The Elasticity of Intergenerational Substitution, Parental Altruism, and Fertility Choice

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Abstract

Dynastic models common in macroeconomics use a single parameter to control the willingness of individuals to substitute consumption both intertemporally, or across periods, and intergenerationally, or across parents and their children. This paper defines the concept of elasticity of intergenerational substitution (EGS), and extends a standard dynastic model in order to disentangle the EGS from the EIS, or elasticity of intertemporal substitution. A calibrated version of the model lends strong support to the notion that the EGS is significantly larger than one. In contrast, estimates of the EIS suggests that it is at most one. What disciplines the identification is the need to match empirically plausible fertility rates for the US. We illustrate the potential role of the EGS in macroeconomics.

Key words: Altruism, Non-separable utility, Intertemporal substitution, Intergenerational substitution, Economic value of a child.

JEL Codes: D10, D64, D91, J13

1 INTRODUCTION

The seminal contributions of Becker and Barro (1988) and Barro and Becker (1989) provided macroeconomists with the endogenous population version of the neoclassical growth model. This contribution was important because it strengthened the central role of the neoclassical growth model in both the growth and business cycles literatures, allowing macroeconomists to study the allocation of consumption and wealth across generations, as well as the dynamics of cohort size. This was all possible while maintaining the infinite horizon representation of the neoclassical growth model, now interpreted as representing a dynasty: a sequence of finitely lived individuals linked by altruism. In this respect, the Barro-Becker model also became a natural framework to study optimal population.

An implication of the dynastic Barro-Becker model is that the intra-personal willingness to substitute consumption across periods is the same as the inter-personal willingness to substitute consumption across generations. More precisely, as we formalize below, the model implies that the elasticity of intertemporal substitution (EIS) is identical to what we call the elasticity of intergenerational substitution (EGS). There is, however, no compelling theoretical or empirical reasons why these two parameters, or margins, need to be identical. This paper generalizes the standard dynastic model to include separate notions of intertemporal and intergenerational substitution. This generalization allows the model to capture the strong intertemporal consumption smoothing motive documented in the literature (Guvenen, 2006), while at the same time allowing for a different degree of consumption smoothing across generations. In fact, our calibration exercises are consistent with an EIS of at most one, and an EGS of at least two. In other words, while the data supports a strong intertemporal consumption smoothing motive, there seems to be much less consumption smoothing across generations.

Our generalized dynastic model aggregates the utility of different generations using a CES representation. This CES representation includes the Barro-Becker model as a special case, one in which the EIS coincides with the EGS. The CES dynastic utility we propose resembles Dixit and Stiglitz's (1977) preferences for varieties. While in their formulation individuals derive utility from consuming a range of potential goods, in our framework parental utility depends on the utility from own consumption and the (lifetime) utility of a number of potential children. In this context, the EGS captures the substitution between parental consumption and children's consumption.

A number of reasons motivate a separate role for the EIS and the EGS in dynastic models. First, we show that the EGS plays a key role in determining the shadow price of a child, or the economic value of a child, in dynamic models of fertility choice. When the optimal fertility choice is an interior solution the economic value of a child equates the present value of all the costs of raising the child. As we discuss below, the imputed value of a child in the United States can be estimated to be \$330,645 for the average low-income family, \$458,351 for typical middle-income family and \$759,674 for a representative high-income family (see Table 1 in Section 3). We show that a large EGS is needed in order to match this range of values of a child. The reason is that the option value of having a child is larger the more inelastic is the willingness to substitute consumption between the parent and the child. We find that if the EGS is lower than one, the inelastic case, then the imputed value of child is much larger, even more than an order of magnitude larger, than what is suggested by the present value cost computation. A similar finding is reported by Murphy and Topel (2006) in a related literature that looks at the value of statistical life for adults. In their case, implausibly large values are obtained when the EIS is lower than one. By separating the EGS from the EIS, the generalized dynastic model can be consistent with both the economic value of a child and the strong intertemporal consumption smoothing motive.

Second, disentangling the EGS from the EIS allows dynastic models to be consistent with the negative fertility-income relationship documented extensively in the empirical literature.[1] For example, Jones and Tertilt (2008) estimate an income elasticity of fertility of about -0.38 using US

[1] See Jones and Tertilt (2008) and Jones, Schoonbroodt and Tertilt (2011) for a survey of the literature.

Census data. In the standard dynastic model, non-homotheticity and a high EIS would be required for the model to replicate a negative fertility-income relationship. As discussed in Cordoba and Ripoll (2016), non-homotheticity is needed to induce a steady-state relationship between income and fertility, and an EIS larger than one for that relationship to be negative. This is the case because the EIS controls the degree of diminishing marginal utility to lifetime parental income. A low EIS would mean that parents run into sharp diminishing marginal utility in their own consumption, and therefore the option value of having a child is larger for richer parents because children provide a way to avoid the decreasing marginal utility. Thus, parents with low EIS would tend to have more children as their income increases. Since the evidence suggests a negative income-fertility relationship, an EIS larger than one would be needed. By introducing the notion of EGS, our generalized dynastic model resolves this tension: it allows for an EIS of at most one as supported by the evidence, and it makes the model consistent with the income-fertility relationship by having the EGS be larger than one.

Third, beyond fertility choice, disentangling the EIS from the EGS may be important to analyze a number of other issues in modern macroeconomics. Since the EGS captures substitution across generations, the study of longer-term issues such as inequality or any policies that involve intergenerational transfers can be more properly analyzed thinking of the EGS instead of the EIS. Models of long-term inequality typically assume that an individual lives for a finite number of years, and that he is replaced by an individual who inherits his assets (Castañeda *et al.*, 2003). This effectively amounts to assuming that the willingness to substitute consumption within the lifetime is the same as across generations. But in this class of models, a strong consumption smoothing motive induces high levels of precautionary savings, more than what it is observed at the left tale of the wealth distribution. A high EGS introduces the possibility that in the long run wealth will be more disperse because intergenerational transfers may not occur at lower levels of parental wealth.

The generalized dynastic model we propose seeks to strengthen the tools of analysis in macroeconomics. The flexibility introduced by the distinction between the EIS and EGS makes this dynastic model consistent with additional stylized facts. While in this paper we mostly focus on facts related to fertility choice, we also illustrate future research avenues in which this class of dynastic models could play a role in modern macroeconomics.

Our benchmark model is a stylized three-period dynastic model of fertility choice. The model features a single parameter, σ, that determines the EIS, and a different one, η, determining the EGS. We calibrate the model to match facts of the income and fertility data across US states. A key task is to provide identification of parameters that capture different aspects of how the utility of the child enters into the utility of the parent. One of these is η, which governs the EGS. We show that η can be identified by requiring the model to match the average US fertility rate. The intuition for the connection between fertility and η is that η determines the economic value of a child and therefore of the incentives to have children. Our benchmark calibration and a number of robustness checks support a value of $EGS = 1/\eta > 1 \geq 1/\sigma = EIS$.

We then show that the basic result, that $EGS > EIS$, is robust to major generalizations of the benchmark model. For this purpose, we extend the model to include a more realistic life cycle of a family, with endogenous consumption, life cycle savings, leisure, labor and fertility. An advantage of the extended richer model is that we are now able to calibrate σ within the model so as to match certain life-cycle stylized facts. In contrast, parameter σ in the benchmark is set to a standard value commonly used in the literature. In the extended model the EIS determines the degree of consumption smoothing over the life cycle of the parents. In the calibration strategy we exploit changes in time use over the life cycle, specifically leisure, in order to determine the EIS. In particular, adults face important changes in leisure with the arrival of children, their departure from the home, and retirement. The EIS is calibrated to match the observed consumption smoothing over the life cycle in the presence of discrete changes in leisure. In contrast, the EGS directly determines the value of a child, making it a key parameter in matching the fertility level. We find that the introduction of leisure into the model makes the case for a separation between the EGS and EIS even stronger. The reason is that leisure makes life even more valuable, and therefore increases the value of a child and the economic incentives to have more children. A higher EGS provides a counterbalancing force, one that makes parental consumption even more valuable.

The remainder of the paper is organized as follows. Section 2 introduces our generalized dynastic preferences and formally defines the EGS as a distinct concept from the EIS. In Section 3 we solve our benchmark (three-period) dynastic model of fertility and calibrate it. We then illustrate the potential of our generalized framework in macroeconomics by extending it to a dynastic life cycle model with leisure in Section 4. Section 5 discusses other potential applications of our framework in modern macroeconomics. Section 6 concludes.

2 THE EGS

In this section we introduce our generalized dynastic preferences and formally define the EGS as a separate concept from the EIS. The framework generalizes the dynastic model of Becker and Barro (1988) and Barro and Becker (1989). Consider the problem of an individual who lives for T periods. Assume the individual derives utility from own consumption and from the utility of his children, if present. We define the lifetime consumption of an individual as a composite consumption C that takes the form

$$C = \begin{cases} \left[\frac{1}{\Omega}\sum_{t=0}^{T}\beta^t c_t^{1-\sigma}\right]^{\frac{1}{1-\sigma}} & \text{if } \sigma > 0 \text{ and } \sigma \neq 1 \\ \exp\left[\frac{1}{\Omega}\sum_{a=0}^{T}\beta^a \ln c_a\right] & \text{if } \sigma = 1 \end{cases}, \tag{1}$$

where $\beta \in (0,1)$ is a discount factor, c_t is consumption, and $\Omega \equiv \sum_{a=0}^{T}\beta^a$. Absent children, C is the only source of utility for an individual. The function defining composite consumption is a CES aggregator with elasticity of substitution $1/\sigma$ and weights β^t. The standard EIS is given by $1/\sigma$. Notice that $C \geq 0$ for all σ.

The lifetime utility of an individual, W, is described by CES preferences

$$W = \left[C^{1-\eta} + \int_0^n \varphi(i)(W_i')^{1-\eta}di\right]^{\frac{1}{1-\eta}}, \tag{2}$$

where n is the number of children, $\varphi(i) \geq 0$ is the weight that the parent attaches to child i, and W_i' is the utility of child i. Positive altruistic weights means that the parent acts as a social planner at the family level, where the implicit weight of the parent is one. This utility representation is non-separable and resembles the Dixit and Stiglitz's (1977) preferences for varieties. While in their representation there is a potential number of varieties consumers demand, here there is a potential number of children parents may have. Parents have n children, each of whom enjoys utility W_i'. The utility of potential children who are not born is implicitly normalized to zero in (2).[2] Children who are not born resemble the varieties of goods that are not consumed under Dixit-Stiglitz. The key new parameter is $\eta \in (0,1)$, which determines the willingness to substitute composite consumption across generations. As we show below, $1/\eta$ is the elasticity of intergenerational substitution or EGS. Restriction $\eta \in (0,1)$ is required to avoid that unborn children, or unconsumed goods in the Dixit-Stiglitz case, drive parental utility to zero. In other words, this restriction guarantees children are not essential.[3]

Assume $\varphi(i) = \alpha\,(1-\varepsilon)\, i^{-\varepsilon}$ and consider the symmetric case with $W_i' = W'$ for all i. Using monotonic transformation $V = W^{1-\eta}/(1-\eta)$, equation (2) can be rewritten as

$$V = \frac{1}{1-\eta} C^{1-\eta} + \alpha n^{1-\varepsilon} V', \tag{3}$$

which is our generalized version of Becker and Barro (1988) that allows for $\eta \neq \sigma$. The Barro-Becker formulation can be obtained by setting $\eta = \sigma$. In this case, $V = \frac{1}{1-\sigma} C^{1-\sigma} + \alpha n^{1-\varepsilon} V'$, which is the standard formulation with a single elasticity. Equation (3) also makes clear that restriction $\eta \in (0,1)$ is required for $V \geq 0$ and $V' \geq 0$, as otherwise adding a positive mass of children would be detrimental to parental utility.

An alternative way to describe preferences (3) is obtained by recursively substituting V'. Under the boundedness condition $lim_{t\to\infty} \alpha^t N_t^{1-\varepsilon} C_t^{1-\eta}/(1-\eta) = 0$, it follows that

$$V = \sum_{t=0}^{\infty} \alpha^t N_t^{1-\varepsilon} \frac{C_t^{1-\eta}}{1-\eta} = \sum_{t=0}^{\infty} \alpha^t N_t^{1-\varepsilon} \frac{\left[\frac{1}{\Omega} \sum_{s=0}^{T} \beta^s c_{t,s}^{1-\sigma}\right]^{\frac{1-\eta}{1-\sigma}}}{1-\eta}, \tag{4}$$

where t is an index for cohorts, s is an index of the individual's age, $n_0 = 1$, and $N_t = \prod_{v=0}^{t-1} n_v$ is the size of cohort t. Again, when $\eta = \sigma$ the standard dynastic Barro-Becker model is obtained.

We now use equation (4) to formally define the EGS. We show that the EIS and the EGS are equal to $1/\sigma$ and $1/\eta$ respectively. The marginal rate of substitution between consumption at age

[2] Utility in the unborn state may seem unusual to many, but it arises naturally in altruistic models with endogenous population because of the need to fully describe the consumption space. An unborn state is analogous to a dead state arising in models of longevity. See, for example, Rosen (1988), Becker, Philipson and Soares (2005), Murphy and Topel (2006), Hall and Jones (2007), and Jones and Klenow (2016). The welfare of the unborn also arises in normative models of endogenous population, as in Golosov, Jones and Tertilt (2007).

[3] Section 5 extends the model to allow for any $\eta > 0$.

s and consumption at age v for an individual is defined as

$$MRS\left(c_v, c_s\right) \equiv \frac{\partial V/\partial c_v}{\partial V/\partial c_s} = \frac{\partial C/\partial c_v}{\partial C/\partial c_s} \frac{C^{-\eta} C^{\sigma} \beta^v c_v^{-\sigma}}{C^{-\eta} C^{\sigma} \beta^s c_s^{-\sigma}} = \beta^{v-s} (c_s/c_v)^{\sigma},$$

and therefore $EIS = EIS\left(c_v, c_s\right) = 1/\sigma$, as it is standard.

The EGS can be defined similarly to the EIS but it relates to the willingness to substitute consumption across different generations rather than across different ages. The marginal rate of substitution between *composite* consumption of generations s and v from the point of view of the head of the dynasty is given by

$$MRS\left(C_v, C_s\right) = \frac{\partial V/\partial C_v}{\partial V/\partial C_s},$$

and the corresponding EGS can be defined as

$$EGS\left(C_v, C_s\right) = \frac{d \ln (C_s/C_v)}{d \ln MRS(C_v, C_s)}. \tag{5}$$

According to this definition, the EGS measures the willingness of the parent to substitute composite consumption across generations s and v, $s \neq v$. Similarly to the EIS, the EGS could be in principle defined only for adjacent generations but, as Proposition 1 below shows, $EGS = 1/\eta$ for any $s \neq v$ when preferences are described by (4).[4] An alternative definition of the EGS that does not involve composite consumption but period consumptions for parents and children is

$$\widetilde{EGS}\left(c_v, c_s'\right) = \frac{\partial \ln (c_s'/c_v)}{\partial \ln (MRS\left(c_v, c_s'\right))}, \tag{6}$$

where c_v is parental consumption at age v and c_s' is children consumption at age s. The partial derivative refers to a change in the c_s'/c_v ratio holding the other consumption ratios constant. This definition includes as a special case the elasticity of substitution between parental and child's consumption in the same period. Specifically, if parents have children at age F, then $\widetilde{EGS}\left(c_{s+F}, c_s'\right)$ is the within period elasticity of substitution between parental and children's consumption. It turns out that $\widetilde{EGS}\left(c_v, c_s'\right) = \widetilde{EGS} = EGS = 1/\eta$ for any feasible v and s'.

Proposition 1. *Suppose the lifetime utility of an individual is described by (4). Then* $EGS\left(C_v, C_s\right) = \widetilde{EGS}\left(c_v, c_s'\right) = EGS = 1/\eta$.

Proof Using (4)

$$MRS\left(C_v, C_s\right) = \frac{\partial V/\partial C_v}{\partial V/\partial C_s} = \frac{\alpha^v N_v^{1-\varepsilon} C_v^{-\eta}}{\alpha^s N_s^{1-\varepsilon} C_s^{-\eta}},$$

[4] The EGS is conceptually and quantitatively different from long term EIS. For example, Biederman and Goenner (2008) allow the degree of intergenerational substitution to vary over the life cycle so that a short term and a long term EIS emerge. They find that the EIS varies over the life cycle, and the EIS seems to be even smaller and below one for longer time horizons. Conceptually, however, the long term EIS still refers to an intertemporal willigness to substitute consumption across time for the same individuals, and therefore it is different from the EGS.

and therefore

$$EGS\left(C_v, C_s\right) = \frac{d\ln(C_s/C_v)}{d\ln MRS(C_v, C_s)} = 1/\eta.$$

Moreover,

$$MRS\left(c_v, c_s'\right) = \frac{\partial V/\partial c_v}{\partial V/\partial c_s'} = \frac{C^{-\eta}C^{\sigma}\beta^{v}c_v^{-\sigma}}{\alpha n^{1-\varepsilon}\left(C'\right)^{-\eta}\left(C'\right)^{\sigma}\beta^{s}\left(c_s'\right)^{-\sigma}}.$$

Since C is constant returns to scale, it can be written as $C = c_v\hat{C}_v$ where $\hat{C}_v$ is homogeneous of degree zero. As a result,

$$MRS\left(c_v, c_s'\right) = \frac{c_v^{-\eta}\hat{C}_v^{-\eta}c_v^{\sigma}\hat{C}_v^{\sigma}\beta^{v}c_v^{-\sigma}}{\alpha n^{1-\varepsilon}\left(c_s'\right)^{-\eta}\left(\hat{C}_v'\right)^{-\eta}\left(c_s'\right)^{\sigma}\left(\hat{C}_s'\right)^{\sigma}\beta^{s}\left(c_s'\right)^{-\sigma}} = \frac{\hat{C}_v^{\sigma-\eta}\beta^{v}}{\alpha n^{1-\varepsilon}\left(\hat{C}_s'\right)^{\sigma-\eta}\beta^{s}}\left(c_v/c_s'\right)^{-\eta}.$$

Consider a change in c_v/c_s' holding all other consumption ratios constant. In that case

$$\widetilde{EGS}\left(c_v, c_s'\right) = \frac{\partial\ln(c_s'/c_v)}{\partial\ln(MRS\left(c_v, c_s'\right))} = 1/\eta$$

as stated. ‖

3 BENCHMARK DYNASTIC MODEL

In this section we specify and calibrate a stylized three-period dynastic model with endogenous fertility using the generalized dynastic preferences in (3). This stylized model illustrates the role of the EGS in the context of fertility choice. We examine a more general life cycle model with leisure in Section 4, and other potential applications of the EGS in Section 5.

3.1 *Model*

Consider an economy in which individuals live for three periods: child, young adult and old adult. Young adults work and raise children, and old adults only consume. Parents decide the consumption of their children. Let b be the total lifetime transfers from a parent to each of his adult children (inter vivos transfers plus bequests). The problem of the young adult is given by

$$V(c_c, b) = \max_{c_y, c_o, n, c_c', b'} \frac{1}{1-\eta}C\left(c_c, c_y, c_o\right)^{1-\eta} + \alpha n^{1-\varepsilon}V(c_c', b'), \tag{7}$$

subject to

$$b + w \geq c_y + c_o/R + n\left(b' + c_c' + \lambda w\right), \tag{8}$$

$$b' \geq 0, \tag{9}$$

$$\overline{n} \geq n \geq 0, \tag{10}$$

where c_c, c_y, and c_o are consumptions as child, young adult and old adult respectively, R is the gross interest rate, w is wage income, and λ is the time cost of raising a child. Equation (8) is the present value budget constraint of a young adult: transfers received from parents plus labor income must cover consumption expenses plus the cost of raising children. The cost of a child includes the time cost for the parent, λw, the consumption of the child, c_c', plus the amount of transfers per child, b'.

Equation (9) is a constraint to intergenerational transfers: parents cannot transfer negative amounts (debt) to their adult children. We also refer to this as the "bequest" constraint. This credit constraint is natural, as it captures the legal and moral reasons precluding parents from enforcing debt obligations on their own adult children. Last, equation (10) limits the number of children to a maximum of $\overline{n} = 1/\lambda$ to guarantee a non-negative labor supply.

Composite consumption, C, is given by

$$C(c_c, c_y, c_o) = \begin{cases} \left[\frac{1}{\Omega}\left(c_c^{1-\sigma} + \beta c_y^{1-\sigma} + \beta^2 c_o^{1-\sigma}\right)\right]^{\frac{1}{1-\sigma}} + \underline{C} & \text{if } \sigma > 0 \text{ and } \sigma \neq 1 \\ \exp\left[\frac{1}{\Omega}\left(\ln c_c + \beta \ln c_y + \beta^2 \ln c_o\right)\right] + \underline{C} & \text{if } \sigma = 1 \end{cases},$$

where $\underline{C} > 0$ is non-market consumption. As discussed in Cordoba and Ripoll (2016), a form of non-homotheticity as well as constraints to intergenerational transfers, as in equation (9), allow deterministic dynastic altruistic models to replicate the observed negative relationship between fertility and income.

For simplicity we assume $\beta R = 1$ so that the model delivers $c_y = c_o$ –adults have a simple flat consumption when young and old. Our results are robust to alternative assumptions that give rise to a more realistic consumption profile over the life cycle (Section 4).

3.1.1 Optimal consumption and transfers

Let θ and μ be the Lagrange multipliers on the budget constraint (8), and the bequest constraint (9) respectively. The first order conditions with respect to c_y, c_o, c_c' and b' are

$$C^{-\eta}\left(C - \underline{C}\right)^{\sigma} \beta c_y^{-\sigma}/\Omega = \theta, \tag{11}$$

$$C^{-\eta}\left(C - \underline{C}\right)^{\sigma} \beta^2 c_o^{-\sigma}/\Omega = \theta/R, \tag{12}$$

$$\alpha n^{1-\varepsilon} \frac{\partial V(c_c', b')}{\partial c_c'} = \theta n, \text{ and} \tag{13}$$

$$\alpha n^{1-\varepsilon} \frac{\partial V(c_c', b')}{\partial b'} + \mu = \theta n / R. \tag{14}$$

The envelope conditions read

$$\frac{\partial V(c_c, b)}{\partial c_c} = C^{-\eta}\left(C - \underline{C}\right)^{\sigma} c_c^{-\sigma}/\Omega, \tag{15}$$

$$V_b(b) = \theta. \tag{16}$$

Equations (11), (12) and the assumption $\beta R = 1$ imply that

$$c_y = c_o. \tag{17}$$

Moreover, equations (14) and (16) can be written as

$$\theta \geq R\alpha n^{-\varepsilon}\theta'. \tag{18}$$

This expression is an intergenerational version of the standard Euler equation, with θ being the marginal utility of a young adult's consumption and $\alpha n^{-\varepsilon}$ an endogenous discount factor, in this case the average degree of altruism which depends on the number of children.

In what follows we focus on a steady state situation. In this case (18) simplifies to $1 \geq R\alpha n^{-\varepsilon}$. If the bequest constraint does not bind (unconstrained case), this expression would hold with equality and the steady state number of children would be $n^* = (R\alpha)^{1/\varepsilon}$. Fertility in this case is a function of the interest rate but independent of any income or level variable such as $\underline{C}$ or wages, which is inconsistent with the documented evidence of a negative relationship between fertility and income (see Jones and Tertilt, 2008). For this reason, from now on we focus on the case in which the bequest constraint binds, which is also the relevant case in the model's calibration.[5]

When the bequest constraint binds, $b = 0$ and the strict inequality $1 > R\alpha n^{-\varepsilon}$ holds. This implies that steady state fertility is larger in the constrained than in the unconstrained case. Combining (13) and (15) it follows that

$$\alpha n^{1-\varepsilon} C'^{-\eta}\left(C' - \underline{C}\right)^{\sigma} c_c'^{-\sigma}/\Omega = \theta n.$$

In steady state this equation and (11) simplify to

$$c_y = G(n)^{1/\sigma} c_c, \tag{19}$$

where $G(n) \equiv n^{\varepsilon}/R\alpha \geq 1$ measures the "tightness" of the bequest constraint or the extent to which consumption during childhood falls below that of the adult period, i.e., $c_c < c_y$. Furthermore, $G(n)$

[5] As we discuss later in the model's calibration, inter vivos transfers and bequests to adult children do occur in the United States, but only a relatively small fraction of adults receive them. In addition, when they occur, they do in small amounts (Altonji *et al.*, 1997). From this perspective, a binding bequest constraint ($b' = 0$) would be a reasonable characterization of the representative or average adult child in the United States.

is increasing in the number of children meaning that in larger families the bequest constraint is tighter due to the fact that the average degree of altruism per child decreases with n. This manifests in the ratio of childhood to adult consumption, c_c/c_y, being lower in larger families. Therefore when the bequest constraint binds the model captures a quality-quantity trade-off between the number of children and the resources parents spend on them during childhood.[6]

Given n, steady state solutions can be obtained using (17), (19), (8) as

$$c_y = c_o = c(n) \equiv \frac{G(n)^{1/\sigma} w\left(1-\lambda n\right)}{n + G(n)^{1/\sigma}\left(1+\beta\right)}, \tag{20}$$

$$c_c = c_c(n) \equiv \frac{1-\lambda n}{n + G(n)^{1/\sigma}\left(1+\beta\right)} w, \tag{21}$$

$$c_c + \lambda w = \frac{1 + G(n)^{1/\sigma}\lambda\left(1+\beta\right)}{n + G(n)^{1/\sigma}\left(1+\beta\right)} w. \tag{22}$$

According to these expressions, the child's consumption decreases with the number of children for two reasons: additional children lower the net income of parents as they reduce parental labor supply; and more children increase the discount per-child due to the decreasing degree of altruism. Furthermore, the total cost of a child, $c_c+\lambda w$, decreases with the number of children because parents reduce consumption per-child while the time cost per-child remains constant. Adult consumption, on the other hand, may be decreasing or increasing in the number of children. A sufficient condition for adult consumption $c(n)$ to be decreasing in the number of children is $\sigma > \varepsilon$, which turns out to be the empirically relevant case.

Given solutions for consumptions, steady state C and V can be written as

$$C(n) = \left[\frac{1}{\Omega}\left(G(n)^{1-1/\sigma} + \beta + \beta^2\right)\right]^{\frac{1}{1-\sigma}} c(n) + \underline{C} \quad \text{and} \quad V(n) = \frac{1}{1-\eta}\frac{C(n)^{1-\eta}}{1-\alpha n^{1-\varepsilon}}. \tag{23}$$

Provided $\sigma > \varepsilon$, the utility the parent derives from own consumption, $C(n)$, is also decreasing in n.

3.1.2 Optimal fertility

We now turn to the fertility choice. The optimality condition for fertility in an interior solution is

$$w\lambda + c_c' = \left(1-\varepsilon\right)\alpha n^{-\varepsilon}\frac{V\left(c_c', b'\right)}{\theta}. \tag{24}$$

The left-hand side of this expression is the marginal cost of a child, which includes the value of parental time cost plus child's consumption. Notice that this also corresponds to the average cost of raising a child, which we use below in the calibration. The right-hand side is the marginal benefit

[6]Notice that $G(n) = 1$ corresponds to the case in which the bequest constraint does not bind, implying $c_c = c_y (= c_o)$.

of the n-th child. The term $V\left(^{\prime}\right)/\theta$ is the welfare of the child measured in parental consumption units, while $(1-\varepsilon)\,\alpha n^{-\varepsilon}$ is the weight that parents attach to the n child.

It is intuitive to think of the marginal benefit of a child as the shadow price of a child, or the economic value of a child, which is given by $VC(n) \equiv (1-\varepsilon)\,\alpha n^{-\varepsilon} V\left(^{\prime}\right)/\theta$. The value of a child to the parent is the welfare of the child measured in parental consumption units, properly weighted by marginal altruism. It can be shown that it also corresponds to the marginal rate of substitution between the number of children and parental consumption. Using (23), the steady-state $VC(n)$ can be written as

$$VC(n) = \frac{1-\varepsilon}{1-\eta}\frac{\alpha n^{-\varepsilon}}{1-\alpha n^{1-\varepsilon}}\frac{C}{(C-\underline{C})^{\sigma}\beta c(n)^{-\sigma}/\Omega}, \tag{25}$$

which highlights the role of η in determining the value of a child. Everything else equal, the higher the EGS (the lower the η), the lower the value of a child. By definition, a high EGS is a high elasticity of substitution between parental and child consumption, but the expression above also links a high EGS with a high marginal rate of substitution between number of children and parental consumption. Interestingly, as $\eta \to 1$ the value of a child becomes infinity, underscoring one of the key role of disentangling the EIS from the EGS. If we consider the standard case with $\sigma = \eta$, equation (25) implies that as $\sigma \to 1$, the value of a child would become implausibly high. In this case, as we show below in the calibration, the value of a child would be too high relative to the marginal cost of raising a child in (24), making it impossible to match the observed fertility in the data. Since $\sigma = 1$ is a common value in quantitative macroeconomics, and a lower bound for available estimates of σ, disentangling η from σ makes it possible to link the value of a child to η rather than to σ. It turns out that matching the observed fertility in the data requires $\eta < 1$.

It is interesting to notice the analogy between the value of a child in (25) and the value of (statistical) life for adults in Murphy and Topel (2006). In their case, as $\sigma \to 1$, the value of life for adults becomes implausibly high relative to the estimates of the value of statistical life in the data. The reason is that as $\sigma \to 1$ the consumption smoothing motive becomes stronger, making it more valuable to extend the life of an adult for additional periods. In our case, as $\eta \to 1$ parents' willingness to substitute consumption between the parent and the child becomes more inelastic, making the option value of a child larger. By separating the EGS from the EIS, the generalized dynastic model can be consistent with both the economic value of a child and the strong intertemporal consumption smoothing motive documented in the quantitative macro literature.

Using (23), the value of a child in (25) can be written as

$$\frac{VC(n)}{c(n)} = \frac{1-\varepsilon}{1-\eta}\frac{G(n)^{1-1/\sigma}+\beta+\beta^{2}}{G(n)-\beta n}\frac{1}{1-\underline{C}/C(n)}, \tag{26}$$

and using (22), (20) and (26), the solution for steady state fertility is characterized by

$$\frac{1+G(n)^{1/\sigma}\lambda\,(1+\beta)}{1-\lambda n} = \frac{1-\varepsilon}{1-\eta}\frac{G(n)+G(n)^{1/\sigma}\beta\,(1+\beta)}{G(n)-\beta n}\frac{1}{1-\underline{C}/C(n)}, \tag{27}$$

which equates the marginal cost and marginal benefit of a child, both as proportion of parental consumption. For the solution to this equation to be optimal, we further need to check that n satisfies $\overline{n} > n > n^*$. We confirm that this is the case in the calibrated model.

For the case in which $\underline{C} = 0$ we can derive explicit necessary and sufficient conditions for optimal fertility n to satisfy $\overline{n} > n > n^*$, which also imply that the bequest constraint is binding. First, at $n = n^*$ we have that the marginal benefit is larger than the marginal cost in equation (27). Since $n^* = (R\alpha)^{1/\varepsilon}$ and $G(n^*) = 1$, then the following parameter restriction is required $n > n^*$ to hold

$$\frac{1+\lambda(1+\beta)}{1+\beta(1+\beta)} \frac{1-\beta(R\alpha)^{1/\varepsilon}}{1-\lambda(R\alpha)^{1/\varepsilon}} < \frac{1-\varepsilon}{1-\eta}.$$

In order to interpret this condition, notice that if $\eta > \varepsilon$, then $\beta > \lambda$ is sufficient for the restriction to be satisfied. The present value of the child's future income is $\beta w = w/R$, while λw is the time cost of raising a child. Therefore if $\beta > \lambda$ children are a net financial gain in the sense that the child's future income is larger than the time cost of raising the child. Absent any constraints to intergenerational transfers, altruistic parents would have the incentive to recover part of the cost of raising children by giving the adult child a negative transfer. This would effectively transfer some of the adult child's income to the parent. Constraint (9) prevents such negative transfers from occurring. Last, it can be shown that since $\overline{n} = 1/\lambda$, condition $\lambda^{1-\varepsilon} > \alpha$ is required to guarantee $\overline{n} > n$.

3.2 *Calibration and results*

In this section we calibrate the model. The main goal is to identify the EGS and assess the extent to which it differs from the EIS.

3.2.1 Fertility data

We use evidence from a cross-section of US states to calibrate the model. This evidence is appropriate for our purpose for several reasons. First, in contrast with cross-sectional international data in which countries are at different stages of the demographic transition, US states have all completed this transition. This feature maps better into our steady-state analysis. Second, cross-state data is better for our purpose than individual-level data because relative income across states is roughly constant, while individual income in any given year does not represent lifetime income. In this respect, the cross-state fertility-income relationship is closer to the one captured in the model. Third, despite the relative convergence in both income and fertility across US states, there is still some cross-sectional variation. Last, the assumption that the interest rate is identical can be better justified across US states than across countries.

Insert Figure 1 around here

Figure 1 displays the total fertility rate versus median household income across US states in 2016. The total fertility rate is from the 2018 National Vital Statistical Report, and it corresponds to the number of births 1,000 women age 15-44 would have in their lifetime if they experienced the births currently occurring at each age. Median household income is from the American Community Survey Brief (US Census Bureau, 2016). Average total fertility in the sample is 1.820 while average median household income is $57,617. The size of the bubbles in Figure 1 represents 2016 population weights from the Statistical Abstract of the US. Taking into account population weights, Figure 1 suggests a slightly negative relationship between fertility and income. Based on this data we estimate an income elasticity of fertility of -0.143 (significant at the 5% level on a population-weighted regression). This elasticity is close to the one estimated by Jones and Tertilt (2008) using individual-level Census data for the most recent cohorts. For instance, for the 1951-1955 cohort, whose average fertility was 2.05 children ever born and average occupational income was $49,378, they estimate an income elasticity of fertility of -0.17. We will use our estimated elasticity as one of the calibration targets.

3.2.2 The costs of raising children

As discussed above, the costs of raising children are fundamental for the identification of the key parameter η. Our calibration requires data on both the goods costs and the time costs of raising children. Recall that in our model c_c effectively corresponds to the present value of the goods costs of raising a child, while λw is the present value of the time costs. We use USDA data from Lino (2012) to compute the goods cost of raising a child. According to the Lino (2012), the typical cost of raising a child born in 2011 from age 0 to 17 for a family of four in the lowest income group is $169,080, while for a family in the middle-income group is $234,900 and for a high-income family is $389,670 in 2011 dollars. These figures include direct parental expenses made on children through age 17 such as housing, food, transportation, health care, clothing, child care, and private expenses in education. Assuming a discount rate of 3%, the corresponding present values of these sums are $132,258 for low income, $183,340 for middle income, and $303,870 for the high income group.

Table 1 presents these goods costs for a "representative family" in each USDA income group. Using the family income brackets from Lino (2012), we select a 2011 income of $43,625 for the representative low-income family; $81,140 for middle income, and $126,435 for a high-income family. The low-income family figure is computed as the average of the following two values: $27,840, which corresponds to the income of a family in which both parents make the federal minimum wage in 2011, and $59,410, which is the upper bound of low-income families from the Lino (2012) classification. The middle-income family number is simply the mid-point of the Lino (2012) interval of $59,410 to $102,280. Last, the high-income "representative" family is computed as the average between $102,280 and $150,000, where the latter corresponds to the 90th percentile of the family income distribution in 2011 according to the US Census Bureau. For each of the representative families we also compute a lifetime household income assuming a 43-year working life span and a 3% real interest rate. From Figure 1 and Table 1, notice that the values of median household income by states fall in between the low and middle-income family groups under the USDA classification. For this reason, we will use the information of these two groups to calibrate the goods costs of raising children.

Insert Table 1 around here

As we discuss in Cordoba and Ripoll (2016), accounting for the time costs of raising children is not trivial. Available estimates are based on time use survey data, but the difficulty of measuring time costs is that in many instances parents multitask, taking care of children as a secondary activity while performing other primary activities. Using the 2003-2006 American Time Use Survey, Guryan *et al.* (2008) find that while mothers spend around 14 hours per week in child care, fathers spend around 7 hours. These correspond to primary time parents spend on children (basic care of children, education, recreation and any travel related to these). If the total time parents spend in the presence of their children is measured (both primary and secondary time), then mothers spend 45 hours per week and fathers spend 30 hours. In a related study, Folbre (2008) uses the 1997 Child Development Supplement of the Panel Survey of Income Dynamics to conclude that the average amount of both passive and active parental-care hours per child (not including sleep) is 41.3 per week for a two-parent household with two children ages 0 to 11. Folbre (2008) also discusses two alternative ways of computing the monetary value of these hours: one uses a child-care worker's wage and the other the median wage. The former method implies that the time cost of raising children is on average around 60% of the total costs (see Table 7.3, p. 135), while the latter implies it is 75%.

In order to compute the time costs for each of the representative families in Table 1 we use the more conservative estimate in which they are about 60% of the total cost of raising a child. As can be seen in Table 1, the present value of the time costs of raising a child is \$198,387 for a low-income family, \$275,011 for a middle-income family, and \$455,804 for a high-income family. Table 1 also presents the total costs of raising a child: \$330,645 for a low-income family, \$458,351 for a middle-income family, and \$759,674 for a high-income family.

It is important to notice that the total costs of raising a child in Table 1 map into the economic value of a child. As discussed before, optimal fertility is decided comparing the marginal cost and the marginal benefit of a child. Since the values in Table 1 correspond to the total marginal cost of a child, they also correspond to the value of a child, or the marginal benefit. As the median household income across US states in Figure 1 ranges from about \$43,000 to \$78,000, our calibrated model should be consistent with a value of child ranging between \$330,645 and \$458,351, which correspond to the total cost of raising a child for low and middle-income families in Table 1. The table also reports the costs of raising a child as a fraction of the lifetime household family income, which are useful for calibration purposes. As Table 1 indicates, for the average two-parent two-child family in the low and middle-income groups in the US, the goods costs of raising each child correspond to 10.8% of the lifetime household income, while time costs are 16.2% and total costs are 26.9%.

3.2.3 Exogenous parameters

Some parameters in the model are set exogenously. We set the length of each of the three periods of life to 25 years: a child consumes with the resources transferred by his parent from ages 0 to 25; young adults have children at age 25 and work until age 50, while old adults consume and retire from age 50 to 75. The annual interest rate is set to 3% which implies a discount factor β of 0.48 per 25-year period.

We set $\sigma = 1$, the typical lowest bound for this parameter in quantitative macro models (Guvenen, 2006). The main purpose of this calibration is to disentangle the EIS from the EGS. With the restriction $\eta \in (0,1)$, setting σ to this lowest bound will be particularly informative regarding how different the EIS and the EGS might be. We do provide alternative calibrations of the model for the larger values of σ in the robustness section.

Parameter ε determines the degree of diminishing altruism. While little is know about this parameter, there is one study that directly estimates it. Dickie and Messman (2004) uses stated-preference data on parental willingness to pay to relieve symptoms in children's acute respiratory illnesses. The distinct feature of this study is that it estimates how parental willingness to pay changes with the number of children in the family. In addition to strongly supporting parental altruism toward their children, the paper estimates an elasticity of the parental willingness to pay with respect to the number of children in the family of -0.288 (see their Table 5, p. 1159).

In order to map this elasticity into our model, and to the extent that health expenditures in treating acute illnesses increases the survival probability of the child, we compute the implied willingness to pay WTP for an increase $\Delta\pi_c$ in survival. In this case the WTP is directly linked to the value of a child and given by[7]

$$WTP(n) = VC(n) \cdot \Delta\pi_c.$$

Since we are only interested in the elasticity of $WTP(n)$ with respect to n, the magnitude of term $\Delta\pi_c$ does not play a role in the value of this elasticity. Using the expression above together with

[7] To derive the $WTP(n)$ formula we write the utility of the parent as

$$V(c_c, b) = \max \frac{1}{1-\eta} C\left(c_c, c_y, c_o\right)^{1-\eta} + \alpha\left(\pi_c n\right)^{1-\varepsilon} V(c_c', b'),$$

where π_c is the survival probability of the child. This implies that the willigness to pay to increase survival by $\Delta\pi_c$ is given by

$$WTP(n) = \frac{\partial V/\partial \pi_c}{\partial V/\partial c_y} \frac{\Delta\pi_c}{n}.$$

Because π_c and n enter symetrically in the altruistic weight function, it turns out that

$$WTP(n) = \frac{\partial V/\partial \pi_c}{\partial V/\partial c_y} \frac{\Delta\pi_c}{n} = \frac{\partial V/\partial n}{\partial V/\partial c_y} \frac{\Delta\pi_c}{\pi_c} = VC(n) \cdot \Delta\pi_c.$$

Since in our model $\pi_c = 1$, evaluating the expression above at $\pi_c = 1$ delivers

$$WTP(n) = VC(n) \cdot \Delta\pi_c.$$

equation (26) we obtain

$$\varepsilon^{WTP}(n) = \frac{\partial WTP}{\partial n}\frac{n}{WTP} = -\varepsilon.$$

We therefore set $\varepsilon = 0.288$. This value suggests a relatively low degree of diminishing marginal altruism. Since we are setting ε exogenously, we provide robustness checks for the value of this parameter below.

3.2.4 Calibrated parameters

The remaining four parameters $[\lambda, \alpha, \eta, \underline{C}]$ are calibrated to four targets. Table 2 reports the results. Although the parameters are jointly calibrated, each one can be more directly related to one of the targets. First, parameter λ is the present value of the time costs of raising a child, λw, as a fraction of parental lifetime income w. We calibrate λ to match this share in the data, which according to Table 1 corresponds to an average of 16.2% for a typical family with two parents and two children in the low and middle-income USDA groups in Lino (2012). Since in our model there is a single parent, and the average fertility in the sample is two children per household, then the average single parent will be raising one child and $\lambda = 0.324$.

Insert Table 2 around here

Second, parameter α, which corresponds to the level parameter in the altruistic weight, has a first-order effect on the goods costs of raising children as a fraction of lifetime parental income, or c_c/w. To see this, using equation (21) write

$$\frac{c_c}{w} = \frac{1-\lambda n}{n + \alpha^{-1/\sigma}\beta^{1/\sigma}n^{\varepsilon/\sigma}(1+\beta)}.$$

Given the exogenous values of R, β and σ, as well as the calibrated value of λ, and given that the average n will be calibrated to a target of $n = 0.921$, then the equation above determines α for a given c_c target. According to Table 1, the average c_c for low and middle-income families is 10.8%. We obtain a calibrated value $\alpha = 0.307$.

Third is our parameter of interest, η. Although η is calibrated jointly with the rest of the parameters, its value is mainly identified from the value of a child. In particular, we choose η so that the model delivers an average fertility of around one child per parent, or more precisely $n = 0.921$, which corresponds to half of the average fertility across US states in Figure 1. Using equation (24), the optimality condition for fertility, together with the expressions for the value of a child in (26), and adult consumption (20), we can write

$$\frac{w\lambda + c_c}{w} = \frac{VC(n)}{w} = \frac{1-\varepsilon}{1-\eta}\frac{G(n)^{1-1/\sigma}+\beta+\beta^2}{G(n)-n/R}\frac{G(n)^{1/\sigma}(1-\lambda n)}{n+G(n)^{1/\sigma}(1+\beta)}\frac{1}{1-\underline{C}/C(n)},$$

where the left-hand side represents the total cost of raising a child as a share of parental lifetime income, and the right-hand side is the value of a child also as a fraction of parental lifetime income.

For $\underline{C} = 0$, and given ε and the calibration targets described for λ, α, the equation above identifies η for a target of $n = 0.921$. Parameter $\underline{C}$ is still to be determined, but as long as $\underline{C}/C(n)$ is small, η will be of first-order importance in determining the value of a child, and through this channel, the fertility level. We obtain a calibrated value of $\eta = 0.285$, or an EGS is 3.5, a large degree of intergenerational substitution.

Last is parameter $\underline{C}$, non-market consumption. We calibrate this parameter in order to match the income elasticity of fertility in our sample, which we computed to be -0.143 (see fitted line in Figure 1). If $\underline{C} = 0$ then fertility would not be related to income in our model. To see that write the optimality condition of fertility in (24) as

$$w\lambda + c_c = \frac{1-\varepsilon}{1-\eta}\frac{G(n)^{1-1/\sigma} + \beta + \beta^2}{G(n) - n/R}\frac{c(n)}{1 - \underline{C}/C(n)}.$$

Notice, using (22) and (20), that both the total cost of raising a child $w\lambda + c_c$ and adult consumption $c(n)$ are proportional to w. Therefore, if $\underline{C} = 0$, then both the marginal cost and the marginal benefit of a child are proportional to lifetime income and fertility choices would be independent of w. Only if $\underline{C} > 0$ there is a link between fertility and income. In fact, this relationship is negative because when w increases the marginal benefit increases less than the marginal cost due to the presence of $\underline{C}/C(n)$. Calibrating $\underline{C}$ in order to target an income elasticity of fertility of -0.143 results in a maximum $\underline{C}/C(n)$ of 4.79% across US states, a small value.[8]

The calibration implies that the bequest constraint binds. Therefore in the calibrated model no bequests or other inter vivos transfers to adult offsprings are given and all expenses on children take place during childhood. This case represents well the average family in the United States. Although inter vivos transfers and voluntary bequests do occur in the US, a relatively small fraction of adults receive them, and they occur in small amounts. For instance, using the 1988 special supplement on transfers between relatives from the PSID, Altonji *et al.* (1997) document that only 23% of adult children (on average 31 years old) receive transfers from parents (on average 59 years old). These are overall small transfers: the mean is \$3,442, and the median is \$951 in 2011 dollars. A similar pattern has been documented for bequests. Using the 1993-1995 Asset and Health Dynamics among the Oldest Old (AHEAD) data, Hurd and Smith (2001) document that most bequests are of little of no value: single descendants at the bottom 30% receive \$2,952, and the average single descendant receives \$14,760 in 2011 dollars. Given the highly skewed wealth distribution in the United States, the occurrence of significant bequests concerns only of a small fraction of the population.[9]

[8] Although the income elasticity of fertilty we estimate is statistically significant, its absolute value is small. Since in the calibrated model US states only differ in household income, this small elasticity implies that the model cannot be expected to explain all the fertility dispersion in the data. However, what is important to notice is that this does not affect the main point of this calibration exercise, which is to illustrate how our main parameter η can be identified from the value of a child that matches average fertility.

[9] It may appear surprising that bequests are minimal in the US, as this may not be the case for other countries. We acknowledge that the US may be different than other countries in this respect. Older adults across countries differ on their portfolio allocation and medical coverage, which will necessarily affect the size of inter vivos transfers and bequests. For instance, in Italy about 56% of bequests are in the form of houses (Bellettini *et al.*, 2017). In contrast, in the US the value of houses and any liquid assets left behind by the deceased parent are used to cover medical or nursing home bills, as well as funeral expenses. Little is left after those payments are accounted for (Hurd

Insert Figure 2 around here

Our calibrated model also has implications for cross-state variations in the value of a child. Figure 2 illustrates this point. Under our benchmark calibration, the average value of a child across US states is \$375,220. The maximum value of a child in the sample is \$517,169 (Maryland) and the minimum is \$264,199 (Mississippi). These values are overall plausible. The median income in all US states falls within the low and middle income brackets. According to Table 1, the value of a child for the average family in these two brackets is \$330,645 and \$458,351 respectively.

3.3 *Robustness and discussion*

We check the robustness of our calibration to alternative values of both σ and ε. For our benchmark value $\varepsilon = 0.288$, we find that as σ increases the calibrated value of η decreases. For instance, for $\sigma = 1.5$, a typical value in quantitative macro, we obtain $\eta = 0.238$. As can be seen in equation (26), a larger σ increases the marginal benefit of a child (value of a child) because the marginal utility of parental consumption falls faster. To counteract this effect and match a given number of children, η has to fall in order to discourage parents from adding new children. The parameter that changes the most as σ increases is $\underline{C}$, which rapidly increases with σ, although the ratio $\underline{C}/C$ only changes slightly. What we conclude is that if anything, raising the value of σ only widens the gap between σ and η, providing additional evidence of a distinct role for the EIS and the EGS.

We also calibrate the model for different values of ε. For our benchmark value $\sigma = 1$, we find that as ε increases, our calibrated value of η also increases. For instance, for $\varepsilon = 0.5$ we obtain $\eta = 0.488$ while for $\varepsilon = 0.8$ we obtain $\eta = 0.8$. As ε increases, the altruistic weight on the marginal child decreases, lowering the demand for children. To counteract this effect and match a given number of children, the EGS has to decrease so that parents are willing to add more children and smooth consumption across more individuals.

We also analyze the consequences of making η approach σ in our benchmark calibration where $\sigma = 1$. This requires abandoning one of the identification targets while other parameters need to be adjusted to match the other moments. If the target on average fertility is abandoned, we find that fertility approaches the maximum feasible as $\eta \to 1$, a prediction inconsistent with the evidence. This is expected because the value of a child, as described by equation (26), explodes as $\eta \to 1$. If the target on goods costs of raising children, c_c/w, is abandoned, then we find that $\alpha \to 0$ as $\eta \to 1$. The model can then match fertility but goods costs of children become just a small fraction of what the data suggest. A low α keeps the value of a child from exploding but also reduces the expenditures of children's consumption. Alternatively, we could exogenously make $\eta \to 1$ while at the same time making ε endogenous to improve the model's fit. According to (26), this requires ε to approach one as well. In that case, the model would require an implausibly strong degree of diminishing altruism. Overall, our calibration exercises support $\eta < 1$, namely an EGS larger than one.

and Smith, 2001). Perhaps in Italy housing is the preferred form of accumulating assets and the coverage of medical expenses for the elder is more comprehensive than in the US. We leave for future work exploring the effect different institutional settings on parental incentives to transfer resources to adult children.

Finally, we analyze what would happen if the restriction $\sigma = \eta$ is imposed and η is calibrated to match the fertility target. We find that for $\varepsilon = 0.288$, the calibrated model requires $\eta < 0$, violating the restriction $\eta \in (0,1)$. To understand why $\eta < 0$ is required, recall that η is identified from the optimal fertility condition, which equates the value of a child to his marginal cost: $VC(n) = MC(n)$. With a low enough ε one might need $\eta < 0$ because the value of a child is much larger than the marginal cost. This implies that for the model to match calibration targets on fertility and consumption, there exists a lower bound for ε below which η would be negative. To see this formally, write $VC(n)$ more extensively as $VC(n, \mathbf{c}; \eta, \varepsilon)$, which makes it explicit that VC depends on the number of children, a consumption sequence $\mathbf{c}$ and parameters η and ε. Given targets for fertility, $\overline{n}$, consumption sequences, $\overline{\mathbf{c}}$, and a value for ε, the value of η must satisfy

$$VC(\overline{n}, \overline{\mathbf{c}}; \eta, \varepsilon) = MC(\overline{n}).$$

Notice that $VC(n, \mathbf{c}; \eta, \varepsilon)$ is increasing in η: it goes to infinite as $\eta \to 1$, decreases as η goes to zero, and reaches a positive lower bound when $\eta = 0$. Therefore, this equation typically has a unique solution. Since $VC(\overline{n}, \overline{\mathbf{c}}; \eta, \varepsilon) = VC(\overline{n}, \overline{\mathbf{c}}; 0, \varepsilon)/(1-\eta)$, the solution for η can be expressed as

$$1 - \eta = \frac{VC(\overline{n}, \overline{\mathbf{c}}; 0, \varepsilon)}{MC(\overline{n})}. \tag{28}$$

In this formulation, $VC(\overline{n}, \overline{\mathbf{c}}; 0, \varepsilon)$ is a lower bound on the value of a child, found by setting $\eta = 0$. For $\eta \in (0,1)$ the restriction $VC(\overline{n}, \overline{\mathbf{c}}; 0, \varepsilon) < MC(\overline{n})$ is needed. This restriction states that when $\eta = 0$ ($EGS = \infty$), the linear case, the value of a child should fall below its marginal costs. Otherwise, the value of a child is too high even for $\eta = 0$ and no feasible value of η can be used to match the fertility target.

As discussed, when the restriction $\sigma = \eta$ is imposed and $\varepsilon = 0.288$, ε is too low (low degree of diminishing altruism) and we obtain $\eta < 0$. In those cases, the minimum feasible ε that allows for $\eta \in (0,1)$ satisfies the condition:

$$VC(\overline{n}, \overline{\mathbf{c}}; 0, \underline{\varepsilon}) = MC(\overline{n}). \tag{29}$$

We then use equation (29) to find the minimum ε for which restriction $\eta \in (0,1)$ is satisfied. We find this value to be $\varepsilon = 0.4$. This value happens to be within the range of other calibrated models in the literature: Birchenall and Soares (2009) calibrate values of ε in the range of 0.4 to 0.6; Doepke (2004) calibrates $\varepsilon = 0.5$; and Manuelli and Seshadri (2009) calibrate $\varepsilon = 0.35$. For the value $\varepsilon = 0.4$ we find the calibrated $\eta = \sigma = 0.133$. Such a calibration poses two important problems. The first one is that the implied EIS of 7.5 conflicts with extensive evidence based on aggregate consumption data, which supports an EIS lower than one. As discussed in Guvenen (2006), the largest EIS that has been either estimated econometrically or calibrated in the context of a model is at most one. For example, Yogo (2004) confirms Hall's (1988) findings that "The EIS is less than 1 and not significantly different from 0 for 11 developed countries" (p. 797). Second, this evidence points to perhaps the major problem with a model that imposes $\eta = \sigma$, namely its lack of robustness. The reason is that estimates of the EIS vary widely, from close to zero to slightly larger than one. Given the uncertainty surrounding σ, a robust fertility model should be able to handle plausible estimates of σ, such as $\sigma = 1$, not just certain values of σ. But as we discussed above, the model with $\sigma = \eta = 1$ fails to match key targets.

To reiterate, the important finding of this exercise is to show that for a reasonable calibration of a generalized dynastic model that disentangles the EIS from EGS, the calibrated EGS turns out to be significantly larger than one and above standard value of the EIS. Our calibration suggest that individuals have a low intertemporal substitution, but a higher intergenerational substitution, a novel result in the literature.

4 DYNASTIC LIFE CYCLE MODEL WITH LEISURE

This section extends the benchmark model to include a more realistic life cycle of altruistic families, endogenous consumption, life cycle savings, leisure, labor, fertility and life span T. This extension allows to check the robustness of our main result, that $EGS > EIS$, to major generalizations of the benchmark model. An advantage of the extended model is that we are now able to calibrate σ within the model to match life-cycle stylized facts. In contrast, σ in the benchmark was set to a standard value commonly used in the literature. The EIS determines the degree of consumption smoothing over the life cycle of the parents. In the calibration strategy we exploit changes in time use over the life cycle, specifically leisure, in order to determine the EIS. In particular, adults face important changes in leisure with the arrival of children, their departure from the home, and retirement. The EIS is calibrated to match the observed consumption smoothing over the life cycle in the presence of discrete changes in leisure. In contrast, the EGS directly determines the value of a child, making it a key parameter in matching the fertility level as in Section 3.

One of the main insights of the analysis in this section is that the introduction of leisure into the model makes the case for a separation between the EGS and EIS even stronger. The reason is that leisure makes life even more valuable, and therefore increases the value of a child and the economic incentives to have more children. A higher EGS provides a counterbalancing force, one that makes parental consumption even more valuable. We are not aware of any other life cycle altruistic models of fertility with leisure in the literature.

The model in this section links to multiple literatures including household consumption over the life cycle, household equivalent scales, consumption smoothing across generations, families in macroeconomics, fertility choice, male and female labor supply, male and female wage differentials over the life cycle, male and female time use, health over the life cycle and retirement. Because of the multiple elements involved, we make some simplifying assumptions in order to focus on the calibration of the EIS and the EGS. In this respect, the exercise in this section can be seen as the first pass of a research agenda on the generalized altruistic framework proposed here. While there is a vast literature on estimating the EIS, we hope a similar literature on estimating the EGS develops.

4.1 *Model*

Consider an economy in which families consist of couples, the "head" and the "wife", and their children. We assume this couple operates as a unitary household and allow for economies of scale at the household level. Children stay at home with the parents until they become adults at age I (for independence). Children enjoy consumption (c) and leisure (l), go to school and do not work. Parents decide their children's consumption and can make non-negative transfers (b) to their adult offsprings. Adults enjoy consumption and leisure, work, save (z), make transfers, have all their children (n) at age F ($\geq I$), retire at age P (for pensioner) and live for T years. There are financial constraints in the form of borrowing and saving limits. Since death is certain at age T there are no unintended bequests. In fact, as we show below, parental transfers in the model only occur when adult offsprings are younger since that is the period when financial constraints are tighter. For tractability, we assume that both the head and the wife enjoy the same consumption and leisure. The differences between the head and wife appear in the budget constraints in the form of differences in wages, labor supply, time spent raising children and time spent in home production.

4.1.1 Lifetime composite consumption

To include leisure and a more complete life cycle, we now define C as

$$C = \begin{cases} \left[\frac{1}{\Omega}\sum_{a=0}^{T}\beta^a H_a x_a^{1-\sigma}\right]^{\frac{1}{1-\sigma}} & \text{if } \sigma > 0 \text{ and } \sigma \neq 1 \\ \exp\left[\frac{1}{\Omega}\sum_{a=0}^{T}\beta^a H_a \ln x_a\right] & \text{if } \sigma = 1 \end{cases}, \tag{30}$$

where

$$x_a = c_a^{\psi} l_a^{(1-\psi)},$$

and $\Omega \equiv \sum_{a=0}^{T}\beta^a H_a$. In this specification a denotes the age, x_a is a composite consumption flow, l_a is leisure, ψ is the share of consumption in the composite consumption flow, $1/\sigma$ is the EIS, and H_a stands for health at age a. H_a is introduced in order match the consumption hump as in Murphy and Topel (2006), and along the lines discussed by Attanasio and Weber (2010). In particular, a decreasing health after age 55 will induce lower levels of consumption and leisure. The definition of Ω ensures that the limit of function C when $\sigma \to 1$ is of the Cobb-Douglas type as described by the second part in equation (30). Finally, we follow Bullard and Feigenbaum (2007) in adopting a Cobb-Douglas specification for the composite consumption flow x.

4.1.2 Couple's utility

We consider only situations in which couples enjoy the same consumption and leisure. This simplification implies that the head and the wife have the same C and their join lifetime utility has the following utilitarian form,

$$W_t = 2\frac{C^{1-\eta}}{1-\eta} + \Phi(n) W_{t+1},$$

where t indexes generations, n now represents *couples* of children and $\Phi(n)$ is the total weight the couple of parents give to their n couples of children. Defining $V_t = W_t/2$ to be the average household utility, then

$$V_t = \frac{C^{1-\eta}}{1-\eta} + \Phi(n) V_{t+1},$$

which corresponds to equation (3) in the previous section.

4.1.3 Household's problem

Let $\mathbf{b} = [b_a]_0^{T-F}$ be the vector of transfers a couple receives from their parents at different ages. We also refer to $\mathbf{b}$ as bequests. These transfers are the only source of consumption for children so that parents have full control over their children's consumption. After age I, the transfers in vector $\mathbf{b}$ correspond to parental transfers to adult offsprings, which can only occur until the offspring is age $T-F$ since parents die at age T.[10]

Let R be gross interest rate, $\overline{h}$ be the total annual available hours, ω_a^i net-of-taxes wages, and χ_a^i time spent in home production at age a for $i = \{\text{head, wife}\}$. The household's problem can be then described compactly as

$$V(\mathbf{b}) = \max_{n,[c_a,l_a,z_a,b'_a]_0^T} \frac{C^{1-\eta}}{1-\eta} + \Phi(n) V\left(\mathbf{b}'\right), \tag{31}$$

subject to

$$b_a + \sum_{i=h,w} \omega_a^i \left(\overline{h} - l_a - \chi_a^i\right) + R z_a \geq \overline{\kappa} c_a + z_{a+1} \quad \text{for } F > a \geq I, \tag{32}$$

$$b_a + \sum_{i=h,w} \omega_a^i \left(\overline{h} - l_a - \lambda_a^i n - \chi_a^i\right) + R z_a + T_a \geq \overline{\kappa} c_a + z_{a+1} + n b'_{a-F} \quad \text{for } T \geq a \geq F, \tag{33}$$

$$b_a = \overline{\kappa} c_a \quad \text{for } I > a \geq 0, \tag{34}$$

$$\overline{z}_a \geq z_a \geq \underline{z}_a \quad \text{for } T \geq a \geq I, \tag{35}$$

$$b'_{a-F} \geq \underline{b}_{a-F} \quad \text{for } T \geq a \geq I, \tag{36}$$

$$\overline{h} - l_a - \lambda_a^i n - \chi_a^i \geq \underline{h}_a \quad \text{for } P > a \geq 0 \text{ and } i = h, w, \tag{37}$$

$$l_a = l_R \equiv \overline{h} - \chi \quad \text{for } T \geq a \geq P \text{ and } i = h, w, \tag{38}$$

[10] Writing the problem with vector $\mathbf{b}$ as the sole state vector saves in notation but it is equivalent to writing the model with a vector of consumptions up to age I and a vector of transfers from age I on as the state vectors.

$$\overline{n} \geq n \geq 0.$$

The first restriction, equation (32), is the budget constraint for adult couples before children are born. $\overline{\kappa} c_a$ are the consumption expenditures of the couple where $\overline{\kappa} \in [0,2]$ is a parameter determining the degree of economies of at the household level. If $\overline{\kappa} = 2$ there are no economies of scale. Restriction (33) is the budget constraint for adult couples once children are born. λ_a^i are time costs of raising children, T_a is retirement income for $P \leq a \leq T$, and nb'_{a-F} are transfers to offsprings. Restriction (34) states that a child's consumption while at home equals the parental transfers divided by $\overline{\kappa}$. To see more clearly the scale economies operating in households with children, substitute (34) into (33) to obtain

$$b_a + \sum_{i=h,w} \omega_a^i \left(\overline{h} - l_a - \lambda_a^i n - \chi_a^i\right) + R z_a \geq \overline{\kappa}\left(c_a + c'_{a-F} n\right) + z_{a+1} \quad \text{for } F + I > a \geq F.$$

Household expenditures are thus given by $\overline{\kappa}\left(c_a + c'_{a-F} n\right)$.[11] Notice that there is generally consumption inequality between parents and their children at any given moment since c_a and c'_{a-F} need not be the same.

Equation (35) are financial constraints, a standard feature of life cycle models that helps match the hump-shaped consumption profile and could be of particular importance for young parents.[12] Equation (36) are transfer constraints preventing parents from extracting resources from their offsprings.[13] Below we show that if bequest constraints do not bind, then borrowing constraints would not bind either. This is because, absent bequest constraints, parents would act as banks for their offsprings, providing resources to the young in exchange for negative transfers later in life. The equations in (37) describe potential lower bounds to the labor supply, $\overline{h} - l_a - \lambda_a^i n - \chi_a^i$. In particular, $\underline{h}_a$ is the lower bound to hours worked. This constraint may be relevant for working parents when children are born.

Let $\left[\theta_a, \underline{\phi}_a, \overline{\phi}_a, \mu_a, \nu_a\right]_{a=I}^{T} \geq 0$ be the Lagrange multipliers on constraints (33), (32), (35), (36) and (37) respectively. The first order conditions with respect to l_a,c_a, z_{a+1}, and b'_{a-F} are given by

$$C^{\sigma-\eta} \frac{\beta^a H_a}{\Omega} x_a^{-\sigma} (1-\psi) \frac{x_a}{l_a} = \theta_a \left[w_a^w + w_a^h + v_a^w + v_a^h\right] \quad \text{for } P \geq a \geq I, \tag{39}$$

$$C^{\sigma-\eta} \frac{\beta^a H_a}{\Omega} x_a^{-\sigma} \psi \frac{x_a}{c_a} = \overline{\kappa} \theta_a \quad \text{for } T \geq a \geq I, \tag{40}$$

$$\theta_a = \theta_{a+1}\left(R + \underline{\phi}_{a+1} - \overline{\phi}_{a+1}\right) \quad \text{for } T \geq a \geq I, \tag{41}$$

[11] $\overline{\kappa}$ multiplies the whole expression because n corresponds to couples of children.

[12] The constraint allows for borrowing as well as saving constraints. Borrowing constraints are standard. Saving constraint are less standard but they improve the fit of the model, as we discuss below. Our main findings do not depend on this feature of the model.

[13] As we discuss in Cordoba and Ripoll (2016) these constraints bind in the empirical relevant case in which the total cost of raising a child is lower than the present value of the child's income. Absent constraints on transfers, fertility will be the highest possible.

$$\Phi(n)\frac{\partial V(\mathbf{b}')}{\partial b'_{a-F}} - \theta_a n + \theta_a \mu_a n = 0 \text{ for } T \geq a \geq F, \tag{42}$$

where the envelope condition with respect to b_a reads

$$\frac{\partial V(\mathbf{b})}{\partial b_a} = \theta_a \text{ for } 0 \leq a \leq T - F. \tag{43}$$

4.1.4 Optimal leisure

Combining (39) and (40) the optimality condition for leisure can be written as

$$\frac{1-\psi}{\psi}\frac{c_a}{l_a} = \frac{1}{\overline{\kappa}}\left[w_a^w + w_a^h + v_a^w + v_a^h\right] \text{ for } P > a \geq I,$$

where $\left[w_a^w + w_a^h + v_a^w + v_a^h\right]/\overline{\kappa}$ corresponds to the shadow price of leisure. If the labor supply constraint (37) does not bind for either the head or the wife, then optimal leisure is given by

$$l_a = \begin{cases} \left[\frac{1-\psi}{\psi}\frac{\overline{\kappa}}{w_a^w + w_a^h}\right] c_a & \text{for } P > a \geq I \\ l_R & \text{for } P \leq a \end{cases},$$

where $\left[w_a^w + w_a^h\right]/\overline{\kappa}$ is the effective average wage for the couple and l_R is (exogenous) leisure upon retirement. Notice that since the head and the wife have the same allocation of leisure and consumption, they face the same shadow price of leisure, which corresponds to the couple's effective average wage.

4.1.5 Consumption smoothing and the role of σ

Let $R_a \equiv R + \underline{\phi}_{a+1} - \overline{\phi}_{a+1}$ be the shadow interest rate at age a. When the borrowing constraint binds $R_a > R$. The following Euler equation is obtained from the optimality condition (40),

$$H_a \frac{x_a^{1-\sigma}}{c_a} = R_{a+1}\left[\beta H_{a+1}\frac{x_{a+1}^{1-\sigma}}{c_{a+1}}\right] \text{ for } T > a \geq I. \tag{44}$$

Notice that when the borrowing constraint binds, then

$$H_a \frac{x_a^{1-\sigma}}{c_a} > R\left[\beta H_{a+1}\frac{x_{a+1}^{1-\sigma}}{c_{a+1}}\right],$$

and the Euler equation does not provide identifying information for the EIS since there is no consumption smoothing. In this case the Euler equation determines the shadow interest rate R_{a+1} rather than the growth rate of consumption. Since the borrowing constraint typically binds for younger adults in life cycle models, as it will be the case in our model too, we can only use the Euler equation later in life to calibrate σ. In particular, we will use the Euler equation at retirement age P for this purpose when savings are positive and leisure jumps. In that case the Euler equation at age $P-1$ can be written as

$$\frac{c_P}{c_{P-1}} = \beta R \frac{H_P}{H_{P-1}} \left[(c_P/c_{P-1})^{\psi} (l_R/l_{P-1})^{1-\psi} \right]^{1-\sigma}. \tag{45}$$

The evidence suggests that consumption either does not or only marginally jumps at retirement (Hurst, 2008), while there is a jump up in leisure from l_{P-1} to l_R. We also confirm using PSID data that there is no significant jump in consumption at retirement. For the equation above to hold one would need $\sigma \simeq 1$ so that the jump on the right-hand side does not conflict with a smooth left-hand side.[14]

An alternative to using the jump in leisure at retirement to identify σ is to find other periods in the life cycle where leisure may also change and individuals are not constrained. An example is the period when all children leave the home. However, data limitations prevent us from pursuing this avenue since we do not have complete life cycle information on all time use from the PSID, while we do know when individuals retire.

4.1.6 Children's consumption

Parents help their offspring smooth their consumption during their lifetime by making transfers. Transfers are strictly positive during childhood but they could also be positive later in life. Using the Envelope condition in (43) and the optimality condition (42), the following intergenerational Euler equation characterizes positive transfers:

$$C^{\sigma-\eta} H_a \frac{x_a^{1-\sigma}}{c_a} = \frac{\Phi(n)}{n} \left[(C')^{\sigma-\eta} \beta^{-F} H_{a-F} \frac{\left(x'_{a-F}\right)^{1-\sigma}}{c'_{a-F}} \right]. \tag{46}$$

According to this equation, between ages F and $F+I$ parents equate their marginal utility at age a (left-hand side) with their children's marginal utility at age $a-F$, weighting the latter by $(\Phi(n)/n)$. In a steady state (46) can be written as:

$$\frac{H_{a-F} x_{a-F}^{1-\sigma} / c_{a-F}}{H_a x_a^{1-\sigma} / c_a} = G(n) (\beta R)^F, \tag{47}$$

where $G(n) \equiv n/(\Phi(n) R^F)$, as in Section 3. The equation above states that the marginal utility of the parent relative to that of the offspring is the same whenever transfers are positive. This equation suggests that parental transfers eventually stop once the offsprings can finance more consumption

[14] We explored the possibility that habit formation could avoid the consumption jump at retirement. In the specification where habits are described by the previous period composite consumption, x, we still find that $\sigma = 1$ is needed to avoid a jump in consumption at retirement.

with their own resources than what parents can provide. Full equalization of marginal utilities between parents and their children is obtained if $G(n) = 1$ and $\beta R = 1$.

Using (44), one also finds that in a steady state[15]

$$\frac{H_{a-F} x_{a-F}^{1-\sigma} / c_{a-F}}{H_a x_a^{1-\sigma} / c_a} = \beta^F \prod_{s=a-F}^{a} R_s.$$

Comparing the last two expressions it follows that

$$G(n) = \prod_{s=a-F}^{a} \frac{R_s}{R}.$$

According to this expression, if $G(n) = 1$ then parents act as banks, offsetting the financial constraints faced by their children. Also, parents would fully smooth children's consumption in the sense that when the child becomes independent at age I, he does not experience a jump in consumption. As we see next, however, $G(n) > 1$ is the relevant case. This gap increases with n, and drives a wedge between parents and children marginal utilities according to Equation (47).

4.1.7 Transfers to adult offspring

Consider now transfers to independent offspring. Using equations (42) and (43) we have that after the child becomes independent, then parental transfers satisfy

$$\theta_a \geq \frac{\Phi(n)}{n} \theta'_{a-F} \quad \text{for } T \geq a \geq F + I,$$

with strict inequality if the bequest constraint binds. The left-hand-side is the marginal cost of transferring resources to the offspring and the right-hand side the marginal benefit. In a steady state this expression simplifies to

$$G(n) R^F \theta_a \geq \theta_{a-F} \quad \text{for } T \geq a \geq F + I,$$

On the other hand, the steady state first order condition for savings is given, according to (41), by $\theta_{a-F} = R_{a-F+1} \theta_{a-F+1}$ or

$$\theta_{a-F} = \theta_a \prod_{s=1}^{F} R_{a-F+s} \quad \text{for } T \geq a \geq F + I.$$

[15] Equation (44) also applies to children who face the most drastic financial constraints: $\overline{z}_a = 0$ and $\underline{z}_a = 0$ for $I > a \geq 0$.

Combining the last two expressions, one obtains

$$G(n) \geq \prod_{s=1}^{F} \frac{R_{a-F+s}}{R} \quad \text{for } T \geq a \geq F+I.$$

Since the left hand side is independent of a then it must be the case that:

$$G(n) \geq \max_{a\in[F+I,T]} \prod_{s=1}^{F} \frac{R_{a-F+s}}{R}. \tag{48}$$

To better understand this expression, it is convenient to consider two cases. First, suppose financial frictions are not binding so that $R_a = R$ for all $a \in [F+I, T]$. In that case, $G(n) \geq 1$ with strict inequality if the transfer constraint is binding for any age. Second, suppose borrowing constraints are binding for at least some ages. In that case $G(n) > 1$. Moreover, if transfers are positive they will be only for those ages in which offsprings are the most constrained, at ages that attain the maximum in equation (48).

In conclusion, if the bequest constraints or borrowing constraints bind then: (i) $G(n) > 1$; (ii) the child has higher marginal utility of consumption than the parent while they live together; and (iii) the adult offspring may receive parental transfer but only when borrowing constraints the tightest.

4.1.8 Optimal fertility

The optimal fertility choice in this model is given by

$$\sum_{a=F}^{T} \frac{\theta_a}{\theta_F} \left[\sum_{i=w,h} \left[\left(w_a^i + \nu_a^i \right) \lambda_a^i \right] + b'_{a-F} \right] = \Phi'(n) \frac{V(\mathbf{b}')}{\theta_F}, \tag{49}$$

which equalizes the marginal cost to the marginal benefit of children. In order to further analyze the marginal cost let $R_{F+1,a} = \prod_{s=F+1}^{a} R_s$ for $a > F$. Then the first order condition for savings, equation (41), implies that

$$\frac{\theta_a}{\theta_F} = \frac{1}{R_{F+1,a}} \text{ for } a > F.$$

Therefore, the marginal cost of children $MC(n)$ can be decomposed between time and goods costs as

$$MC(n) = MC_{time} + MC_{goods} = \sum_{a=F}^{T} \frac{1}{R_{F+1,a}} \sum_{i=w,h} \left[\left(w_a^i + \nu_a^i \right) \lambda_a^i \right] + \sum_{a=F}^{T} \frac{1}{R_{F+1,a}} b'_{a-F}. \tag{50}$$

According to this expression, the marginal cost of a child for a constrained parent is lower than for an unconstrained parent since the former discount the future at a higher rate, that is, $R_{F+1,a} >$

$R^{a-(F+1)}$. In addition, a binding minimum labor hour constraint ($\nu_a^i > 0$) increases the cost of raising children since wages do not fully capture the value of the time invested in children.

The economic value of a child corresponds to the marginal benefit in equation (49). Using (31) and (40), and assuming $\Phi(n) = \alpha n^{1-\varepsilon}$, this value can be written in a steady state as

$$\frac{VC(n)}{c_F} = \Phi'(n)\,\frac{V(\mathbf{b}')}{\theta_F} = \frac{1-\varepsilon}{1-\eta}\frac{\alpha n^{-\varepsilon}}{1-\alpha n^{1-\varepsilon}}\frac{\overline{\kappa}\Omega}{\psi\beta^F H_F}\left(\frac{C}{x_F}\right)^{1-\sigma}. \tag{51}$$

This equation is a generalization of (26) in Section 3. Notice that $VC(n)$ includes the lifetime utility the child derives from his individual consumption and leisure because $\Omega C^{1-\sigma} = \sum_{a=0}^{T}\beta^a H_a x_a^{1-\sigma}$. This version of the model generates significantly higher economic values of children relative to standard fertility models in the literature which typically abstract from leisure.

4.2 *Data*

In order to calibrate the model we use information from the Panel Study of Income Dynamics (PSID) to obtain a series of stylized features of the life cycle of a family in the US.

4.2.1 PSID sample

The PSID is a longitudinal panel with information on household income, wealth, consumption, demographics, hours worked and labor earnings for heads and wives. In addition, the Child Development Survey (CDS) of the PSID provides information on the time spent raising children. We use the sample period 1999-2013 (biennial) in order to utilize the upgraded consumption measure, one that accounts for about 70% of the consumption measured in the Consumption Expenditure Survey (Charles *et al.*, 2006). As it is standard practice, we only use the Survey Research Center data and drop Survey of Economic Opportunity component of the PSID.

To be consistent with the model, we only include in our sample married or cohabiting couples that remain intact during the observed period 1999-2013. We use marital status and change in marital status to select this sample.[16] We also eliminate the top and bottom 0.1% of the wealth distribution (Blundell *et al.*, forthcoming). The resulting sample includes 5,030 households and 23,114 observations. For about 35% of the households we have at least 7 observations, while for 48% we have at least 5. Virtually all sample heads are males. A substantial fraction of female wives work, with 86% in the labor force for at least some part of the observed period, and only 17% of them exiting the labor force for at least some time when kids are born. Finally, children of up to age 17 are present in 62% of the households.

Table 3 presents descriptive statistics for the PSID sample, which align well with other samples in the literature. Starting with hours worked, the median participating female wife in our sample works about 200 less hours a year than the median male head, who works 2,015 hours. There is

[16] If a couple in our sample divorces or separates during the observed period, we drop the subsequent observations since the head and wife are no longer an intact couple. We lose few observations under this criteria.

a sizable wage gap, with median male hourly wages of $26 and female wages of $17. We compute hourly wages directly by using labor earnings (including labor income from business) and hours worked.

Insert Table 3 around here

Median family income in the sample is $87,500, higher than the 2016 US median of $57,617. This can be attributed to our sample selection of intact married couples, which is the one that best reflects the model. Median total wealth, which includes home equity, is $198,720. Since the median age of household heads in our sample is 52, our median wealth is also higher than the median net worth for US households in a similar age bracket (about $90,000 in 2011 –US Census).

Our measure of consumption includes non-durables and services. We construct this measure by adding the following PSID categories: total food expenditure (home, away and delivery); housing (rent, insurance and utility, and excluding mortgages and property taxes); transportation (gasoline, auto insurance, vehicle repair, parking, bus, cab and other, and excluding vehicle loan and down payment); education; childcare; and health care. As in Blundell *et al.* (forthcoming), we impute homeowners rent as 6% of the home value reported by the households. Median consumption in the sample is $36,187, which would correspond to about $51,695 if measured in the Consumption Expenditure Survey (Charles *et al.*, 2006).

Turning to some sample demographic characteristics, the median male head has 14 years of schooling, and the female wife 13 years. This relatively high education levels are reflected in late childbearing age, with the male head at 32 years of age and the female at 30. Last, among couples with children, the median number is 2 kids.

4.2.2 Life-cycle profiles

We use our PSID sample to construct life cycle profiles of hourly wages and labor supply for heads and wives, as well as household consumption. Hourly wages are exogenous in the calibration, while the profiles of labor supply and consumption are used to compute calibration targets. In order to generate these profiles we proceed as follows. Starting first with hourly wages, we run a pooled regression of (log) wages (separately for head and wife) against year dummies, as well as demographic characteristics (state of residence, race and schooling).[17] We restrict the sample to individuals ages 20 to 75. Omitted categories are: year 2011, white race, and the state of Pennsylvania (PA). We normalize the residuals of the wage regression using the median schooling (of the head or the wife). We then plot the implied hourly wage profile by averaging the residuals by age. Figure 3 shows the results. The dots are averages by age and the fitted lines are age polynomials. At age 20 the hourly wage for a white male head with median schooling in PA in 2011 is around $15. This wage peaks at around $30 at age 50. In addition to the wage gap between male heads and female wives, we find a flatter wage profile for the latter. A white female wife with the median years of schooling in PA in 2011 is around $12 at age 20 and it peaks to $18 at age 50. We use these fitted wages as exogenous profiles in the calibration.

[17] Data are weighted using the lastest longitudinal family weight and errors are clustered at the household level.

Insert Figure 3 around here

We follow a similar procedure to construct labor supply profiles as shown in Figure 3. The only difference with respect to wages is that when we fit the age polynomials for hours worked (for heads and wife), we also control for the number of children in the household, the number of adults, a dummy for whether or not children are present, and a dummy for retirement. We follow Bullard and Feingembaun (2007) and classify as retired individuals older than 65 who work less than 10 hour a week. We use these additional controls to construct fitted profiles that are consistent with the representative couple in our model. This representative couple has the median years of schooling and the median number of children. In the model, all children are born at the same time when the parents are age F and leave the home together when the parents are age $F + I$. We choose $F = 32$, which corresponds to the median childbearing age in our sample, and $I = 22$.

As can be seen in Figure 3, the average hours worked for male heads between ages 30 and 55 is about 2,100 hours, while it is about 1,500 for female wives. Notice how in this age range, the profile for males is flatter than that of females. The fitted age profiles for the representative couple in our model show how male heads slightly increase hours worked when children arrive, while female wives decrease labor supply. We calibrate our model to match these labor supply profiles. Since we assume the leisure of heads and wives is the same, the difference in labor supplies are attributed to child care and home production, with the latter being a residual in the calibration. Last, the coefficient on the retirement dummy is significant and is reflected on the fitted age profiles as a marked drop in labor supply at age 65.

Figure 3 also includes the profile of household consumption of non-durables and services. In constructing this profile we follow the same procedure outlined for the case of hours worked, but we include couples of all ages. The dots on the graph correspond to the averages by age, where controls have only been introduced for time, race, state of residence and schooling, but not for family size. The consumption profile traced by the dots is hump-shaped starting at around \$24,000 at age 20 and peaking at \$38,000 at age 55. Since PSID data captures about 70% of the consumption categories, actual total consumption would be \$34,285 for age 20 and \$54,285 at age 55. This represents a peak to through ratio of 1.58. As in the case of hours worked, the fitted polynomial is drawn to capture the representative couple in the model with two children arriving when the parents are age 32. We estimate an increase in household consumption of 17% when the two children are born. This estimate is consistent with the 7.5% increase in household consumption that results from the arrival of the first child as estimated by Blundell *et al.* (forthcoming) using the PSID (their Table 4). Last, we find that there is no statistically significant change in household consumption at retirement. We use this estimated consumption change at retirement together with the growth rate of consumption between ages 20 and 55, and the growth rate after 55 as target moments in the calibration.

4.3 *Calibration and results*

We calibrate our model to various features of the life cycle of a typical couple in the PSID.

4.3.1 Exogenous parameters

Table 4 lists the exogenous parameters in our calibration. We follow Birchenall and Soares (2009) and set the interest rate $r = 3\%$, a typical value in the demographics literature. Recall that $\overline{\kappa}$ represents economies of scale in consumption. The consumption literature has used household-equivalence scales to transform household expenditure data for families of different sizes into per capita consumption services. These equivalence scales only provide information about the household's technology to transform expenditures into consumption services, but tell nothing about potential consumption inequality within the family. We therefore use information on equivalence scales to calibrate economies of scale at the family level, a technological parameter. In particular, we follow Fernandez-Villaverde and Krueger (2007) and use the equivalence scale for households of size two to set $\overline{\kappa} = 1.34$ in the calibration (their Table 1, p. 554).

Insert Table 4 around here

We set $\overline{h}$, which is the number of available hours in a year to $5,000$. We compute this number by taking the $8,760$ total hours in a year and subtracting the average time spent in sleeping, eating and personal care. According to Aguiar and Hurst (2007) this average time was about 3,760 hours (72 hours per week) in 2003 (their Table III, p. 977). We set $T = 80$.

In the model, at age a the head spends $\lambda_a^h n$ hours raising children, and the wife spends $\lambda_a^w n$. We calibrate the model to $n = 1$ (2 children per couple) and set λ_a^h and λ_a^w exogenously for the case of the typical household with two parents and two children. As discussed above, Guryan *et al.* (2008) find that the average mother spend around 14 hours per week in primary child care (728 hours a year), while the average father spend around 7 hours (364 hours a year). These numbers go up to 45 and 30 hours a week (or 2,340 and 1,560 hours a year) respectively if secondary time is taken into account. Folbre (2008) argues in favor of counting both primary and secondary time. Here we compromise between these two approaches and assign 1,340 hours of childcare a year for the wife and 660 for the head (husband). Under this calibration the wife contributes 67% of the total childcare time.[18] This is consistent with averages reported by others in the literature (Blundell *et al.*, forthcoming). Last, wage profiles ω_a^h and ω_a^w for the head and the wife are as reported in Figure 3.

4.3.2 Calibrated parameters

Table 5 reports the results of our calibration. Parameter σ is calibrated so that there is no jump in consumption at retirement. As discussed above, this is consistent with our estimates of the fitted consumption profile in Figure 3 and other results in the literature (Hurst, 2008; Aguiar and Hurst, 2005). Consistent with our discussion of equation (45), we set $\sigma = 1$ to prevent a consumption jump at retirement.

Insert Table 5 around here

[18] In computing the numerical model solution, we let childcare hours decline gradually to capture the fact that older children are less time-intensive.

Parameter ψ, which is the share of consumption in the composite good x is calibrated to match the average hours worked by make heads between ages 30 and 55. As observed in Figure 3, male heads' labor supply is quite stable within this age range and averages around 2,100 hours. We obtain $\psi = 0.4$. Parameter β is calibrated to match the average growth of consumption between ages 22 and 55. This results in $\beta = 0.962$, a typical value.

Health index H_a is calibrated so that the annual consumption growth rate after age 55 fits the data. The resulting H_a is plotted in Figure 4. Normalizing $H_{20} = 1$ we obtain that, for instance, $H_{80} = 0.67$.

Parameter α, which corresponds to the level parameter of the altruistic function is informative of the goods costs of raising one child. We use the USDA estimates of the goods costs of raising one child in Lino (2012) as presented in Table 1. We choose the middle-income group because the median household income in our sample of \$87,500 falls into this bracket. The computations in Lino (2012) correspond to children ages 0 to 17 and do not include the costs of attending college. These are hard to compute because they vary widely, from \$17,131 per year at the typical 4-year public college (tuition, room and board), to \$38,589 per year in private institutions (Lino, 2012, p. 22). Since we assume children leave the home at age $I = 22$ and in our PSID sample parents are relatively well educated and wealthy, we assume parents pay for college. Since most enrollment in college in the US is in public institutions, we add a present-value cost of \$38,526 to the total costs of raising a child. This corresponds to college costs of \$17,131 per year for ages 18 to 21. Therefore, the present value of the goods costs of raising one child is set to \$221,866 (in 2011 dollars). We then calibrate α to match a value of \$211,866, which results in $\alpha = 0.22$.

Two parameters remain to be determined, ε and η. We initially chose to exogenously set $\varepsilon = 0.288$ as discussed in Section 3, and to calibrate η to match the median number of children per household, which is two in the PSID sample ($n = 1$). However, we faced the issue that for this value of ε the calibrated η was a negative number. Intuitively, this result arises because the inclusion of leisure into the utility significantly increases the value of a child making unfeasible for the model to account for a fertility of $n = 1$ with a calibrated degree of diminishing altruism of $\varepsilon = 0.288$. This issue is even more severe if the restriction $\eta = \sigma$ is imposed.

To better understand the issue, remember that the economic value of a child is positively related to η (see equation 51): it goes to infinite as $\eta \to 1$, decreases as η goes to zero, and reaches a positive lower bound when $\eta = 0$. As in equation (28), we can calculate this lower bound for the value of a child when $\eta = 0$. In the case of the life cycle model with leisure, the lower bound for the economic value of a child, $VC\left(n, \mathbf{c}, \mathbf{l}; \eta, \varepsilon\right)$, is given by

$$VC\left(1, \mathbf{c}, \mathbf{l}; 0, \varepsilon\right) = \frac{\alpha\left(1-\varepsilon\right)}{1-\alpha} \frac{\overline{\kappa}}{\psi \beta^F H_F} \sum_{a=0}^{T} \beta^a H_a \left(\frac{c_a^{\psi} l_a^{(1-\psi)}}{c_F^{\psi} l_F^{(1-\psi)}} \right)^{1-\sigma},$$

which can be easily computed by setting the target $n = 1$, using the observed allocations of consumption ($\mathbf{c}$) and leisure ($\mathbf{l}$) from the data (which the model matches well as we show below), and given the parameters already calibrated (α, $\overline{\kappa}$, ψ, β, H_a, σ). We find that $VC\left(1, \mathbf{c}, \mathbf{l}; 0, 0.288\right)$

is around \$1.2 million, while $MC(1)$ is around \$400,000. Thus, for $\varepsilon = 0.288$ and for the lowest possible value of $\eta = 0$, the marginal benefit of children in the model is much larger than the marginal cost, a possibility already discussed in Section 3.3.[19] The economic value of a child tends to be much larger when leisure is taken into account.

We are not aware of any other altruistic models of fertility choice in which leisure enters utility. As a result, the range of values of ε for which the restriction $0 < \eta < 1$ is satisfied is smaller. In the model with leisure, the minimum required ε satisfies the condition

$$VC\left(1, \mathbf{c}, \mathbf{l}; 0, \underline{\varepsilon}\right) = MC\left(1\right),$$

which is the analogous to equation (29). Separating σ from η is even more important when leisure is introduced because this restriction increases the economic value of the child.

For the calibrated parameters and model allocations we find that $\varepsilon \in [0.67, 1)$. These values of ε imply a stronger degree of diminishing altruism than what the estimate of Dickie and Messman (2004) suggests. They are also larger than calibrated values of ε in the literature. For example, Birchenall and Soares (2009) calibrate values of ε in the range of 0.4 to 0.6, Doepke (2004) calibrates $\varepsilon = 0.5$, and Manuelli and Seshadri (2009) calibrate $\varepsilon = 0.35$. However, none of these models includes leisure in the utility function. At the low end of the feasible interval for ε, where diminishing altruism is as mild as possible and $\varepsilon \in [0.67, 0.80]$, the calibration implies that η falls in the range $\eta \in [0.0, 0.407]$. What we can conclude from this calibration is that η might be closer to zero, and is definitely far from one. This confirms the result obtained in Section 3 that $EGS = 1/\eta > 1 \geq 1/\sigma = EIS$.

4.3.3 Results and discussion

Figure 4 portrays the model's predictions for the median representative family in the sample over the life cycle. It includes the growth rate of consumption, health index H_a, and hours worked for heads and wives. The model overall tracks the data well over the life cycle.

Insert Figure 4 around here

Figure 5 plots the predictions regarding time use over the life cycle, in particular leisure, child-care, and homework hours for heads and wives. Although we do not have life cycle profiles for these variables from the PSID, the model's predictions are reasonable. For example, Aguiar and Hurst (2007) report average weekly hours of leisure to be 35 in 2003, which corresponds to 1,820 hours a year (their leisure measure 1 in Table III). Our model is consistent with this level during prime age. Homework hours are computed as a residual in the model.

Insert Figure 5 around here

[19] We tried a number of alternatives seeking to reduce $VC\left(1, \mathbf{c}, \mathbf{l}; 0, \varepsilon\right)$ when $\varepsilon = 0.288$, but calibration targets, such as requiring the model to match average leisure time, impose a tight discipline and leave little room to affect this value. Imperfect altruism, so that for example parents do not care about children's leisure, could reduce the problem.

Figure 5 also plots the predictions regarding saving. Our model does not include mortality risk nor health shocks, which explain the behavior of saving after retirement. The calibrated model predicts that the median representative couple does not leave bequests at the end of life. This is consistent with Di Nardi *et al.* (2016) who document that bequests motives are large only for the richest people. We do not expect the addition of mortality risk to affect our calibration of the EGS. As Di Nardi *et al.* (2016) show, in the presence of mortality risk and deteriorating health in old age, medical spending becomes the most important factor in explaining saving during retirement.

Although our model does reasonably well fitting the life cycle data in a number of dimensions, there is room for improvement. We leave additional refinements of this model for future work. We see the model in this section as the initial step of a research agenda that integrates the life cycle into our generalized dynastic model.

5 FURTHER EXTENSIONS

We close this paper providing additional illustrations of the scope of our framework beyond altruistic models of fertility choice. We provide a few extensions in order to illustrate other contexts in which disentangling the EGS from the EIS might be useful. In particular, we suggest how to extend our framework to include infant mortality risk, to allow for $\eta > 1$, and to analyze long-term inequality in a model with idiosyncratic risk.

5.1 *EGS and the coefficient of risk aversion*

Those familiar with the Epstein-Zin-Weil (EZW) preferences from Epstein and Zin (1989) and Weil (1990), may find a resemblance between these and our formulation, and may wonder whether our framework reduces to a relabeling of EZW preferences. Although non-separability is a feature of both EZW and our preferences, they are conceptually quite different. EZW preferences disentangle aversion to risk from aversion to deterministic consumption fluctuations. In the absence of risk, the EZW formulation collapses into the standard formulation. This is not the case with our preferences. The framework we presented above does not model risk. Our preferences disentangle aversion to two types of *deterministic* fluctuations in consumption: temporal variation and intergenerational variation.

In order to illustrate the relationship between EZW and our preferences, we now introduce child mortality risk into our model and combine EZW utility with our approach to disentangle three parameters: the EIS, the EGS and the coefficient of relative risk aversion (CRRA). Infant mortality is a potentially important determinant of fertility choices.[20] In order to introduce risk, it is convenient to utilize the representation of our preferences in equation (2). Recall that lifetime utility W is non-negative so that zero is a lower bound, a property that we use shortly. Consider now the possibility that the lifetime utility of the child is a random variable, $\tilde{W}$. Let $\mu\left(\tilde{W}\right)$ denote the certainty equivalent operator. In particular, Epstein and Zin (1989) as well as Weil (1990)

[20] See, among others, Doepke (2005) and Jones and Schoonbroodt (2010).

consider a particular CRRA operator $\mu(\tilde{W}) = \left(E\tilde{W}^{1-\rho}\right)^{1/(1-\rho)}$ where $\rho \geq 0$ is the coefficient of relative risk aversion. For example, $\rho = 0$ means that parents are neutral to risks associated to their children's welfare. Following EZW, when certainty equivalent $\mu\left(\tilde{W}\right)$ is what the parent perceives as the utility of his child, preferences can be described by

$$W = \left[C^{1-\eta} + \Phi(n)\mu\left(\tilde{W}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

Suppose now infant mortality is the only risk. In particular, let π be the survival probability of a newborn. In that case, $\mu\left(\tilde{W}\right) = \left[\pi\left(W'\right)^{1-\rho} + (1-\pi)\,D\right]^{1/(1-\rho)}$ where D is the imputed utility in case of death. To simplify, suppose $D = 0$ which means that being alive is always better than not, $W' \geq D = 0$. Furthermore, if the death of a child is not so painful as to eliminate all enjoyment of having children, then the additional assumption $\rho \in (0, 1)$ is required. In other words, if $\rho > 1$ so that parents are significantly risk averse, then $\mu\left(W'\right)$ would be zero whenever $D = 0$. Parental welfare simplifies to

$$W = \left[C^{1-\eta} + \Phi(n)\pi^{(1-\eta)/(1-\rho)}\left(W'\right)^{1-\eta}\right]^{1/(1-\eta)}.$$

In order to relate the expression above to our earlier formulation in (3), assume that $\Phi(n) = \alpha n^{1-\varepsilon}$ and rewrite preferences in terms of V rather than W to obtain

$$V = \frac{1}{1-\eta}\left(\sum\nolimits_{t=0}^{T}\beta^t c_t^{1-\sigma}\right)^{\frac{1-\eta}{1-\sigma}} + \alpha n^{1-\varepsilon}\pi^{(1-\eta)/(1-\rho)}V'.$$

These preferences are an extension of our framework that disentangles three different concepts: the $EIS = 1/\sigma$, $EGS = 1/\eta$ and the $CRRA = \rho$. The expected utility model is the special case $\eta = \rho$, while if $\sigma = \eta = \rho$ would imply additive separability across time, generations, and states. Finally, if $1 - \varepsilon = (1-\eta)/(1-\rho)$ then parents only care about the number of surviving children, πn, which provides microfoundations to the simplifying assumption made in the literature.[21]

5.2 *EGS less than one*

Our benchmark formulation assumes $\eta \in (0, 1)$, and the calibration shows this assumption is not binding. We now show that it is simple to relax this assumption. Consider again the representation of our preferences in (2). As mentioned above, they are a monotonic transformation of our benchmark preferences and are strictly non-negative for any η, not just for $\eta \in (0, 1)$. Despite W being positive, it is still true that parental welfare is decreasing in the number of children, n, when $\eta > 1$. Therefore, in that case the optimal number of children would be zero. This result, however, is due to the implicit assumption that the welfare of the unborn individual is zero (see Cordoba and Ripoll, 2011). The following generalized version of (2) makes this point clear. Suppose there is a number of potential children, n_p. Let W be the welfare of an individual if born, and D if

[21] See, for instance, Jones and Schoonbroodt (2010).

unborn. D is what parents perceive, or impute, is the welfare of the unborn. This is analogous, but not the same, to the perceived utility in case of dead. As in the previous example, D could be normalized to zero so that altruistic parents perceive potential children are better off being born than unborn.[22] In this case parental preferences are

$$W = \left[C^{1-\eta} + \Phi(n) \left(W' \right)^{1-\eta} + \left(\Phi(n_p) - \Phi(n) \right) D^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{52}$$

Equation (2) is a special case of (52) that requires $D = 0$. Notice that if $\eta > 1$ and $D = 0$ then $W = 0$. In this case, the small degree of substitutability between utilities means that if one individual receives zero utility then parents utility is also zero. To make an analogy with the theory of the firm, if W is production and the inputs are the utilities of individuals, then $\eta > 1$ means that all inputs are essential. To avoid this implication when assuming $D = 0$ requires the restriction $\eta \in (0, 1)$ as in the benchmark. But if $D > 0$ then parental utility increases with the number of children, for any $\eta > 0$, as long as $W \geq D$. Allowing for $D = 0$ and calibrating the model with preferences (52) would still require a low D and $\eta \in (0, 1)$, as otherwise the model would not be able to match the value of a child as discussed in Section 3.

5.3 *EGS in Bewley models*

The EGS is a potentially important determinant of long-run inequality. There is evidence from Menchik (1979) and Sholz and Seshadri (2009) that family size affects the ability of parents to accumulate wealth. There is also evidence of substantial intergenerational persistence in earnings, income and wealth (Mulligan, 1997; Lee and Solon, 2009). While in existing models of inequality the EIS determines precautionary saving, the EGS may play an important role in determining intergenerational precautionary savings and long-run inequality. Preliminary evidence of this can be found in Cordoba, *et al.* (2016). While their analysis does not disentangle the EGS from the EIS, they do find that a low curvature in the utility function is necessary to match both the fertility-income relationship as well as intergenerational savings and the mass of adult children who receive zero transfers and bequests from their parents.

Cordoba, *et al.* (2016) analyze a two-period (child and adult) Barro-Becker model of fertility embedded into a Bewley framework. Since individuals in their model only consume when adults, the framework does not allow for a distinction between the EIS and the EGS. Adult children receive transfers from the parent and draw an earning ability shock that is conditional on that of the parent. In the context of their model, the curvature of the utility function determines consumption smoothing across generations. They find that the following ingredients are needed in order for the model to replicate persistent inequality: a low utility curvature (larger than one elasticity of substitution), a non-negative transfer constraint, an exponential altruistic function, diminishing marginal time costs of raising children, and discrete number of kids. Extending Cordoba, *et al.* (2016) to incorporate a life cycle model along the lines of the model in Section 4 is left for future

[22] One way, although not the only way, to rationalize abortion by altruistic parents would occur when $D > W$.

work.

6 CONCLUDING COMMENTS

The EIS has always played an important role in most macroeconomic models, determining both decisions within the lifetime of an individual, as well as across generations. This key role is in part due to the artifact that existing models assume the EIS and EGS to be identical. Once these concepts are disentangled, some of the roles previously played by the EIS now belong to the EGS. For instance, we have shown how the EGS is a key determinant of the long-term fertility rate and might be also key to understand long-run inequality. There are also instances in which the EGS is likely to play an important role in the short term. For instance, at the business cycles frequency, the EGS determines how a shock to the family budget, say an unemployment shock or winning the lottery, affects expenditures in children and, in particular, investments in their education and human capital formation.

Our paper is the first to formally model a distinction between intertemporal and intergenerational substitution. The generalized dynastic framework we propose easily allows to associate a single parameter with the EIS, and a different one with the EGS. The simplicity of our preferences provides a useful and general framework for analyzing intergenerational issues. We expect this framework to introduce a new perspective, and to be useful in analyzing a number of interesting and relevant questions in macroeconomics.

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