Trading Dynamics with Private Buyer Signals in the Market for Lemons

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Abstract

We present a dynamic model of trading under adverse selection in which a seller sequentially meets buyers, each of whom receives a noisy signal about the quality of the seller’s asset and offers a price. We fully characterize the equilibrium trading dynamics and show that buyers’ beliefs about the quality of the asset can either increase or decrease over time, depending on the initial level. This result demonstrates how the introduction of private buyer signals enriches the set of trading patterns that can be accommodated within the framework of dynamic adverse selection, thereby broadening its applicability. We also examine the economic effects of search frictions and the informativeness of buyers’ signals in our model and discuss the robustness of our main insights in multiple directions.

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1 Introduction

Buyers often draw inferences about the quality of an asset from its duration on the market. In the real estate market, a long time on the market is typically interpreted as bad news (see, e.g., Tucker et al., 2013; Dube and Legros, 2016). This is arguably the reason why some sellers reset

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their days on the market by relisting their properties without any major repairs or renovations. In the labor market, it is well-known that unemployment duration affects a worker’s reemployment probability and reservation wage (see, e.g., Imbens and Lynch, 2006; Shimer, 2008). This duration dependence is often attributed to the “non-employment stigma,” which refers to the phenomenon that employers interpret a long unemployment spell as a bad signal about the worker’s productivity and, therefore, are reluctant to hire such a worker. Intriguingly, despite their intuitive appeal, these types of negative inferences have not garnered clear empirical support, with mostly “mixed and controversial” evidence (Ljungqvist and Sargent, 1998). Furthermore, most dynamic models of adverse selection, which seem the most natural framework to address such an inference problem, generates the opposite prediction, that average quality increases over time.

We present a dynamic model of adverse selection that generates multiple dynamic patterns of trade. In particular, in our model, delay can be either good news or bad news about the quality of an asset, depending on market conditions. The trading environment is a familiar one: a seller has private information about the quality of her indivisible asset, which can be either high or low. Buyers arrive sequentially, observe the seller’s time-on-the-market, and make price offers. Our innovation is to introduce private buyer signals into this canonical environment: each buyer receives a private and imperfectly informative signal about the quality of the asset. Notice that such signals are often available to potential buyers in real markets, as they can be generated by common (home) inspections or (job) interviews. We show that this simple and plausible innovation suffices to enrich the set of dynamic trading patterns that can be accommodated within the framework of dynamic adverse selection.

In order to understand when, and why, delay is perceived as good news or bad news, notice that there are three sources of delay in our model. First, delay could be just because of search frictions, that is, a seller may have been unlucky and not met any buyer yet. If this is the main source for delay, buyers’ inferences about the quality of an asset should be independent of the seller’s time-

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1This is a common practice, but its harmful effects are well-recognized. Blanton (2005) compares it to “resetting the odometer on a used car.” The real estate listing service in Massachusetts decided to prevent the practice in 2006.

2There is an agreement over the negative relationship between duration and unconditional job-finding probability. However, it is not clear whether it is due to “true” duration dependence or unobserved heterogeneity, that is, whether each individual’s performance is indeed affected by his/her duration or not. Much effort has been put in to separate “true” duration dependence from unobserved heterogeneity. See Heckman and Singer (1984) for a fundamental econometric problem. Recent studies utilize a natural experiment (e.g., Tucker et al., 2013) or a field experiment (e.g., Oberholzer-Gee, 2008; Kroft et al., 2013; Eriksson and Rooth, 2014) in order to circumvent the identification problem.

3See Evans (1989); Vincent (1989, 1990); Janssen and Roy (2002); Deneckere and Liang (2006); Hörner and Vieille (2009) for some seminal contributions. In all of these papers, average quality increases over time. Daley and Green (2012) consider a model in which public news about the quality accumulates over time. Due to noise in the (Brownian) news process, buyers’ beliefs about the quality fluctuate over time. However, the expected quality weakly rises over time for the same reason as in other (deterministic) models. Note that there are several other theories for duration dependence, including deprecating human capital models (e.g., Acemoglu, 1995), duration-based ranking models (e.g., Blanchard and Diamond, 1994), and varying search intensity models (e.g., Coles and Smith, 1998; Lentz and Tranaes, 2005).
on-the-market. Second, delay might be caused by adverse selection. A high-quality seller, due to her higher reservation value, is more willing to wait for a high price than a low-quality seller. In this case, delay conveys good news about the quality of the asset. Finally, previous buyers might have decided not to purchase after observing an unfavorable attribute. If this is the main driving force, then delay would be interpreted as bad news and buyers get more pessimistic about the quality of the asset over time. Our model intertwines these three forces and consequently accommodates different forms of trading dynamics.

We show that whether delay is good or bad news depends on an asset’s initial reputation (i.e., buyers’ prior beliefs about the quality of the asset). If an asset enjoys a rather high reputation initially, the asset’s reputation, conditional on no trade, declines over time, while if an asset starts out with a low reputation, then the asset’s reputation improves over time. To understand these opposing patterns, first note that the higher an asset’s reputation is, the more likely buyers are to offer a high price. This implies that while enjoying a high reputation, even a low-quality seller would have a strong incentive to hold out for a future chance of a high price and, therefore, be reluctant to accept a low price. In this case, trade can be delayed only when buyers are unwilling to offer a high price despite the asset’s high reputation, which is the case when they receive sufficiently unfavorable inspection outcomes. Since a low-quality asset is more likely to generate such inspection outcomes, the asset is deemed less likely to be of high quality, the longer it stays on the market. In the opposite case when an asset suffers from a low reputation, a low-quality seller would be willing to settle for a low price, while a high-quality seller would still insist on a high price in order to recoup his higher cost. Since a high-quality asset would stay on the market relatively longer than a low-quality asset, the asset’s reputation improves over time.

This result contributes to the existing literature mainly in two ways. First, it broadens the applicability of the theory of dynamic adverse selection. As introduced at the beginning, delay is perceived as bad news in several markets. Our analysis offers a simple and natural mechanism through which such negative inferences arise in this framework. Second, it provides a potential resolution for mixed empirical results. Whether delay is good news or bad news depends on market conditions. Therefore, it is natural that different studies report different empirical results. This finding further suggests that it may be fruitful to shift the focus of empirical study from a general qualitative question (whether delay is good news or bad news) to more sophisticated and quantitative ones (such as what market factors affect buyer inferences under what conditions, as exemplified by Kroft et al. [2013] and Eriksson and Rooth [2014]).

By incorporating multiple dynamic patterns of trade, our model creates a potential for obtaining new insights regarding the effects of certain policies or changes in the economic environment. Indeed, we show that in our model, the economic effects of increasing the informativeness of buyers'
signals, which can be interpreted as enhancing asset transparency, are in general ambiguous. If the initial reputation is rather low and, therefore, buyers’ beliefs increase over time, then an increase in the informativeness of buyers’ signals speeds up trade and also increases seller surplus. However, if the asset’s reputation is initially rather high and declines over time, then the same change can slow down trade and be harmful to market participants. This latter result holds precisely because delay can be caused by a lack of good signal realizations and, therefore, bad news about the quality. If buyers’ signals become more informative, then delay becomes a stronger indication of low quality, which reduces later buyers’ incentives to offer a high price and, therefore, adversely affects trade.

The role of search frictions in equilibrium trading dynamics also deserves elaboration. First, although search frictions are neutral to the direction of the evolution of beliefs, they affect the speed of the evolution. Buyers can never exclude the possibility that the seller has been so unfortunate that no buyer has contacted her yet. This forces buyers’ beliefs to change gradually. Second, they indirectly influence the direction of the evolution of beliefs through their impact on the equilibrium structure. In particular, a reduction in search frictions makes the decreasing pattern more prevalent: the threshold initial reputation level decreases as search frictions reduce. Finally, search frictions are responsible only for a portion of delay: even if search frictions are arbitrarily small, the expected time to trade remains bounded away from zero. This is similar to the persistence of delay in other models of dynamic adverse selection, but differs in that it holds despite the fact that each buyer generates a constant amount of information and, therefore, an arbitrarily large amount of exogenous information is instantaneously generated about the quality of the asset in the search-frictionless limit.

Related Literature

Most existing studies on dynamic adverse selection focus on the implications of the difference in different types’ reservation values and, therefore, feature only increasing beliefs. One notable exception is Taylor (1999). He studies a two-period model in which the seller runs a second-price auction with a random number of buyers in each period and the winner conducts an inspection, which can generate a bad signal only when the quality is low. He considers several settings that differ in terms of the observability of first-period trading outcomes (in particular, inspection outcome and reservation (list) price history) by second-period buyers. In all settings, buyers assign a

\footnote{It is common wisdom that asset (corporate) transparency improves market efficiency by facilitating socially desirable trade. Such beliefs have been reflected in recent government policies, such as the Sarbanes-Oxley Act passed in the aftermath of the Enron scandal and the Dodd-Frank Act passed in the aftermath of the recent financial crises, both of which include provisions for stricter disclosure requirements on the part of sellers. Presumably, the main goal of such policies is to help buyers assess the merits and risks of financial assets more accurately. This naturally corresponds to an increase in the informativeness of buyers’ signals in our model.}
lower probability to the high quality in the second period than in the first period (that is, buyers’ beliefs decline over time). Despite various differences in modeling, the logic behind the evolution of beliefs is similar to our declining beliefs case: trade occurs only when the winner receives a good signal, and the high type is more likely to generate a good signal than the low type. Therefore, the asset remaining in the second period is more likely to be the low type. However, the opposite form of trading dynamics (in which buyers’ beliefs increase over time) is absent in his model. In addition, he addresses various other economic problems, such as the dynamics of reserve (list) prices and the effects of the observability of first-period reserve price and inspection outcome, while we focus on better understanding equilibrium trading dynamics.

Two papers consider an environment similar to ours. Lauermann and Wolinsky (2016) investigate the ability of prices to aggregate dispersed information in a setting where, just like in our model, an informed player (buyer in their model) faces an infinite sequence of uninformed players, each of whom receives a noisy signal about the informed player’s type. Zhu (2012) studies a similar model, interpreted as an over-the-counter market, with an additional feature that the informed player can contact only a finite number of uninformed players. In both studies, in contrast to our model, uninformed players have no access to the informed player’s trading history. In particular, uninformed players do not observe the informed player’s time-on-the-market. This induces uninformed players’ beliefs and strategies to be necessarily stationary (i.e., their beliefs do not evolve over time). To the contrary, the evolution of uninformed players’ beliefs and the resulting trading dynamics are the main focus of this paper.

Daley and Green (2012) study the role of exogenous information (“news”) about the quality of an asset in a setting similar to ours. The most crucial difference from ours is that news is public information to all buyers. This implies that buyers do not face an inference problem regarding other buyers’ signals, making their trading dynamics distinct from ours. Similarly to us, they also explore the effects of increasing the quality of news and find that it is not always efficiency-improving. However, the mechanism leading to the conclusion is different from ours. In particular, the negative effect of increased informativeness stems from the strengthening of buyers’ negative inferences about other buyers’ signals in our model, while in Daley and Green (2012), it is due to its impact on the incentive of the high-type seller to wait for good news.

The rest of the paper is organized as follows. We formally introduce the model in Section 2 and

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5Prior to Taylor (1999), this “screening” mechanism was discussed by Vishwanath (1989) and Lockwood (1991). However, they do not investigate the working of the mechanism in a full-blown strategic setting: in Vishwanath (1989), (stochastic) price offers are exogenously generated, while in Lockwood (1991), trade takes place only at one price, which is equal to the reservation value common to all worker types.

6This is due to his assumption that there are no gains from trade of a low-quality asset. In this case, buyers have no incentive to offer a price that can be accepted only by the low type, and thus the low type can not trade faster than the high type. In the online appendix (Section B), we consider the comparable case and show that the same result holds in our model.
provide a full characterization in Section 3. We analyze the effects of changing the informativeness of buyers’ signals in Section 4 and study the role of search frictions in Section 5. In Section 6, we demonstrate the robustness of our main insights in three dimensions: the number of seller types, the bargaining protocol, and the market structure. In Section 7, we conclude by providing several empirical implications and suggesting some directions for future research.

The Model

2.1 Physical Environment

A seller wishes to sell an indivisible asset. Time is continuous and indexed by \( t \in \mathbb{R}_+ \). The time the seller comes to the market is normalized to 0. Potential buyers arrive sequentially according to a Poisson process of rate \( \lambda > 0 \). Upon arrival, each buyer receives a private signal about the quality of the asset and offers a price to the seller. If the seller accepts the price, then they trade and the game ends. Otherwise, the buyer leaves, while the seller waits for the next buyer. All players discount future payoffs at rate \( r > 0 \).

The asset is either of low quality \((L)\) or of high quality \((H)\). If the asset is of quality \( a = L, H \), then the seller obtains flow payoff \( r c_a \) during her possession of the asset, while a buyer, once he acquires it, receives flow payoff \( rv_a \) indefinitely. The asset is more valuable to all players when its quality is high than when it is low: \( c_L < c_H \) and \( v_L < v_H \). In addition, there are always gains from trade: \( c_L < v_L \) and \( c_H < v_H \). However, the quality of the asset is private information of the seller, and adverse selection is severe in the sense that there is no price that always ensures trade: \( v_L < c_H \). Finally, it is commonly known that the asset is of high quality with probability \( \hat{q} \) at time 0.

Each buyer’s signal \( s \) takes one of two values, \( l \) or \( h \). For each \( a = L, H \) and \( s = l, h \), we let \( \gamma_s^a \) denote the probability that each buyer receives signal \( s \) from the type-\( a \) asset. Without loss of generality, we assume that \( \gamma_h^H > \gamma_l^H \) (equivalently, \( \gamma_l^L < \gamma_h^L \)), so that buyers assign a higher probability to the asset being of high quality when \( s = h \) than when \( s = l \). We also assume that \( \gamma_s^a > 0 \) for any \( a = L, H \) and \( s = l, h \), so that no signal perfectly reveals the underlying type of the asset. See Section 4.1 for the limiting equilibrium outcomes as \( \gamma_l^H \) or \( \gamma_h^H \) tends to 0.

We assume that buyers observe (only) how long the asset has been up for sale (i.e., time \( t \)).

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7In many models of dynamic adverse selection, attention is restricted to the case where \( \hat{q} \) is so small (e.g., \( \hat{q} v_H + (1 - \hat{q}) v_L < c_H \)) that some inefficiency (delay) is unavoidable, that is, it cannot be an equilibrium that trade always occurs with the first buyer. We do not impose such a restriction, because the decreasing dynamics, which is the novel outcome of this paper, emerges only when \( \hat{q} \) is not sufficiently small. As explained in Section 5, the exact condition differs from, but is related to, the familiar inequality between \( \hat{q} v_H + (1 - \hat{q}) v_L \) and \( c_H \).

8There is a sizable literature that studies the role of uninformed players’ (buyers’) information about the history in dynamic games with incomplete information. For example, in a closely related model to ours (but without buyer
This enables us to focus on our main economic question, namely the relationship between time-on-the-market and economic variables. It also has a notable technical advantage. For any \( t \), there is a positive probability \( (e^{-\lambda t}) \) that no buyer has arrived and trade has not occurred. This means that there are no off-equilibrium-path public histories and, therefore, buyers’ beliefs at any point in time can be derived through Bayes’ rule.

### 2.2 Strategies and Equilibrium

The offer strategies of buyers are represented by a Lebesgue-measurable right-continuous function \( \sigma_B : \mathbb{R}_+ \times \{l, h\} \times \mathbb{R}_+ \to [0, 1] \), where \( \sigma_B(t, s, p) \) denotes the probability that the buyer who arrives at time \( t \) and receives signal \( s \) offers price \( p \) to the seller. The offer acceptance strategy of the seller is represented by a Lebesgue-measurable right-continuous function \( \sigma_S : \{L, H\} \times \mathbb{R}_+ \times \mathbb{R}_+ \to [0, 1] \), where \( \sigma_S(a, t, p) \) denotes the probability that the type-\( a \) seller accepts price \( p \) at time \( t \). An outcome of the game is a tuple \((a, t, p)\), where \( a \) denotes the seller’s type, \( t \) represents the time of trade, and \( p \) is the transaction price. All agents are risk neutral. Given an outcome \((a, t, p)\), the seller’s payoff is given by \((1 - e^{-rt})c_a + e^{-rt}p\). The buyer who trades with the seller receives \( v_a - p \), while all other buyers obtain zero payoff.

We study perfect Bayesian equilibria of this dynamic trading game. Let \( q(t) \) represent buyers’ beliefs that the seller who has not traded until \( t \) is the high type. In other words, \( q(t) \) is the belief held by the buyer who arrives at time \( t \) prior to his inspection. A tuple \((\sigma_S, \sigma_B, q)\) is a perfect Bayesian equilibrium of the game if the following three conditions hold.

(i) Buyer optimality: \( \sigma_B(t, s, p) > 0 \) only when \( p \) maximizes a buyer’s expected payoff conditional on signal \( s \) and time \( t \), that is,

\[
p \in \operatorname{argmax}_p q(t)\gamma_H\sigma_S(H, t, p') (v_H - p') + (1 - q(t))\gamma_L\sigma_S(L, t, p') (v_L - p').
\]

(ii) Seller optimality: \( \sigma_S(a, t, p) > 0 \) only when \( p \) is weakly greater than the type-\( a \) seller’s continuation payoff at time \( t \), that is,

\[
p \geq E_{\tau, p'} [(1 - e^{-r(\tau - t)})c_a + e^{-r(\tau - t)}p'|a, t],
\]
where \( \tau(\geq t) \) and \( p' \) denote the random time and price, respectively, at which trade takes place according to the strategy profile \((\sigma_S, \sigma_B)\). \( \sigma_S(a, t, p) = 1 \) if the inequality is strict.

(iii) Belief consistency: \( q(t) \) is derived through Bayes’ rule, that is,

\[
q(t) = \frac{\tilde{q}e^{-\lambda t}\sum_{s} \gamma_{H}(f \sigma_B(x, s, p)\sigma_S(H, x, p)dp)}{\tilde{q}e^{-\lambda t}\sum_{s} \gamma_{H}(f \sigma_B(x, s, p)\sigma_S(H, x, p)dp)} + (1 - \tilde{q})e^{-\lambda t}\sum_{s} \gamma_{L}(f \sigma_B(x, s, p)\sigma_S(L, x, p)dp)}
\]

2.3 Preliminaries

Let \( p(t) \) denote the low-type seller’s reservation price (i.e., the price which the low-type seller is indifferent between accepting and rejecting) at time \( t \). We restrict attention to strategy profiles in which each buyer offers either \( c_H \) or \( p(t) \) at each point in time. This restriction incurs no loss of generality. First, for the same reasoning as in the Diamond paradox, buyers never offer a price strictly above \( c_H \). This implies that the high-type seller’s reservation price is always equal to her reservation value \( c_H \) and in equilibrium she accepts \( c_H \) with probability 1. Note that, due to the difference in flow payoffs \((c_L < c_H)\), \( p(t) \) is always smaller than \( c_H \): \( p(t) \leq \int_{0}^{\infty}((1 - e^{-rt})c_L + e^{-rt}c_H)d(1 - e^{-\lambda t}) < c_H \) for any \( t \). Second, it is strictly suboptimal for any buyer to offer a price strictly between \( p(t) \) and \( c_H \). Finally, if in equilibrium a buyer offers a losing price (strictly below \( p(t) \)), then it suffices to set his offer to be equal to \( p(t) \) and specify the low type’s acceptance strategy \( \sigma_S(L, t, p(t)) \) to reflect her rejection of the buyer’s losing offer. This adjustment is feasible because the low-type seller is indifferent between accepting and rejecting \( p(t) \).

The fact that all buyers offer either \( c_H \) or \( p(t) \) implies that \( p(t) \) depends only on the rate at which the low type receives offer \( c_H \). This is because the low-type seller is indifferent between accepting and rejecting \( p(t) \) at any point in time and, therefore, her reservation price can be calculated as if

\( ^9 \)Formally, let \( \overline{p} \) denote the supremum among all equilibrium prices buyers offer in this game. Suppose \( \overline{p} \geq c_H \). Then, the best case scenario for the high-type seller is to receive \( \overline{p} \) with probability 1 from the next buyer. This means that her reservation price at any point in time cannot exceed

\[
\int_{0}^{\infty}((1 - e^{-rt})c_H + e^{-rt}\overline{p}) d(1 - e^{-\lambda t}) = \frac{rc_H + \lambda \overline{p}}{r + \lambda}.
\]

Since no buyer has an incentive to offer more than \((rc_H + \lambda \overline{p})/(r + \lambda), \overline{p} \leq (rc_H + \lambda \overline{p})/(r + \lambda) \). On the other hand, due to search frictions (i.e., \( \lambda < \infty \)), \((rc_H + \lambda \overline{p})/(r + \lambda) \leq \overline{p} \). Therefore, it must be that \( \overline{p} = c_H \). Intuitively, search frictions endow each buyer with some monopsony power. If \( \overline{p} > c_H \), then each buyer can undercut the price to \((rc_H + \lambda \overline{p})/(r + \lambda) \) and still make sure that the offer is accepted. Knowing that no buyer would offer \( \overline{p} \) and the highest price offer would be \((rc_H + \lambda \overline{p})/(r + \lambda) \), each buyer can undercut the price even further. This process continues indefinitely as long as \( \overline{p} > c_H \). Consequently, in equilibrium \( \overline{p} \) cannot exceed \( c_H \).
she would accept only $c_H$\textsuperscript{10}. Formally, given any buyer strategy $\sigma_B$, $p(t)$ is given by

$$p(t) = \int_t^\infty ((1 - e^{-r(x-t)})c_L + e^{-r(x-t)}c_H) d \left(1 - e^{-\lambda \int_t^x \sigma_B(y,s,c_H) dy}\right).$$

Clearly, $p(t)$ increases if buyers offer $c_H$ more frequently and decreases if they do so less frequently.

We let $\dot{q}(t)$ and $\dot{p}(t)$ denote the right derivatives of $q(t)$ and $p(t)$, respectively, that is\textsuperscript{11}

$$\dot{q}(t) = \lim_{\Delta \to 0^+} \frac{q(t + \Delta) - q(t)}{\Delta} \quad \text{and} \quad \dot{p}(t) = \lim_{\Delta \to 0^+} \frac{p(t + \Delta) - p(t)}{\Delta}.$$

Both are straightforward to derive from the general equations for $q(t)$ and $p(t)$ above:

$$\dot{q}(t) = -q(t)(1-q(t))\lambda \left(\sum_s \gamma_H^s \sigma_B(t, s, c_H) - \sum_s \gamma_L^s (\sigma_B(t, s, c_H) + \sigma_B(t, s, p(t))\sigma_S(L, t, p(t)))\right),$$

and

$$\dot{p}(t) = r(p(t) - c) - \lambda \sum_s \gamma_L^s \sigma_B(t, s, c_H)(c_H - p(t)).$$

In the main text, we restrict attention to the case where search frictions are sufficiently small. Precisely, we maintain the following assumption:

**Assumption 1**

$$v_L < \int_0^\infty ((1 - e^{-rt})c_L + e^{-rt}c_H) d \left(1 - e^{-\lambda \gamma_L^h}\right) = \frac{rc_L + \lambda \gamma_L^h c_H}{r + \lambda \gamma_L^h} \Leftrightarrow \lambda \gamma_L^h > \frac{r(v_L - c_L)}{c_H - v_L}.$$ 

This assumption ensures that if all subsequent buyers offer $c_H$ whenever $s = h$, then the low-type seller’s reservation price $p(t)$ exceeds $v_L$ (buyers’ willingness-to-pay for a low-quality asset). Clearly, it is necessary that $\gamma_L^h > 0$ and, conditional on that, the inequality holds when $\lambda$ is sufficiently large. We focus on this case, because otherwise, as formally shown in the online appendix (Section A), the model exhibits only the familiar increasing dynamics.

\textsuperscript{10}This does not rule out the possibility that the low-type seller accepts $p(t)$. Indeed, as shown shortly, in the unique equilibrium of our model, she does accept $p(t)$ after certain histories. We simply exploit the fact that the seller’s acceptance decision is not observable to future buyers and, therefore, her reservation price is independent of whether she accepts $p(t)$ or not. This property fails, for example, if rejected prices are observable to future buyers.

\textsuperscript{11}The continuous-time specification and the presence of search frictions guarantee that the equilibrium objects $p(\cdot)$ and $q(\cdot)$ evolve continuously over time (Lemma 6 in the appendix) and both $\dot{q}(t)$ and $\dot{p}(t)$ are well-defined. The qualifier “right” is due to the fact that, as shown in Propositions 1 and 2, the left and the right derivatives of $q(t)$ do not coincide at a finite number of points.
3 Equilibrium Characterization

In this section, we characterize the unique equilibrium of the dynamic trading game. We begin by showing that there exists a belief level $q^*$ and the corresponding equilibrium strategy profile such that buyers’ beliefs $q(t)$ stay constant once they reach $q^*$, even though buyers continue to condition their offers on their signals. We then show that, unless buyers are so optimistic about the seller’s type at the beginning of the game (i.e., $\hat{q}$ is so large) that it is an equilibrium for buyers to always offer $c_H$, there is a unique equilibrium in which buyers’ beliefs continuously converge to $q^*$, whether starting from above or below. We also explain how these results generalize when there are more than two signals.

3.1 Stationary Path

We explicitly construct the stationary equilibrium strategy profile in which $q(t)$ stays constant at $q^*$ even though buyers do not always offer $c_H$. The following lemma, which is useful in later analysis as well, implies that a necessary condition for $q(t)$ to stay constant is $p(t) = v_L$.

Lemma 1 In equilibrium, if $p(t) < v_L$, then the low-type seller accepts $p(t)$ with probability 1 and, therefore, $\dot{q}(t) \geq 0$. If $p(t) > v_L$, then trade takes place only at $c_H$ and, therefore, $\dot{q}(t) \leq 0$. In both cases, the inequality holds strictly as long as the probability of trade conditional on buyer arrival is strictly between 0 and 1.

Proof. See the appendix.

Intuitively, if $p(t) < v_L$, then the buyer can make sure that the low-type seller trades by offering slightly more than $p(t)$. In this case, the low-type seller trades as long as there is a buyer, while the high-type seller insists on $c_H$. Therefore, no trade (delay) is more likely when the seller is the high type. If $p(t) > v_L$, then trade occurs only at $c_H$, because no buyer would pay more than $v_L$, knowing that it would be accepted only by the low type. Since buyers are more willing to offer $c_H$ when $s = h$ than when $s = l$ and the high type generates signal $h$ more frequently than the low type, the high type is more likely to trade than the low type and $q(t)$ decreases over time.

Let $\rho_L$ denote the constant rate at which the low-type seller receives offer $c_H$ on the stationary path. Then, her reservation price is given by

$$p(t) = c_L + \int_t^\infty e^{-rx}(c_H - c_L)d(1 - e^{-\rho_L(x-t)}) = \frac{rc_L + \rho_L c_H}{r + \rho_L}.$$  

For $p(t) = v_L$, it must be that

$$\rho_L = \frac{r}{c_H - v_L}.$$
In other words, \( p(t) \) remains equal to \( v_L \) if the low-type seller receives offer \( c_H \) at a constant rate of \( \rho_L = r(v_L - c_L)/(c_H - v_L) \).

Assumption \( 1 \) (which is equivalent to \( \lambda \gamma^h_L > \rho_L \)) implies that buyers must randomize between \( c_H \) and \( p(t) \) conditional on \( s = h \). Precisely, it is necessary and sufficient that conditional on \( s = h \), buyers offer \( c_H \) with probability

\[
\sigma^*_B \equiv \frac{\rho_L}{\lambda \gamma^h_L} = \frac{r(v_L - c_L)}{\lambda \gamma^h_L(c_H - v_L)}. \tag{1}
\]

Clearly, buyers must be indifferent between offering \( c_H \) and \( v_L \) upon receiving signal \( h \). This implies that no buyer offers \( c_H \) when \( s = l \) because it would give him a negative payoff.

We now determine \( q^* \), using buyers’ indifference conditional on \( s = h \). Consider a buyer who has prior belief \( q^* \) and receives signal \( h \). By Bayes’ rule, his belief updates to

\[
\frac{q^* \gamma^h_H}{q^* \gamma^h_H + (1 - q^*) \gamma^h_L}.
\]

At this belief, the buyer must be indifferent between offering \( c_H \) and offering \( p(t) = v_L \). Therefore,

\[
q^* \gamma^h_H(v_H - c_H) + (1 - q^*) \gamma^h_L(v_L - c_H) = (1 - q^*) \gamma^h_L(v_L - p(t)) = 0 \iff \frac{q^*}{1 - q^*} = \gamma^h_L \frac{c_H - v_L}{v_H - c_H}. \tag{2}
\]

It remains to pin down the probability that the low-type seller accepts \( v_L \), which we denote by \( \sigma^*_S \). We use the fact that \( q(t) \) is time-invariant if and only if the two seller types trade at an identical rate. The high type accepts only \( c_H \). Therefore, given buyers’ offer strategies, her trading rate is equal to \( \lambda \gamma^h_H \sigma_B \). If the low type accepts \( v_L \) with probability \( \sigma^*_S \), then her trading rate is equal to \( \lambda(\gamma^h_L \sigma_B + (1 - \gamma^h_L \sigma_B) \sigma^*_S) \). The equilibrium value of \( \sigma^*_S \) must equate the two rates, that is,

\[
\lambda \gamma^h_H \sigma^*_B = \lambda(\gamma^h_L \sigma^*_B + (1 - \gamma^h_L \sigma_B) \sigma^*_S) \iff \sigma^*_S = \frac{(\gamma^h_H - \gamma^h_L) \sigma^*_B}{1 - \gamma^h_L \sigma^*_B}. \tag{3}
\]

The following lemma summarizes all the findings on the stationary path. Note that the strategy profile is constructed so as to satisfy all players’ incentive constraints and, therefore, is an equilibrium.

**Lemma 2** Let \( q^* \) be the value defined by equation (2). Then, \( q^* \) is the unique belief level that supports an equilibrium in which for all \( t \geq 0 \), (i) \( q(t) = q^* = \hat{q} \) and (ii) the probability of trade conditional on buyer arrival is strictly between 0 and 1. In the unique equilibrium,

- all buyers offer \( c_H \) with probability \( \sigma^*_B \) and \( v_L \) with probability \( 1 - \sigma^*_B \) conditional on \( s = h \) and \( v_L \) with probability 1 conditional on \( s = l \), and
the low-type seller accepts $v_L$ with probability $\sigma_S^*$, where $\sigma_B^*$ and $\sigma_S^*$ are the values given by equations (1) and (3).

Equation (3) well describes how the “skimming” effect (which stems from the fact that the low type has a lower reservation price than the high type and, therefore, drives up $q(t)$) and the signal effect (which originates from the fact that the high type generates good signals more frequently than the low type and, therefore, pushes down $q(t)$) manifest themselves and how they are balanced on the stationary path. The skimming effect is reflected in the fact that $\sigma_S^* > 0$ (the low type accepts not only $c_H$ but also $p(t)$), while the signal effect is materialized in the inequality $\gamma_H^h \sigma_B^* > \gamma_L^h \sigma_B^*$ (the high type is more likely to receive $c_H$ than the low type). On the stationary path, $\sigma_B^*$ and $\sigma_S^*$ are such that the two effects cancel each other out and $q(t)$ remains constant.

3.2 Equilibrium Dynamics

We now construct an equilibrium for each value of $\hat{q} \neq q^*$. We first consider the case where $\hat{q} < q^*$. In this case, early buyers are not willing to offer $c_H$ even when the inspection outcome is $h$, because whenever $q(t) < q^*$,

$$\frac{q(t)\gamma_H^hv_H + (1 - q(t))\gamma_L^hv_L}{q(t)\gamma_H^h + (1 - q(t))\gamma_L^h} < \frac{q^*\gamma_H^hv_H + (1 - q^*)\gamma_L^hv_L}{q^*\gamma_H^h + (1 - q^*)\gamma_L^h} = c_H.$$  

This implies that as long as buyers’ beliefs remain in this range (i.e., until $q(t)$ reaches $q^*$), the high-type seller never trades, while $p(t) < v_L$ and, therefore, the low-type seller trades at rate $\lambda$. Given this observation and Lemma 2, an equilibrium can be immediately constructed, as formally stated in the following proposition.

**Proposition 1** If $\hat{q} < q^*$, then there is an equilibrium in which

- whenever $t < t^*$, the buyer offers $p(t)$ regardless of his signal, the low-type seller accepts $p(t)$ with probability 1, $p(t)$ increases according to $\dot{p}(t) = r(p(t) - c_L)$ with the terminal condition $p(t^*) = v_L$, and $q(t)$ increases according to $\dot{q}(t) = q(t)(1 - q(t))\lambda$, and

- whenever $t \geq t^*$, the players play as in Lemma 2 and $q(t) = q^*$,

where $t^*$ is defined by

$$q^* = \frac{\hat{q}}{\hat{q} + (1 - \hat{q})e^{-\lambda t^*}}.$$  

**Proof.** The result that the strategy profile is an equilibrium follows from the fact that $p(t) < v_L$ (which ensures that the low-type seller trades at rate $\lambda$) and $q(t) < q^*$ (which implies that the buyer offers $p(t)$) whenever $t < t^*$.  


Now consider the case where \( \hat{q} > q^* \). In order to distinguish between the case when buyers offer \( c_H \) regardless of their signal and the case when they do so only when \( s = h \), let \( \overline{q} \) be the value such that

\[
\frac{\gamma^L_H v_H + (1 - \overline{q}) \gamma^L_L v_L}{\overline{q} \gamma^L_H + (1 - \overline{q}) \gamma^L_L} = c_H \iff \overline{q} = \frac{\gamma^L_L c_H - v_L}{\gamma^L_H v_H - c_H}.
\]

In words, a buyer with prior belief \( \overline{q} \) obtains zero expected payoff if he offers \( c_H \) conditional on \( s = l \).

If \( \hat{q} > \overline{q} \), then there is an equilibrium in which each buyer offers \( c_H \) regardless of his inspection outcome and, therefore, \( q(t) \) stays constant at \( \hat{q} \). In this equilibrium, the low type seller’s reservation price \( p(t) \) remains constant at \((rc_L + \lambda c_H)/(r + \lambda)\), which strictly exceeds \( v_L \). This observation verifies that buyers’ offer strategies are indeed optimal.

Next, suppose that \( \hat{q} \in (q^*, \overline{q}) \). By the definitions of \( q^* \) and \( \overline{q} \), it is natural that buyers offer \( c_H \) if and only if \( s = h \) until \( q(t) \) reaches \( q^* \). Given this offer strategy, \( p(t) > v_L \), because the low-type seller receives \( c_H \) at rate \( \lambda \gamma^h_L > \rho_L \) until \( t^* \) and at rate \( \rho_L \) thereafter. This implies that trade occurs only at \( c_H \), which conversely justifies buyers’ offer strategies. It also follows that \( q(t) \) decreases according to

\[
\dot{q}(t) = -q(t)(1 - q(t))\lambda(\gamma^h_L - \gamma^h_L) < 0 \text{ while } q(t) \in (q^*, \overline{q}).
\]

As in the previous case, this allows us to explicitly calculate the length of time it takes for \( q(t) \) to reach \( q^* \), which in turn can be used to derive \( p(t) \).

**Proposition 2** Let \( \overline{q} \) be the value given in equation (4). If \( \hat{q} > \overline{q} \), then it is an equilibrium that all buyers offer \( c_H \) regardless of their signal. In this case, \( p(t) = (rc_L + \lambda c_H)/(r + \lambda) \) and \( q(t) = \hat{q} \) for any \( t \geq 0 \). If \( \hat{q} \in (q^*, \overline{q}) \), then there is an equilibrium in which

- whenever \( t < t^* \), the buyer offers \( c_H \) if \( s = h \) and \( p(t) \) if \( s = l \), the low-type seller accepts only \( c_H \), \( p(t) \) decreases according to \( \dot{p}(t) = r(p(t) - v_L) - \lambda \gamma^h_L (c_H - p(t)) \) with the terminal condition \( p(t^*) = v_L \), and \( q(t) \) decreases according to equation (5), and

- whenever \( t \geq t^* \), the players play as in Lemma 2 and \( q(t) = q^* \),

where \( t^* \) is defined by

\[
q^* = \frac{\hat{q}e^{-\lambda \gamma^h_L t^*}}{\hat{q}e^{-\lambda \gamma^h_L t^*} + (1 - \overline{q})e^{-\lambda \gamma^h_L t^*}}.
\]

**Proof.** The result that the strategy profile for \( \hat{q} \in (q^*, \overline{q}) \) is an equilibrium follows from the fact that \( p(t) > v_L \) (which ensures that trade occurs only at \( c_H \)) and \( q(t) \in (q^*, \overline{q}) \) (which implies that the buyer is willing to offer \( c_H \) if and only if \( s = h \)) for any \( t < t^* \).
Figure 1: The evolution of buyers’ beliefs for different initial values of $\hat{q}$. The left panel is for the case with informative buyer signals ($\gamma^h_H = \gamma^l_L = 2/3$), while the right panel is for the case with no (or uninformative) buyer signals ($\gamma^h_H = \gamma^l_L = 1/2$). The other parameter values used for both panels are $c_L = 0, v_L = 1, c_H = 2, v_H = 3, r = 0.25$, and $\lambda = 1$. We note that the dashed curve in the left panel is not linear in the decreasing region.

Figure 1 depicts three typical paths of buyers’ beliefs both for the case with informative buyer signals (left) and for the benchmark case without buyer signals (right). If $\hat{q}$ is sufficiently large, then all buyers offer $c_H$ regardless of their signal and, therefore, $q(t)$ stays constant at $\bar{q}$ (the horizontal solid line above $\bar{q}$ in the left panel). If $\hat{q}$ is rather low, buyers’ beliefs increase over time (the weakly increasing solid curve in the left panel). This dynamics prominently arises in the absence of buyer signals, as shown in the right panel, and is well-understood in the literature (e.g., Deneckere and Liang, 2006; Hörner and Vieille, 2009; Kim, 2017): the low-type seller, due to her lower reservation value, accepts a wider range of prices than the high-type seller. Therefore, delay (no trade) is more likely when the quality is high, and $q(t)$ increases over time. If $\hat{q} \in (q^*, \bar{q})$, then buyers’ beliefs decrease over time (the weakly decreasing dashed curve in the left panel). This dynamics is in contrast to most existing work on dynamic adverse selection and precisely due to the introduction of private buyer signals, as is clear from the comparison between the two panels. At such beliefs, the low-type seller is optimistic about her prospect of receiving an offer of $c_H$ and unwilling to trade at a low price, similarly to the high type. Therefore, trade takes place only at a high price. Since buyers offer a high price only with a good inspection outcome, the high type trades at a higher rate than the low type. This drives down buyers’ beliefs over time, offsetting the usual skimming effect.
3.3 Equilibrium Uniqueness

Propositions \ref{prop:equilibrium_1} and \ref{prop:equilibrium_2} present an equilibrium for each value of $\hat{q}$, except for the case where $\hat{q} = \overline{q}$. If $\hat{q} = \overline{q}$, then there is a continuum of equilibria. As for the case where $\hat{q} > \overline{q}$, it is an equilibrium that all buyers offer $c_H$ regardless of their signal: each buyer obtains a strictly positive expected payoff if $s = h$ but zero expected payoff if $s = l$. There is another equilibrium which is analogous to the equilibrium when $\hat{q} \in (q^*, \overline{q})$: buyers offer $c_H$ if and only if $s = h$ and trade occurs only at $c_H$ until $q(t)$ reaches $q^*$. In addition, we can construct a continuum of equilibria by taking a convex combination of these two equilibria.\footnote{Specifically, for any fixed $\tilde{t} \geq 0$, it is an equilibrium that buyers switch their behavior from the first equilibrium to the second one at $\tilde{t}$: buyers offer $c_H$ regardless of their signal until $\tilde{t}$ but start conditioning on their signal from $\tilde{t}$. Note that it is not an equilibrium to switch from the second type to the first type, because $q(t)$ falls below $\hat{q} = \overline{q}$ as soon as buyers employ the strategy of offering $c_H$ only when $s = h$.}

The following result states that except for the above knife-edge case, there is a unique equilibrium. In other words, if $\hat{q} \neq \overline{q}$, then the equilibrium presented in Propositions \ref{prop:equilibrium_1} and \ref{prop:equilibrium_2} is the unique equilibrium in our model.

**Theorem 1** Unless $\hat{q} = \overline{q}$, there exists a unique equilibrium.

**Proof.** See the appendix.

The equilibrium construction above utilizes the existence of a unique stationary path (Lemma \ref{lem:stationary_path}) and the interval-partitional equilibrium structure (that buyers’ offer strategies can be described with respect to two cutoff beliefs, $\underline{q}(= q^*)$ and $\overline{q}$). Moreover, it is immediate that the constructed equilibrium is the unique equilibrium given the stationary equilibrium behavior in Lemma \ref{lem:stationary_path} and requiring the interval-partitional structure. This means that equilibrium uniqueness would follow once it is shown (i) that there is a unique equilibrium when $q(t) = q^*$ (i.e., the strategy profile in Lemma \ref{lem:stationary_path} is the unique equilibrium when $\hat{q} = q^*$) and (ii) that any equilibrium necessarily takes an interval-partitional structure. Both results derive from the following lemma.

**Lemma 3** In any equilibrium, $q(t) \leq q^*$ if, and only if, $p(t) \leq v_L$.

**Proof.** See the appendix.
Since the left-hand side depends on both $q(t)$ and $p(t)$, even when $q(t)$ is higher (which lowers the left-hand side and raises the right-hand side), if $p(t)$ is significantly lower (which increases the left-hand side), then he would be more reluctant to offer $c_H$, which, in turn, would justify lower $p(t)$. The main thrust of our proof of Lemma 3 is to show that such a possibility, which cannot be ruled out with a local argument, is not consistent with the long-run (global) dynamics. For example, if $q(t) > q^*$ but $p(t) < v_L$ then, by Lemma 1, $q(t)$ must increase over time. However, it cannot be increasing forever because the inequality above cannot hold if $q(t)$ is sufficiently large.

Lemma 3 combined with Lemma 1 implies that if $q(t)$ hits $q^*$, it must stay constant thereafter: if $q(t)$ becomes smaller than $q^*$, then $p(t) < v_L$ and, therefore, $q(t)$ returns back to $q^*$. Likewise, if $q(t)$ goes above $q^*$, then $p(t) > v_L$, which pushes $q(t)$ back to $q^*$. Then, by construction, the strategy profile in Lemma 1 is the unique equilibrium when $q(t) = q^*$. Lemma 3 also implies that a buyer never offers $c_H$ if $q(t) < q^*$ and offers $c_H$ as long as it yields a positive payoff if $q(t) > q^*$. Combining the latter with the fact that a buyer’s expected payoff by offering $c_H$ is increasing in $q(t)$, it follows that it is optimal for a buyer to offer $c_H$ conditional on $s = h$ if and only if $q(t) > q^*$ and conditional on $s = l$ if and only if $q(t) > q^*$. All together, these imply that any equilibrium is interval-partitional, completing the uniqueness argument.

### 3.4 Beyond Binary Signals

The assumption of binary signals allows us to illustrate the effects of private buyer signals in the simplest way possible but is not crucial for any qualitative aspect of the model. In this subsection, we illustrate how our equilibrium characterization can be generalized when there are more than two signals. We defer some details of construction as well as the formal statements of the results to the online appendix.

#### 3.4.1 $N$ Signals

Suppose that each buyer receives a signal from a finite set $S = \{s_1, ..., s_N\}$ and that the signal structure satisfies the usual monotone likelihood ratio property, so that $s_{n+1}$ is a stronger indicator of high quality than $s_n$ for any $n = 1, ..., N - 1$. For each $a = L, H$, let $\Gamma_a(s_n)$ denote the cumulative probability that each buyer receives a signal weakly below $s_n$. Even in this general model, for a generic set of parameter values, there continues to exist a unique equilibrium, which exhibits the same qualitative properties as the unique equilibrium of the baseline binary-signal model.

The (generically unique) equilibrium can be described by an integer $n^*$ and a finite partition \( \{q_{N+1} = 0, q_N, \ldots, q_1, q_0 = 1\} \). Here, $n^*$ is the integer that identifies the cutoff signal on the
stationary path and is determined by

\[ \lambda(1 - \Gamma_L(s_{n^*})) < \rho_L = \frac{r(v_L - c_L)}{c_H - v_L} < \lambda(1 - \Gamma_L(s_{n^*-1})). \]

These inequalities mean that the low-type seller’s reservation price \( p(t) \) falls short of \( v_L \) if all subsequent buyers employ the strategy of offering \( c_H \) if and only if \( s > s_{n^*} \) but exceeds \( v_L \) if their strategy is to do so if and only if \( s \geq s_{n^*} \). Since the low-type seller’s reservation price must be equal to \( v_L \) on the stationary path (for the same reason as in the baseline model), it follows that \( s_{n^*} \) must serve as the cutoff signal: each buyer offers \( c_H \) with probability 1 if \( s > s_{n^*} \) and appropriately randomizes between \( c_H \) and \( v_L \) if \( s = s_{n^*} \). In turn, this allows us to pin down the stationary belief level \( q^* (= q_{n^*}) \), using the requirement that a buyer must be indifferent between offering \( c_H \) and \( v_L \) conditional on belief \( q^* \) and signal \( s_{n^*} \).

When \( q(t) \neq q^* = q_{n^*} \), buyers’ offer strategies are defined with respect to the partition \( \{q_{N+1} = 0, q_N, ..., q_1, q_0 = 1\} \): if \( q(t) \in (q_{n+1}, q_n) \), then the buyer offers \( c_H \) if and only if \( s > s_n \) (see the example in Figure 2). Combined with the inequalities above, this implies that the low-type seller’s reservation price exceeds \( v_L \) if \( q(t) \) is larger than \( q^* \) and falls short of \( v_L \) if \( q(t) \) is smaller than \( q^* \). In turn, these together determine how buyers’ beliefs \( q(t) \) evolve over time: if \( q(t) \in (q_{n+1}, q_n) \) for \( n \geq n^* \), then the low type accepts both \( p(t) \) and \( c_H \), while the high type trades if and only if \( s > s_n \), and thus

\[ q(t + dt) = \frac{q(t)e^{-\lambda(1-\Gamma_H(s_n))dt}}{q(t)e^{-\lambda(1-\Gamma_H(s_n))dt} + (1 - q(t))e^{-\lambda dt}}, \]

which yields

\[ \dot{q}(t) = q(t)(1 - q(t))\lambda\Gamma_H(s_n) > 0. \]
If \( q(t) \in (q_{n+1}, q_n) \) for \( n = 1, \ldots, n^* - 1 \), then both seller types trade if and only if \( s > s_n \), and thus

\[
q(t + dt) = \frac{q(t)e^{-\lambda\Gamma_H(s_n)dt}}{q(t)e^{-\lambda\Gamma_H(s_n)dt} + (1 - q(t))e^{-\lambda\Gamma_L(s_n)dt}},
\]

which implies

\[
\dot{q}(t) = q(t)(1 - q(t))\lambda(\Gamma_H(s_n) - \Gamma_L(s_n)) < 0.
\]

In both cases, \( q(t) \) eventually converges to \( q^* \), just as in the baseline model.

The cutoff beliefs, \( q_N, \ldots, q_1 \), can be determined analogously to the baseline model. One complication is that, whereas the cutoffs above \( q^* \) (i.e., \( q_{n^* - 1}, \ldots, q_1 \)) can be found independently of \( p(t)(> v_L) \), the cutoffs below \( q^* \) (i.e., \( q_N, \ldots, q_{n^* + 1} \)) must be jointly determined with \( p(t) \): each \( q_n \) is determined by the requirement that a buyer with belief \( q_n \) must be indifferent between offering \( c_H \) and \( p(t) \) conditional on signal \( s_n \). If \( p(t) > v_L \), then it is simply not accepted and, therefore, the indifference condition is independent of \( p(t) \). To the contrary, if \( p(t) < v_L \), then the buyer obtains a positive expected payoff even with \( p(t) \) and, therefore, \( q_n \) depends on \( p(t) \). In fact, this complication arises even in the baseline model when Assumption I is violated (because \( q < q^* = \overline{q} \)). We explain how to recursively construct the cutoffs below \( q^* \) in the online appendix (Section A for the baseline model and Section C for the general finite-signal model).

### 3.4.2 A Continuum of Signals

Now suppose that each buyer’s signal is drawn from the interval \( S = [\underline{s}, \overline{s}] \) according to the type-dependent cumulative distribution function \( \Gamma_a \) with density \( \gamma_a \) and the monotone likelihood ratio property holds (i.e., \( \gamma_H(s)/\gamma_L(s) \) is strictly increasing in \( s \)).

Equilibrium characterization proceeds just as in the general finite case above. Let \( s^* \) be the unique value in \( S \) such that

\[
\lambda(1 - \Gamma_L(s^*)) = \rho_L = \frac{r(v_L - c_L)}{c_H - v_L}.
\]

Given \( s^* \), the stationary path can be fully constructed using buyers’ indifference between \( c_H \) and \( p(t) = v_L \) conditional on \( s^* \) (which pins down \( q^* \)) and belief invariance (which allows us to identify \( \sigma^*_h \)). Given the characterization of the unique stationary path, one can also show that \( q(t) \) gradually converges to \( q^* \), whether from above or from below, by applying Lemmas I and 3, both of which extend to this case without modification. To be specific, let \( s(t) \) denote the cutoff signal above which buyers offer \( c_H \) at time \( t \). If \( q(t) > q^* \), then trade occurs only at \( c_H \) (because \( p(t) > v_L \)), and thus

\[
q(t + dt) = \frac{q(t)e^{-\lambda(1-\Gamma_H(s(t)))dt}}{q(t)e^{-\lambda(1-\Gamma_H(s(t)))dt} + (1 - q(t))e^{-\lambda(1-\Gamma_L(s(t)))dt}},
\]
which leads to
\[ \dot{q}(t) = q(t)(1 - q(t))\lambda(\Gamma_H(s(t)) - \Gamma_L(s(t))) < 0. \]

If \( q(t) < q^* \), then the low type accepts both \( p(t)(< v_L) \) and \( c_H \), and thus
\[ q(t + dt) = \frac{q(t)e^{-\lambda(1-\Gamma_H(s(t)))dt}}{q(t)e^{-\lambda(1-\Gamma_H(s(t)))dt} + (1 - q(t))e^{-\lambda dt}}, \]
which yields
\[ \dot{q}(t) = q(t)(1 - q(t))\lambda\Gamma_H(s(t)) > 0. \]

Although this alternative specification has an advantage of purifying buyers’ offer strategies (i.e., all buyers, including those on the stationary path, play a simple cutoff strategy), the characterization of buyers’ offer strategies when \( q(t) < q^* \) is significantly more complicated. As explained also for the finite-signal case, if \( q(t) < q^* \) then \( p(t) < v_L \) and, therefore, buyers’ offer strategies cannot be separately identified from \( p(t) \). Unlike in the finite case (where \( s(t) \) is a step function), \( s(t) \) varies continuously and, therefore, the three relevant equilibrium functions, \( s(t) \), \( q(t) \), and \( p(t) \), can be characterized only by the following system of equations (together with the law of motion for \( q(t) \) above):

- Each buyer is indifferent between \( c_H \) and \( p(t) \) conditional on \( s = s(t) \), and thus
  \[ \frac{q(t)}{1 - q(t)} = \frac{\gamma_L(s(t))}{\gamma_H(s(t))} \frac{c_H - p(t)}{v_H - c_H}. \]

- The low-type seller’s reservation price changes over time according to
  \[ r(p(t) - c_L) = \lambda(1 - \Gamma_L(s(t)))(c_H - p(t)) + \dot{p}(t), \]

Although a closed-form solution is not available, it can be shown that all equilibrium properties from the finite case carry over. In particular, if \( \hat{q} < q^* \), then in equilibrium both \( p(t) \) and \( q(t) \) are necessarily increasing, while \( s(t) \) is decreasing (meaning that buyers offer \( c_H \) more frequently), over time. See the online appendix for a formal analysis.

4 Informativeness of Buyers’ Signals

In this section, we analyze the effects of varying the informativeness of buyers’ signals. In particular, we study how an increase in the informativeness, which presumably helps mitigate information asymmetry in the market, affects market efficiency and seller surplus. For the former, we consider
the expected delay to trade, because inefficiency takes the form of delay in our dynamic environment.\footnote{An alternative is to consider expected social surplus from trade of each type, that is, $E[e^{-r\tau_a}(v_a - c_a)]$, where $\tau_a$ denotes the random time of trade when the seller’s type is $a$. We do not separately consider this alternative criterion, because it involves effectively the same arguments and leads to analogous economic conclusions.} For the latter, we focus on the low-type seller’s expected payoff $p(0)$, because the high-type seller never obtains a strictly positive expected payoff.

For each $a = L, H$, we let $\tau_a$ denote the random time at which the type-$a$ seller trades and $F_a$ denote the corresponding distribution function, so that $F_a(t)$ is the probability that the type-$a$ seller trades before $t$. Note that the seller leaves the market only when she trades and, therefore, the hazard rate $f_a(t)/(1 - F_a(t))$ coincides with the type-$a$ seller’s trading rate at $t$. In addition, we let $t^*(\hat{q})$ denote the length of time it takes for $q(t)$ to travel from $\hat{q}$ to $q^*$ in the unique equilibrium of the game, whether $\hat{q} < q^*$ or not.

### 4.1 Blackwell Informativeness

In our model, an inspection technology is described by a matrix

$$
\Gamma = \begin{pmatrix}
\gamma^L_L & \gamma^L_H \\
\gamma^H_L & \gamma^H_H
\end{pmatrix}
= \begin{pmatrix}
1 - \gamma^h_L & 1 - \gamma^h_H \\
\gamma^h_L & \gamma^h_H
\end{pmatrix}.
$$

Applying Blackwell’s notion of informativeness (Blackwell, 1951) to our model, $\Gamma$ is more informative than $\Gamma'$ if there exists a non-negative (Markov) matrix $M = (m_{ij})_{2 \times 2}$ such that each row sums to 1 (i.e., $\sum_j m_{ij} = 1$) and $\Gamma' = M \Gamma$. The following result shows that Blackwell informativeness is fully summarized by the likelihood ratios in our model with binary signals.\footnote{In general, Blackwell informativeness regulates only the likelihood ratios of the two extreme signals, one with the lowest ratio and the other with the highest ratio (see, e.g., Ponssard, 1975). Most results in this section go through unchanged even with more than two signals and the resulting weaker implication on the likelihood ratios. See an earlier version of this paper for such a general treatment.}

**Lemma 4** $\Gamma$ is more informative than $\Gamma'$, in the sense of Blackwell (1951), if and only if the likelihood ratio conditional on $l$ is smaller, while that conditional on $h$ is larger, under $\Gamma$ than under $\Gamma'$, that is,

$$
\frac{\gamma^H_H}{\gamma^H_L} \leq \frac{\gamma'^H_H}{\gamma'^H_L} \quad \text{and} \quad \frac{\gamma^h_H}{\gamma^h_L} \geq \frac{\gamma'^h_H}{\gamma'^h_L}.
$$

**Proof.** See the appendix. $\blacksquare$

Intuitively, a more informative signal allows a decision-maker to take the right action (e.g., offering $c_H$ to the high type and $p(t)$ to the low type) with a higher probability. This means that a more informative signal should bring the decision-maker’s posterior closer to 0 or 1, depending on its realization. In other words, a signal is more informative if it induces more dispersed posterior
beliefs. Lemma 4 stems from the fact that dispersion of posterior beliefs is determined by the likelihood ratios: given prior belief $q(t)$ and signal $s$, the posterior is given by

$$
q(t, s) = \frac{q(t) \gamma^s_H}{q(t) \gamma^s_H + (1 - q(t)) \gamma^s_L} \Leftrightarrow \frac{q(t, s)}{1 - q(t, s)} = \frac{q(t) \gamma^s_H}{1 - q(t) \gamma^s_L}.
$$

One immediate but crucial implication of Lemma 4 is that as $\Gamma$ becomes more informative, the two cutoff beliefs, $q^*$ and $\overline{q}$, move in opposite directions (see Figure 3) and the seller trades faster on the stationary path, as formally stated in the following result.

**Corollary 1** If $\Gamma$ becomes more informative, then $q^*$ decreases, while $\overline{q}$ increases. In addition, the seller’s trading rate on the stationary path, denoted by $\rho$, increases.

**Proof.** The results are immediate from Lemma 4 and the following closed-form solutions:

$$
\frac{q^*}{1 - q^*} = \frac{\gamma^h_L c_H - v_L}{\gamma^h_H v_H - c_H}, \quad \overline{q} = \frac{\gamma^l_H c_H - v_L}{\gamma^l_H v_H - c_H}, \quad \text{and} \quad \rho = \frac{\gamma^h_H r(v_L - c_L)}{\gamma^L_H c_H - v_L}.
$$

where the first two come from equations (2) and (4), while the last follows from the fact that $\rho = \lambda \gamma^h_H \sigma^*_B$ (the high type’s trading rate on the stationary path) and equation (1).

To understand this result, recall that $q^*$ is the point at which a buyer obtains zero expected payoff when he offers $c_H$ conditional on $s = h$, and $\overline{q}$ is the corresponding point conditional on $s = l$. If $\Gamma$ becomes more informative, then $h$ becomes a stronger signal of high quality, while $l$ becomes a weaker signal of high quality. That is, the buyer becomes more confident about the quality of the asset conditional on $s = h$ but less confident conditional on $s = l$. This pushes down $q^*$ but drives up $\overline{q}$, because the prior belief level $q^*$ necessary for the buyer to break even with $c_H$ is inversely related to the likelihood ratio $\gamma^s_H / \gamma^s_L$, that is,

$$
q^* \gamma^s_H (v_H - c_H) + (1 - q^*) \gamma^s_L (v_L - c_H) = 0 \Leftrightarrow \frac{q^*}{1 - q^*} = \frac{1}{\gamma^s_H / \gamma^s_L} \frac{c_H - v_L}{v_H - c_H}.
$$

For the result on the seller’s trading rate $\rho$ on the stationary path, notice that an increase in the informativeness of $\Gamma$ makes the high type generate signal $h$ even more frequently relative to the low type. This, together with the fact that the rate at which the low type receives $c_H$ on the stationary path must remain unchanged at $\rho_L = r(v_L - c_L)/(c_H - v_L)$, implies that the high type’s trading rate (at price $c_H$) increases. Since the seller’s trading rate is independent of her type on the stationary path, the low-type seller also trades faster.
Figure 3: The effects on the evolution of buyers’ beliefs when \( \Gamma \) becomes more informative, from \( \gamma_H^h = \gamma_L^l = 2/3 \) (the dashed lines, \( q^* \), and \( q \)) to \( \gamma_H^h = \gamma_L^l = 3/4 \) (the solid lines, \( q^{*'} \), and \( q' \)). The other parameter values used are identical to those for Figure 1.

4.2 Pessimistic Initial Beliefs

If \( \hat{q} < q^* \) then, as shown in Proposition 1, buyers offer \( p(t) \) even with \( s = h \) and the low-type seller accepts \( p(t) \) with probability 1 until \( t^*(\hat{q}) \) (i.e., until \( q(t) \) reaches \( q^* \)). After \( t^*(\hat{q}) \), both seller types trade at rate \( \rho \) and \( p(t) \) stays constant at \( v_L \). Therefore,

\[
p(0) = c_L + e^{-rt^*(\hat{q})}(p(t^*(\hat{q}))-c_L) = c_L + e^{-rt^*(\hat{q})}(v_L-c_L)
\]

and

\[
q(t) = \frac{\hat{q}}{\hat{q}+(1-\hat{q})e^{-\lambda t}} \text{ for all } t < t^*(\hat{q}).
\]

An increase in the informativeness of \( \Gamma \) affects this dynamic outcome in two ways. First, since \( q^* \) falls (by Corollary 1 above), \( t^*(\hat{q}) \) decreases (see the left panel of Figure 3). Second, since \( \rho \) increases (again by Corollary 1), both seller types trade faster after \( t^*(\hat{q}) \). The following result is immediate once these effects are applied to the equilibrium outcome.

**Proposition 3** Suppose that \( \hat{q} < q^* \). If \( \Gamma \) becomes more informative, then \( \tau_H \) decreases in the sense of first-order stochastic dominance, \( \mathbb{E}[	au_L] \) decreases, and \( p(0) \) increases.

**Proof.** See the appendix. \( \blacksquare \)

Proposition 3 is fairly intuitive. An increase in the informativeness of buyers’ signals reduces
information asymmetry in the market. This makes buyers become less reluctant to offer $c_H$ when they receive a good signal and, therefore, start offering $c_H$ from an earlier time. This is beneficial to the low-type seller, who receives $c_H$ at the constant rate of $\rho_L$ (independent of $\Gamma$) on the stationary path. Since the high-type seller’s trading rate on the stationary path increases (by Corollary 1), this also means that the high-type seller clearly trades faster. At the same time, such an adjustment increases the low-type seller’s incentive to reject $p(t)$ and wait for $c_H$, which is why $\tau_L$ does not decrease in the sense of first-order stochastic dominance: in the left panel of Figure 3, the low-type seller’s trading rate increases from $\tau_L$ to $\tau_L^*$ because Blackwell informativeness disciplines $c_L$ unambiguously decreases because this indirect negative effect cannot outweigh the direct positive effect of trading faster on the stationary path.

4.3 Optimistic Beliefs

Now we consider the case when $\hat{q} \in (q^*, \bar{q})$. In this case, as shown in Proposition 2 until $t^*(\hat{q})$ (i.e., until $q(t)$ reaches $q^*$), buyers offer $c_H$ if $s = h$ and $p(t) > v_L$ if $s = l$. Since trade occurs only at $c_H$, buyers’ beliefs decrease according to

$$q(t) = q e^{-\lambda \gamma^H_t t}$$

$$\frac{\gamma^H_H c_H - v_L}{\gamma^H_H v_H - c_H} = \frac{q^*}{1 - q^*} = \frac{\hat{q}}{1 - \hat{q}} e^{-\lambda \gamma^H_t t^*(\hat{q})} + \frac{\hat{q}}{1 - \hat{q}} e^{-\lambda \gamma^L_t t^*(\hat{q})} \Leftrightarrow e^{\lambda (\gamma^H_H - \gamma^L_L) t^*(\hat{q})} = \frac{\hat{q}}{1 - \hat{q}} \frac{\gamma^H_H v_H - c_H}{\gamma^L_L c_H - v_L}.$$

With optimistic beliefs, $\Gamma$ determines not only the length of the convergence path $t^*(\hat{q})$ but also each seller type’s trading rate $\lambda \gamma^a_t$ on the path. This implies that the information content of time-on-the-market is also influenced by a change in $\Gamma$. The evolution of buyers’ beliefs, however, is determined by the difference $\gamma^H_H - \gamma^L_L$, not by the ratio $\gamma^H_H / \gamma^L_L$, as shown in the equation for $q(t)$ above. This suggests that without further restrictions, various different results may emerge depending on how we vary $\Gamma$, because Blackwell informativeness disciplines $\gamma^H_H / \gamma^L_L$ but not $\gamma^H_H - \gamma^L_L$ in general. For instance, if both $\gamma^H_H$ and $\gamma^L_L$ decrease, then $\gamma^H_H - \gamma^L_L$ can fall when $\gamma^H_H / \gamma^L_L$ rises.

In what follows, in order to further discipline variations in $\Gamma$ and get clean insights, we focus on the symmetric signal structure such that $\gamma^H_L = \gamma^H_H = \gamma$ for some $\gamma \in (1/2, 1)$, that is,

$$\Gamma = \left( \begin{array}{cc} \gamma & 1 - \gamma \\ 1 - \gamma & \gamma \end{array} \right).$$

Naturally, $\gamma$ measures the informativeness of buyers’ signals: $\Gamma$ is more informative, in the sense
of Blackwell (1951), if and only if \( \gamma \) is higher. In addition, both the likelihood ratio \( \gamma^h_H/\gamma^h_L = \gamma/(1 - \gamma) \) and the difference \( \gamma^h_H - \gamma^h_L = 2\gamma - 1 \) always increase in \( \gamma \).

Under the symmetry restriction, the equation for \( q(t) \) above simplifies to

\[
q(t) = \frac{\hat{q}}{q + (1 - \hat{q})e^{\lambda(2\gamma - 1)t}}.
\]

Clearly, for any \( t < t^*(\hat{q}) \), \( q(t) \) decreases in \( \gamma \). Intuitively, when \( \hat{q} \in (q^*, \overline{q}) \), delay is mainly caused by the failure to generate signal \( h \) and \( q(t) \) reflects the expected difference in the frequency of signal \( h \) between the two seller types. If this difference grows due to an increase in the informativeness of \( \Gamma \), then delay becomes a stronger indicator of low quality and, therefore, \( q(t) \) decreases faster (see the right panel of Figure 3).

The equation for \( t^*(\hat{q}) \) above reduces to

\[
e^{\lambda(2\gamma - 1)t^*(\hat{q})} = \frac{\hat{q} - 1 - q^*}{1 - \hat{q}} = \frac{\hat{q}}{1 - q^*} \frac{\gamma}{1 - \gamma} \frac{v_H - c_H}{c_H - v_L}.
\]

The left-hand side captures the effect of the speed of belief evolution (i.e., \( \lambda(\gamma^h_H - \gamma^h_L) \)) on \( t^*(\hat{q}) \), while the right-hand side reflects the distance between \( \hat{q} \) and \( q^* \). As \( \gamma \) increases, \( q(t) \) falls faster, which shortens \( t^*(\hat{q}) \). In the meantime, as shown in Corollary \( 1 \) \( q^* \) decreases and, therefore, becomes further apart from \( \hat{q} \), which lengthens \( t^*(\hat{q}) \) (see the right panel of Figure 3). In general, \( t^*(\hat{q}) \) can both increase or decrease in \( \gamma \). The following lemma provides a necessary and sufficient condition under which \( t^*(\hat{q}) \) increases in \( \gamma \).

**Lemma 5** Let \( \hat{q}^* \equiv (c_H - v_L)/(v_H - v_L) \in [q^*, \overline{q}] \). If \( \hat{q} \leq \hat{q}^* \), then \( t^*(\hat{q}) \) increases in \( \gamma \). Otherwise, there exists \( \gamma(\hat{q}) \in (1/2, 1) \) such that \( t^*(\hat{q}) \) increases in \( \gamma \) if and only if \( \gamma > \gamma(\hat{q}) \).

**Proof.** See the appendix.

Intuitively, if \( \hat{q} \) is close to \( q^* \), then \( t^*(\hat{q}) \) is close to 0. In this case, a marginal change of the speed of belief evolution over \([0, t^*(\hat{q})] \) has a negligible impact, while a decrease in \( q^* \) has the first-order effect on \( t^*(\hat{q}) \). Therefore, \( t^*(\hat{q}) \) increases in \( \gamma \). If \( \hat{q} \) is considerably larger than \( q^* \), then the relative strength of the two effects depends on \( \gamma \), because the marginal effect of the speed of convergence (captured by the term \( 2\gamma - 1 \)) is independent of \( \gamma \), while that of \( q^* \) (captured by the term \( \gamma/(1 - \gamma) \)) increases in \( \gamma \). Therefore, \( t^*(\hat{q}) \) decreases in \( \gamma \) if \( \gamma \) is close to 1/2 but increases if \( \gamma \) is close to 1. In Figure 4, \( t^*(\hat{q}) \) decreases in \( \gamma \) if and only if \( (\gamma, \hat{q}) \) lies above the dashed line.

In order to understand the economic effects of these changes, first consider \( \tau_H \). By Proposition
the high-type seller’s trading rate is given as follows:

\[
\frac{f_H(t)}{1 - F_H(t)} = \begin{cases} 
\lambda \gamma & \text{if } t < t^*(\hat{q}), \\
\rho & \text{if } t \geq t^*(\hat{q}).
\end{cases}
\]

An increase in $\gamma$ raises the high-type seller’s trading rates both on the convergence path ($\lambda \gamma$) and on the stationary path ($\rho$), where the latter follows from Corollary 1. Since $\lambda \gamma > \rho$, if it also increases $t^*(\hat{q})$, then the overall effect is clear: $\tau_H$ decreases in the sense of first-order stochastic dominance. If $t^*(\hat{q})$ decreases, instead, the overall effect is ambiguous. Still, since all the variables change continuously, it is natural that $E[\tau_H]$ increases as long as $t^*(\hat{q})$ does not decrease sufficiently fast.

Now consider the low-type seller’s expected payoff $p(0)$. Recall that $p(0)$ depends only on the rate at which the low type receives $c_H$. Letting $\rho_L(t)$ denote the rate at each $t$, by Proposition 2

\[
\rho_L(t) = \begin{cases} 
\lambda(1 - \gamma) & \text{if } t < t^*(\hat{q}), \\
\rho_L = r(v_L - c_L)/(c_H - v_L) & \text{if } t \geq t^*(\hat{q}).
\end{cases}
\]

In contrast to the high type’s corresponding rates, $\lambda(1 - \gamma)$ falls in $\gamma$, and $\rho_L$ is independent of
Since \( \lambda(1 - \gamma) > \rho_L \), an increase in \( \gamma \) clearly lowers \( p(0) \) if it decreases \( t^*(\widehat{q}) \). Otherwise, the overall effect is ambiguous, but \( p(0) \) would decrease as long as \( t^*(\widehat{q}) \) does not increase so fast that the negative effect due to lower \( \lambda(1 - \gamma) \) outweighs the positive effect due to higher \( t^*(\widehat{q}) \).

For the effects on \( \tau_L \), recall that the low-type seller’s trading rate is given as follows, again by Proposition \([2] \):

\[
\frac{f_L(t)}{1 - F_L(t)} = \begin{cases} 
\lambda(1 - \gamma) & \text{if } t < t^*(\widehat{q}), \\
\rho & \text{if } t \geq t^*(\widehat{q}).
\end{cases}
\]

An increase in \( \gamma \) lowers \( \lambda(1 - \gamma) \) but raises \( \rho \) (by Corollary \([1] \)). Therefore, regardless of whether \( t^*(\widehat{q}) \) increases or decreases, the overall effect is ambiguous: \( \gamma_L \) does not change in the sense of first-order stochastic dominance. Nevertheless, it is clear that if \( \widehat{q} \) is so close to \( q^* \) that \( t^*(\widehat{q}) \) is sufficiently small, then the former negative effect is dominated by the latter positive effect, and thus \( E[\tau_L] \) decreases. In the opposite case when \( \widehat{q} \) is considerably larger than \( q^* \), the former effect can be significant and dominate the latter effect, in which case \( E[\tau_L] \) increases.

We summarize the results so far in the following proposition. Roughly, it states that improving the informativeness of buyers’ signals may be harmful to efficiency and seller surplus when \( \widehat{q} > \overline{q}^* = (c_H - v_L)/(v_H - v_L) \), which is the case when trade is fully efficient in the absence of buyer signals and, therefore, typically excluded in other models of dynamic adverse selection.

**Proposition 4** Suppose that \( \widehat{q} \in (q^*, \overline{q}) \) and consider the symmetric signal structure such that \( \gamma^h_H = \gamma^l_L = \gamma \) for some \( \gamma \in (1/2, 1) \). If \( \widehat{q} \) is sufficiently close to \( q^* \), then both \( E[\tau_L] \) and \( E[\tau_H] \) decrease, while \( p(0) \) increases, in \( \gamma \). If \( \widehat{q} \) is sufficiently close to \( \overline{q} \) and \( \gamma \) is sufficiently close to \( 1/2 \), then both \( E[\tau_L] \) and \( E[\tau_H] \) increase, while \( p(0) \) decreases, in \( \gamma \).

**Proof.** See the appendix. \( \blacksquare \)

For the intuition, recall that if \( \widehat{q} \in (q^*, \overline{q}) \), then time-on-the-market \( t \) contains negative information about the seller’s type: a seller’s availability reflects how unlikely she is to generate signal \( h \) (the signal effect), not how much she insists on a high price (the skimming effect). When \( \Gamma \) becomes more informative, this negative information contained in \( t \) is amplified, which makes buyers more pessimistic and, therefore, weakens their incentives to offer a high price. When \( \widehat{q} \) is close to \( \overline{q} \) (in which case \( t^*(\widehat{q}) \) is significant), this negative effect is particularly strong and may even outweigh the general positive effects of more informative signals. If that happens, market efficiency deteriorates and the seller loses out.

Our efficiency result is particularly related to Daley and Green (2012), who study the effects of introducing public news in a model with competitive buyers. They find that introducing news necessarily improves efficiency if a static lemons condition holds (translated as \( v_L < c_H \) in our model) but weakly reduces efficiency if the condition fails. This is qualitatively consistent with our
result that introducing private buyer signals (i.e., increasing $\gamma$ from $1/2$) contributes to efficiency if and only if $\hat{q} < \hat{q}^*$. The mechanism behind their result is different from ours: their result is driven by the high-type seller’s incentive to wait for more favorable public news, which strengthens as news quality improves, not by buyers’ inferences about previous buyers’ signals. Nevertheless, both results highlight the subtle role of informative signals in the market for lemons and call for caution on the conventional wisdom that transparency necessarily helps market efficiency.

5 The Role of Search Frictions

In this section, we investigate the role of search frictions in our dynamic trading environment. In particular, we study whether, and how, an increase in the arrival rate of buyers $\lambda$ can improve market efficiency and seller surplus.\(^{15}\)

An increase in $\lambda$ has a direct positive effect on both efficiency and seller surplus: if the players’ strategies were to remain unchanged, then trade would occur faster and the low-type seller would obtain a higher expected payoff. However, the players do adjust their strategies in response. In particular, the low-type seller becomes more willing to wait for $c_H$, which induces buyers to adjust their offer behavior accordingly. In order to systematically assess the overall effects, we separately consider the effects on the stationary path (after $q(t)$ reaches $q^*$) and those on the convergence path (before $q(t)$ reaches $q^*$).

Recall that the seller’s trading rate on the stationary path (which is by definition independent of the seller’s type) is given by

$$\rho = \lambda \gamma^h \sigma^*_B = \lambda \gamma^h \rho_L = \frac{\gamma^h}{\gamma^L} \frac{r(v_L - c_L)}{c_H - v_L}.$$  

Notice that $\rho$ is independent of $\lambda$, that is, an increase in $\lambda$ has no effect on the seller’s trading rate on the stationary path. Technically, this is because $p(t) = v_L$ on the stationary path, which can be sustained only when $\gamma^h_L \sigma^*_B$ (the probability that each buyer offers $c_H$ to the low-type seller) proportionally decreases as $\lambda$ increases. Intuitively, an increase in $\lambda$ strengthens the low-type seller’s incentive to reject $p(t)$ and wait for $c_H$, which weakens buyers’ incentives to offer $c_H$. In equilibrium, buyers decrease their probability of offering $c_H$ up to the point where $p(t)$ remains equal to $v_L$. Naturally, the stationary belief $q^*$ is also independent of $\lambda$, because it is determined by the requirement that a buyer must break even with offer $c_H$ conditional on belief $q^*$ and signal $h$, for which the buyer arrival rate $\lambda$ is irrelevant.

\(^{15}\) Palazzo (2017) conducts a related exercise. He considers an environment in which the seller must incur explicit search costs $c$ in order to meet (sample) another buyer and shows that the lemons problem is more severe when $c$ is sufficiently small (because it is when the low-type seller has a strong incentive to pool with the high-type seller). He proposes a budget balanced mechanism that can mitigate the problem.
In order to evaluate the impact on the convergence path, recall from Propositions 1 and 2 that $t^*(\hat{q})$ satisfies the following equation in each case: if $\hat{q} < q^*$ then

$$q^* = \frac{\hat{q}}{\hat{q} + (1 - \hat{q})e^{-\lambda t^*(\hat{q})}} \Leftrightarrow \lambda t^*(\hat{q}) = \log \left( \frac{\hat{q} - 1 - q^*}{q - q^*} \right),$$

while if $\hat{q} \in (q^*, \overline{q})$ then

$$q^* = \frac{\hat{q}e^{-\lambda \gamma^H t^*(\hat{q})}}{\hat{q}e^{-\lambda \gamma^L t^*(\hat{q})} + (1 - \hat{q})e^{-\lambda \gamma^L t^*(\hat{q})}} \Leftrightarrow \lambda t^*(\hat{q}) = -\frac{1}{\gamma^H} \log \left( \frac{\hat{q} - 1 - q^*}{1 - \hat{q}} \right).$$

In either case, $\lambda t^*(\hat{q})$ is independent of $\lambda$, that is, $t^*(\hat{q})$ proportionally decreases as $\lambda$ increases. This means that when $\lambda$ increases, the probability that each seller type trades on the convergence path remains constant but, since $t^*(\hat{q})$ decreases, trade occurs faster on average.

Combining these two results leads to the following conclusion: the indirect effect associated with an increase in $\lambda$ cannot outweigh the direct positive effect and, therefore, an increase in $\lambda$ improves both market efficiency and seller surplus.

**Proposition 5** If $\lambda$ increases, then both $\tau_L$ and $\tau_H$ decrease in the sense of first-order stochastic dominance and $p(0)$ increases.

**Proof.** From the characterization results in Section 3 if $\hat{q} < q^*$ then

$$\frac{f_L(t)}{1 - F_L(t)} = \begin{cases} \lambda & \text{if } t < t^*(\hat{q}), \\ \rho & \text{otherwise,} \end{cases}$$

and

$$\frac{f_H(t)}{1 - F_H(t)} = \begin{cases} 0 & \text{if } t < t^*(\hat{q}), \\ \rho & \text{otherwise,} \end{cases}$$

while if $\hat{q} > q^*$ then for both $a = L, H$,

$$\frac{f_a(t)}{1 - F_a(t)} = \begin{cases} \lambda \gamma^H_a & \text{if } t < t^*(\hat{q}), \\ \rho & \text{otherwise.} \end{cases}$$

Using the fact that both $\lambda t^*(\hat{q})$ and $\rho$ are independent of $\lambda$, one can directly show that for both $a = L, H$, and whether $\hat{q} < q^*$ or $\hat{q} \in (q^*, \overline{q})$, $F_a(t)$ strictly decreases in the sense of first-order stochastic dominance as $\lambda$ increases. The payoff result can be established by showing that the distribution of the random time by which the low-type seller receives $c_H$ also decreases in $\lambda$ in the sense of first-order stochastic dominance, whose proof is analogous to the one above.

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16This seems inconsistent with a common empirical finding in the literature on online vs. offline markets, namely that the lemons problem tends to be more severe in online markets than in offline markets (see, e.g., [Wolf and Muhanna, 2005], [Jin and Katz, 2007], [Overby and Jap, 2009]). However, the empirical patterns are likely to be driven by market segmentation (i.e., different sellers choosing different markets) and, therefore, do not negate our result on $\lambda$. 

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Figure 5: The effects of increasing the arrival rate of buyers from $\lambda = 1.5$ (dashed) to $\lambda' = 2$ (solid) on the rate at which the low-type seller trades (left) and on the cumulative probability with which she trades (right). The two shaded areas in the left panel are of equal size. The dotted line in the right panel is for the case where $\lambda$ is sufficiently large ($\lambda = 20$). The parameter values used for this figure are $c_L = 0$, $v_L = 1$, $c_H = 2$, $v_H = 3$, $r = 0.35$, $\gamma_H = \gamma_L = 2/3$, and $\tilde{q} = 0.55$.

Figure 5 illustrates the logic behind Proposition 5. An increase in $\lambda$ does not affect the seller’s trading rate ($\rho$) and the low-type seller’s expected payoff ($p(t) = v_L$) on the stationary path. However, it shortens the length of time it takes for $q(t)$ to reach $q^*$. When $\tilde{q} \in (q^*, \bar{q})$, this means that the seller receives $c_H$ more frequently at earlier times (see the left panel of Figure 5). This reduces the expected delay to trade and, due to discounting, increases seller surplus. When $\tilde{q} < q^*$, an increase in $\lambda$ directly speeds up trade of the low-type seller before $t^*(\tilde{q})$. In addition, since $t^*(\tilde{q})$ decreases, the seller starts receiving $c_H$ earlier, which implies both that the high type also trades faster and that the low-type seller’s expected payoff $p(0) = c_L + e^{-rt^*(\tilde{q})}(v_L - c_L)$ increases.

Proposition 5 is in stark contrast to a result by Fuchs and Skrzypacz (2017). They consider a discrete-time model with competitive buyers and show that under a regularity condition, the most infrequent trading (restricting trade to take place only at time 0) maximizes expected gains from trade. Their result follows because a seller who needs to wait longer until the next trading opportunity has a weaker incentive to delay trade and, therefore, is more willing to trade. This effect is present in our model as well, reflected in the fact that $p(t)$ increases in $\lambda$. The main difference lies in the flexibility of timing design. Whereas we vary only the Poisson arrival rate $\lambda$, they consider a more general timing design problem in which, effectively, buyers’ arrival times can be chosen by the designer. In our model, the arrival time of the first buyer, as well as the frequency of subsequent arrivals, depends on $\lambda$, which offsets the aforementioned effect of increased $\lambda$ and ultimately leads to the conclusions in Proposition 5.
We note that although an increase in $\lambda$ improves market efficiency, it does not eliminate inefficiency even in the limit as $\lambda$ tends to infinity. For any $\tilde{q} < q^*$, $t^*(\tilde{q})$ approaches 0 and the seller trades almost immediately with a positive probability. However, as shown above, the trading rate on the stationary path $\rho$ is independent of $\lambda$ and, therefore, real-time delay persists even as $\lambda$ grows unboundedly. The dotted line in the right panel of Figure 5 illustrates this limiting outcome. When $\lambda$ is sufficiently large, $F_a(t)$ reaches a point at which $q(t) = q^*$ almost immediately. However, its hazard rate $f_a(t)/(1 - F_a(t))$ remains constant at $\rho$ thereafter, which generates real-time delay.

6 Robustness

Our model is parsimonious in various dimensions. This allows us to deliver our main insights in a particularly simple fashion as well as analyze the effects of key policy variables. However, it also raises the question of the robustness of our findings. In this section, we show that our main insights continue to hold when the model is modified in three important dimensions. We briefly introduce our exercises and discuss key ideas, while relegating all formalities to the online appendix.

Three seller types. In the online appendix (Section D), we analyze the case of three seller types, where the asset can be either of low quality ($L$), of middle quality ($M$), or of high quality ($H$). We show that, under certain regularity assumptions, there exists a unique stationary path, identified by a belief vector $q^* = (q_L^*, q_M^*, q_H^*)$, and for any initial belief $\tilde{q}$ (such that it is not an equilibrium that all buyers offer $c_H$), one can construct an equilibrium that converges to $q^*$.

Figure 6 depicts how buyers’ beliefs evolve over time in the three-type case. In Area $a = L, M, H$, trade occurs only at $p_a(t)$ (the type-$a$ seller’s reservation price). Its effects on the evolution of buyers’ beliefs, however, differ across different areas. $p_L(t)$ is accepted only by the low type. Therefore, in Area L, $q_L/q_M$ and $q_L/q_H$ decrease over time, while $q_M/q_H$ stays constant. $p_M$ is accepted by the low type and the middle type but is more likely to be offered to the middle type than to the low type. Therefore, in Area M, $q_M/q_L$, $q_M/q_H$, and $q_L/q_H$ decrease over time. Finally, $p_H(t)$ is accepted by all three types. In Area H, since $p_H(t)$ is offered only with a good signal, $q_H/q_L$, $q_H/q_M$, and $q_M/q_L$ decrease over time. Eventually, $q(t)$ arrives at one of the three dividing curves. Thereafter, trade occurs at two relevant prices. The players’ trading strategies are such that $q(t)$ converges to $q^*$ following the path.

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17We follow the standard interpretation of a two-dimensional simplex. Each vertex corresponds to buyers’ degenerate beliefs. For example, the top vertex ($L$) is the point at which buyers assign probability 1 to the event that the seller is the low type. The probability that the seller is of type $a$ is constant on any straight line that is parallel to the line between buyers’ degenerate beliefs for the other types. For example, the probability of the low type is constant on any horizontal line. The probability of a particular type decreases along the line that connects from buyers’ degenerate beliefs for that type to the center of their degenerate beliefs for the other two types.
Figure 6: The evolution of buyers’ beliefs and their equilibrium offer strategies with three types. $q_{ab}^*$ represents the equilibrium stationary belief level in the model with types $a$ and $b$ only, while $p_a(t)$ denotes the type-$a$ seller’s reservation price at each point.

The underlying economic forces are, reassuringly, similar to those for the two-type case. If buyers initially assign a large probability to the low type (Area L), then they offer only $p_L(t)$, which is accepted only by the low type. Therefore, delay is mainly attributed to higher types’ resistance to accept a low price, and thus the reputation of the asset improves over time. If the initial probability of the high type is relatively large (Area H), then buyers offer $p_H(t)$, unless they observe a particularly bad signal. Therefore, delay mainly conveys negative information about the quality of the asset, and thus the reputation deteriorates over time. When the initial probability of the middle type is large (Area M), delay is interpreted as a mixture of these two effects. On the one hand, it indicates the high type’s unwillingness to trade at a mediocre price $p_M(t)$, thereby increasing the probability of the high type. On the other hand, it also suggests the possibility that all previous buyers have received sufficiently bad signals about the quality of the asset, thereby increasing the relative probability of the low type as well.

**Alternative bargaining protocols.** In our main model, (uninformed) buyers make price offers to the (informed) seller. Although this bargaining protocol is most commonly adopted in the literature, it exhibits some properties that may be considered undesirable or implausible. In particular, the high(est) seller type never obtains a positive payoff and, therefore, does not play a strategic role in the model. In the online appendix (Section E), we demonstrate that the central lessons from our main model are not subject to this particular property by analyzing three alternative bargaining
protocols, each of which yields a strictly positive expected payoff to the high-type seller.\footnote{We consider the following three bargaining protocols: (i) a simultaneous announcement game \cite{Wolinsky1990, Blouin2001, Blouin2003}, (ii) a simplified version of random proposal bargaining \cite{Compte2010, Lauermann2016}, and (iii) the case where the (informed) seller makes price offers \cite{Lauermann2011, Gerardi2014, Palazzi2017}.} Specifically, for each bargaining protocol, we construct an equilibrium whose behavior closely resembles the unique equilibrium of our model, in that there exists a stationary belief level $q^*$ such that $q(t)$ converges to $q^*$, whether starting from below or above $q^*$. Despite some quantitative differences, the underlying economic forces are identical to those for our main model, and thus we omit a detailed discussion.\footnote{For example, in the equilibrium we construct for the case where the seller makes price offers, $q(t)$ converges to $q^*$ only asymptotically. See Figure 2 in the online appendix.}

**Competitive market structure.** In our main model, the seller faces at most a single buyer at each time. In other words, each meeting is bilateral and, therefore, each buyer possesses temporary monopsony power. This is another driving force for the Diamond paradox \cite{Diamond1982}. In the online appendix (Section F), we introduce instantaneous buyer competition into our model and show that our main insights continue to hold under that competitive market structure.\footnote{Specifically, we examine the case where a fixed number of buyers arrive simultaneously, observe a common signal, and offer prices competitively. This specification is tractable, because it permits a reduced-form approach about buyers’ offer strategies, as in, e.g., \cite{Daley2012} and \cite{Fuchs2015}. }

As for alternative bargaining protocols, this variation produces some distinct equilibrium properties. For example, the competitive price depends on buyers’ (evolving) beliefs and, therefore, varies over time and may exceed $c_H$ (see Figure 3 in the online appendix). Even when $\hat{q}$ is so small that only the low-type seller may trade, the (offered) competitive price strictly exceeds the low-type seller’s reservation price. Nevertheless, the main dynamic equilibrium properties remain unchanged. There exists a unique stationary belief level $q^*$, and buyers’ beliefs $q(t)$ increase over time if $q(t) < q^*$ and decrease over time if $q(t) > q^*$.

### 7 Conclusion

We conclude by providing some empirical implications and potential directions for future research.

#### 7.1 Empirical Implications

Our model environment is stylized, abstracting away from many important details in real markets. As always, a certain degree of abstraction is unavoidable to obtain clean and fundamental economic insights. On the other hand, taking such a model to data requires additional steps to account for various factors that are not present in the model. For instance, brokers and list prices play
an important role in the real estate market (see, e.g., Horowitz 1992; Merlo and Ortalo-Magne, 2004; Hendel et al., 2009). Unemployment durations are also affected by other factors, such as skill depreciation and worker discouragement (see, e.g., Pissarides 1992; Gonzalez and Shi 2010). Nevertheless, our model generates some novel and robust predictions regarding market outcomes, some of which are potentially testable. It is beyond the scope of this paper to develop a complete empirical strategy. We provide a list of potentially testable predictions of our model and discuss each of them briefly.

As shown in Sections 3 and 6, buyers’ beliefs and (reservation) prices typically move in the same direction. In what follows, we say that the equilibrium trading dynamics exhibits the increasing (decreasing) pattern if they increase (decrease) over time.

**Prediction 1** The trading dynamics exhibits the decreasing pattern if an asset’s initial reputation is high and the increasing pattern if the initial reputation is low.

This prediction restates our main result. Its simplicity is desirable for empirical purposes. One potential obstacle lies in the difficulty of measuring (initial) reputations. Although this is a non-trivial task in itself, there typically exist observable characteristics that are related to an asset’s (seller’s) reputation and, therefore, can be used to construct a reputation variable. For instance, the neighborhood, vintage, building company, and owner history provide information about a property’s quality. Similarly, a worker’s education and prior employment histories would affect his reputation in the labor market.

Our next prediction links the pattern of trading dynamics to the evolution of trading probability (equivalently, volume) over time.

**Prediction 2** If the trading dynamics exhibits the decreasing (increasing) pattern, then the overall trading probability also decreases (increases) initially.

The result follows from the fact that the frequency with which buyers offer \( c_H \) is an increasing function of their beliefs. If \( \hat{q} > q^* \), then trade occurs only at \( c_H \) until \( q(t) \) reaches \( q^* \). This immediately implies that trade occurs less frequently over time. If \( \hat{q} < q^* \), then the low-type seller trades at a constant rate of \( \lambda \) until \( q(t) \) reaches \( q^* \), while the trading rate of the high-type seller weakly increases. This co-movement of trading pattern and trading probability does not extend into the full time horizon: it is valid along the convergence path, but not at the moment of convergence (i.e., when \( q(t) \) reaches \( q^* \)). The trading rate of the high-type seller is always monotone over time. However, when \( \hat{q} < q^* \) (i.e., the increasing pattern), the trading rate of the low type jumps down from \( \lambda \) to \( \rho \) at the end of the convergence path. If \( \hat{q} > q^* \) (i.e., the decreasing pattern), then the trading rate changes from \( \lambda \gamma^*_L \) to \( \rho = \lambda \gamma^*_H \sigma^*_B \). Depending on the parameter values, the latter can exceed the former.
We now relate the pattern of trading dynamics to three market characteristics.

**Prediction 3** The decreasing pattern is more likely to arise with little gains from trade of low quality (i.e., relatively small \( v_L - c_L \)), small search frictions (i.e., large \( \lambda \)), and a good inspection technology (i.e., informative buyer signals).

Each of these observations is immediate from the three main characterization sections (Sections 3-5). These results could be useful in interpreting both cross-sectional data and time series. For example, if both the search technology and the inspection technology have improved over time, then the decreasing pattern is more likely to arise in recent data than in old data.

Our final prediction is concerned with the relationship between trading dynamics and the nature of inspection. Specifically, we compare the case when inspection is mainly about finding a fatal flaw (red flag) and the case when inspection may reveal a particular merit of the asset (green flag). Formally, we compare the following two cases: for \( \gamma, \epsilon > 0 \),

<table>
<thead>
<tr>
<th>Red flag</th>
<th>Green flag</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_L = \gamma ), ( \gamma_H = 1 - \gamma )</td>
<td>( \gamma_L = 1 - \epsilon ), ( \gamma_H = 1 - \gamma )</td>
</tr>
<tr>
<td>( \gamma_L = 1 - \gamma ), ( \gamma_H = 1 - \epsilon )</td>
<td>( \gamma_L = \epsilon ), ( \gamma_H = \gamma )</td>
</tr>
</tbody>
</table>

If \( \epsilon \) is sufficiently small, then \( l \) (red flag) is a sufficiently informative signal about low quality in the case represented by the left-hand column, while \( h \) (green flag) is a sufficiently good signal about high quality in the case represented by the right-hand column. Assuming that \( \gamma \) is not particularly large (precisely, \( \lambda(1 - \gamma) > \rho_L \)), the following prediction is straightforward to obtain from the characterization results in Section 3 and Section A in the online appendix. See Section G in the online appendix for formal arguments.

**Prediction 4** The decreasing pattern is more likely to arise when the inspection technology is of a red-flag kind than when it is of a green-flag kind.

### 7.2 Directions for Future Research

Our findings suggest several directions for future research. One of our maintained assumptions is that buyers’ offers are private and not observable to future buyers. Hörner and Vieille (2009) show that it is a crucial assumption and the equilibrium dynamics dramatically changes if buyers’ offers are public. In particular, with public offers, “bargaining impasse” can arise. As also suggested by Hörner and Vieille (2009), it could be interesting to allow for buyer inspection in the model with public offers and investigate its impact on bargaining outcome. Our model is a dynamic trading model with random search, but there is a growing literature on directed search with adverse selection (see, e.g., Guerrieri et al., 2010; Guerrieri and Shimer, 2014; Chang, 2017). Introducing
buyer inspection into models of directed search might also lead to interesting insights or predictions. Finally, we assume, crucially, that buyers are short-lived. This assumption can be relaxed in various ways. For example, one can consider a market environment in which there are many sellers and buyers and all agents go through sequential search until they trade (see, e.g., Wolinsky, 1990; Blouin and Serrano, 2001; Moreno and Wooders, 2010). Such a model, whether stationary or non-stationary, would embed our model into a market setting and endogenize buyers’ outside options. Another possibility is to introduce and endogenize buyers’ optimal timing decisions (i.e., when to arrive and make an offer to the seller). Clearly, it would influence the informational content of time-on-the-market and, therefore, make buyers’ inference problems even more intriguing.

Appendix: Omitted Proofs

**Proof of Lemma 1.** If \( p(t) < v_L \), then in equilibrium the low-type seller accepts \( p(t) \) with probability 1: otherwise, the buyer could offer a slightly higher price than \( p(t) \), which would increase the low-type seller’s acceptance probability to 1 and, therefore, give a strictly higher expected payoff to the buyer. Since \( \sigma_S(L, t, p(t)) = 1 \), the law of motion for \( q(t) \) is given by

\[
\dot{q}(t) = -q(t)(1 - q(t))\lambda \left( \sum_s \gamma_H^s \sigma_B(t, s, c_H) - 1 \right) \geq 0.
\]

Notice that \( \dot{q}(t) > 0 \) as long as \( \sum_s \gamma_H^s \sigma_B(t, s, c_H) < 1 \) (i.e., the probability that the high type trades conditional on buyer arrival is less than 1).

If \( p(t) > v_L \), then trade takes place only at \( c_H \): \( p(t) \) is accepted only by the low-type seller, but no buyer would be willing to pay more than \( v_L \) for a low-quality asset. Therefore, such \( p(t) \) must be rejected in equilibrium. In this case, \( q(t) \) evolves according to

\[
\dot{q}(t) = -q(t)(1 - q(t))\lambda \sum_s (\gamma_H^s - \gamma_L^s) \sigma_B(t, s, c_H).
\]

For the buyer’s optimality, he should offer \( c_H \) with a higher probability when \( s = h \) than when \( s = l \) (i.e., \( \sigma_B(t, h, c_H) \geq \sigma_B(t, l, c_H) \)). Combining this with the fact that \( \gamma_H^h > \gamma_L^h \), it follows that \( \dot{q}(t) \leq 0 \) and the inequality holds strictly as long as \( \sigma_B(t, l, c_H) \neq 1 \) (i.e., unless buyers always offer \( c_H \) and, therefore, the probability of trade conditional on buyer arrival is equal to 1).

\[\Box\]

**Lemma 6** Given any strategy profile (such that all buyers’ offers are between \( c_L \) and \( v_H \)), \( q(\cdot) \) and \( p(\cdot) \) are continuous.

**Proof.** Fix \( t \) and consider \( \Delta > 0 \). \( q(t) \) increases fastest when the low-type seller trades whenever a buyer arrives (i.e., at rate \( \lambda \)), while the high-type seller does not trade at all and decreases fastest when the opposite holds. Therefore,

\[
\frac{q(t)e^{-\lambda\Delta}}{q(t)e^{-\lambda\Delta} + (1 - q(t))} \leq q(t + \Delta) \leq \frac{q(t)}{q(t) + (1 - q(t))e^{-\lambda\Delta}}.
\]

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As \( \Delta \) tends to 0, both bounds converge to \( q(t) \). Therefore, \( \lim_{\Delta \to 0} q(t + \Delta) = q(t) \). An analogous argument can be used also to show that \( \lim_{\Delta \to 0} q(t - \Delta) = q(t) \).

Given \( p(t + \Delta) \), \( p(t) \) is maximized when the low-type seller receives the highest possible price \( v_H \) whenever a buyer arrives between \( t \) and \( t + \Delta \) and minimized when she does not receive any offer between \( t \) and \( t + \Delta \). Therefore,

\[
e^{-r\Delta}(p(t + \Delta) - c_L) \leq p(t) - c_L \leq \int_0^\Delta e^{-r(s-t)}(v_H - c_L)d(1 - e^{-\lambda(s-t)}) + e^{-r\Delta}(p(t + \Delta) - c_L).
\]

From these bounds, it follows that \( \lim_{\Delta \to 0} p(t + \Delta) = p(t) \). Again, an analogous argument can be used also to show that \( \lim_{\Delta \to 0} p(t - \Delta) = p(t) \).

**Proof of Lemma 3.** We establish the result in three steps.

(1) If \( q(t) < q^* \), then \( p(t) < v_L \).

Suppose \( q(t) < q^* \), but \( p(t) > v_L \). First we establish that there exists \( t' > t \) such that \( p(t') < v_L \). Suppose, towards a contradiction, that for all \( t' > t \), \( p(t') > v_L \). Then, by Lemma 1, \( q(\cdot) \) cannot be strictly increasing and, therefore, \( q(t') < q^* \) for all \( t' > t \). This implies that trade can occur neither at \( c_H \) (because \( q(t') < q^* \)) nor at \( p(t') \) (because \( p(t') > v_L \)). But then, \( p(t) = c_L < v_L \), which is a contradiction. Let \( t' = \inf\{t'' > t | p(t'') \leq v_L\} \), so that for any \( x \in (t, t') \), \( p(x) > v_L \). By Lemma 1, for any such \( x \), \( q(x) \leq q(t) < q^* \) and, therefore, the buyer never offers \( c_H \). This, together with \( p(t') \leq v_L \), implies that \( p(t) < v_L \), which is a contradiction.

Now suppose \( q(t) < q^* \), but \( p(t) = v_L \). By continuity of \( q(\cdot) \), there exists \( \varepsilon > 0 \) such that for all \( t'' \in (t, t + \varepsilon) \), \( q(t'') < q^* \). Since no buyer offers \( c_H \) between \( t \) and \( t + \varepsilon \), \( v_L = p(t) = (1 - e^{-re})c_L + e^{-re}p(t + \varepsilon) < p(t + \varepsilon) \). This means that \( q(t + \varepsilon) < q^* \) while \( p(t + \varepsilon) > v_L \), which was ruled out above.

(2) If \( q(t) > q^* \), then \( p(t) > v_L \).

Suppose \( q(t) > q^* \), but \( p(t) < v_L \). First we establish that there exists \( t' > t \) such that \( p(t') \geq v_L \). Suppose, towards a contradiction, that for all \( t' > t \), \( p(t') < v_L \). Then, letting \( \rho_H(t) \) denote the rate at which type-\( a \) seller trades at time \( t \), we have \( \rho_L(t') = \lambda \) for all \( t' > t \). Moreover, by Lemma 1, \( q(\cdot) \) is non-decreasing over time, which implies that there exists \( q_\infty \) such that \( \lim_{t \to \infty} q(t) = q_\infty \). Such convergence can occur only if \( \lim_{t \to \infty} \rho_H(t)/\rho_L(t) = 1 \). Since \( \rho_L(t') = \lambda \) for all \( t' > t \), this implies that \( \lim_{t \to \infty} \rho_H(t) = \lambda \). Since the high type trades only at price \( c_H \), this implies that the unconditional (on signal realization) probability of each seller offering \( c_H \) converges to 1, which in turn implies that conditional on each signal, this probability approaches 1. But then, \( p(t) \to (rc_L + \lambda c_H)/(r + \lambda) > v_L \), where the inequality follows by Assumption 1 which is a contradiction. Let \( t' = \inf\{t'' \geq t | p(t'') \geq v_L\} \), so that for any \( t'' \in (t, t') \), \( p(t'') < v_L \). By Lemma 1, and also noting that buyers always offering \( c_H \) leads to a contradiction to \( p(t) < v_L \), for all \( t'' \in (t, t') \), we have \( q(t'') > q(t) > q^* \). Consider a buyer arriving at such \( t'' < t' \) with \( s = h \). When \( t'' \) is sufficiently close to \( t' \), this buyer’s payoff from offering \( p(t'') \) is almost 0, as \( p(t'') \) is almost \( v_L \), by continuity of \( q(\cdot) \). In contrast, such a buyer’s payoff from offering \( c_H \) is bounded away from 0, since \( q(t'') > q(t) > q^* \). Therefore, there exists \( t'' < t' \) such that for all \( t'' \in (t'', t') \), a buyer arriving at \( t'' \) with \( s = h \) offers \( c_H \) with probability 1. Then,

\[
p(t'') = c_L + \int_{t''}^{t'} e^{-rx}(c_H - c_L)d(1 - e^{-\lambda s}) + e^{-(r + \lambda h)(t'' - t')}v_L - c_L > v_L.
\]
where the last inequality follows by Assumption 1. This is a contradiction establishing that if $q(t) > q^*$, then $p(t) \geq v_L$.

Now suppose $q(t) > q^*$, but $p(t) = v_L$. By continuity of $q(\cdot)$, there exists $\varepsilon > 0$ such that for all $t' \in (t - \varepsilon, t + \varepsilon)$, $q(t') > q^*$. Next we claim that there exists $t'' \in (t, t + \varepsilon)$ such that $p(t'') < v_L$. Suppose not. Then any buyer arriving at $t'' \in (t, t + \varepsilon)$ offers $c_H$ at least when $s = h$, as this generates a positive expected payoff while offering $p(t'') \geq v_L$ generates a non-positive expected payoff. But then, by Assumption $\mathbb{1}$, $p(t) > v_L$, which is a contradiction. It then follows that there exists $t'' \in (t, t + \varepsilon)$ such that $p(t'') < v_L$ while $q(t'') > q^*$, whose possibility was ruled out above.

(3) If $q(t) = q^*$, then $p(t) = v_L$.

Suppose that $q(t) = q^*$ but $p(t) > v_L$. Without loss of generality, let $t$ be the first time at which trade occurs with a positive probability. The buyer at $t$ with $s = l$ makes a losing offer since offering $p(t)$ or $c_H$ generates a negative expected payoff. Then, the probability of trade at time $t$ is strictly less than 1. Thus, by Lemma 1, $q(t) < 0$. By continuity of $q(\cdot)$ and $p(\cdot)$, there exists $t'$ such that $q(t') < q^*$ and $p(t') > v_L$, which was ruled out above. Now suppose that $q(t) = q^*$ but $p(t) < v_L$. In this case, the buyer strictly prefers offering $p(t)$ to $c_H$, regardless of his signal. By Lemma $\mathbb{1}$, $q(t)$ must be decreasing at $t$. By continuity of $q(\cdot)$ and $p(\cdot)$, there exists $t'$ such that $q(t') > q^*$ but $p(t') < v_L$, which was also ruled out above.

**Proof of Theorem 1**

We first argue that if $q(t) = q^*$ after any history, then for all $t' > t$, $q(t') = q^*$. It suffices to show that $\dot{q}(t) = 0$ whenever $q(t) = q^*$. For a contradiction, suppose $\dot{q}(t) > 0$ (respectively, $\dot{q}(t) < 0$). Then, by continuity of $q(\cdot)$ there exists $\varepsilon$ such that for all $t' \in (t, t + \varepsilon)$, $q(t') > q^*$ (respectively, $q(t') < q^*$). Moreover, for all such $t'$, by Lemma $\mathbb{5}$ we have $p(t') > v_L$ (respectively, $p(t') < v_L$), which implies by Lemma $\mathbb{1}$ that $\dot{q}(t') \leq 0$ (respectively, $\dot{q}(t') \geq 0$). This implies that $\lim_{\varepsilon' \to 0} q(t + \varepsilon') = q(t + \varepsilon/2) > (\lim_{\varepsilon' \to 0} q(t + \varepsilon/2) <) q^*$, contradicting the continuity of $q(\cdot)$. Hence we have established that if $q(t) = q^*$ after any history, then for all $t' > t$, $q(t') = q^*$. Then, the construction described in Section 3.1 uniquely pins down the subsequent equilibrium behavior.

Now, by the previous argument and the continuity of $q(\cdot)$, if $\hat{q} < q^*$, then $q(t) \leq q^*$ for all $t$. Let $t^* = \inf\{t | q(t) = q^*\}$ if $\{t | q(t) = q^*\} \neq \emptyset$. Otherwise, set $t^* = \infty$. Then, by Lemma $\mathbb{3}$, for any $t < t^*$, $p(t) < v_L$. Moreover, by the definition of $q^*$, for any $t < t^*$ the buyer is not willing to offer $c_H$. Then the buyer at $t < t^*$ offers $p(t)$ regardless of his signal. Then, the low-type seller’s reservation price is uniquely pinned down as in Proposition $\mathbb{1}$ and $t^* = t^*$ as defined in the same Proposition. Again by the above argument and continuity of $q(\cdot)$, if $\overline{q} > q^*$, then $\overline{q} > q(t) \geq q^*$ for all $t$. Let $t^* = \inf\{t | q(t) = q^*\}$ if $\{t | q(t) = q^*\} \neq \emptyset$. Otherwise, set $t^* = \infty$. Then, by Lemma $\mathbb{3}$, for any $t < t^*$, $p(t) > v_L$. Then, for any $t < t^*$ trade never takes place at $p(t)$. Then by the definitions of $\overline{q}$ and $q^*$, the buyer at $t < t^*$ offers $c_H$ if and only if the realized signal is $h$. Then, the low-type seller’s reservation price is uniquely pinned down as in Proposition $\mathbb{2}$ and $t^* = t^*$ as defined in the same proposition.

Finally, assume $\hat{q} > \overline{q}$. We show that for all $t$, $q(t) > \overline{q}$. Suppose for a contradiction that there exists $t$ with $q(t) \leq \overline{q}$. Then, by continuity of $q(\cdot)$, there exists a minimum $t^*$ such that $q(t^*) = \overline{q}$. For all $t < t^*$, $q(t) > \overline{q}$, and thus $p(t) > v_L$ (by Lemma $\mathbb{3}$). Then, for such $t$, it is optimal for the buyer to offer $c_H$ regardless of his signal. This means that both seller types trade at the same rate and, therefore, $q(t) = \hat{q} > \overline{q}$ until $t^*$, which contradicts the continuity of $q(\cdot)$ at $t^*$. This argument also establishes that if $\hat{q} > \overline{q}$, then $q(t) = \hat{q}$ for all $t$ and therefore the equilibrium strategies are
necessarily as claimed. ■

**Proof of Lemma 4.** If \( \Gamma \) is more informative than \( \Gamma' \), then there exists a non-negative Markov matrix \( M \) such that \( \Gamma' = M \Gamma \). Without loss of generality, let

\[
M = \begin{pmatrix}
1 - \varepsilon_l & \varepsilon_l \\
\varepsilon_h & 1 - \varepsilon_h
\end{pmatrix}.
\]

Then,

\[
\Gamma' = \begin{pmatrix}
\gamma^u_L & \gamma^u_H \\
\gamma^h_L & \gamma^h_H
\end{pmatrix} = M \Gamma = \begin{pmatrix}
(1 - \varepsilon_l)\gamma^l_L + \varepsilon_l\gamma^h_L & (1 - \varepsilon_l)\gamma^l_H + \varepsilon_l\gamma^h_H \\
\varepsilon_h\gamma^l_L + (1 - \varepsilon_h)\gamma^h_L & \varepsilon_h\gamma^l_H + (1 - \varepsilon_h)\gamma^h_H
\end{pmatrix}.
\]

It follows that

\[
\frac{\gamma^u_L}{\gamma^u_H} = \frac{(1 - \varepsilon_l)\gamma^l_L + \varepsilon_l\gamma^h_L}{(1 - \varepsilon_l)\gamma^l_L + \varepsilon_l\gamma^h_L} = \frac{(1 - 2\varepsilon_l)\gamma^l_L + \varepsilon_l\gamma^h_L}{(1 - 2\varepsilon_l)\gamma^l_L + \varepsilon_l\gamma^h_L} \geq \frac{\gamma^l_H}{\gamma^l_L},
\]

and

\[
\frac{\gamma^h_L}{\gamma^h_H} = \frac{\varepsilon_h\gamma^l_L + (1 - \varepsilon_h)\gamma^h_L}{\varepsilon_h\gamma^l_L + (1 - \varepsilon_h)\gamma^h_L} = \frac{\varepsilon_h + (1 - 2\varepsilon_h)\gamma^h_L}{\varepsilon_h + (1 - 2\varepsilon_h)\gamma^h_L} \leq \frac{\gamma^h_H}{\gamma^h_L}.
\]

The inequalities are due to the fact that \( \gamma^l_H/\gamma^l_L < 1 < \gamma^h_H/\gamma^h_L \) (MLRP).

Now suppose that \( \gamma^l_H/\gamma^l_L \leq \gamma^u_H/\gamma^u_L \) and \( \gamma^h_H/\gamma^h_L \geq \gamma^u_H/\gamma^u_L \). Then, there exist \( \varepsilon_l \) and \( \varepsilon_h \) such that \( \varepsilon_l, \varepsilon_h \in [0, 1] \) and

\[
\frac{\gamma^l_H}{\gamma^l_L} = \frac{(1 - \varepsilon_l)\gamma^l_L + \varepsilon_l\gamma^h_L}{(1 - \varepsilon_l)\gamma^l_L + \varepsilon_l\gamma^h_L} \quad \text{and} \quad \frac{\gamma^h_L}{\gamma^h_H} = \frac{\varepsilon_h\gamma^l_L + (1 - \varepsilon_h)\gamma^h_L}{\varepsilon_h\gamma^l_L + (1 - \varepsilon_h)\gamma^h_L}.
\]

It then suffices to set

\[
M = \begin{pmatrix}
1 - \varepsilon_l & \varepsilon_l \\
\varepsilon_h & 1 - \varepsilon_h
\end{pmatrix}.
\]

■

**Proof of Proposition 3.** By Proposition 1,

\[
F_L(t) = \begin{cases}
1 - e^{-\lambda t} & \text{if } t < t^*(\hat{q}), \\
1 - e^{-\lambda t^*(\hat{q}) - \rho(t-t^*(\hat{q}))} & \text{otherwise},
\end{cases}
\quad \text{and} \quad
F_H(t) = \begin{cases}
0 & \text{if } t < t^*(\hat{q}), \\
1 - e^{-\rho(t-t^*(\hat{q}))} & \text{otherwise}.
\end{cases}
\]

The result for \( \tau_H \) is immediate from the fact that \( \rho \) increases, while \( t^*(\hat{q}) \) decreases (which implies \( F_H(t) \) weakly increases at any \( t \)). \( \tau_L \) does not decrease in the sense of first-order stochastic dominance because \( F_L(t) \) decreases when \( t = t^*(\hat{q}) \). Consider the expected value of \( \tau_L \):

\[
E[\tau_L] = \int_0^{t^*(\hat{q})} td(1 - e^{-\lambda t}) + e^{-\lambda t^*(\hat{q})} \int_{t^*(\hat{q})}^\infty td(1 - e^{-\rho(t-t^*(\hat{q}))}) = \frac{1 - e^{-\lambda t^*(\hat{q})}}{\lambda} + \frac{e^{-\lambda t^*(\hat{q})}}{\rho}
\]

The second term \( e^{-\lambda t^*(\hat{q})}/\rho \) is independent of \( \Gamma \), because

\[
\rho = \frac{\gamma^h_L}{\gamma^h_L} r(v_L - c_L) \quad \text{and} \quad e^{-\lambda t^*(\hat{q})} = \frac{\hat{q}}{1 - \hat{q}} \frac{1 - q^*}{q^*} = \frac{\hat{q}}{1 - \hat{q}} \gamma^h_L c_H - v_L.
\]

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Then, the desired result (that $E[\tau]$ decreases as $\Gamma$ becomes more informative) follows from the fact that the first term $(1 - e^{-\lambda t(\tilde{q})})/\lambda$ increases in $t^*(\tilde{q})$. The result on $p(0)$ is straightforward from the fact that $p(0) = c_L + e^{-\lambda t^*(\tilde{q})}(v_L - v_L)$.

Proof of Lemma 5. From equation (7),

\[ e^{2\lambda t^*(\tilde{q})} = \left( \frac{C}{1 - \gamma} \right)^{\frac{\gamma - \gamma_{1/2}}{1 - \gamma}}, \]

where

\[ C = \frac{\tilde{q}}{1 - \tilde{q}} \frac{v_H - c_H}{c_H - v_L}. \]

Then,

\[ \frac{de^{2\lambda t^*}}{d\gamma} = \left( C \frac{\gamma}{1 - \gamma} \right)^{\frac{-1}{1 - \gamma}} \left( -\frac{1}{(\gamma - 1/2)^2} \log \left( C \frac{\gamma}{1 - \gamma} \right) + \frac{1}{\gamma - 1/2} \frac{1}{\gamma(1 - \gamma)} \right). \]

In order to find the condition under which $dt^*(\tilde{q})/d\gamma > 0$, it suffices to find the condition for

\[ \frac{\gamma - 1/2}{\gamma(1 - \gamma)} > \log C + \log \left( \frac{\gamma}{1 - \gamma} \right). \]  \hspace{1cm} (11)

Define a function $H : [1/2, 1] \rightarrow \mathcal{R}$ so that

\[ H(\gamma) = \frac{\gamma - 1/2}{\gamma(1 - \gamma)} - \log \left( \frac{\gamma}{1 - \gamma} \right). \]

Then, $H(1/2) = 0$ and

\[ H'(\gamma) = \left( \gamma - \frac{1}{2} \right) \frac{\gamma^2 - (1 - \gamma)^2}{\gamma^2(1 - \gamma)^2} > 0 \text{ whenever } \gamma > \frac{1}{2}. \]

It then follows that inequality (11) holds for any $\gamma > 1/2$ if $C \leq 1$. If $C > 1$, then there exists $\gamma(\tilde{q})$ such that $H(\gamma) > \log C$ (and, therefore, $t^*(\tilde{q})$ increases in $\gamma$) if and only if $\gamma > \gamma(\tilde{q})$. Notice that $\tilde{q}^*$ is defined to be the value such that $C = 1$ when $\tilde{q} = \tilde{q}^*$.

Proof of Proposition 4. For notational simplicity, we denote $t^*(\tilde{q})$ simply by $t^*$. Recall that if $\tilde{q} \in (q^*, \tilde{q})$, then

\[ F_L(t) = \begin{cases} 1 - e^{-\lambda t(1 - \gamma)} & \text{if } t < t^*, \\ 1 - e^{-\lambda(1 - \gamma)t^* - \rho(t - t^*)} & \text{otherwise}, \end{cases} \]

and $F_H(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < t^*, \\ 1 - e^{-\lambda t^* - \rho(t - t^*)} & \text{otherwise}, \end{cases}$

and

\[ \rho = \frac{\gamma}{1 - \gamma} \frac{r(v_L - c_L)}{c_H - v_L}. \]
From these color blue and using the definition of $C$ in the proof of Lemma 5, we get

\[
E[\tau_H] = P_H(\gamma) \left[ \frac{1}{\rho} - \frac{1}{\lambda \gamma} \right] + \frac{1}{\lambda \gamma},
\]

\[
E[\tau_L] = P_L(\gamma) \left[ \frac{1}{\rho} - \frac{1}{\lambda (1 - \gamma)} \right] + \frac{1}{\lambda (1 - \gamma)},
\]

and

\[
p(0) = (1 - D(\gamma))(\bar{P}(\gamma) - v_L) + v_L,
\]

where

\[
P_H(\gamma) = \left( \frac{1 - \gamma}{C - \gamma} \right)^{\frac{1}{\gamma - 1}}, P_L(\gamma) = \left( \frac{1 - \gamma}{C - \gamma} \right)^{\frac{1}{\gamma - 1}}, D(\gamma) = \left( \frac{1 - \gamma}{C - \gamma} \right)^{A(\gamma)},
\]

with $A(\gamma) = (\rho + \lambda (1 - \gamma))/(\lambda (2 \gamma - 1))$, and $\bar{P}(\gamma) = (r c_L + \lambda (1 - \gamma) c_H)/(\rho + \lambda (1 - \gamma))$. Therefore,

\[
\frac{\partial E[\tau_H]}{\partial \gamma} = \frac{\partial P_H(\gamma)}{\partial \gamma} \left( \frac{1}{\rho} - \frac{1}{\lambda \gamma} \right) + P_H(\gamma) \left( - \frac{\partial \rho}{\partial \gamma} \frac{1}{\rho^2} - \frac{1}{\lambda \gamma^2} \right) - \frac{1}{\lambda \gamma^2},
\]

\[
\frac{\partial E[\tau_L]}{\partial \gamma} = \frac{\partial P_L(\gamma)}{\partial \gamma} \left( \frac{1}{\rho} - \frac{1}{\lambda (1 - \gamma)} \right) + P_L(\gamma) \left( - \frac{\partial \rho}{\partial \gamma} \frac{1}{\rho^2} - \frac{1}{\lambda (1 - \gamma)^2} \right) + \frac{1}{\lambda (1 - \gamma)^2},
\]

\[
\frac{\partial p_0}{\partial \gamma} = - \frac{\partial D(\gamma)}{\partial \gamma} (\bar{P}(\gamma) - v_L) + \frac{\partial \bar{P}(\gamma)}{\partial \gamma} (1 - D(\gamma)).
\]

Finally, we note that

\[
\frac{\partial P_H(\gamma)}{\partial \gamma} = - \frac{1}{2 \gamma - 1} P_H(\gamma) \left( \log(P_H(\gamma)) + \frac{1}{1 - \gamma} \right),
\]

\[
\frac{\partial P_L(\gamma)}{\partial \gamma} = - \frac{1}{2 \gamma - 1} P_L(\gamma) \left( \frac{\log(P_L(\gamma))}{1 - \gamma} + \frac{1}{\gamma} \right),
\]

and

\[
\frac{\partial D(\gamma)}{\partial \gamma} = - \frac{1}{2 \gamma - 1} D(\gamma) \left( \log(D(\gamma)) \frac{2 \rho + \lambda}{\rho + \lambda (1 - \gamma)} + \frac{r + \lambda (1 - \gamma)}{\lambda (1 - \gamma)} \right).
\]

First consider the case when $\hat{q}$ is sufficiently close to $\bar{q}$. As $\hat{q} \rightarrow \bar{q}$, $P_H(\gamma), P_L(\gamma), D(\gamma) \rightarrow 1$. Therefore,

\[
\frac{\partial E[\tau_H]}{\partial \gamma} \rightarrow - \frac{1}{2 \gamma - 1} \left( \frac{1}{\rho} - \frac{1}{\lambda \gamma} \right) - \frac{\partial \rho}{\partial \gamma} \frac{1}{\rho^2}.
\]

Then, the claim for $\tau_H$ follows by noting that $\partial \rho/\partial \gamma > 0$ and $\rho < \lambda \gamma$. Also,

\[
\frac{\partial E[\tau_L]}{\partial \gamma} \rightarrow - \frac{1}{2 \gamma - 1} \left( \frac{1}{\rho} - \frac{1}{(1 - \gamma) \lambda} \right) - \frac{\partial \rho}{\partial \gamma} \frac{1}{\rho^2},
\]

which simplifies to

\[
- \frac{1}{(1 - \gamma)(2 \gamma - 1)} \left( \frac{1}{\rho} - \frac{1}{\lambda} \right).
\]
Then, the claim for $\tau_L$ follows because $\rho < \lambda$. Finally,

$$\frac{\partial p(0)}{\partial \gamma} \rightarrow \frac{1}{2\gamma-1} \frac{r + \lambda(1 - \gamma)}{\lambda\gamma(1 - \gamma)} (\dot{P}(\gamma) - v_L).$$

The result follows simply by $\dot{P}(\gamma) > v_L$.

Next, consider the case when $\hat{q}$ is sufficiently close to $\bar{q}$ and $\gamma$ is sufficiently close to $1/2$. As $\hat{q} \rightarrow \bar{q}$, we have $1/C \times (1 - \gamma)/\gamma \rightarrow ((1 - \gamma)/\gamma)^2$. Then, when also $\gamma \rightarrow 1/2$, we have

$$P_H(\gamma) \rightarrow \exp(-2), P_L(\gamma) \rightarrow \exp(-2), D(\gamma) \rightarrow \exp(-2(2r + \lambda)/\lambda).$$

The results for $\tau_H$ and $\tau_L$ follow because $\frac{\partial E[\tau_L]}{\partial \gamma}, \frac{\partial E[\tau_L]}{\partial \gamma} \rightarrow +\infty$ with positive coefficients (note that for the case of $\tau_L$, the coefficient is positive if and only if $r(v_L - c_L) < (1/2)\lambda(c_H - v_L)$, which is exactly requirement (1), when $\gamma = 1/2$) and all other terms remain finite. For the result on $p(0)$, we note that since $(1 - D(\gamma)\dot{P})/\partial \gamma$ remain finite in these limits, and $\dot{P} - v_L > 0$, it is sufficient to argue that $\partial D(\gamma)/\partial \gamma \rightarrow +\infty$. Substituting the expression for $\rho$ and rearranging terms, we find that the coefficient of the term $1/(2\gamma - 1)$ in the expression for $\partial D(\gamma)/\partial \gamma > 0$ approaches $2\frac{2r + \lambda}{\lambda} > 0$, establishing that $\partial D(\gamma)/\partial \gamma \rightarrow +\infty$ as $\gamma \rightarrow 1/2$ and $\hat{q} \rightarrow \bar{q}$.

References


