combining the probability of declaring bankruptcy with non-exempt resources, $\pi_m$, with the conditional expected value of a marginal dollar in those states, $E_{m}\left[\beta u^{'}(c^{P}_{D})/u^{'}(c_{0})\right]$, according to their own stochastic discount factor. The households’ valuation of the additional dollar of resources kept in bankruptcy captures the marginal benefit of a more lenient bankruptcy procedure and formalizes the notion that high exemptions improve households’ ability to insure.

Two properties of the environment considered here that are crucial to derive such simple characterization. The first property is that households borrow and default optimally. This fact allows us to set to zero the welfare impact of exemption changes on the total amount borrowed $d_{b}$ and the probability of bankruptcy $d_{\pi}$, since these have a second-order impact on households’ welfare. The second property is that lenders’ welfare is invariant to exemption changes. This fact eliminates the need to account for the direct effect of exemption changes on lenders’ behavior on households’ welfare. For instance, the value of deadweight losses associated with bankruptcy is a direct input into the determination of $\frac{\partial q_{0}}{\partial m}$, as well as any other factor that determines lenders’ credit supply decision.

Several insights emerge from the characterization established in Proposition 1. The following corollary, which provides the foundation for the empirical application in Section 4, formulates a central insight of the paper.

**Corollary 1.** (Sufficient statistics) Proposition 1 implies that measures of four observable or recoverable variables are sufficient to determine whether a bankruptcy exemption level is optimal or should be increased or decreased. These variables are i) households’ debt position, $b_{1}$, ii) the sensitivity of the credit supply schedule to a change in the exemption level, $\frac{\partial q_{0}}{\partial m}$, iii) the probability of filing for bankruptcy while claiming the full exemption, $\pi_{m}$, and iv) the conditional expected value of a dollar, using the households’ stochastic discount factor, in states in which bankrupt households claim the full exemption, $E_{m}\left[\beta u^{'}(c^{P}_{D})/u^{'}(c_{0})\right]$.

While the abstract tradeoff between intertemporal smoothing and insurance can be traced back to Zame (1993) and Dubey et al. (2005), Corollary 1 shows that it is sufficient to observe a small number of variables to exactly quantify that tradeoff. Equation (12) provides a simple test, expressed as a function of potentially observable variables, for whether it is optimal to increase or decrease the bankruptcy exemption, starting from a given level. Section 3 shows that a modified version of Equation (12) that accounts for dynamics and heterogeneity in a more general environment shares the same set of sufficient statistics.

It is worth highlighting that the only sufficient statistic that needs to be computed by running a regression is the sensitivity of the credit supply to exemption changes. The remaining variables can be directly measured in the data without any transformation. The key object of interest to compare the marginal benefit from increasing $m$, which accrues at date 1, to the date 0 cost, is the
households’ valuation of resources in bankruptcy states relative to borrowing states. Under CRRA preferences, measures of consumption in bankruptcy relative to measures of consumption when borrowing are sufficient to construct such valuation measures.

The value of the marginal welfare change \( \frac{dW}{dm} \) is exact and that does not contain any approximation error. In order to assess the welfare impact of non-local changes, one can integrate over the relevant exemptions. Formally, an increase the exemption level from \( m_0 \) to \( m_1 \) is associated with a welfare change given by

\[
W(m_1) - W(m_0) = \int_{m_0}^{m_1} \frac{dW}{dm} (\tilde{m}) \, d\tilde{m},
\]

where \( \frac{dW}{dm}(\tilde{m}) \) is given by

\[
\frac{dW}{dm} = u'(c_0) \frac{\partial q_0}{\partial mb_1} + \beta \int_{\tilde{s}} \tilde{s} \, u'(c_D^1(s)) \, dF(s)
\]

and defined for each exemption level \( \tilde{m} \in [m_0, m_1] \). Similarly, we can characterize the fixed point that determines \( m^* \) by setting \( \frac{dW}{dm} = 0 \). For instance, in the baseline model, the optimal exemption satisfies

\[
m^* = \frac{\Pi m}{\Lambda \frac{c_q}{c_q + b_1 n_0} - \Lambda \frac{c_0}{c_0 + b_1 n_0} - \beta u'(c_D^1) dF(s)}.
\]

where \( \Lambda \equiv \frac{q_0 b_1}{\ln(1+r)} \) and \( \Pi m \equiv \int_{\tilde{s}} \tilde{s} \frac{c_D^1(s)}{c_0} \, dF(s) \). However, it is not possible to solve for the optimal exemption level \( m^* \) as a function of primitives. Instead, by repeatedly applying Equation (13), under appropriate regularity conditions, one would eventually find the optimal bankruptcy exemption level in a given economy. Although the equation characterizing \( m^* \) must hold at the optimum, it does not provide a characterization of \( m^* \) as a function of primitives, because all right hand side variables are endogenous to the exemption level. Alternatively, one could make direct assumptions on how the sufficient statistics vary with \( m \) and then solve for a fixed point. Consistently with the empirical implementation in Section 4, I focus on marginal welfare changes, which can be credibly measured.

**Advantages of the approach** Proposition 1 advances the understanding of how to set bankruptcy exemptions optimally in two ways. First, the sufficient statistics can be measured directly to construct empirical counterparts of the marginal welfare change \( \frac{dW}{dm} \). This approach motivates the empirical application conducted in Section 4. Importantly, since the exact specification of many

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7The condition \( \frac{dW}{dm} = 0 \) is necessary but not sufficient to find the optimal exemption. As usual in problems with continuous distributions of shocks, the convexity of the planning problem is in general not guaranteed. I discuss sufficient conditions for convexity and show that the model is well-behaved numerically for standard primitives in the Appendix.
primitives becomes irrelevant provided that the sufficient statistics can be measured directly, finding credible estimates of these variables becomes the ideal approach to determine the optimal exemption level. My results highlight the importance of improving the measurement of the variables identified in Corollary 1. This is the form in which the results of the paper are more powerful, since they encompass a much larger class of models that could be characterized analytically or solved computationally.

Alternatively, the characterization of Proposition 1 can be used to discipline and improve the understanding of theoretical and structural modeling. This approach motivates the numerical simulation conducted in Section 2.4, which links primitives to the optimal exemption through the sufficient statistics identified in Corollary 1. In richer models, one could set the sufficient statistics as targets to discipline a model’s calibration, which would guarantee that the model predictions for the welfare impact of exemption changes are locally exact.\(^8\)

The main drawback of the approach developed in this paper when compared to solving a fully specified model is the inherently local nature of the measurement exercise. Although, as just described above, one could in principle construct measures of the relevant sufficient statistics for any value of the exemption level and integrate over them to find global results, this is challenging to do in practice. Consequently, extrapolating marginal welfare changes far away from observed exemption levels may require to calibrate a fully specified quantitative model.

2.4 Numerical Simulation: Behind the Sufficient Statistics

Because every element of Equation (12) is endogenous, it is not easy to establish a direct link between the primitives of the model and the desirability of increasing or decreasing the exemption level. Before generalizing the results and implementing them empirically, I numerically solve the baseline model for different parameter specifications. The goal of this exercise is to illustrate the link between different model primitives and sufficient statistics and how these influence the optimal exemption policy.\(^9\) This exercise highlights how by providing an intermediate link between primitives and welfare assessments, the approach developed in this paper is helpful to improve our understanding of complex structural models.

Given the simplicity of the baseline model, it would be ill-advised to try to conduct a serious calibration. I present here instead a numerical illustration of the underlying relevant channels. A credible empirical application based on the

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\(^8\)Livshits (2015) describes existing normative structural work on bankruptcy by stating that: “the welfare assessments of bankruptcy regimes and reforms have been quite wide-ranging”. There is scope to also use the sufficient statistics identified in this paper to distinguish the channels through which different quantitative papers have reached different conclusions.

\(^9\)A reader mostly interested in the measurement of the sufficient statistics can jump to Section 4 at little cost.
sufficient statistics identified in the context of the general model is developed in Section 4.

To numerically solve the model, I assume that households have constant relative risk aversion utility (CRRA) preferences, that is, \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \), with relative risk aversion coefficient \( \gamma \equiv -c \frac{u''(c)}{u'(c)} \). I assume that the distribution \( F(s) \) of date 1 endowments is a log-normal with parameters of the underlying normal denoted by \( \mu \) and \( \sigma \). Parameters values, chosen to best illustrate the qualitative features of the model, are included in the description of each figure.

**Comparative Statics through Sufficient Statistics** I include in Appendix ?? a graphical illustration of each of the relevant objects that determine the equilibrium, including lenders’ credit supply and households’ default and borrowing decisions, as well as social welfare. However, in the text, I focus on two illustrative comparative statistics exercises. First, I describe through the lens of Proposition 1 how changes in the level of deadweight losses \( (1-\delta) \) affect the determination of the optimal exemption level. Second, I use the same approach to describe how changes in the volatility of households’ date 1 endowments \( (\sigma) \) affect the determination of the optimal exemption level. It is possible to carry out similar comparative statics exercise for other parameters of the model.

The top plots in Figure 3 illustrate how the optimal exemption changes with the recovery rate \( \delta \) (left plot) and the volatility of households’ date 1 endowments \( \sigma \) (right plot).\(^{10}\) Figure 3 shows that higher recovery rates \( \delta \) (lower deadweight losses in bankruptcy) and more volatile endowments are associated with higher optimal exemptions. The bottom plots in Figure 3 show that households are worse off when deadweight losses are large (\( \delta \) is low) and when volatility is high, for any value of exemptions. These plots show that there is greater scope for welfare improvements through exemption changes when deadweight losses are small and when volatility is large, a similar insight that can be drawn from the top plots in Figure 4. The bottom plots also illustrate the potential drawback of the local approach developed in this paper. For instance, when \( m \) is sufficiently low, the welfare function is close to flat, even though the exemption level is far from optimal.

The next three rows of plots in Figure 4 allow us to decompose the marginal welfare change into marginal benefits and costs. Figure 4 shows that the credit supply is less sensitive to exemption changes everywhere when \( \delta = 0.9 \) relative to when \( \delta = 0.1 \). This should be intuitive, since when recovery rates are high, the total amount recovered by lenders after default varies less, reducing the sensitivity to exemption changes. This fact reduces the marginal cost of increasing the exemption level. Interestingly, since lenders pass the higher recovery rate to households through more attractive borrowing conditions, households borrow more in equilibrium (\( b_1 \) is higher when \( \delta = 0.9 \)). On the one hand, the increase in borrowing increases the marginal cost of borrowing, but only substantially for high levels of \( m \). On the other hand, the increase in borrowing, through the

\(^{10}\) Given that the distribution of \( n_1(s) \) is log-normal, varying \( \sigma \) also affects the mean of households’ date 1 endowment, but this effect is quantitatively insignificant.
Figure 3: Comparative Statistics $m^*(\delta)$ and $m^*(\sigma)$

**Note:** The left panel of Figure 3 shows how the optimal exemptions $m^*$, defined in Equation (14), varies as a function of the recovery rate by lenders in case of default $\delta$ (the complement of the dead-weight loss). The single period is meant to represent three years, with parameters $\beta = 0.92^3$, $r^f = 1.04^3 - 1$, $\gamma = 2$, $n_0 = 1$, $\mu = 0.15$, $\sigma = 0.18$, $\bar{s} = 0.2$, and $T = 2$. I consider a range for the recovery rate $\delta$ given by $\delta \in [0, 0.9]$. The right panel of Figure 3 shows how the optimal exemptions $m^*$, defined in Equation (14), varies as a function of the volatility of the distribution of date 1 endowments ($\sigma$). The single period is meant to represent three years, with parameters $\beta = 0.92^3$, $r^f = 1.04^3 - 1$, $\gamma = 2$, $\delta = 0.75$, $n_0 = 1$, $\mu = 0.2$, $\bar{s} = 0.2$, and $T = 2$. I consider a range for the volatility given by $\sigma \in [0, 0.26]$. 
Figure 4: Marginal Welfare Change and Marginal Cost/Benefit Decomposition

Note: Each row of plots in Figure 4 respectively shows, from top to bottom: i) the normalized social welfare change as a function of $m$, defined in Equation (12), ii) the normalized marginal benefit, $\frac{\partial q_0}{\partial m}$, and cost, $\beta \int_{s} \frac{\psi'(c')}{\psi(c_0)} dF(s)$, iii) the components of the normalized marginal benefit, $\frac{\partial q_0}{\partial m}$ and $b_1$, and iv) the components of the normalized marginal cost, $\pi_m$ and $E_m$. All left figures have parameters $\beta = 0.92$, $r = 1.043$, $\gamma = 2$, $\delta = \{0.1, 0.9\}$, $\nu_0 = 1$, $\mu = 0.2$, $\sigma = 0.18$, $s = 0.2$, and $s_c = 2$. All right figures have parameters $\beta = 0.92$, $r = 1.043$, $\gamma = 2$, $\delta = 0.75$, $\nu_0 = 1$, $\mu = 0.2$, $\sigma = \{0.1, 0.26\}$, $s = 0.2$, and $s_c = 2$. 

- Marginal Welfare Change
- Normalized Marginal Benefit/Cost
- Marginal Cost Decomposition
- Marginal Benefit Decomposition
moral hazard effect, further increases the marginal benefit of borrowing, since households default more often. The marginal conditional expected valuation of a dollar, 

\[ \mathbb{E}_m \left[ \frac{\beta u'(\epsilon^I_m)}{u'(\epsilon^0_m)} \right] \]

is constant in both scenarios. Overall, the reduction in the magnitude of the credit supply sensitivity \( \left| \frac{\partial q_0}{\partial m} \right| \) associated with high recovery rate \( \delta \) dominates the other effects, pushing towards a higher optimal exemption level.

When considering variation in households’ endowment volatility \( \sigma \), the bottom plot shows that households default more often while claiming the full exemption when volatility is high. Defaulting more frequently increases the marginal benefit of increasing the exemption level for almost every value of \( m \). On the other hand, when \( \sigma \) is high, households borrow less for low values of \( m \), but at some point start to borrow more than when \( \sigma \) is low. It is also the case that the sensitivity of credit supply \( \left| \frac{\partial q_0}{\partial m} \right| \) is higher and increasing in \( m \) for low-to-intermediate exemption levels, which increases the marginal cost of increasing exemptions. Overall, the increase in the marginal benefit of higher exemptions caused by the increase in bankruptcies dominates, making a higher exemption level optimal when households’ endowment volatility is higher.

3 General Environment

This section shows that the insights derived from the baseline model described in Section 2 are robust and apply more generally. I now explicitly consider a dynamic environment in which a cross-section of households with time- and state-dependent utility make endogenous labor supply decisions and have access to a rich set of investment and borrowing opportunities. The environment considered here provides the foundation for the empirical application in Section 4.

3.1 Environment

Households There are \( i = 1, \ldots, |I| \) different types of households in the economy. Every household lives for \( T \leq \infty \) periods, where \( t = 0, 1, \ldots, T \). Households can invest in a set \( J \) of assets, indexed by \( j = 1, \ldots, |J| \). Households can also borrow using a set \( K \) of contracts, indexed by \( k = 1, \ldots, |K| \). Households, which maximize expected utility of consumption and leisure, have the option to declare bankruptcy, as described below.

It is convenient to formulate the households’ problem recursively. I denote the set of endogenous state variables for a type \( i \) household by \( \Psi_i = \{ a_i, b_i, x_i \} \), where \( a_i \) denotes a \( |J| \times 1 \) vector of assets (individual elements are denoted by \( a^j_i \)), \( b_i \) denotes a \( |K| \times 1 \) vector of liabilities (individual elements are denoted by \( b^k_i \)), and \( x_i \) denotes a vector of other endogenous non-financial state variables. The vector \( x_i \) may include households’ human capital stock or a bankruptcy indicator, among other variables, and evolves according to a par-
with an obligation term debt. Each asset considers one-period liabilities – see the Appendix for how to account for long-term debt. Each asset \( j \) has a payoff \( z_j(s_i) \), while every liability \( k \) is associated with an obligation \( z_k(s_i) \), which depends on the realization of the state \( s_i \). For instance, a debt contract can be modeled as a given liability with a constant repayment \( z_k(s_i) = \sum_{\Omega} W_k, \forall s_i \).

The flow utility of type \( i \) household in period \( t \) is given by \( u_{i,t}(c_{i,t}, h_{i,t}; \Omega_t) \), where \( c_{i,t} \) and \( h_{i,t} \) respectively denote consumption and hours worked. The net income of a household of age \( t \) and type \( i \) is given by \( y_{i,t}(x_i, s_i) + w_{i,t}(x_i, s_i) h_{i,t} \). The term \( y_{i,t}(x_i, s_i) \) corresponds to the net non-financial non-labor related income. If positive, it can correspond, for instance, to an inheritance, while, if negative, it can correspond to uninsured medical bills, legal bills, costs of divorce, or unplanned children expenses. The term \( w_{i,t}(x_i, s_i) h_{i,t} \) corresponds to households’ labor income, where the wage \( w_{i,t}(x_i, s_i) \) can be a function of households’ past choices through \( x_i \).

At the beginning of period \( t \), each household decides whether to repay his liabilities or to default. We can express the value function of a type \( i \) household of age \( t \) at the stage in which the bankruptcy decision is made as follows

\[
V_{i,t}(\Omega_t; m) = \max \left\{ V_{i,t}^D(\Omega_t; m), V_{i,t}^N(\Omega_t; m) \right\},
\]

where the functions \( V_{i,t}^D(\cdot) \) and \( V_{i,t}^N(\cdot) \) respectively denote continuation values after declaring bankruptcy and after repaying outstanding liabilities. The value of repaying outstanding liabilities is given by

\[
V_{i,t}^N(\Omega_t; m) = \max u_{i,t}(c_{i,t}, h_{i,t}; \Omega_t) + \beta_i E_t [V_{i,t+1}(\Omega_t'; m)],
\]

subject to the following budget constraint

\[
\sum_j a_{i,t+1}^j + c_{i,t} = y_{i,t}(x_i, s_i) + w_{i,t}(x_i, s_i) h_{i,t} + \sum_j z_j(s_i) a_{i,t}^j - \sum_k z_k(s_i) b_{i,t}^k + \sum_k q_{i,t}^k(\Psi_i', \Omega_t, m) b_{i,t+1}^k,
\]

where \( q_{i,t}(\Psi_i', \Omega_t, m) \), whose determination is described below, denotes the price of liability \( k \). The value of declaring bankruptcy is given by

\[
V_{i,t}^D(\Omega_t; m) = \max u_{i,t}(c_{i,t}, h_{i,t}; \Omega_t) + \beta_i E_t [V_{i,t+1}(\Omega_t'; m)],
\]

subject to the following budget constraint

\[
\sum_j a_{i,t+1}^j + c_{i,t} = W(m; a_i, x_i, s_i) - f(\Omega_t),
\]

where \( W(m; \cdot) \), which takes the form \( W(m; \cdot) = \min \{m, \cdot\} \) with a second argument that is independent of \( m \), denotes the net amount of wealth at the disposal of a household after bankruptcy, and \( f(\Omega_t) \geq 0 \) models a filing fee,
meant to account for administrative and attorney fees. In principle Equation (18) allows bankrupt households to save or borrow after bankruptcy. The dependence of \( f(\Omega_i) \) on state variables captures that the complexity and cost of bankruptcy may vary with households’ asset composition. The function \( W(m; a_i, x_i, s_i) \) can take different forms depending on the type of exemptions considered. For instance, if \( m \) corresponds to a wildcard exemption that applies to a household’s overall portfolio, \( W(\cdot) \) can be expressed as in Equation (19). Alternatively, if \( m \) corresponds to a homestead exemption that applies to a single asset (home equity, modeled here in net terms as asset \( j = 1 \)), \( W(\cdot) \) can be expressed as in Equation (20):

\[
W(m; a_i, x_i, s_i) = \min \left\{ m, \max \{ y_{i,t}(x_i, s_i), 0 \} + w_{i,t}(x_i, s_i) \right\} \quad \text{(Wildcard)}
\]

\[
W(m; a_i, x_i, s_i) = \min \{ m, z_1(s_i) a_i^1 \} \quad \text{(Homestead)}
\]

I discuss below how this general environment captures many features considered to be relevant in the context of personal bankruptcy design.

**Lenders** The total amount of credit raised by a household \( i \) in period \( t \) is given by \( \sum_k q_{i,t}^k(\Psi_i', \Omega_i, m) b_{i,t+1}^k \). As in the baseline model, as long as this object is well behaved, and lenders’ profits are invariant to the exemption level, there is no need to make further assumptions on lenders’ behavior. However, to study a fully specified environment, I assume that lenders are risk neutral, perfectly competitive, and that they require a given rate of return \( 1 + r^l \). I also assume that there is a deadweight loss of \( 1 - \delta \) per unit of resources transferred in bankruptcy. Under these assumptions, we can express the supply of credit available to household \( i \) as follows

\[
\sum_k q_{i,t}^k(\Psi_i', \Omega_i, m) b_{i,t+1}^k = \frac{\delta \int_{D_{t+1}} W^C(m; a'_i, x'_i, s'_i) dF(s'_i|\Omega_i) + \int_{N_{t+1}} \sum_k z_k(s'_i) b_{i,t}^k dF(s'_i|\Omega_i)}{1 + r^l},
\]

where \( W^C(\cdot) \), which depends on the exact formulation of the bankruptcy procedure (e.g., homestead vs. wildcard, as described above), accounts for the repayment received by lenders in bankruptcy, and the sets \( D_{t+1} \) and \( N_{t+1} \) denote the default regions one-period forward, which are determined in equilibrium as a function of households choices \( \Omega'_i \) and the realization of exogenous shocks. In order for Proposition 2 to be valid, it is important that lenders have the ability to condition their credit supply schedule on the relevant state variables \( \Psi'_i \) and that household \( i \)’s credit supply schedules do not depend on the actions of other households, although credit supply schedules could depend on characteristics of the population of households.\(^{11}\)
Equilibrium definition  Given an exemption level $m$, an equilibrium is defined as a set of consumption, $c_{i,t}(\Omega_i;m)$, hours worked, $h_{i,t}(\Omega_i;m)$, assets, $a_{i,t}(\Omega_i;m)$, liabilities, $b_{i,t}(\Omega_i;m)$, other endogenous choices, $x_{i,t}(\Omega_i;m)$, default decisions, and credit supply schedules $q_{i,t}^k(\Psi'_i,\Omega_i;m)$, $\forall k$, for each household $i$ such that i) households make optimal decisions, internalizing the credit supply schedules, and ii) credit supply schedules offered by lenders satisfy a zero-profit condition.

3.2 Main Results

The characterization of the equilibrium is similar to the baseline model. Given households’ future default decisions, it is possible to determine credit supply schedules, which determine households’ choices. Given the dynamic nature of the model, now households’ bankruptcy and borrowing decisions have an important forward-looking component. Instead of focusing on the positive predictions of the model, which does not have a tractable analytical representation and which would rely on numerical simulations, I directly introduce the counterpart of Proposition 1 in this more general environment.

Proposition 2. (Directional test for a change in the exemption level $m$: general model) Let’s denote by $W_{i,\tau} = V^{N}_{i,\tau}(\Omega_i;m)$ the value function of a given household $i$ that has not declared bankruptcy in period $t$. The welfare change induced by a marginal change in the bankruptcy exemption $m$ for that household is given by

$$
\frac{dW_{i,\tau}}{dm} = \sum_{t=\tau}^{T} \beta_t^{t-\tau} \pi^{N}_{i,t} \mathbb{E}^{N}_{t} \left[ \frac{\partial u_{i,t}}{\partial c_{i,t}} \frac{\partial \sum_{k} q_{i,t}^k b_{i,t+1}^k}{\partial m} \right] + \sum_{t=\tau}^{T} \beta_t^{t-\tau+1} \pi^{D}_{i,t+1} \mathbb{E}^{D}_{t} \left[ \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial W_{i+1}}{\partial m} \right],
$$

where $\pi^{N}_{i,t}$ and $\pi^{D}_{i,t}$ respectively denote the equilibrium repayment and default probabilities of the household in period $t$ given period $\tau$ state variables, $\mathbb{E}^{N}_{t} [\cdot]$ and $\mathbb{E}^{D}_{t} [\cdot]$ denote the conditional expectations on repayment and default states of period $t$ random variables given period $\tau$ state variables, and $\frac{\partial W_{i+1}}{\partial m}$ denotes the partial derivative of $W(m; a_i, x_i, s_i)$ in period $t+1$.

Proposition 2 extends the results from the baseline model and shows that the nature of the sufficient statistics is the same in this more general environment. The marginal welfare change induced by a change in the exemption level is given by an appropriately weighted sum of the costs associated with the increase in rates over the set of states in which households are borrowers and the benefits in the form of higher resources available in the states in which households’ file for bankruptcy with non-exempt resources. The derivation of Proposition 2 exploits the fact that $\frac{\partial W}{\partial m}$ takes the value of one whenever bankrupt households claim the households only interact through the equilibrium credit supply schedule, the sensitivity of the credit supply schedule accounts for all the relevant information to assess the impact of exemption changes on households’ welfare. See Section 5 for how to account for general equilibrium interactions.
full exemption but is zero otherwise. This feature implies that households value a marginal increase in the exemption as a dollar only in those states in which they claim the full exemption, regardless of whether that dollar is consumed, saved, or invested.\footnote{This consideration can only arise in a dynamic model. It would be incorrect to value the marginal gain of increasing the exemption level by a dollar using the sensitivity of households’ equilibrium consumption to exemption changes.} Importantly, the nature of the sufficient statistics is the same as in the baseline model, although in a dynamic model it is necessary to also measure the likelihood of alternative future scenarios.

Several insights emerge by considering the more general case. First, the precise determination of the region(s) in which households decide to default does not change the optimal exemption formula, because households default optimally. In general, forward-looking households internalize the option value of waiting before defaulting, which crucially depends on whether shocks are temporary or permanent. These considerations, which prevent a simple characterization of households default decisions, do not modify the set of relevant sufficient statistics. Similarly, changes in hours worked and in other endogenous choice variables do not affect $\frac{dW_{i,t}}{dm}$. Second, all marginal benefits and costs associated with an exemption change are valued according to households’ own stochastic discount factor. When households’ flow utility is separable in consumption, measures of households’ consumption along with preference parameters are sufficient to appropriately value costs and benefits over time and across states. Third, the response of credit supply schedules to exemption changes still contains the relevant information to assess the marginal cost of exemption changes. As I show in the Appendix, this is also the case in environments in which households may be pooled or excluded from borrowing markets. Finally, one could derive a formula for the optimal exemption level $m^*$ that must be satisfied at the optimum, as in Equation (14). This expression averages marginal benefits/losses across periods and states but, as in the baseline model, cannot be calculated using exclusively local information.

### 3.3 Additional Remarks

The environment studied in this section is rich enough to capture the most relevant institutional features for the determination of households’ bankruptcy decisions, in particular, those often included in quantitative bankruptcy models.

**General Preferences.** Households’ preferences and risk attitudes in the model are potentially time- and state-dependent. This generality accommodates multiple phenomena. First, households can experience non-pecuniary costs associated with bankruptcy, consistently with White (1998) and Fay et al. (2002). These authors find that the fraction of households that would benefit from bankruptcy, based on pecuniary considerations, far exceeds the fraction of households who actually file for bankruptcy. This discrepancy is often attributed to stigma, social pressure, or hounding by lenders. Second, households can derive non-pecuniary benefits by holding a given asset. This is particularly relevant in the context of housing, which is an asset with a non-negligible consumption
dimension. Third, households preferences can depend on past variables, as in habit models. Overall, given the other sufficient statistics, preferences only affect the characterization of the marginal welfare change through their effect on how households value marginal gains and losses over time/ across states. For instance, this model can account for a time-varying risk premium that weights marginal costs and benefits differently over the business cycle. To simplify the exposition, I have restricted my attention to expected utility preferences, although the Appendix includes an extension with Epstein-Zin preferences.

**General Income Dynamics.** The shocks that determine households’ income processes can be mapped to health shocks, employment shocks, and family shocks (e.g., childbirths, spousal separations, etc.), which existing literature has found to be relevant drivers of borrowing and bankruptcy decisions. The model also allows for shocks to income to be permanent, transitory, or a mixture of both. This is important, since Livshits et al. (2007) show that the nature of income shocks — whether they are transitory or permanent — as well as lifecycle borrowing motives play a critical role on households decisions. Taxation and social insurance programs that affect borrowing and bankruptcy decisions can also be accommodated by reinterpreting \( y_{i,t}(x_i, s_i) \). Finally, note that households in this model can invest in their own human capital, which can be mapped to an endogenous state variable in \( x_i \) that affects households’ labor income.

**Chapter 7 vs. Chapter 13.** When a period in the model is sufficiently long, default in the model can be interpreted as a Chapter 7 or a Chapter 13 bankruptcy. As described in Appendix A, Chapter 7 bankrupt debtors give up all unsecured assets above predetermined exemption levels, in exchange for an immediate discharge of liabilities, while Chapter 13 debtors are able to keep their assets, in exchange for proposing a repayment plan. Importantly, the repayment plan must give creditors at least the same amount they would receive under Chapter 7, so the level of bankruptcy exemptions is a key determinant of Chapter 13 repayment plans. Therefore, if a period in the model is taken to be two years or longer, it is reasonable to abstract from modeling the exact details of monthly repayment plans, and simply interpret the function \( W(m; a_i, x_i, s_i) \) as the amount of net wealth available to households after filing for bankruptcy and committing to their repayment plan. Not accounting for Chapter 13 bankruptcies would underestimate the marginal benefit of exemptions in the empirical application, since virtually all Chapter 13 bankruptcies are asset bankruptcies in which marginal changes to the exemption level should affect the marginal amount of wealth kept by households in bankruptcy.

**Forced vs. Strategic Default.** The distinction between forced default, in which households declare bankruptcy because they do not have enough resources to pay back, and strategic default, in which household declare bankruptcy despite having enough resources to pay back their debt, often plays a prominent role in informal discussions about bankruptcy. The results of this paper show that this distinction is not relevant for the ex-ante optimal determination of optimal exemptions. From a normative perspective, it is only relevant whether households default and whether they claim the full exemption, but not whether
default is forced or strategic.

**Informal Default/Renegotiation.** Instead of formally filing for bankruptcy, debtors can cease payments without filing, entering into “informal bankruptcy” (Dawsey and Ausubel, 2004). If all parties anticipate and price the possibility of informal bankruptcy appropriately, and the law of motion of unpaid debts is independent of the exemption level, Proposition 2 would remain unchanged. Explicitly modeling a renegotiation process in which borrowers and lenders bargain over the terms of a new contract after default may require to account for the additional benefits of changing exemptions in those states, since households’ outside option depends directly on the exemption level. Whether this is the case depends on the details of the bargaining protocol. If lenders have full bargaining power, one may conjecture that allowing for this form of bargaining is irrelevant for the results of the paper, but there is scope to study further how bargaining in informal bankruptcy affects the determination of optimal exemptions.

**Filing Fees.** Filing and legal fees can determine in practice whether a debtor files for bankruptcy and under which chapter. Albanesi and Nosal (2015) document that the increase in attorney costs caused BAPCPA reduced total filings and increased the share of Chapter 13 bankruptcies. The formulation of filing fees in Equation (18) does not explicitly model the possibility of failing to file for bankruptcy due to a lack of liquid assets. Explicitly modeling a law of motion for unpaid debts such that a household needs enough liquid assets to file for bankruptcy would not affect Proposition 2.

**Contract Selection.** This paper follows the tradition of general equilibrium with incomplete markets, imposing no restrictions on the shape of contracts used while taking the set of available contracts as a primitive. I extend the results to allow households to endogenize the shape of contracts in the Appendix. As long as households optimally choose the shape of the contracts traded or the choice of contracts is invariant to the exemption level (for instance, debt contracts are observed in high and low exemption economies), Proposition 2 remains valid.

**Interpretation of Planning Problem.** In practice, bankruptcy exemptions are set by jurisdictions, not by private parties. This may reflect the complexity of coordinating borrowers with multiple lenders, especially in the case of multiple contracts. The problem solved in this paper can be interpreted as that of a jurisdiction setting exemptions to maximize ex-ante social welfare. Given that lenders make zero profit, it can also be interpreted as the problem of borrowers who optimally choose exemption levels ex-ante under commitment.

### 4 Empirical Results

The approach developed in this paper to determine whether it is optimal to increase or decrease exemptions is most useful when combined with direct measures of the sufficient statistics identified. With that goal, I now merge multiple data sources to recover empirical estimates of the identified sufficient statistics. I proceed in three steps. First, I describe how to implement empirically the directional test of the general model derived in Section 3. Second, I describe the
data sources used to measure each of the sufficient statistics. Finally, I proceed to measure and combine all sufficient statistics in order to report estimates of marginal welfare changes by state and income quintile.

The empirical estimates of the sufficient statistics identified in this paper using data between 2008 and 2016 imply that there are welfare gains from increasing exemption levels across all income groups in all considered U.S. states. This result is due to the fact that the estimated sensitivity of credit supply to exemption levels, which directly modulates the cost of increasing exemption levels, is an order of magnitude smaller than the insurance benefits in case of bankruptcy.

4.1 Empirical Implementation

The empirical implementation is designed to recover measures of marginal welfare changes induced by exemption changes in the cross-section of households for different states. To reach that objective, and disciplined by the available data, I now operationalize the theoretical results derived in Proposition 2 as follows.

First, I assume that households have identical time-separable CRRA utility of consumption with discount factor $\beta$ and constant relative risk aversion coefficient $\gamma$, and that they face the same form of uncertainty. Second, I take $T \to \infty$, and assume that households’ value function admits a recursive representation. Both assumptions, when combined with a discrete representation of households’ state space, yield the following result.

**Proposition 3. (Empirical implementation)** Given households’ discount factor $\beta$ and risk aversion coefficient $\gamma$, it is possible to compute the $S \times 1$ vector

$$\frac{dW}{dm} = \left( \frac{\partial Q}{\partial m}(\Omega^1), \frac{\partial Q}{\partial m}(\Omega^2), \ldots, \frac{\partial Q}{\partial m}(\Omega^S) \right)'$$

of normalized welfare changes for households that transition between $S$ possible states as follows

$$\frac{dW}{dm} = A^{-1}F,$$

where $F$ is a vector of dimension $S \times 1$ and $A$ is a matrix of dimension $S \times S$, given by

*(Flow Vector)* $F = Q_m + \beta \text{diag}(\Pi_{N \rightarrow m} \times G'_{N \rightarrow m})$

*(Transition Matrix)* $A = I_S - \beta (\Pi_{N \rightarrow N} \odot G_{N \rightarrow N}) - \beta^2 (\Pi_{N \rightarrow D} \odot G_{N \rightarrow D}) \times (\Pi_{D \rightarrow N} \odot G_{D \rightarrow N})$,

where $Q_m = \left( \frac{\partial Q}{\partial m}(\Omega^1), \frac{\partial Q}{\partial m}(\Omega^2), \ldots, \frac{\partial Q}{\partial m}(\Omega^S) \right)'$ is a vector of dimension $S \times 1$ that measures marginal welfare costs, given by $\frac{\partial Q}{\partial m}(\Omega) = \sum_k \frac{\partial q_k}{\partial m}(\Omega) h^k(\Omega)$, $\Pi = \{\Pi_{N \rightarrow N}, \Pi_{N \rightarrow D}, \Pi_{D \rightarrow N}, \Pi_{N \rightarrow m}\}$, denotes a set matrices of dimension $S \times S$. I denote standard matrix multiplication by $\times$, element-wise matrix multiplication (Hadamard product) by $\odot$, and matrix transposition by $X'$, for a given matrix $X$. I denote an identity matrix of dimension $S$ by $I_S$. 

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that define transition probabilities between no default states (N), default states (D), and default states in which household claim the full exemption (m), while \( G = \{ G_N \rightarrow N, G_N \rightarrow D, G_D \rightarrow N, G_N \rightarrow m \} \) denotes matrices of dimension \( S \times S \) that define households’ relative valuations between the same set of states.

Proposition 3 provides a recursive representation of normalized welfare changes for the general environment.\(^{14}\) This recursive representation decomposes the welfare impact of changing exemptions into a flow vector \( F \), which maps directly to the characterization of the baseline two-period model, and a transition matrix \( A \), which accounts for dynamics and households’ valuations across states.

Given Proposition 3, constructing empirical estimates of the normalized welfare changes boils down to finding empirical counterparts of the elements in \( A \) and \( F \). The vector \( F \) exclusively depends on measures of marginal cost \( Q_m \) and on the benefits of declaring bankruptcy while claiming the full exemption one period forward, given by \( \beta \text{diag}(\Pi_{N \rightarrow m} \times G'_{N \rightarrow m}) \), exactly as in the baseline model. The matrix \( A \) exclusively depends on valuation matrices \( G \) and transition probabilities \( \Pi \), whose element-wise products can be interpreted as shadow Arrow-Debreu prices (risk-neutral probabilities) from the perspective of a given household.

The ideal dataset to implement Proposition 3 would contain detailed information regarding i) households’ balance sheets, in particular different types of liabilities, ii) income and other relevant individual state variables, for instance, health, education status, or past default history, iii) expenditure measures, iv) whether a household files for bankruptcy with or without non-exempt assets, and v) interest rate schedules for all forms of credit given households’ characteristics and how these depend on exemption levels. Only the sensitivity of credit supply schedules to exemptions requires information on lenders’ behavior. The remaining sufficient statistics can be collected in longitudinal household surveys. Given the available data sources, I describe my approach to measurement next.

4.2 Measurement Approach and Data Description

Here I describe how to construct empirical counterparts of the sufficient statistics from existing data sources. I map a period in the model to a three-year period in the data, as in Livshits et al. (2007) and I restrict the analysis to the period 2008-2016. This choice is motivated by the availability of data, since the dataset with the universe of bankruptcy filings starts in 2008, and by the need to avoid the period surrounding the implementation of BAPCPA (2005).

I use households’ income as the single state variable, classifying households by income quintiles in each state. While one could in principle use a multidimensional set of state variables that includes households’ income, assets, liabilities, and other characteristics, this would reduce the number of observations per

\(^{14}\)I focus on recovering normalized welfare changes, \( dW^{N} \). The purpose of normalizing \( dW^{N} \) by households’ marginal utility is to compute money-metric measures in dollars. That is, the vector \( d\hat{W} \) computes the net present values of marginal welfare benefits and costs in different states, according to households’ own stochastic discount factor.
Table 1: Empirical Counterparts

<table>
<thead>
<tr>
<th>Measured Variable</th>
<th>Data Source</th>
<th>Maps Into</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest-Rate Sensitivity</td>
<td>RateWatch, Exemptions Dataset, Regional Controls (ACS/BLS/FHFA)</td>
<td>$Q_m$</td>
</tr>
<tr>
<td>Households’ Liabilities</td>
<td>PSID</td>
<td>$Q_m$</td>
</tr>
<tr>
<td>Households’ Expenditure</td>
<td>PSID</td>
<td>$G$</td>
</tr>
<tr>
<td>Income Transition Matrix</td>
<td>PSID</td>
<td>$\Pi$</td>
</tr>
<tr>
<td>Prob. of Bankruptcy</td>
<td>ID-FJC</td>
<td>$\Pi_{N\rightarrow m}$, $\Pi_{N\rightarrow D}$</td>
</tr>
<tr>
<td>Prob. of No-Asset Bankruptcy</td>
<td>ID-FJC</td>
<td>$\Pi_{N\rightarrow m}$</td>
</tr>
</tbody>
</table>

Note: Table 1 schematically describes the data sources used to calculate each sufficient statistic required to compute $\frac{dV}{dm}$, as described in Proposition 3.

state, with the associated loss of statistical precision. Consequently, I estimate marginal cost and benefit measures by income quintile, as well as transition and valuation matrices between income quintiles. I define income quintiles by state using data from the 2013 five-year estimates of the American Community Survey (ACS).

As discussed above, specifying preference parameters is only relevant to discount over time and to account for different relative valuations across states. Only two parameters are required: households’ discount factor $\beta$ and households’ risk aversion coefficient $\gamma$. Consistently with existing quantitative literature, I set $\beta = (0.94)^3$ and $\gamma = 2$ as baseline parameters, although I discuss the sensitivity of the results to both choices.

I briefly describe here the data used to construct measures of $A$ and $F$, along with a few relevant sample restrictions. Appendix B contains a more detailed description of the sources of data, as well as summary statistics. I apply the same methodology to the 38 states whose homestead exemptions are not unlimited and whose state-level exemptions are not dominated by federal exemptions.

First, to compute default probabilities for households with different income levels and to calculate the fraction of no-asset bankruptcies state-by-state, I use data on the universe of bankruptcy filings provided by the U.S. Department of Justice, through the Integrated Database of the Federal Justice Center (ID-FJC). The ID-FJC contains information regarding households’ income when filing for bankruptcy, which allows us to construct estimates of the probability of bankruptcy and no-asset bankruptcy by income quintiles for each state. This information serves as an input to calculate $\Pi_{N\rightarrow m}$ and $\Pi_{N\rightarrow D}$.

Second, to estimate transition matrices across income quintiles at the state level, I rely on the Panel Study of Income Dynamics (PSID), a bi-annual longitudinal survey with information on expenditure, income, and wealth collected from a representative sample of households. I use the PSID waves between 2007 and 2015 to construct measures of liabilities (auto and personal loans) by income quintile for each state, which is a necessary input to compute $Q_m$. I also use the
PSID to estimate by maximum likelihood a transition matrix between different income quintiles, which is a direct input into the calculation of all matrices in the set $\Pi$. Finally, I use PSID’s consumption expenditure data to construct the valuation matrices in the set $G$. The main limitation of the PSID is that it lacks information on whether households have filed for bankruptcy. For that reason, I estimate a single transition matrix between income quintiles, which I combine with the probability of bankruptcy and no-asset bankruptcy from the ID-FJC dataset to calculate transition matrices into bankruptcy with and without non-exempt assets, as described in the Appendix. Similarly, because I do not observe consumption measures conditional on bankruptcy, I estimate a single valuation matrix between income quintiles and states.

Third, to estimate the sensitivity of credit supply schedules to exemptions changes, I combine data from RateWatch, a data collection company that gathers quarterly interest rate data on different debt products from banks and credit unions at the branch level, with manually collected information on exemption levels over time. I also use data on income, unemployment, and house prices at the county level from the American Community Survey (ACS), the Bureau of Labor Statistics (BLS), and the Federal Housing Authority (FHFA) as regional controls. Given that the information on exemption levels is annual, I aggregate all variables to an annual frequency, so the units of observation are average yearly rates set by a given branch of a financial institution.

I proceed to recover measures of the sensitivity of credit supply schedules to exemption changes by running regressions of interest rates offered by banks to lenders on exemption levels and other controls. The baseline empirical specification used to recover the desired sensitivity is

$$\log (1 + r_{it}) = \alpha_i + \alpha_t + \psi m_{st} + \theta X_{st} + \varepsilon_{ist},$$

(R1)

where $r_{it}$ denotes the net interest rate offered by banks to lenders in a given category, $\alpha_i$ denotes a branch fixed-effect, $\alpha_t$ denotes a time fixed-effect, $m_{st}$ denotes the exemption level applicable in a given state in a given period, and $X_{st}$ denotes regional controls. The use of branch-fixed effects for a fixed product category isolates changes in credit supply schedules, since this regression controls by the type and quantity of the loan and the time invariant lender characteristics, as emphasized below. The identification of the parameter of interest $\psi$ is driven by changes in exemption levels over time. Severino and Brown (2017), who run similar regressions to study the impact of exemptions on total credit volume and interest rates over the period 1999-2005, provide empirical support for the hypothesis that exemption changes can be treated as plausibly exogenous.

$^{15}$For simplicity, I estimate transition matrices only with observations of households who do not migrate between states in a given period. It is possible to extend the methodology to account for migration between states.

$^{16}$These regressions describe for the first time the relation between interest rates and exemptions in the post-BAPCPA period. See Severino and Brown (2017) for a full empirical study of how total credit and interest rates vary with exemption changes before the implementation of BAPCPA, with an emphasis on justifying a causal interpretation of estimates.
Table 2: Credit Supply Sensitivity

<table>
<thead>
<tr>
<th></th>
<th>Personal (1)</th>
<th>Personal (2)</th>
<th>Personal (3)</th>
<th>Auto (4)</th>
<th>Auto (5)</th>
<th>Auto (6)</th>
<th>Mortgage (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exemption (homestead)</td>
<td>0.0071∗</td>
<td>0.0044</td>
<td>0.0058∗</td>
<td>0.0054</td>
<td>−0.0001</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0041)</td>
<td>(0.0079)</td>
<td>(0.0025)</td>
<td>(0.0037)</td>
<td>(0.0009)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exemption (personal)</td>
<td>0.0185</td>
<td>0.0240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0538)</td>
<td>(0.0201)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>0.0004</td>
<td>0.0005</td>
<td>0.0003∗</td>
<td>0.0003</td>
<td>0.0003∴</td>
<td>0.0004∴</td>
<td>0.0004∴</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0004)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>House Prices</td>
<td>−0.00002∗</td>
<td>−0.00002∗</td>
<td>−0.00002***</td>
<td>0.000005</td>
<td>0.000004</td>
<td>0.000002</td>
<td>0.000001</td>
</tr>
<tr>
<td></td>
<td>(0.00001)</td>
<td>(0.00001)</td>
<td>(0.000003)</td>
<td>(0.000003)</td>
<td>(0.000003)</td>
<td>(0.000003)</td>
<td>(0.000001)</td>
</tr>
<tr>
<td>Income</td>
<td>0.0023</td>
<td>0.0024</td>
<td>0.000002</td>
<td>0.0004</td>
<td>0.0001</td>
<td>0.0008</td>
<td>−0.0007</td>
</tr>
<tr>
<td></td>
<td>(0.0017)</td>
<td>(0.0017)</td>
<td>(0.00018)</td>
<td>(0.0009)</td>
<td>(0.0009)</td>
<td>(0.0011)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Branch FE</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>County FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Observations</td>
<td>16,255</td>
<td>16,255</td>
<td>16,255</td>
<td>18,349</td>
<td>18,349</td>
<td>18,349</td>
<td>7,732</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.8348</td>
<td>0.8348</td>
<td>0.3614</td>
<td>0.8885</td>
<td>0.8884</td>
<td>0.4473</td>
<td>0.8452</td>
</tr>
</tbody>
</table>

Note: Table 2 reports the results of running Regression R1 using personal unsecured loans’ interest rates, auto loans’ interest rates, and mortgage rates (thirty-year fixed rate mortgage) as the dependent variable. Exemptions are measured in millions of dollars. Interest rates are yearly averages from RateWatch, county unemployment rate is from the BLS, county house prices are from the FHFA, and county median household income is from the ACS. Standard errors, in parentheses, are clustered at the state level, which allows for serial correlation in the error term.

***Significant at the 1 percent level. **Significant at the 5 percent level. *Significant at the 10 percent level.
Before describing the empirical estimates, it is important to discuss the conceptual link between the regression coefficient $\psi$ and its model counterpart $\partial q^k / \partial m$ (equivalently $\partial \log(1+r^k) / \partial m$). First, the model implies that the relevant object of interest is the elasticity of credit supply, $\partial q^k / \partial m$, not the equilibrium response of the price of credit (interest rates), $dq^k / dm = \partial q^k / \partial m + \partial q^k / \partial b^k db^k$, which would include the change in households’ borrowing decisions, $db^k / dm$. Since Regression R1 is run for a given credit product, which fixes the amount of credit $b^k$ offered to a given household with a given creditworthiness, the recovered estimate is exactly measuring a credit supply elasticity. Second, it is possible that a change in the exemption level changes the composition of borrowers – in terms of unobservables – that borrow using a specific product in a given branch. In other words, even though Regression R1 is run for a given credit product that conditions on borrowers’ observed creditworthiness, the composition of the set of borrowers that use that product may still change, which would affect the rate offered by lenders. In the Appendix, I analyze an extension of the model with pooling and exclusion that shows that, in that case, the right credit supply elasticity to recover is the one that incorporates these selection effects. Intuitively, from the perspective of a given borrower, the ability to borrow is modulated by the change in interest rates, regardless of whether this change comes from pooling and selection on unobservables or not. Finally, note that the ideal empirical design to recover $\partial q^k / \partial m$ would directly observe the credit supply schedule offered by lenders to a given borrower, defined by a given set of state variables that includes individual characteristics and borrowing choices, and computes its slope in the direction of $m$.

Table 2 reports the outcomes of Regression R1 for three different dependent variables: interest rates on personal unsecured loans, interest rates on auto loans, and interest rates on mortgages. It shows that increases in homestead exemption levels are associated with increases in interest rates charged by lenders for personal unsecured loans and auto loans (columns 1 and 4), but not for mortgages (column 7). Columns 2 and 5 show that the effect on rates offered of changes in personal exemptions has the same sign and even higher point estimates as the effect of changes in homestead exemptions, but these are not statistically significant. Columns 3 and 6 show that including branch fixed effects instead of county fixed effects accounts for a substantial share of the total $R^2$. This finding implies that branch-specific characteristics explain a substantial part of the total variation in offered interest rates. Motivated by the finding that mortgage rates are not sensitive to exemption levels, I parsimoniously disregard mortgage liabilities when calculating marginal welfare changes. This is a natural finding, since mortgages are a form of secured credit whose pricing is less linked to the level of exemptions. This finding also motivates the use of one-period credit in most of the paper.

In terms of magnitudes, increasing the homestead bankruptcy exemption by $100k$ is associated with an increase in the interest rate charged on unsecured credit of roughly 7 basis points, with a standard deviation of 4 basis points,
as shown by the first column in Table 2. Increasing the personal exemption by $100k is associated with an increase in rates on unsecured credit of roughly 19 basis points (second column), with a standard deviation of 54 basis points. In the context of auto loans, increasing the bankruptcy exemption by $100k is associated with an increase in the interest rate charged of 6 basis points (fourth column), with a standard deviation of 2.5 basis points. The same increase for personal exemptions is associated with an increase in rates of 24 basis points, with a standard deviation of 20 basis points. It is natural to find a smaller quantitative effect in auto loans relative to unsecured credit, since these loans are partly collateralized. As I will show next, these estimates, which directly determine the marginal cost of changing exemption levels are quantitatively small relatively to estimate benefits of increasing exemptions. This would be the case even if we used estimates one or two standard deviations higher than the highest point estimates.\footnote{The fact that interest rates are not too sensitive to changes in the leniency of bankruptcy procedures is consistent with the aggregate time-series evidence generated by the introduction of BAPCPA, which does not show a sharp change in interest rates after its implementation.}

Given the amount of data available, the main limitation of this approach to recover credit supply sensitivities is that it only yields a single set of credit supply sensitivities for all states and all income quintiles. Because, as I show next, the estimated sensitivities are an order of magnitude lower than needed to counterbalance estimated marginal benefits, the broader empirical conclusions of the paper are not affected by the lack of richer credit supply elasticity estimates.

It is worth making two final remarks on the empirical implementation. First, if households happened to trade assets that directly reveal their willingness to pay for a unit of consumption in bankrupt states, it would be possible to calculate valuation matrices without making any parametric assumption on households preferences. Second, having access to a longer time series would allow us to include richer life-cycle dynamics into the model – adding age as a state variable – as well as calculating more refined valuation matrices – for instance, including an aggregate state to account for time-varying risk premia.

4.3 Main Results: Estimates of Marginal Welfare Change

Following the procedure just described, I use Proposition 3 to combine estimates of flow vectors $F$ and transition matrices $A$ for each U.S. state. The baseline empirical findings are based on the point estimates for the credit supply sensitivities for personal unsecured credit and auto loans from Columns 1 and 4 of Table 2. As described above, I use $\beta = (0.94)^3$ and $\gamma = 2$ as baseline preference parameters.

Figure 5 reports the average across income quintiles of the marginal welfare change associated with a one-dollar increase in the exemption level as a function of current exemption values state-by-state. Figure 5 implies that all states find it welfare improving on average to increase exemption levels, although there is substantial cross-sectional variation. The empirical relation between average exemption gains and current exemption levels turns out to be negative, with
Figure 5: Marginal Welfare Changes and Exemption Levels

**Note:** Figure 5 shows average normalized marginal welfare changes associated with a one-dollar increase in the exemption level by state. Average normalized welfare changes are calculated as an unweighted sum of normalized marginal welfare changes across different income quintiles, calculated following Proposition 3. The determination of income quintiles by state uses income data from the 2013 five-year ACS. The figure includes 36 U.S. states. It excludes states with unlimited exemption levels, states with dominated state exemptions, as well as two states for which some income quintile is not populated in the PSID data (Rhode Island and North Dakota). Preference parameters are $\beta = (0.94)^3$ and $\gamma = 2$. Exemption levels are in dollars as of 2016.
average welfare gains found to be larger in southern states (Tennessee, Alabama, Georgia, and Louisiana).

Figures 7 and 8 show that marginal welfare gains are indeed positive for all quintiles across all states.\(^{18}\) In principle, we could have found negative marginal welfare changes for some quintiles or states. The explanation for the general desirability of increasing exemptions is that the estimates of credit supply elasticities seem to be an order of magnitude lower than the potential benefits associated with enjoying a marginal dollar when bankrupt. Two reasons explain why those southern states seem to benefit the most from increasing exemptions. First, the probabilities of filing for bankruptcy in those states are large. Second, the probabilities of filing an asset case (claiming the full exemption) in those states are substantially larger than in other states. Table 9 illustrates these differences comparing the states of Alabama and Massachusetts. It should not be surprising that exemption limits bind more often in states with low exemptions levels. A final mechanism that increases the estimated gains for those states is the larger than average disparity in consumption across income quintiles, which increases the value of insurance when including dynamic considerations.

Table 3 reports summary statistics of the distributions of marginal welfare changes quintile by quintile for flow welfare changes \(F\) and normalized welfare gains \(\frac{\partial W}{\partial m}\). As expected, normalized welfare gains are larger than flow measures, since they account for future gains. It is worth highlighting that the gains and losses vary across income quintiles. Interestingly, flow welfare gains tend to be higher for households in middle income quintiles, and marginal welfare gains are often higher for households in middle and high income quintiles. Two forces explain these results. First, households in lower quintiles are less likely to file for bankruptcy with non-exempt assets, so they do not benefit at the margin from having access to higher exemptions. This is also partly because they borrow less. Also, households in the higher quintiles unconditionally declare bankruptcy slightly less often. Second, middle and higher income households find large gains from insurance, given that they anticipate the possibility of moving towards a lower income quintile. On the contrary, households currently in the first quintile value less the insurance role of exemptions, since they do not have much to insure against: they are already in a bad scenario.

Quantitatively, the average estimate of normalized marginal welfare changes of 48 basis points implies that an increase in the exemption level of $50,000 is associated with a welfare gain of roughly $240 (roughly 0.4% of average household income in the U.S.). For households in the highest income quintile, this number can be as high as $645 dollars in some states (90th percentile in the distribution of states), or as low as $315 (10th percentile in the distribution of states). For households in the lowest income quintile, this number can be as high as $395 (90th percentile) or as low as $190 (10th percentile). In a

\(^{18}\)Note that my conclusion is different from both Chatterjee et al. (2007) and Livshits et al. (2007), who find that less lenient bankruptcy procedures improve welfare. Overall, existing quantitative literature has found an ambiguous relation between leniency in bankruptcy and welfare. See Livshits (2015) for a recent survey that highlights how welfare assessments may be sensitive to model parametrizations.
Table 3: Marginal Welfare Changes

<table>
<thead>
<tr>
<th>Quintile</th>
<th>( \frac{d\tilde{W}}{dx} ) (Marginal Welfare Change)</th>
<th>( F ) (Flow Change)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
</tr>
<tr>
<td>First</td>
<td>0.0038</td>
<td>0.0034</td>
</tr>
<tr>
<td>Second</td>
<td>0.0050</td>
<td>0.0046</td>
</tr>
<tr>
<td>Third</td>
<td>0.0044</td>
<td>0.0039</td>
</tr>
<tr>
<td>Fourth</td>
<td>0.0046</td>
<td>0.0038</td>
</tr>
<tr>
<td>Fifth</td>
<td>0.0063</td>
<td>0.0045</td>
</tr>
<tr>
<td>Overall</td>
<td>0.0048</td>
<td>0.0041</td>
</tr>
</tbody>
</table>

Note: Table 3 reports summary statistics of the distribution of normalized marginal welfare changes by state associated with a one-dollar increase in the exemption level. Average normalized welfare changes are calculated as an unweighted sum of normalized marginal welfare changes across different income levels, calculated following Proposition 3. The determination of income quintiles by state uses income data from the 2013 five-year ACS. The figure includes 36 U.S. states. It excludes states with unlimited exemption levels, states with dominated state exemptions, as well as two states for which some income quintile is not populated in the PSID data (Rhode Island and North Dakota). Preference parameters are \( \beta = (0.94)^3 \) and \( \gamma = 2 \).

Few states, average normalized welfare gains can be substantial on average. In Tennessee and Alabama, a $50,000 increase in exemptions is valued at roughly $2000 (roughly 4% of average household income). These numbers should be interpreted as households’ willingness to pay for a change in exemptions. Because households’ incomes differ across quintiles, the relative amount of income that a household is willing to give up may still be higher for households in low income quintiles. In aggregate terms, a back-of-the-envelope calculation for the U.S. economy (with roughly 125 million households) implies that a $50,000 increase in exemptions is valued at $240 × 125m = $30bn. While the average estimates are non-negligible, these results suggest that some regions may greatly benefit from exemption changes.

Finally, I explore the sensitivity of the results to varying the inputs that go into calculating marginal welfare changes. First, I explore how the results vary when considering alternative values for credit supply elasticities. In Table ?? in the Appendix, I report summary statistics of the distribution of normalized marginal welfare changes when taking the highest point estimates for exemption sensitivities and augmenting them by one standard deviation. Even in that case, average marginal welfare changes remain positive, although they are smaller in magnitude. Second, I explore the role of risk aversion for the estimates. Values of \( \gamma \) lower than 2 yield similar estimates to the baseline parametrization. By assuming values of \( \gamma \) greater or equal than 2 – Table ?? sets \( \gamma = 4 \) – it is possible to find moderately larger measures of welfare gains associated with higher exemption levels. The impact of increasing risk aversion is stronger for households in high income quintiles, who have the possibility of experiencing large consumption drops. Finally, although I do not report specific results in the paper, varying the discount factor \( \beta \) within a reasonable range has barely
any impact on the results.

5 Additional Channels

Three relevant channels not considered so far call for augmenting the characterization of marginal welfare changes. First, I consider the problem of unsophisticated households who make financial decisions under distorted beliefs. Second, I consider the possibility that lenders have market power. Finally, I show how to account for the general equilibrium impact of exemption changes.

Two main takeaways emerge from the study of these new channels. First, even though each extension calls for augmenting the expression to compute marginal welfare changes, the main forces highlighted in this paper do not vanish and still provide the starting point to study the optimality of changing exemptions. Second, in these environments, exemption changes must be considered along with other policies. In particular, policies that restrict financial decisions of unsophisticated households’, that tax lenders’ profits, or that force households to internalize pecuniary and aggregate demand externalities should be jointly implemented with exemptions changes. When those policies are in place, optimal exemptions can be set according to the marginal benefits and costs studied in this paper.

Belief Distortions Throughout the paper, both households and lenders hold correct beliefs about the uncertainty they face. While this is a natural starting point, households sometimes fail to make optimal financial decisions. I now modify the baseline model to assume that households make borrowing choices using a different set of beliefs from the planner who computes social welfare – similar arguments apply when households face preference distortions. Formally, households maximize

$$\max u(c_0) + \beta \mathbb{E}[u(c_1(s))]$$

subject to date 0 and date 1 budget constraints, defined in Equations (3), (4), (5). The expectation $\mathbb{E}[\cdot]$ is taken over a perceived distribution of date 1 endowments $\tilde{F}(\cdot)$, with the same support as $F(\cdot)$, which denotes the distribution used by the planner to compute social welfare. For simplicity, I assume that lenders and the planner share the same set of beliefs and that there are no distortions at date 1. This guarantees that the planner and the households agree on when to default ex-post. Formally, social welfare from the planner’s perspective corresponds to $W(m)$, defined as

$$W(m) = u(c_0) + \beta \mathbb{E}[u(c_1(s))]$$

19See for instance Campbell (2016) on the importance of correcting for behavioral biases and Gennaioli and Shleifer (2018) for a compendium of theoretical and empirical work that highlights the importance of individual belief formation to understand individual and aggregate decisions. Dávila (2014) shows how to design corrective policies in the context of financial market trading when investors have distorted beliefs. Nakajima (2012, 2017) studies bankruptcy in models with temptation preferences.
where consumption, borrowing, and default choices are selected by households. The only difference between Equations (23) and (24) is that households’ and the planner assess differently the likelihood of date 1 states. Households optimally borrow according to

\[ u'(c_0) \frac{\partial Q_0}{\partial b_1} = \beta \int \hat{s} \ u'(c_1^N(s)) \ d\hat{F}(s) \]

while the marginal welfare change associated with varying exemptions from the planner’s perspective is now given by

\[ \frac{dW}{dm} = u'(c_0) \frac{\partial q_0}{\partial m} b_1 + \beta \int \hat{s} \ u'(c_1^P(s)) \ dF(s) \]

\[ + \beta \left( \int \hat{s} \ u'(c_1^N(s)) \ dF(s) - \int \hat{s} \ u'(c_1^N(s)) \ dF^i(s) \right) \frac{db_1}{dm} \]

Three conclusions can be drawn from Equation (25). First, the impact of an exemption change on borrowing decisions, \( \frac{db_1}{dm} \), has now a first-order effect on welfare, since the planner perceives that households borrow too much or too little. Interestingly, only beliefs over states in which households repay determine the belief distortion. Second, the planner would find optimal to correct households’ borrowing choices, for instance, through a borrowing cap. In that case, Proposition 1 would remain valid, while the belief distortion in Equation (25) determines the Pigovian correction to households’ borrowing. Finally, belief distortions that affect households’ bankruptcy decisions or lenders’ credit supply determination will have an independent impact on social welfare. These fascinating issues are outside the scope of this paper.

**Lenders’ Market Power** Assuming perfect competition among lenders implies that there is no need to account for lenders’ welfare separately. While perfect competition is a standard assumption in quantitative bankruptcy models – see, for instance, the arguments in Chatterjee et al. (2007) – there exists evidence consistent with the presence of market power in lending markets. See the classic contribution of Ausubel (1991) or more recent ones by Corbae and D’Erasmo (2019) and Scharfstein and Sunderam (2013).

It is possible to relax the assumption of perfect competition and to account explicitly for lenders’ welfare. Formally, starting from the baseline model and preserving the risk neutrality assumption allows us to map lenders’ utility to the net present value of lenders’ profits, which can be expressed as a function of households borrowing choices and the exemption level as follows

\[ W^\ell = \Pi^\ell (b_1, m) = \frac{\delta}{1 + r^\ell} \max \left\{ u(s) - m, 0 \right\} dF(s) + b_1 \int dF(s) \]

\[ - Q_0 (b_1, m) \],

where \( Q_0 (b_1, m) \) denotes the total amount lent at date 0. Regardless of how \( Q_0 (b_1, m) \) is determined, social welfare, equal weighted in dollars, can now be

\[ \text{(25)} \]
calculated as
\[
\frac{dW}{dm} = \frac{dW^t}{dm} + \frac{\partial q_0}{\partial m} b_1 + \pi_m \mathbb{E}_m \left[ \frac{\beta u'(c^1_{D})}{u'(c_0)} \right] + \frac{dW^L}{dm}. \tag{26}
\]

where the new term simply accounts for how a change in exemptions affects the net present value of lenders’ profits.

Two conclusions emerge when interpreting Equation (26). First, the crucial assumption made so far is that lenders profits are invariant to the level of \( m \), not necessarily equal to zero.\(^{21}\) For instance, fixed costs of entry or limit pricing strategies are consistent with lenders that make positive profits but whose profits are insensitive to the exemption level. Second, even when lenders make positive profits, the sensitivity of the credit supply schedule to exemptions remains as the relevant sufficient statistic. In practice, pass-through measures of how exemption changes translate into interest rates may be affected by the form of competition in the lending market, but the sensitivity of the credit supply schedule to exemptions still accounts for the impact of lenders’ behavior on households’ welfare.

**General Equilibrium Externalities** The paper has abstracted from the impact of exemption changes on aggregate variables, including asset prices, wages, and demand-determined output. In general, changing exemptions may affect the prices of goods and assets. These effects have first-order welfare consequences since, by construction, the economy features incomplete markets. After augmenting the baseline model to include an asset held by households which trades at a price \( p_0 \) and \( p_1 (s) \) at dates 0 and 1, it is possible to express the marginal welfare change associated with varying the exemption level as follows
\[
\frac{dW}{dm} = u'(c_0) \frac{\partial q_0}{\partial m} b_1 + \beta \int_{D_m} u'(c^1_D (s)) dF(s) + u'(c_0) \frac{dp_0}{dm} (a_1 - a_0) + \beta \int_{D_y} u'(c^1_D (s)) \frac{dp_1}{dm} a_1 dF(s) + \beta \int_{N} u'(c^1_N (s)) \frac{dp_1}{dm} a_1 dF(s), \tag{27}
\]

where \( D_m \) denotes the set of realizations in which households default claiming the full exemption, \( D_y \) denotes the set of states in which default but do not claim the full exemption, and \( N \) denotes the no default states.

The pecuniary effects in Equation (27) are a manifestation of distributive pecuniary externalities, using the language of Dávila and Korinek (2018). Similar effects emerge under pecuniary externalities that operate through binding constraints, like collateral externalities. Likewise, the expression for the marginal

\(^{21}\)One may conjecture that increasing exemptions improves households’ outside option when bargaining with lenders. This may plausibly lower banks’ profits and the social benefit from increasing exemptions. However, other mechanisms may be at play, so studying the response of lenders’ profits to exemption changes becomes a relevant empirical question.
welfare change would need to be augmented to account for aggregate demand externalities in environments with price rigidities and constrained monetary policy, as shown in the Appendix.

6 Conclusion

This paper develops a methodology, using measurable sufficient statistics, to assess the desirability of varying bankruptcy exemption levels in a wide variety of environments. Measures of households’ debt, of credit supply elasticities to exemption changes, of the likelihood of declaring bankruptcy, and of the relative valuation of resources across states, which under standard preferences can be calculated using consumption changes, are sufficient to determine whether a bankruptcy exemption level is optimal for a given economy, or should be increased or decreased. Other features of the environment need not be specified once these variables are measured. Empirical estimates of the sufficient statistics identified in this paper using U.S. data between 2008 and 2016 imply that increasing existing exemption levels improves overall welfare, although there is substantial variation regarding the magnitude of potential gains across U.S. states and income quintiles. These estimates provide the first cross-sectional assessment of whether exemptions should be increased or decreased for different U.S. states.

The approach developed in this paper has the potential to spur further research on both quantitative structural modeling and microeconometric work on bankruptcy. Comparing the sufficient statistics identified in this paper among different quantitative models of bankruptcy can shed light on the channels through which different assumptions on primitives affect optimal exemptions in each model. Refining the measures of the sufficient statistics identified in this paper can improve the accuracy of welfare assessments. Looking forward, a better understanding of i) how belief distortions and other forms of household unsophistication affect financial decisions, ii) how exemption changes affect the competitive structure of the lending sector, and iii) how pecuniary and aggregate-demand externalities interact with exemption levels will allow us to better assess bankruptcy reforms.
References


APPENDIX

A Institutional Background

Bankruptcy in the U.S. in modern times is shaped by the Bankruptcy Code, enacted by Congress in 1978 and amended multiple times since then. In the U.S., debtors can choose between two different personal bankruptcy procedures: Chapter 7, often called “liquidation” bankruptcy, and Chapter 13, often called “repayment” bankruptcy. Regardless of the chapter chosen, all collection efforts by creditors stop when an individual files for bankruptcy. See Skeel (2001) for a historical account of the evolution of the U.S. bankruptcy system and Claessens and Klapper (2005) for an overview of bankruptcy procedures around the world.

Under Chapter 7, bankrupt debtors give up all unsecured assets above predetermined exemption levels, in exchange for discharging most types of liabilities. There are certain types of liabilities that are rarely dischargeable in bankruptcy, notoriously student loans and tax obligations. After a Chapter 7 bankruptcy, debtors are not obliged to use future income to repay pre-bankruptcy debts, a feature often described as “fresh start”. Although there are federal bankruptcy exemptions, states have adopted their own exemption levels, so bankruptcy exemptions vary widely across states as a result. In some states, debtors can decide whether to use federal or state exemption levels. The most significant exemptions are the wildcard, personal, and homestead exemptions.

Under Chapter 13, bankrupt debtors are able to keep their assets, but they must propose a repayment plan. Repayment plans typically involve using a proportion of the debtor’s future income to repay outstanding debts monthly, usually over a three-to-five-year period. The law prescribes that the repayment plan must give creditors at least the same amount they would receive under Chapter 7, so the level of bankruptcy exemptions is very relevant for Chapter 13 bankruptcies. In practice, it is often the case that repayment plans involve monthly payments that exactly match the non-exempt amount. Because of their complexity, attorney fees are higher in Chapter 13 cases relative to Chapter 7 cases. Porter et al. (2017) document that average attorney fees as of 2015 for

22A popular online resource explains how bankruptcy exemptions affect a Chapter 13 repayment plan as follows: “First, it’s important to understand that creditors must receive the same or more in a Chapter 13 case as they would in a Chapter 7 case. Therefore, if the Chapter 7 trustee would have been able to sell property for the benefit of creditors in a Chapter 7 case, then the creditors should at least get that same amount in a Chapter 13 case. So how do you figure out the amount creditors should receive? It’s pretty easy actually. It’s the value of the debtor’s non-exempt property. A Chapter 13 debtor must pay to keep the property that would otherwise have been lost had the debtor filed for Chapter 7 bankruptcy.” Another popular source describes the role of exemptions under Chapter 13 as follows: “Even though debtors do not actually turn over their property in a Chapter 13 case, exemptions are still just as important as they are in a Chapter 7 case in valuing and distributing assets to creditors to satisfy debt.”
Chapter 7 bankruptcies are roughly $1,200, while Chapter 13 fees are on average higher than $3,000, although Chapter 13 fees can be paid over time as part of the repayment plan.

The Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005 is the last major change to the bankruptcy code. Most significantly, it introduces a “means test” that debtors must pass in order to file under Chapter 7. If a debtor’s income is over a certain threshold, the debtor may not be eligible to file under Chapter 7, leaving only the possibility of filing under Chapter 13. This legislative change makes it harder for high-income debtors to take advantage of large exemption levels, forcing them to file a repayment plan under Chapter 13. In addition to the means test, this reform also increases the filing fees and, through increased reporting requirements, the legal fees associated with filing for bankruptcy.

B Detailed Data Description

This Appendix provides additional discussion of the multiple sources of data used in the application in Section 4. Although the model should be interpreted in real terms, given the possibility of introducing biases by using inadequate price deflators and the minimal amount of inflation materialized over the period studied, I work with undeflated nominal variables. Similar results obtain when deflating all nominal variables with a consumer expenditure price index.

Bankruptcy Filings Data  I use data from bankruptcy filings provided by the U.S. Department of Justice, through the Integrated Database of the Federal Justice Center (ID-FJC). This data and its associated documentation can be downloaded from https://www.fjc.gov/research/idb/. The ID-FJC database makes available all petitions filed under the Bankruptcy Code between October 2007 and September 2017. In addition to the date of filing, this dataset includes the filing chapter, the nature of the debt (consumer vs. business), the debtor’s zip code, as well as measures of assets, liabilities, and monthly income, as reported by debtors when filing. Importantly for the approach developed in this paper, the dataset contains a variable (no-asset) that determines whether there will be funds available for distribution to creditors. Bankruptcy cases in which the value of non-exempt assets is less than the level of exemptions, so that there are no non-exempt resources left to pay unsecured creditors, are denominated “no-asset” bankruptcies.

I restrict the analysis to bankruptcies filed between January 2008 and December 2016 (nine calendar years). I restrict my attention to personal bankruptcies filed under Chapters 7 and 13, whose evolution is illustrated in Figure 6. Note that 96.5% of all bankruptcy filings in the U.S. are of personal/non-business nature. When using debtor’s financial information, I disregard observations without zip code information, without no-asset information, and for which financial information (assets, liabilities, income) takes negative values. This process of data harmonization reduces the number of observations by roughly 9%, so the
Figure 6: Evolution of Personal Bankruptcy Filings (2001-2017)

Note: The left panel of Figure 6 shows the yearly number of personal bankruptcy filings in the U.S. from 2001 until 2017, as reported by the U.S. Department of Justice through the Integrated Database of the Federal Justice Center (ID-FJC). The right panel shows the fraction of Chapter 7 and Chapter 13 filings over the same period. See Livshits et al. (2010, 2016) for a systematic analysis of the long-run evolution of consumer bankruptcies, starting in 1970.
Table 4: Bankruptcy Filings by Chapter and No-Asset Status

<table>
<thead>
<tr>
<th></th>
<th>Chapter 7</th>
<th>Chapter 13</th>
<th>Both Chapters</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-Asset Cases</td>
<td>67.55%</td>
<td>0.11%</td>
<td>67.66%</td>
</tr>
<tr>
<td>Asset Cases</td>
<td>3.12%</td>
<td>29.21%</td>
<td>32.33%</td>
</tr>
<tr>
<td>Both Cases</td>
<td>70.67%</td>
<td>29.32%</td>
<td></td>
</tr>
</tbody>
</table>

Note: Table 4 reports the share of bankruptcies by chapter and by “no-asset” denomination in the U.S. from 2008 until 2016, as reported by the U.S. Department of Justice through the Integrated Database of the Federal Justice Center. A bankruptcy filing is defined as an asset case when “there will be funds available for distribution to unsecured creditors, after any exempt property is excluded”. As confirmed by personal communication with the ID-FJC database managers, it is likely that the negligible fraction of reported no-asset Chapter 13 bankruptcies are due to record-keeping inaccuracies, since Chapter 13 bankruptcies involve repayments to creditors.

The final dataset used to carry out the calculations in Section 4 and to construct Tables 4 and 5 contains 9.38 million bankruptcy filings.

Figure 6 shows the yearly number of total personal bankruptcies in the U.S., decomposed in terms of Chapter 7 and Chapter 13 bankruptcies. The increase in the number of bankruptcies between 2008 and 2010 is partly motivated by the experienced cyclical downturn and by the fact that there were an unusually large number of bankruptcies in 2005, right before the implementation of BAPCPA, which one can argue lowers the number of bankruptcies filed in 2008. After 2010, there is a downward trend in the number of personal bankruptcies in the United States. Most bankrupt borrowers file under Chapter 7, accounting for roughly 69% of filings over the period studied, as shown in Table 4. Figure 6 also shows that the share of Chapter 13 bankruptcies is steadily rising in the most recent years.

Table 5 shows summary statistics of assets, liabilities, and current income as reported by bankrupt debtors between 2008 and 2016. The measure of debtor’s net worth is constructed as the difference between assets and liabilities. As expected, most bankrupt debtors have negative net worth.

Bankruptcy Exemptions Data I manually gathered and compiled a dataset with exemption levels by state over time. To do so, I relied on multiple editions of a popular bankruptcy filing book, Elias et al. (2015), as well as on state bankruptcy codes and press articles. This dataset contains homestead exemption levels and personal exemptions, the latter calculated by aggregating motor vehicle, wildcard, tools of trade, and cash or bank deposit exemptions. The dataset also includes whether bankrupt debtors in a given state have the option to use the federal limits, whether they can double their homestead or personal exemptions if married, and whether the applicable homestead exemption is unlimited. Since the majority of bankruptcies are filed jointly (roughly 70%, according to the ID-FJC), I use exemption levels for married couples in
Table 5: Summary Statistics of Bankruptcy Filing Information

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Pctl(10)</th>
<th>Median</th>
<th>Pctl(90)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Worth</td>
<td>-73977</td>
<td>478464</td>
<td>-187425</td>
<td>-37537</td>
<td>12764</td>
<td>9383793</td>
</tr>
<tr>
<td>Assets</td>
<td>127948</td>
<td>287923</td>
<td>3720</td>
<td>65442</td>
<td>314232</td>
<td>9383793</td>
</tr>
<tr>
<td>Liabilities</td>
<td>201925</td>
<td>481458</td>
<td>24199</td>
<td>119837</td>
<td>441378</td>
<td>9383793</td>
</tr>
<tr>
<td>Income</td>
<td>41940</td>
<td>334532</td>
<td>4966</td>
<td>35578</td>
<td>79029</td>
<td>9383793</td>
</tr>
</tbody>
</table>

Note: Table 5 reports summary statistics on net worth, total assets, total liabilities, and yearly income of bankrupt filers in the U.S. from 2008 until 2016, as reported by the U.S. Department of Justice through the Integrated Database of the Federal Justice Center. It provides information on the mean, median, standard deviation, as well as the 10th and 90th percentiles of each of the variables. All variables are measured in dollars and truncated to be smaller than $100 million.

the empirical implementation in Section 4 and to construct Table 6.
Table 6 shows summary statistics of homestead, personal, and total bankruptcy exemptions across states in 2008 and in 2016. I exclude from the sample states with unlimited exemptions. I also exclude the five states (Hawaii, Kentucky, Michigan, New Jersey, and Pennsylvania) that allow debtors to choose between state and federal exemption levels and for whose state exemption levels are lower than federal exemption levels. Similar results obtain when one attributes the federal exemption level to these states. Figure ?? in the Online Appendix graphically illustrates the evolution of exemption levels over the period studied. The average level of exemptions moves from roughly $113k to $171k over the period, with homestead exemptions accounting for the major part of total exemptions.

Interest Rate Data The interest rate data necessary to find the sensitivity of credit supply schedules to exemption changes is gathered by RateWatch, a data collection company that tracks interest rate quotes at the branch level for a large number of financial institutions in the United States.23 Both regulators (FDIC and Federal Reserve) and financial institutions regularly use this data to monitor the behavior of regulated entities or competitors. This is the only proprietary dataset used in the paper, and can be purchased through http://www.rate-watch.com/. The unit of observation are branch-level rates quoted for a specific loan product. I focus on interest rates quoted between 2008 and 2016 on unsecured personal loans, auto loans, and fixed-rate mortgages, since these loan classes map directly to the variables on debt positions from the PSID. Table 7 shows summary statistics of quoted rates on unsecured personal loans, auto loans, and mortgages. As expected, average rates on personal unsecured credit rates are higher (11%) than average rates on auto and mortgage loans (4%).

23This dataset also includes information on deposit rates, used for instance in Egan et al. (2017) and Drechsler et al. (2017).
Table 6: Summary Statistics of Bankruptcy Exemption Levels

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Pctl(10)</th>
<th>Median</th>
<th>Pctl(90)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homestead (2008)</td>
<td>113493</td>
<td>139083</td>
<td>10000</td>
<td>63750</td>
<td>300000</td>
<td>38</td>
</tr>
<tr>
<td>Personal (2008)</td>
<td>17641</td>
<td>10650</td>
<td>6280</td>
<td>15150</td>
<td>28945</td>
<td>38</td>
</tr>
<tr>
<td>Total (2008)</td>
<td>131134</td>
<td>141787</td>
<td>29290</td>
<td>88150</td>
<td>324080</td>
<td>38</td>
</tr>
<tr>
<td>Homestead (2016)</td>
<td>147499</td>
<td>151811</td>
<td>27893</td>
<td>97375</td>
<td>423000</td>
<td>38</td>
</tr>
<tr>
<td>Personal (2016)</td>
<td>24109</td>
<td>13292</td>
<td>11700</td>
<td>20300</td>
<td>35960</td>
<td>38</td>
</tr>
<tr>
<td>Total (2016)</td>
<td>171608</td>
<td>156038</td>
<td>45560</td>
<td>122250</td>
<td>448840</td>
<td>38</td>
</tr>
</tbody>
</table>

Note: Table 6 reports summary statistics for the level of personal, homestead and total state level exemptions (defined as the sum of personal and homestead exemptions) for the years 2008 and 2016. It provides information on the mean, median, standard deviation, as well as the 10th and 90th percentiles of the distribution of exemptions across states. Exemptions are expressed in dollars as of the relevant year, 2008 or 2016. The summary statistics include 38 states, but exclude the seven states (in addition to DC) for which homestead exemptions are unlimited and the five states in which households can opt for federal exemptions that are uniformly higher than state exemption levels.

Debt, Income, and Expenditure Data

The empirical implementation of the results of this paper requires the knowledge of liabilities (personal debt, mortgage debt, auto debt) and expenditure information. The Panel Study of Income Dynamics is the one longitudinal study that contains detailed information on the required variables. I download information for the waves 2007-2015 and use households as the relevant unit of observation, automatically merging the different waves through the PSID Data Center data extraction procedure. I define total expenditure (consumption) as the sum of total food expenditure, utilities, telephone, and internet expenditure, transportation expenditure, education and childcare expenditure, and clothing, trips, and other recreation expenditure. I construct a measure of auto loans by combining loans on first and second vehicles. I construct a measure of personal loans by adding up the value of credit card, medical, legal, and other debt. I use total family income as the relevant income measure.

Table 8 shows summary statistics of the PSID information on households during the waves 2007, 2009, 2011, 2013, and 2015. Median yearly income is $48,000, with median expenditure given by roughly $22,000. I weight observations using the PSID core/immigrant family longitudinal weights. There is substantial variation in the number of available observations per state, with a median of 660 observations per state.

Regional Data

The county-level controls used in Regression R1 come from different sources. Unemployment rates at the county level correspond to annual averages from the Local Area Unemployment Statistics released by the Bureau of Labor Statistics (BLS), which can be downloaded from https://www.bls.gov/lau/.
Table 7: Summary Statistics of Interest Rates and Regional Controls

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Pctl(10)</th>
<th>Median</th>
<th>Pctl(90)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal Unsecured Rate</td>
<td>0.11</td>
<td>0.03</td>
<td>0.07</td>
<td>0.10</td>
<td>0.16</td>
<td>313799</td>
</tr>
<tr>
<td>Auto Loan Rate</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
<td>207982</td>
</tr>
<tr>
<td>Mortgage Fixed Rate</td>
<td>0.04</td>
<td>0.01</td>
<td>0.04</td>
<td>0.04</td>
<td>0.05</td>
<td>26236</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>6.85</td>
<td>2.53</td>
<td>4.20</td>
<td>6.50</td>
<td>10.00</td>
<td>32914</td>
</tr>
<tr>
<td>Median Income</td>
<td>51348.86</td>
<td>13064.15</td>
<td>39086.50</td>
<td>48927.00</td>
<td>66667.00</td>
<td>32914</td>
</tr>
<tr>
<td>House Price Index</td>
<td>387.86</td>
<td>213.78</td>
<td>182.62</td>
<td>352.81</td>
<td>645.85</td>
<td>32914</td>
</tr>
</tbody>
</table>

Note: Table 7 reports summary statistics (mean, median, standard deviation, as well as the 10th and 90th percentiles of each of the variables) for the levels of net interest rates offered across branches in the U.S. between 2008 and 2016 on personal unsecured loans, auto loans on two-year old cars, and thirty-year fixed-rate mortgages. It also presents summary statistics for the unemployment rate (BLS), median income (ACS), and the FHFA county house price index.

House prices at the county level correspond to the House Price Index released by the Federal Housing Finance Agency, which relies on sales or refinancings of single-family homes whose mortgages have been purchased or securitized by Fannie Mae or Freddie Mac. This series can be downloaded from https://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index.aspx. Income and population data at the county and state levels come from five-year estimates of the American Community Survey (ACS), which can be downloaded from https://factfinder.census.gov. While the other series are available from 2008 until 2016, the series for income at the county level is only available starting in 2010, which reduces the number of available observations for the regressions when income measures are added as a control. Table 7 shows summary statistics of all the variables.
Table 8: Summary Statistics of Households’ Personal Information

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Pctl(10)</th>
<th>Median</th>
<th>Pctl(90)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>67512</td>
<td>98266</td>
<td>10957</td>
<td>48050</td>
<td>136500</td>
<td>43673</td>
</tr>
<tr>
<td>Expenditure</td>
<td>26236</td>
<td>19776</td>
<td>7500</td>
<td>22360</td>
<td>47993</td>
<td>43673</td>
</tr>
<tr>
<td>Personal Debt</td>
<td>6961</td>
<td>41226</td>
<td>0</td>
<td>0</td>
<td>17000</td>
<td>43673</td>
</tr>
<tr>
<td>Auto Loans</td>
<td>4522</td>
<td>10087</td>
<td>0</td>
<td>0</td>
<td>18700</td>
<td>43673</td>
</tr>
<tr>
<td>Mortgage Principal</td>
<td>50272</td>
<td>98429</td>
<td>0</td>
<td>0</td>
<td>175000</td>
<td>43673</td>
</tr>
</tbody>
</table>

Note: Table 8 presents summary statistics for the sample of 43673 households obtained from the PSID over the period 2007-2015. All variables are measured in dollars for the current year. I restrict the sample to observations with annual expenditure greater than $100 and negative income values smaller in magnitude than $100,000. I weight observations using the PSID core/immigrant family longitudinal weights.

C Empirical Results

This section contains two sets of results. First, it includes measures of bankruptcy probabilities by income level for two representative states: Alabama and Massachusetts. It also includes an average valuation matrix, to give a sense of the magnitudes involved. Second, it includes two figures representing marginal normalized welfare changes, \( \frac{dV}{dm} \), in Figure 7, and normalized flow welfare changes, \( F \), in Figure 8.

Average Valuation Matrix and Representative Default Probabilities

Table 9 reports probabilities of default in a given year by income quintile for two representative states with low and high exemption levels: Alabama and Massachusetts. Table 10 reports averages across the final set of 36 states considered of their valuation matrix across income quintiles. Quantitatively, a household currently in the third income quintile values a dollar if he transitions to the fourth quintile next period at 63 cents, and he transitions to the second quintile at $1.77.
### Table 9: Default Probabilities

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Alabama</th>
<th>Massachusetts</th>
<th>Alabama</th>
<th>Massachusetts</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>0.0157</td>
<td>0.0068</td>
<td>0.6115</td>
<td>0.0934</td>
</tr>
<tr>
<td>Second</td>
<td>0.0228</td>
<td>0.0081</td>
<td>0.6359</td>
<td>0.1413</td>
</tr>
<tr>
<td>Third</td>
<td>0.0199</td>
<td>0.0060</td>
<td>0.5996</td>
<td>0.2433</td>
</tr>
<tr>
<td>Fourth</td>
<td>0.0115</td>
<td>0.0039</td>
<td>0.5933</td>
<td>0.3898</td>
</tr>
<tr>
<td>Fifth</td>
<td>0.0041</td>
<td>0.0007</td>
<td>0.6527</td>
<td>0.6817</td>
</tr>
</tbody>
</table>

**Note:** Table 9 reports two examples of yearly bankruptcy probabilities by income quintiles for the states of Alabama and Massachusetts, chosen as examples of states associated with high and low marginal welfare gains from increasing exemption levels. These values are constructed combining data on the universe of bankruptcies from the Department of Justice through the Integrated Database of the Federal Justice Center (ID-FJC) with total population data by state from the 2013 five-year ACS. The determination of income quintiles by state uses income data also from the 2013 five-year ACS.

### Table 10: Average Valuation Matrix

<table>
<thead>
<tr>
<th></th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
<th>Fifth</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>1</td>
<td>0.47</td>
<td>0.27</td>
<td>0.17</td>
<td>0.09</td>
</tr>
<tr>
<td>Second</td>
<td>2.64</td>
<td>1</td>
<td>0.59</td>
<td>0.38</td>
<td>0.20</td>
</tr>
<tr>
<td>Third</td>
<td>4.56</td>
<td>1.77</td>
<td>1</td>
<td>0.63</td>
<td>0.33</td>
</tr>
<tr>
<td>Fourth</td>
<td>7.35</td>
<td>2.93</td>
<td>1.65</td>
<td>1</td>
<td>0.53</td>
</tr>
<tr>
<td>Fifth</td>
<td>14.31</td>
<td>5.91</td>
<td>3.35</td>
<td>2.04</td>
<td>1</td>
</tr>
</tbody>
</table>

**Note:** Table 10 reports the average across the final set 36 states of their valuation matrix $G$. The determination of income quintiles by state uses income data from the 2013 five-year ACS. Households’ risk aversion is $\gamma = 2$. 
Figure 8: Flow Marginal Welfare Changes by Income Quintile

**Note**: Figure 8 shows the flow normalized marginal welfare changes by income quintiles by state, which corresponds to the vector $F$, defined in the statement of Proposition 3. The figure includes 36 U.S. states. It excludes states with unlimited exemption levels, states with dominated state exemptions, as well as two states for which some income quintile is not populated in the PSID data (Rhode Island and North Dakota). First (fifth) quintile corresponds to the bottom (top) 20% of the income distribution. Preference parameters are $\beta = (0.94)^3$ and $\gamma = 2$. 
Figure 7: Marginal Welfare Changes by Income Quintile

Note: Figure 7 shows the normalized marginal welfare changes by income quintiles by state, calculated using Equation (22). The figure includes 36 U.S. states. It excludes states with unlimited exemption levels, states with dominated state exemptions, as well as two states for which some income quintile is not populated in the PSID data (Rhode Island and North Dakota). First (fifth) quintile corresponds to the bottom (top) 20% of the income distribution. Figure 5 in the text reports unweighted averages across quintiles by states of marginal welfare changes. Preference parameters are $\beta = (0.94)^3$ and $\gamma = 2$. 
D Proofs and Derivations

The Online Appendix provides a more detailed characterization of the equilibrium in Section 2, as well as additional proofs and derivations of results in the remaining sections.

Section 2

Proposition 1. (Directional test for a change in the exemption level \(m\))

Proof. Given households’ indirect utility for a given level of \(m\), defined in Equation (6), and exploiting households’ optimal default and borrowing choices, we can express \(\frac{dW}{dm}\) as follows

\[
dW \frac{dm}{dm} = u'(c_0) \frac{\partial Q_0(b_1,m)}{\partial m} + \beta \int_\hat{s}^{\tilde{s}} u'(c_1^n(s)) dF(s)
\]

\[
+ \left[ u'(c_0) \frac{\partial Q_0(b_1,m)}{\partial b_1} - \beta \int_{\hat{s}}^{\tilde{s}} u'(n(s) - b_1) dF(s) \right] \frac{db_1}{dm}
\]

\[
+ \beta \left[ u(m) f(\hat{s}) \frac{d\hat{s}}{dm} - u(m) f(\tilde{s}) \frac{d\tilde{s}}{dm} \right]
\]

where \(\tilde{s} = \max \{m, \hat{s}\}\). Therefore,

\[
\frac{dW}{dm} = u'(c_0) \frac{\partial q_0}{\partial m} b_1 + \beta \int_{\hat{s}}^{\tilde{s}} u'(c_1^n(s)) dF(s).
\]

Dividing by \(u'(c_0)\), we can express the normalized welfare change as follows

\[
\frac{dW}{u'(c_0)} = \frac{\partial Q_0(b_1,m)}{\partial m} + \beta \int_{\hat{s}}^{\tilde{s}} \frac{u'(c_1^n(s))}{u'(c_0)} dF(s) = \frac{\partial Q_0(b_1,m)}{\partial m} + \beta \int_{\hat{s}}^{\tilde{s}} dF(s) \frac{\int_{\hat{s}}^{\tilde{s}} \frac{u'(c_1^n(s))}{u'(c_0)} dF(s)}{\int_{\hat{s}}^{\tilde{s}} dF(s)},
\]

which exactly corresponds to Equation (12) in the paper.

Section 3

Proposition 2. (Directional test for a change in the exemption level \(m\): general model)

Proof. I use the definitions of households’ value functions when making the default/repayment decision, in Equation (15), and after repayment and defaulting,
in Equations (16) and (17). Exploiting the envelope theorem, the marginal welfare change in the value function of households $i$ in period $t$ after repaying and defaulting can be expressed as follows:

$$\frac{dV_i^{N}}{dm}(\Omega_i; m) = \frac{\partial u_{i,t}(c_{i,t}, h_{i,t}; \Omega_i)}{\partial c_{i,t}} \sum_k \beta_i b_{i,t}^k (\Psi_{i,t}^k, \Omega_i, m) b_{i,t+1}^k + \beta_i \frac{dE_t[V_{i,t+1}(\Omega_i; m)]}{dm}$$

$$\frac{dV_i^{P}}{dm}(\Omega_i; m) = \frac{\partial u_{i,t}(c_{i,t}, h_{i,t}; \Omega_i)}{\partial c_{i,t}} \frac{\partial W(m; a_i, x_i, s_i)}{\partial m} + \beta_i \frac{dE_t[V_{i,t+1}(\Omega_i; m)]}{dm},$$

where the expression for $\frac{dE_t[V_{i,t+1}(\Omega_i; m)]}{dm}$ can be written as

$$\frac{dE_t[V_{i,t+1}(\Omega_i; m)]}{dm} = \pi_i^D \frac{dV_{i,t+1}^{P}(\Omega_i; m)}{dm} + \pi_i^N \frac{dV_{i,t+1}^{N}(\Omega_i; m)}{dm},$$

$$f_{N_{t+1}} \frac{dV_{i,t+1}^{N}(\Omega_i; m)}{dm} dF(x_{t+1}|\Omega_i) = f_{N_{t+1}} \frac{dV_{i,t+1}^{N}(\Omega_i; m)}{dm} dF(x_{t+1}|\Omega_i).$$

(28)

At any given period and state the partial derivative of the function $W(\cdot)$ takes the form

$$\frac{\partial W}{\partial m} = \begin{cases} 1, & \text{if } W(\cdot) = m \\ 0, & \text{otherwise.} \end{cases}$$

Starting in period $t$, we can recursively substitute in the expression for $\frac{dV_i^{N}}{dm}(\Omega_i; m)$, after suppressing the argument of most functions to simplify notation, to find that

$$\frac{dV_i^{N}}{dm} = \frac{\partial u_{i,t}}{\partial c_{i,t}} \sum_k \frac{\partial q_{i,t}^k}{\partial m} b_{i,t+1}^k + \beta_i \pi_i^{N_{t+1}} \sum_k \frac{\partial u_{i,t+1}}{\partial m} \frac{dV_{i,t+1}^{N}(\Omega_i; m)}{dm} + \beta_i \pi_i^{N_{t+1}} \frac{dW_{i,t+1}}{dm} + \beta_i \pi_i^{N_{t+1}} \frac{dV_{i,t+1}^{N}(\Omega_i; m)}{dm}.$$ 

where I denote by $W_{i,t+1}$ the function $W(\cdot)$ evaluated in period $t + 1$. Further substitution implies that

$$\frac{dV_i^{N}}{dm} = \frac{\partial u_{i,t}}{\partial c_{i,t}} \sum_k \frac{\partial q_{i,t}^k}{\partial m} b_{i,t+1}^k + \beta_i \pi_i^{N_{t+1}} \sum_k \frac{\partial u_{i,t+1}}{\partial m} \frac{dV_{i,t+1}^{N}(\Omega_i; m)}{dm} + \beta_i \pi_i^{N_{t+1}} \frac{dW_{i,t+1}}{dm} + \beta_i \pi_i^{N_{t+1}} \frac{dV_{i,t+1}^{N}(\Omega_i; m)}{dm}.$$ 

$$+ \beta_i \pi_i^{N_{t+1}} \sum_k \frac{\partial q_{i,t+1}^k}{\partial m} b_{i,t+2}^k + \beta_i \pi_i^{N_{t+1}} \frac{dW_{i,t+2}}{dm} + \beta_i \pi_i^{N_{t+1}} \frac{dV_{i,t+2}^{N}(\Omega_i; m)}{dm}.$$ 

$^{24}$Equation (28) would feature an additional term if households’ law of motion $F(\cdot)$ depended directly on i) the exemption level $m$ or ii) other households’ choices, that is, in general equilibrium.
Iterating forward one more period implies that

\[
\frac{dV_{i,N}}{dm} = \frac{\partial u_{i,t}}{\partial c_{i,t}} \sum_k \frac{\partial q_i^k}{dm} b^{k}_{i,t+1} + \beta_i \pi_{t+1}^N \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \sum_k \frac{\partial q_i^k}{dm} b^{k}_{i,t+2} + \beta_i \pi_{t+1}^D \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial W_{i,t+1}}{dm} + \beta_i^2 \pi_{t+1}^D \mathbb{E}_t^{D} \left[ \frac{\partial u_{i,t+2}}{\partial c_{i,t+2}} \sum_k \frac{\partial q_i^k}{dm} b^{k}_{i,t+3} + \beta_i \frac{dE_{t+2}}{dm} V_{i,t+3} (\Omega'; \omega) \right] + \beta_i^2 \pi_{t+1}^N \mathbb{E}_t^{N} \left[ \frac{dV_{i,t+2}}{dm} (\Omega'; \omega) + \pi_{t+1}^N (\Omega'; \omega) \left[ \frac{dV_{i,t+2}}{dm} (\Omega'; \omega) \right] \right],
\]

and so on. Consequently, changing time indexes and starting in period \( \tau \), we can express the marginal welfare change for a household \( i \) that starts from a no-default state from the perspective of period \( \tau \) as follows,

\[
\frac{dW_{i,\tau}}{dm} = \sum_{t=\tau}^{T} \beta_i^{t-\tau} \pi_{t,\tau}^N \mathbb{E}_t^{N} \left[ \frac{\partial u_{i,t}}{\partial c_{i,t}} \frac{\partial q_i^k}{dm} b^{k}_{i,t+1} \right] + \sum_{t=\tau}^{T} \beta_i^{t-\tau+1} \pi_{t,\tau}^D \mathbb{E}_t^{D} \left[ \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial W_{i,t+1}}{dm} \right],
\]

where \( \pi_{t,\tau}^N \) and \( \pi_{t,\tau}^D \) respectively denote equilibrium repayment and default probabilities for a given household in period \( t \) given period \( \tau \) state variables, and \( \mathbb{E}_t^{N} [\cdot] \) and \( \mathbb{E}_t^{D} [\cdot] \) denote the conditional expectations on repayment and default states of period \( t \) random variables given period \( \tau \) state variables. If households had access to credit markets in the same period in which they default, the second term in Equation (21) needs to be augmented and would read as

\[
\sum_{t=\tau}^{T} \beta_i^{t-\tau+1} \pi_{t,\tau}^D \mathbb{E}_t^{D} \left[ \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \left( \frac{\partial W_{i,t+1}}{dm} + \frac{\partial \sum_k q_i^k b^{k}_{i,t+2}}{dm} \right) \right].
\]

Section 4

Proposition 3. (Empirical Implementation)

Proof. Starting from Equation (18), given that households have homogeneous time-invariant preferences and that \( T \to \infty \), it is possible to drop individual \( i \) and time \( t \) subscripts, to express \( \frac{dV}{dm} (\Omega) \) recursively as follows

\[
\frac{dV}{dm} (\Omega) = u' (c (\Omega)) \frac{\partial Q}{dm} (\Omega) + \beta \mathbb{E}_t^{D} \left[ \frac{dV}{dm} (\Omega') \right] + \beta \mathbb{E}_t^{N} \left[ \frac{dV}{dm} (\Omega') \right],
\]

where the flow marginal cost term \( \frac{\partial Q}{dm} (\Omega) \) is given by

\[
\frac{\partial Q}{dm} (\Omega) = \sum_k \frac{\partial q_i^k}{dm} (\Omega) b^{k'} (\Omega).
\]

Similarly, we can express \( \frac{dV}{dm} (\Omega') \) as follows

\[
\frac{dV}{dm} (\Omega') = u' (c (\Omega')) \frac{\partial W}{dm} (\Omega') + \beta \int N \frac{dV}{dm} (\Omega'^{\prime}) dF (\Omega'^{\prime}|\Omega').
\]
After normalizing both expressions by the relevant state marginal utilities, we can rewrite them as

\[
\frac{dV_N}{dm}(\Omega) = \frac{\partial Q}{\partial m}(\Omega) + \beta \int_D u'(c(\Omega')) \frac{dV}{dm}(\Omega') dF(\Omega'|\Omega) + \beta \int_N u'(c(\Omega')) \frac{dV_N}{dm}(\Omega') dF(\Omega'|\Omega). 
\]

When combined, we find a single functional equation that exclusively depends on the values of \(\frac{dV_N}{dm}(\Omega)\) for different states \(\Omega\), given by

\[
\frac{dV_N}{dm}(\Omega) = \frac{\partial Q}{\partial m}(\Omega) + \beta \int_D u'(c(\Omega')) \frac{dV}{dm}(\Omega') dF(\Omega'|\Omega) + \beta \int_N u'(c(\Omega')) \frac{dV_N}{dm}(\Omega') dF(\Omega'|\Omega). 
\]

In order to work with more intuitive expressions in matrix form, from now on I explicitly state whether a given variable (e.g., consumption) is in a default or no default state. This change to a more explicit notation allows us to express \(\frac{dV_N}{dm}(\Omega)\) as follows

\[
\frac{dV_N}{dm}(\Omega) = \frac{\partial Q}{\partial m}(\Omega) + \beta \int_D u'(c(\Omega; D)) \frac{dV}{dm}(\Omega') dF(\Omega'|\Omega; D) + \beta \int_N u'(c(\Omega; N)) \frac{dV_N}{dm}(\Omega') dF(\Omega'|\Omega; N). 
\]

After discretizing the state space, we will be able to solve a system of equations that characterizes normalized welfare changes \(\frac{dV_N}{dm}(\Omega)\) for different values of \(\Omega\). I now proceed to discretize the state space by assuming that there is a finite set of \(S\) possible states, denoted by \(\Omega \in \{\Omega^1, \Omega^2, \ldots, \Omega^S\}\). The stacked vector of normalized marginal welfare changes for households that haven’t defaulted for different state variables, denoted by \(\frac{dV}{dm}\) and given by

\[
\frac{dV}{dm} = V_m^N = \left( \frac{dV_N}{dm}(\Omega^1), \ldots, \frac{dV_N}{dm}(\Omega^S) \right)',
\]

can be expressed as the solution to

\[
V_m^N = F + \beta (\Pi_N \circ G_N) \times V_m^N + \beta^2 (\Pi_D \circ G_N) \times (\Pi_D \circ G_D) \times V_m^N, 
\]

(29)
where $\times$ denotes standard matrix multiplication, $\odot$ denotes element-wise matrix multiplication (Hadamard product), and where $X'$ for a matrix $X$ denotes matrix transposition. I now describe each of the elements of Equation (29). The vector $F$ is given by

$$F = Q_m + \beta \text{diag} (\Pi_{N\rightarrow m} \times G_{N\rightarrow m}') ,$$

where each element of $Q_m = \left( \frac{\partial Q}{\partial m} (\Omega^1), \frac{\partial Q}{\partial m} (\Omega^2), \ldots, \frac{\partial Q}{\partial m} (\Omega^S) \right)'$, given by $\frac{\partial Q}{\partial m} (\Omega) = \sum_k \frac{\partial q_k}{\partial m} (\Omega) b_k (\Omega)$, captures the marginal flow cost associated with higher interest rates. The matrix $\Pi_{N\rightarrow m}$ corresponds to the transition matrix into bankruptcy claiming the full exemption, while $G_{N\rightarrow m}$ corresponds to relative valuations towards those states, with each element given by $u'(c(\Omega')) \frac{u'(c(\Omega))}{u'(c(\Omega'))}$. We can similarly define transition matrices between no-default ($N$) and bankruptcy ($D$), as those given by $\Pi_{N\rightarrow N}$, $\Pi_{N\rightarrow D}$, and $\Pi_{D\rightarrow N}$. In the same way, we can define relative valuation matrices between these scenarios, as in $G_{N\rightarrow m}$, $G_{N\rightarrow N}$, $G_{N\rightarrow D}$, and $G_{D\rightarrow N}$. I provide explicit characterizations of these objects for the specific application implemented in the paper in the Online Appendix.

The solution to the system for $V^N_m$ defined in Equation (29) is therefore given by

$$\frac{dW_m}{dm} = V^N_m = A^{-1} F ,$$

where $F$ is described above and $A$ is given by

$$A = I - \beta (\Pi_{N\rightarrow N} \odot G_{N\rightarrow N}) - \beta^2 (\Pi_{N\rightarrow D} \odot G_{N\rightarrow D}) \times (\Pi_{D\rightarrow N} \odot G_{D\rightarrow N}) ,$$

which corresponds to Equation (22) in the text, and concludes the proof. □