The Analytics of SVARs: A Unified Framework to Measure Fiscal Multipliers

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Abstract

Do tax cuts and spending increases stimulate output? Studies that identify fiscal shocks using structural vector autoregressions (SVAR) have reached different conclusions. In this paper, we show analytically that this lack of consensus reflects different assumptions on the fiscal rules that—by relating tax and spending policies to macroeconomic conditions—determine the identification of fiscal shocks and the associated fiscal multipliers. We then propose a new identification strategy based on a proxy SVAR that uses non-fiscal instruments to directly estimate the parameters of the fiscal rules. We find that spending increases stimulate output more than tax cuts.

Key words: Fiscal policy; Identification; Structural vector autoregressions
JEL Codes: E62; C52

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1 Introduction

Governments often use fiscal policy to stabilize economic activity in response to macroeconomic shocks. During the Great Recession, for example, the U.S. Congress passed the *American Recovery and Reinvestment Act*, which introduced increases in public spending and cuts in taxes, a fiscal stimulus amounting to 6 percent of GDP (CBO, 2010). Similar fiscal packages were approved and implemented in the euro area—within the framework of the *European Economic Recovery Plan*—as well as in other economies such as Canada and Japan.

Despite the frequent use of fiscal policy for stabilization purposes, the size of fiscal multipliers—the response of output following an *exogenous* fiscal policy intervention—is the subject of a long-standing debate in academic and policy circles. As Perotti (2007) notes in his survey of the literature, "[P]erfectly reasonable economists can and do disagree on the basic theoretical effects of fiscal policy and on the interpretation of existing empirical evidence." In fact, in the empirical literature, which relies heavily on structural vector autoregressions (SVARs), there is substantial disagreement on the size—and even the sign—of fiscal multipliers, the key metric used to gauge the efficacy of fiscal policy.[1] As shown by Caldara and Kamps (2008) and Chahrour et al. (2012), this wide range of fiscal multiplier estimates is caused by differences in the identification of the underlying fiscal shocks.

The main contribution of this paper is to develop a unified analytical framework that can be used to compare the sign and magnitude of fiscal multipliers implied by commonly used identification schemes. To do so, we characterize the systematic component of fiscal policy by rules relating policy instruments (i.e., government spending and taxes) to macroeconomic conditions. We show how these rules—which depend on parameters with clear economic interpretation—also identify the corresponding exogenous policy interventions, that is, the fiscal policy shocks. We then use this one-to-one mapping between the systematic component of fiscal policy and the fiscal shocks to derive an analytical relationship between the parameters characterizing the fiscal policy rules and the fiscal multipliers.

As a starting point for our analysis, we use simple fiscal rules under which taxes and government spending can systematically respond, within a quarter, only to changes in output. We find that—using

[1] Using structural VARs, Blanchard and Perotti (2002) find that spending multipliers are larger than tax multipliers, while Mountford and Uhlig (2009) reach the opposite conclusion. By contrast, studies using the so-called narrative approach, which combines VARs with exogenous changes in taxation and public spending identified from records of policy deliberations, typically estimate larger tax multipliers (Romer and Romer, 2010; Mertens and Ravn, 2011, 2013) and smaller spending multipliers (see Ramey and Shapiro, 1998; Eichenbaum and Fisher, 2005). Ramey (2016) elegantly summarizes this literature.

a canonical VAR model estimated on U.S. data—small differences in the systematic response of fiscal policy to output imply large differences in estimated tax and spending multipliers. We then show how existing identification schemes impose, either explicitly or implicitly, different restrictions on the systematic response of fiscal policy to output. These differences account for the wide range of multiplier estimates reported in the literature. Importantly, we find that allowing for more general fiscal rules—in which fiscal instruments are allowed to systematically respond to several macroeconomic variables—has modest effect on results, suggesting that the simple rules capture the essence of the identification problem.

Our analytical framework characterizes the set of policy rules and fiscal multipliers that are consistent with the data. For instance, taking spending policy as an example, we prove that the relationship between the size of the systematic response to output in the policy rule and the spending multiplier is negative.[2] Specifically, if the systematic response to output in the rule is set to zero, the associated spending multiplier is about 1; this configuration is selected by the Blanchard and Perotti (2002) identification. Conversely, if the systematic response to output is positive and large, the spending multiplier is small; this configuration is selected by the Mountford and Uhlig (2009) identification.

The key to understand this relationship between the systematic component and the spending multiplier is the positive comovement between government spending and output observed in U.S. data. Any identification approach implicitly identifies a rule for government spending that decomposes this positive comovement into the fraction explained by the government spending shock, and the fraction explained by the remaining shocks in the VAR. The latter fraction is determined by the size of the systematic component of fiscal policy. If the systematic component is set to zero, government spending does not respond contemporaneously to other shocks in the VAR. Thus, all the positive comovement observed in the data must be generated by the government spending shock and, by implication, the spending multiplier is positive and large. In contrast, as the size of the systematic component is increased, the other shocks in the VAR explain an increasingly larger fraction of the comovement between spending and output. Consequently, to match the data, the spending multiplier has to become smaller. If the systematic component of government policy is sufficiently large, the government spending shock does not move output at all—a zero spending multiplier—because the correlation between output and spending is fully accounted for by the other shocks in the VAR. A similar reasoning applies to the identification of tax shocks.[3]

[2] This relationship holds for a large set of models that encompasses existing identification strategies. The full characterization is provided in Section 3.

[3] The analysis of the tax multiplier also includes the proxy SVAR identification of Mertens and Ravn (2014), which uses

The second contribution of this paper is to estimate the parameters of the tax and spending policy rules by employing an instrumental variables approach, the so-called proxy SVAR identification. The novelty of our approach lies in the use of non-fiscal instruments to directly estimate the parameters of the fiscal rules, addressing the endogeneity problem arising from the two-way interaction between economic activity and fiscal policy. In our preferred specification of the policy rules, fiscal variables can respond to output, inflation, and the interest rate. We instrument these macroeconomic variables by the Fernald (2012) measure of total factor productivity adjusted for factor utilization, the Hamilton (2003) series of oil shocks, and the Romer and Romer (2004) series of monetary policy shocks. We provide evidence that the instruments are both relevant and orthogonal to exogenous changes in fiscal policy, and hence yield unbiased estimates of the parameters of the fiscal rules.[4]

We find that the systematic response of taxes to output is positive and large, while the systematic response of government spending to output is mildly negative. Our estimates of the fiscal rules imply positive tax and spending multipliers for about five years after the fiscal shocks, with spending increases providing more stimulus than tax cuts. For the first year after the shock, the median tax multipliers range between 0.5 and 0.7, while the spending multipliers range between 1 and 1.3.

The remainder of the paper is organized as follows. In Section 2, we derive the analytical relationship between the systematic component of fiscal policy and the fiscal multipliers. In Section 3, we apply the analytical framework to characterize the identification of fiscal rules in a standard VAR model. In Section 4, we estimate the coefficients of the fiscal rules using a proxy SVAR identified using non-fiscal instruments. In Section 5, we discuss the robustness of the results obtained under the proxy SVAR identification. In Section 6, we conclude.

2 Econometric Methodology

In this section, we first characterize the systematic component of fiscal policy in a SVAR by rules that relate the policy instruments to macroeconomic conditions. We then show that the identification of the systematic component is equivalent to the identification of fiscal shocks. We then derive an analytical relationship

the Romer and Romer (2010) narrative series of tax shocks as an instrument for the unobserved tax shocks in the VAR.

[4]Importantly, the empirical specification of the VAR model is designed to isolate unanticipated fiscal shocks, thus controlling for the anticipated fiscal policy shocks studied, among others, by Ramey (2011) and Leeper et al. (2013). In the baseline model, we follow Giannone and Reichlin (2006) and include variables—such as prices and the interest rate—that react to signals about future changes in policy. In a robustness exercise, we directly include in the VAR measures of anticipated fiscal policy shocks.

between the parameters characterizing the systematic component and the implied fiscal multipliers. For ease of exposition, in this section we study the identification of one fiscal shock, and the policy instrument is either tax revenue tr_t or government spending g_t.[5] However, in Appendix A we extend the framework to jointly identify shocks to taxes and spending, which involves making an assumption on the interaction between tr_t and g_t.

2.1 The SVAR Model

Consider the following VAR, written in structural form:

$$y_t' A_0 = \sum_{\ell=1}^{\overline{\ell}} y_{t-\ell}' A_\ell + c + e_t', \quad \text{for } 1 \leq t \leq T, \tag{1}$$

where y_t is an $n \times 1$ vector of endogenous variables, e_t is an $n \times 1$ vector of structural shocks, A_ℓ is an $n \times n$ matrix of structural parameters for $0 \leq \ell \leq \overline{\ell}$ with A_0 invertible, c is a $1 \times n$ vector of intercepts, $\overline{\ell}$ is the lag length, and T is the sample size. The vector e_t is Gaussian with a mean of zero and covariance matrix I_n (the $n \times n$ identity matrix). The model described in Equation (1) can be written in compact form as

$$y_t' A_0 = x_t' A_+ + e_t', \quad \text{for } 1 \leq t \leq T, \tag{2}$$

where $x_t = [y_{t-1}', \ldots, y_{t-\overline{\ell}}', 1]'$ and $A_+ = [A_1', \ldots, A_{\overline{\ell}}', c']'$. The reduced-form representation of this model is given by

$$y_t' = x_t' \Phi + u_t', \quad u_t \sim \mathcal{N}(0, \Sigma). \tag{3}$$

The reduced-form and structural representations of the model are linked through

$$\Sigma = (A_0 A_0')^{-1}, \, \Phi = A_+ A_0^{-1}. \tag{4}$$

Thus, the relationship between the reduced-form residuals and the structural parameters is:

$$u_t = (A_0^{-1})' e_t. \tag{5}$$

The impulse response functions (IRFs) of the endogenous variables to the structural shocks are defined

[5] The same strategy is adopted in Blanchard and Perotti (2002) and Mountford and Uhlig (2009).

as follows. Let (A_0, A_+) represent arbitrary structural parameters. Then, the IRF of the j-th variable to the i-th structural shock at a finite horizon h, corresponds to the element in row j and column i of the matrix $L_h(A_0, A_+)$.

The identification of the structural shocks e_t requires to impose restrictions on the structural model. The reason is that there are many possible combinations of structural parameters (A_0, A_+) that yield the same reduced-form parameters (Φ, Σ). Previous work has used different strategies to identify fiscal shocks: some identification schemes impose restrictions directly on (A_0, A_+), while others impose them on the impulse responses $L_h(A_0, A_+)$. Moreover, the choice of the elements of (A_0, A_+) and $L_h(A_0, A_+)$ to restrict—and consequently the number of restrictions to impose—varies across studies.

These differences have made the comparison of identification schemes a particularly difficult task. In the next subsection, we show that we can compare different identification schemes by using the fiscal policy rule implied by the SVAR. Any scheme that identifies fiscal shocks identifies also a fiscal rule, irrespective of whether restrictions are imposed directly on the coefficients of the rule or on other objects in the model.

2.2 The Fiscal Policy Rule

To study the identification of fiscal policy shocks, we need to select the elements of e_t that represent these shocks. We consider the partition $e_t = [e'_{p,t}, e'_{np,t}]'$, where $e_{p,t}$ is a $k \times 1$ vector of policy shocks and $e_{np,t}$ is a $n - k \times 1$ vector of non-policy shocks. In this section, we assume that $k = 1$. As discussed in Leeper et al. (1996), specifying $e_{p,t}$ is equivalent to specifying the equation that characterizes the fiscal policy behavior:

$$y'_t A_{0,1} = x'_t A_{+,1} + e_{p,t}, \quad \text{for } 1 \leq t \leq T, \tag{6}$$

where $A_{0,1}$ and $A_{+,1}$ denote the first column of A_0 and A_+, respectively. We can rewrite Equation (6) in the form of a fiscal policy rule as follows:

$$y_{p,t} \quad = \quad y'_{np,t}\psi_0 + \sum_{\ell=1}^{\bar{\ell}} y'_{t-\ell}\psi_\ell + \omega_p e_{p,t}, \quad \text{for } 1 \leq t \leq T, \tag{7}$$

where $\psi_0 = -a_{0,1np}/a_{0,11}$ is a $n-k \times 1$ vector of contemporaneous elasticities, $\psi_\ell = a_{\ell,1\cdot}/a_{0,11}$ are $n \times 1$ vectors of lagged elasticities, $\omega_p = 1/a_{0,11}$ is a constant that scales the size of the policy shock, with $a_{\ell,ij}$ denoting the ijth element of A_ℓ. The first two terms on the right-hand side of Equation (7) describe the systematic component of fiscal policy, characterizing how the fiscal instrument at time t responds to contemporaneous and lagged movements in the variables included in the model.

In our analysis we take the estimation of the reduced-form model (3) as given, and concentrate on the problem of identifying the fiscal shocks. Thus, we can use Equation (5) to rewrite the fiscal policy rule in terms of reduced-form residuals:

$$u_{p,t} = \psi_0 u_{np,t} + \omega_p e_{p,t}, \quad \text{for } 1 \leq t \leq T. \tag{8}$$

For given (Φ, Σ), the formulations of the fiscal policy rules in Equation (7) and Equation (8) are equivalent because (i) the reduced-form residuals u_t embed information about lags of y_t and (ii) the lagged structural coefficients that enter in the rule can be recovered using the relationship $\Phi = A_+ A_0^{-1}$. Hence, the vector of contemporaneous elasticities ψ_0 fully characterizes the systematic component of fiscal policy.

Equation (8) shows that, for an arbitrary vector of contemporaneous elasticities ψ_0, we can recover the fiscal shock $e_{p,t}$. Moreover, for an arbitrary $e_{p,t}$, we can use a two-step procedure to invert this relationship and recover the contemporaneous elasticities ψ_0. In the first step, we regress the reduced-form residuals of the non-policy variables $u_{np,t}$ on $e_{p,t}$:

$$u_{np,t} = \xi_0 e_{p,t} + \Omega_{np} e_{np,t}, \tag{9}$$

where $\Omega_{np}\Omega_{np}'$ is the covariance of $\tilde{e}_{np,t} = \Omega_{np} e_{np,t}$. The residuals $\tilde{e}_{np,t}$ in Equation (9) are, by construction, uncorrelated to the policy shock $e_{p,t}$. Hence, in the second step we use $\tilde{e}_{np,t}$ as instruments for $u_{np,t}$ to estimate the contemporaneous elasticities in Equation (8). The resulting system of instrumental variable regressions is just identified as we have $n-k$ instruments in $\tilde{e}_{np,t}$ to estimate the $n-k$ coefficients in ψ_0.

Thus, the identification of the fiscal policy shock $e_{p,t}$ is equivalent to the identification of the coefficients that characterize the systematic component in the fiscal policy rule. It follows that we can use the fiscal rule to study identification schemes that impose restrictions directly on the coefficients of the rule or on the coefficients of other equations in the model or on IRFs. This makes our framework general as it allows to encompass all identification schemes used in the literature.

In the next sections, for ease of exposition and to build intuition, our analysis concentrates on a simple rule in which the fiscal variable can respond contemporaneously only to output:

$$u_{p,t} = \psi^p_{gdp} u_{gdp,t} + \omega_P e_{P,t}, \quad \text{for } 1 \leq t \leq T, \tag{10}$$

where ψ^p_{gdp} is the element of ψ_0 associated to output. Importantly, since the output residual $u_{gdp,t}$ can in principle react to all shocks in the system, the simple fiscal rule allows the policy residual $u_{p,t}$ to respond to all structural shocks, but only through their effects on output. In the next subsection, under this simple fiscal rule, we derive an analytical expression that relates ψ^p_{gdp} to fiscal multipliers. This relationship enhances the understanding of the theoretical restrictions used in the literature to identify fiscal shocks.

As we show in Section 3.3, allowing for more general fiscal rules—in which the policy instruments can respond contemporaneously to other macroeconomic variables and can interact with each other—has a modest effect on the identification of fiscal shocks, suggesting that the simple rules capture the essence of the identification problem. Moreover, these simple fiscal rules are consistent with the evidence that, at quarterly frequency, the contemporaneous elasticities ψ_0 are likely to reflect automatic stabilizers, which work mostly through output (Blanchard and Perotti, 2002). Consequently, automatic stabilizers are typically measured by ψ^p_{gdp}, the contemporaneous elasticities of tax revenues and government spending to GDP (e.g. Follette and Lutz, 2010).

2.3 Fiscal Multipliers

To assess the efficacy of fiscal policy in stimulating economic activity, we compute fiscal multipliers. In particular, we follow Blanchard and Perotti (2002) and define the fiscal multiplier at horizon h as the response of output at horizon h to an unanticipated policy shock of size \$1.

Since the identification of the fiscal shock depends uniquely on the contemporaneous elasticities ψ_0 and on the reduced-form parameters (Φ, Σ), the multipliers are also functions of these same coefficients. Equation (5) shows that we can compute the impact IRFs $L_0(A_0, A_+) = (A_0^{-1})'$ by projecting the structural shocks on the reduced-form residuals. We can rewrite $e_{p,t}$ in terms of reduced-form residuals and the contemporaneous elasticities using the fiscal rule (8) and express the *impact* responses of all variables to the fiscal shock in terms of the elasticities ψ_0 and of the covariance matrix of reduced-form residuals Σ.[6]

The analytical expression for the impact fiscal multiplier (M_0) derived under the simple rule is:[7]

$$M_0(\psi^p_{gdp}, \Sigma) \equiv \frac{L_0(A_0, A_+)_{gdp,p}}{\omega_p} = \frac{\sigma_{p,gdp} - \psi^p_{gdp}\sigma^2_{gdp}}{(\psi^p_{gdp}\sigma_{gdp})^2 + \sigma^2_p - 2\psi^p_{gdp}\sigma_{p,gdp}}, \tag{11}$$

where σ^2_{gdp}, σ^2_p, and $\sigma_{p,gdp}$ are elements of Σ that describe the covariance between the reduced-form residuals for output ($u_{gdp,t}$) and the policy variable ($u_{p,t}$). We use this equation to characterize how the identification of the policy rule coefficient ψ^p_{gdp} determines the fiscal multiplier M_0. The elements of Σ define the set of policy rule coefficients and impact multipliers that are consistent with the data. Equation (11) reveals that, under the simple rule, this set depends on the elements of Σ related to output and the policy variable, rather than on all elements of Σ. The reason is the following. The impact multiplier measures the covariance between the output and policy residuals $u_{gdp,t}$ and $u_{p,t}$ conditional on the occurrence of a policy shock. In contrast, the systematic component of fiscal policy ψ^p_{gdp} determines the covariance between $u_{gdp,t}$ and $u_{p,t}$ conditional on the occurrence of non-policy shocks.[8] By construction, these two conditional covariances add up to the unconditional covariance $\sigma_{p,gdp}$ observed in the data. Hence, by varying the systematic component of fiscal policy ψ^p_{gdp}, we vary the size of the conditional covariance between $u_{gdp,t}$ and $u_{p,t}$ explained by the non-policy shocks. Consequently, to match the data, the conditional covariance generated by the policy shock—and thus the impact multiplier M_0—has to adjust.

The assumption of a simple fiscal rule greatly simplifies the analysis. Under a general fiscal rule, the size of the conditional covariance between $u_{gdp,t}$ and $u_{p,t}$ explained by the non-policy shocks depends (i) on the systematic response of fiscal policy to all non-policy variables and (ii) on the cross-correlations between output and the policy variable on the one hand and the remaining non-policy variables on the other. Moreover, the insights from the analysis of the impact multiplier help to understand the behavior of fiscal multipliers at longer horizons.

The following proposition formalizes key properties of the relationship between ψ^p_{gdp} and M_0.

Proposition 1. *Under the simple rule, $M_0(\psi^p_{gdp}, \Sigma)$ has the following properties:*

[6] Moreover, by using the relationship $\Phi = A_+A_0$, we can express the IRFs to a fiscal shock at any horizon, $L_h(A_0, A_+)_{\cdot,p}$, as a function of ψ_0 and (Φ, Σ). The derivations of the analytical expressions for the fiscal multipliers are reported in Appendix A.

[7] To compute fiscal multipliers, we divide the IRFs of output by ω_p—the standard deviation of the fiscal shock—and by a scaling factor that we set to the sample mean of the ratio p/gdp. We derive the analytical expression for ω_p using Equation (8). We omit p/gdp from Equation (11) as it is only a scaling factor.

[8] See Equation (10): ψ^p_{gdp} determines the response of $u_{p,t}$ to a change in $u_{gpd,t}$ and, consequently, the covariance between $u_{p,t}$ and $u_{gdp,t}$ generated by the non-policy shocks.

1. *$M_0(\psi^p_{gdp}, \Sigma) = 0$ if and only if $\psi^p_{gdp} = \sigma_{p,gdp}/\sigma^2_{gdp} \equiv \psi^{p,chol}_{gdp}$.*

2. *$M_0(\psi^p_{gdp}, \Sigma)$ has a global minimum and a global maximum obtained at $\psi^{p,min}_{gdp}$ and $\psi^{p,max}_{gdp}$, respectively.*

3. *It is strictly decreasing for $\psi^p_{gdp} \in \left[\psi^{p,max}_{gdp}, \psi^{p,min}_{gdp}\right]$.*

Proof. See Appendix A □

In the next section, we use Equation (11) and Proposition 1 to study the identification of fiscal shocks in a standard VAR model estimated in the literature. Importantly, Proposition 1 ensures that the key properties of the relationship between the systematic component of the fiscal rule and the fiscal multiplier hold in any VAR model, and do not depend on the details of the estimation of the reduced-form model.

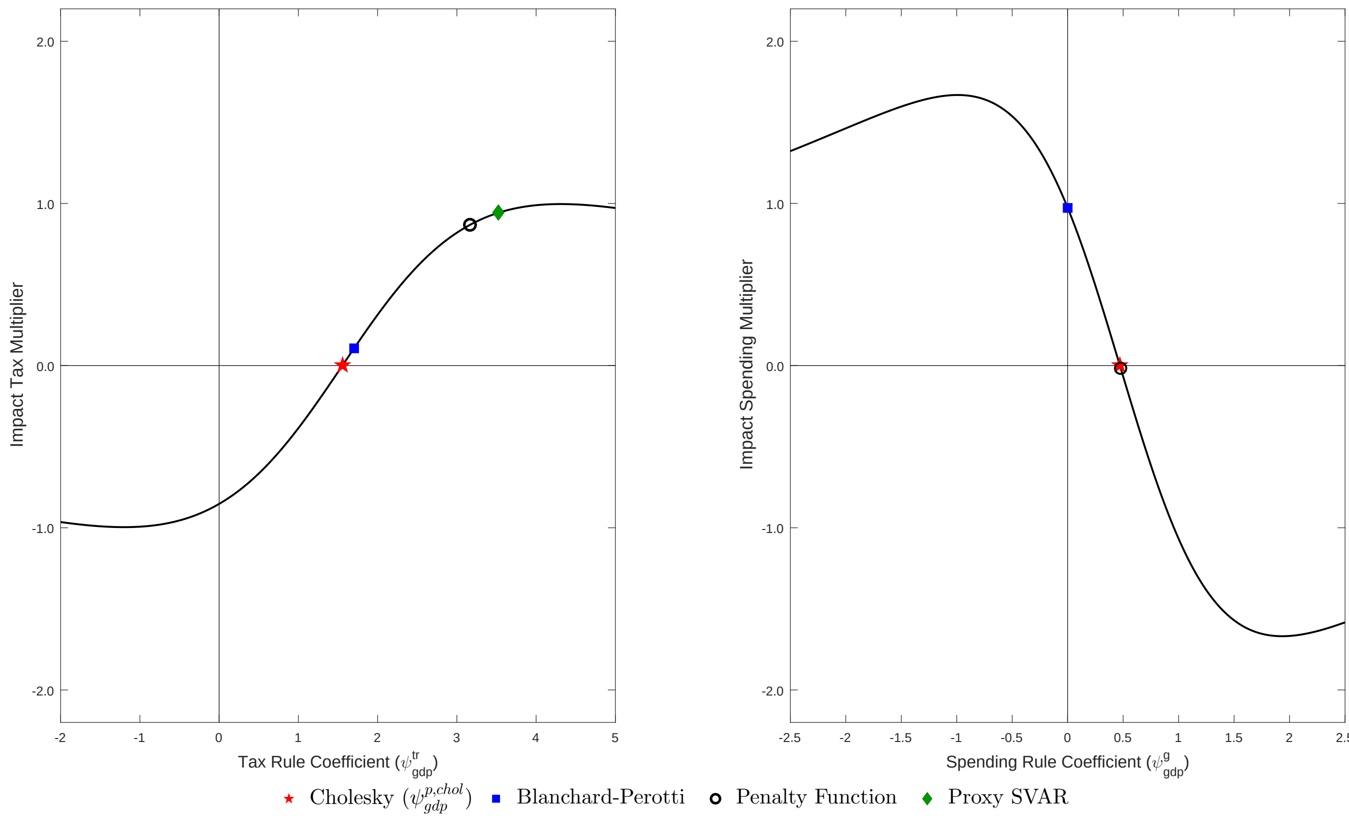
3 Results

In this section, we apply our theoretical framework to study the identification of fiscal policy shocks in a standard fiscal VAR model. In subsection 3.1 we study the relationship between the systematic component of tax and spending policies and the impact multipliers under the simple rules. In subsection 3.2 we discuss how existing identification schemes impose different restrictions on the systematic response of fiscal policy to output. In these two subsections, for illustrative purposes, we use reduced-form parameters evaluated at their OLS estimates. In subsection 3.3 we show that the results regarding the estimation of the impact multipliers under the simple fiscal rules extend to dynamic multipliers estimated under more general rules, and for the full Bayesian estimation of the model.[9]

Our baseline VAR model consists of the following five endogenous variables: real per-capita federal tax revenue (tr_t); government spending, defined as the sum of government consumption and investment (g_t); gross domestic product (gdp_t), consumer price inflation (π_t); and the 3-month T-bill rate (r_t). For the first three variables, we take the natural logarithm and we extract a deterministic trend by OLS regression. We estimate the model on quarterly data for the United States from 1950 to 2006. The choice of the sample is dictated by the availability of the narrative series of tax shocks discussed below. The VAR includes four lags of the endogenous variables and a constant. We impose a Minnesota prior on the

[9] This exposition strategy based on OLS estimates is used also by Uhlig (2005) and Baumeister and Hamilton (2015) to isolate the impact of identification—as opposed to sampling uncertainty—on inference. As discussed below, since we use a weakly informative prior, the contemporaneous elasticities and multipliers evaluated at the OLS estimates are very similar to those evaluated at the posterior median.

Figure 1: Impact Fiscal Multipliers as Functions of Fiscal Rule Coefficients (Simple Fiscal Rules)



Note: The solid lines plot the relationship between the impact fiscal multiplier and the systematic response of fiscal policy to output under the simple fiscal rule described in Equation (11). The left panel plots the relationship when the policy variable is tax revenue and the policy intervention is a tax cut, while the right panel plots it when the policy variable is government spending and the policy intervention is a spending increase. Σ is evaluated at the OLS estimate. See text for details.

reduced-form VAR parameters by using dummy observations (Del Negro and Schorfheide, 2011) and select the hyper-parameters that govern the prior distributions to impose a relatively weak prior.

In the remainder of the paper, we study policy shocks that are meant to stimulate economic activity, and hence we compare the effects of exogenous tax *cuts* to those of exogenous government spending *increases*.

3.1 Impact Fiscal Multipliers Under the Simple Fiscal Rules

Under the simple fiscal rules, differences in the identification of the systematic component of tax and spending policies lead to a wide range of fiscal multipliers.

Figure 1 shows this result. The left panel displays all combinations of impact tax multipliers and contemporaneous elasticities of taxes to output (ψ^{tr}_{gdp}) that are consistent with the data, as defined by Equation (11).[10] Similarly, the right panel plots all combinations of impact spending multipliers and contemporaneous elasticities of government spending to output (ψ^{g}_{gdp}) that are consistent with the data.

The black line in the left panel shows that the set of admissible impact tax multipliers is large, ranging between -1 and 1. Positive impact tax multipliers are associated with tax rules that feature a large systematic component. By contrast, negative tax multipliers are associated with tax rules that feature a small systematic component. Importantly, under the simple fiscal rules, only points on the line are consistent with the data. For instance, the data reject structural models that feature a large systematic component and a low tax multiplier.

The black line in the right panel shows that the set of admissible impact spending multipliers is large and ranges between -1.7 and 1.7. Positive impact spending multipliers are associated with government spending rules that feature a small (even negative) systematic component, while negative multipliers are associated to rules that feature a large and positive response to output.

In the remainder of this subsection, we discuss the economic intuition behind the relationship between the systematic component and the impact multipliers described above and characterized in Proposition 1, using tax policy as example. To this end, it is important to note that the unconditional covariance between tax revenue and output residuals ($\sigma_{tr,gdp}$) is positive. Since the unconditional covariance between government spending and output residuals ($\sigma_{g,gdp}$) is also positive, a similar intuition applies also to government spending policy.

To understand this relationship, it is useful to start by inspecting the value of ψ^{tr}_{gdp} that imposes a zero impact tax multiplier. This elasticity, which corresponds to $\psi^{tr,chol}_{gdp}$ in Proposition 1 and is represented by the red pentagram in Figure 1, implements a Cholesky identification that orders output before taxes in the VAR. This identification sets up a rule so that the non-policy shocks explain all the unconditional covariance between the tax revenue and output residuals.

For $\psi^{tr}_{gdp} > \psi^{tr,chol}_{gdp}$, the tax multiplier is positive for the following reason. Compared to the Cholesky identification, the implied tax rule is such that the non-policy shocks generate a larger conditional covariance between the tax revenue and output residuals, which is also larger than the unconditional covariance observed in the data. Thus, for the SVAR to match the data, the tax shock needs to generate a nega-

[10]Since we define the tax multiplier as the output response to a tax cut, we multiply Equation (11) by -1. This implies a reflection of the function over the x-axis.

tive conditional covariance between the output and tax residuals. Hence, the output response to a tax cut—which is zero under the Cholesky identification—has to be positive.

By contrast, for $\psi_{gdp}^{tr} < \psi_{gdp}^{tr,chol}$, the tax rule embeds a weaker systematic component compared to the Cholesky identification. Consequently the non-policy shocks generate a conditional covariance between tax revenue and output residuals that is smaller than the covariance observed in the data. Thus, to match the data, the tax shock has to generate a positive covariance, implying that the impact tax multiplier is negative (a tax cut induces a decline in output).

3.2 Restrictions on Fiscal Rules

The elasticity $\psi_{gdp}^{p,chol}$ is a first example of how to map restrictions on impulse responses (a Cholesky decomposition) into restrictions on the coefficients of the fiscal rules. We next derive the fiscal rules implied by three identification schemes that either impose restrictions directly on the coefficients of the systematic component—the Blanchard and Perotti (2002) approach—or on impulse responses to fiscal and non-fiscal shocks—the penalty function approach and the proxy SVAR approach.

3.2.1 Blanchard-Perotti Approach

The Blanchard-Perotti (BP) approach imposes numerical restrictions directly on the coefficients of the fiscal rules. In particular, Blanchard and Perotti (2002) rely on the methodology developed by the Organization for Economic Co-operation and Development (OECD) to estimate ψ_{gdp}^{tr} by using micro data and making assumptions based on institutional information about the U.S. tax and transfer systems. In our implementation of the BP approach—based on the more detailed estimation framework developed for the U.S. by Follette and Lutz (2010)—we set ψ_{gdp}^{tr} to 1.7. The OECD and other U.S. agencies assume that $\psi_{gdp}^{g} = 0$, as government consumption and investment do not have built-in automatic stabilizers.

The blue squares in Figure 1 depict the systematic components of taxes and government spending, as well as the associated tax and spending multipliers implied by the BP approach. The impact tax multiplier is 0.1, while the impact spending multiplier is about 1. The small tax multiplier can be explained by the similarity between the elasticity used in the BP identification and $\psi_{gdp}^{chol,tr}$, the elasticity implied by a Cholesky ordering that attributes all contemporaneous co-movements between taxes and output to non-tax shocks. By contrast, the large spending multiplier is due to the choice of the opposite Cholesky ordering, which attributes all contemporaneous co-movements between government spending and output to the government spending shock.

3.2.2 Penalty Function Approach

The penalty function approach—developed initially by Faust (1998) and Uhlig (2005)—has been used by Mountford and Uhlig (2009), MU henceforth, to identify tax and government spending shocks. Under this approach, structural shocks are identified using a criterion that each shock should maximize the impulse responses of some target variables over a pre-specified horizon. When applied to the identification of multiple shocks, this approach requires a sequential identification of the shocks.

As a result, following MU, we implement the penalty function identification in two steps. First, we search for an innovation that maximizes the responses of output and tax revenue over a one-quarter horizon, under the constraint that the sign of both responses has to be positive. This optimization step identifies what MU label a *business cycle shock*. In the second step, we search for an innovation that maximizes the response of a fiscal policy indicator over the same horizon and that is orthogonal to the business cycle shock identified in the first step. We also restrict the contemporaneous interaction between the two fiscal instruments to zero, a restriction that we relax in the next section.[11] When the policy variable is tax revenue, MU label this shock a *tax shock*; when the policy variable is government spending, MU label this shock a *government spending shock*. Under this implementation of the penalty function approach, the resulting fiscal rules are simple. In economic terms, this identification reflects MU's view that the bulk of business cycle fluctuations are unrelated to fiscal shocks. Since the business cycle shock is the only non-policy shock that can move taxes and government spending, its identification pins down the systematic response of fiscal policy.

The black circles in Figure 1 depict the systematic components of taxes and government spending, as well as the associated tax and spending multipliers implied by the penalty function approach. The large impact tax multiplier—about 0.9—is due to the strong systematic component embedded in the tax rule, characterized by an elasticity ψ_{gdp}^{tr} of about 3.2. This large systematic response is consistent with the design of the penalty function identification. In Appendix B we prove that—in a bivariate model in output

[11] This implementation of the penalty function approach differs from the MU implementation in two dimensions. First, MU identify a business cycle shock *and* a monetary policy shock before identifying the two fiscal shocks. In the next subsection we report results under this alternative identification, which implies fiscal rules that feature a contemporaneous systematic response to inflation and the interest rate. Second, MU identify a business cycle shock by also targeting the response of private consumption and investment. However, as we show in Caldara and Kamps (2012), the inclusion of these two variables—which are highly correlated with output and tax revenue—has modest effects on the estimated fiscal multipliers. Note that this approach does not impose orthogonality between the two fiscal shocks.

and tax revenue—the penalty function identifies an elasticity ψ_{gdp}^{tr} that maximizes the covariance between $u_{gdp,t}$ and $u_{tr,t}$ explained by the business cycle shock, which results in the selection of a large systematic component. We also prove that—under this identification approach—the impact tax multiplier is always positive, and consequently the penalty function approach is not agnostic about the sign of the multiplier.

The impact spending multiplier under the penalty function approach is zero. Since the output and government spending reduced-form residuals are positively correlated, the business cycle shock mechanically—by increasing output—induces an increase in government spending. Thus, this approach is not agnostic about the systematic response of government spending to output, but implicitly restricts ψ_{gdp}^{g} to be positive, about 0.5 in Figure 1.[12] Note that the identification of the government spending shock implied by the penalty function approach is nearly identical to a Cholesky identification that orders output before government spending, the opposite ordering compared to the BP approach.

3.2.3 Proxy SVAR Approach

The last methodology we study is the narrative approach. In particular, we build on the proxy SVAR implementation proposed by Stock and Watson (2012) and Mertens and Ravn (2013). The analysis of this approach requires inverting the steps we followed to study the BP and the penalty function approaches: the narrative approach estimates directly the fiscal multiplier. Hence, in our analysis we take the estimated multiplier from the proxy SVAR as given, and we compute the systematic component of fiscal policy that generates the unconditional covariance between output and the policy variable observed in the data. This characterization of the systematic component allows us to compare the narrative approach to the other identification approaches.[13]

We assume that a narrative measure of fiscal shocks, denoted by $m_{p,t}$, is a proxy for the unobserved structural shock $e_{p,t}$ in the SVAR. In addition, we assume that the proxy $m_{p,t}$ satisfies

$$\mathbb{E}[m_{p,t}e'_{p,t}] = \gamma, \tag{12}$$

$$\mathbb{E}[m_{p,t}e'_{np,t}] = 0. \tag{13}$$

[12]See Caldara and Kamps (2012) for an analytical characterization of this result.

[13]In an independent study, Mertens and Ravn (2014) adopt a similar strategy to reconcile SVAR and narrative estimates of tax multipliers.

Equation (12) states that the proxy $m_{p,t}$ is correlated with the unobserved policy shock of interest; Equation (13) states that the proxy is uncorrelated with the remaining non-policy shocks, denoted by $e_{np,t}$. These two conditions are very similar to those required of an instrument in an IV regression.

As shown in Mertens and Ravn (2013), under these conditions, the proxy can be used to directly identify the IRFs to the shock of interest. Here, instead, using similar derivations to those discussed in Section 2.2, we use the proxy to identify the coefficients of the fiscal policy rule. To this end, we regress the reduced-form residuals of the non-policy variables $u_{np,t}$ on the proxy:

$$u_{np,t} = \xi_0^{proxy} m_{p,t} + \tilde{e}_{np,t}, \tag{14}$$

where ξ_0^{proxy} represents the vector of impact responses to a policy shock proxied by $m_{p,t}$, and $\tilde{e}_{np,t} = \Omega_{np,t} e_{np,t}$. Hence, the impact fiscal multiplier M_0 is the coefficient (appropriately scaled) in ξ_0^{proxy} associated with the residual $u_{gpd,t}$. Thus, the implementation of the proxy SVAR approach does not require to impose restrictions on the systematic component of fiscal policy to estimate the fiscal multiplier.

However, we can compute the coefficients of the systematic component of fiscal policy that—for a given estimate of the impact multiplier—generate the covariance Σ observed in the data. To this end, note that in Equation (9), the residuals $\tilde{e}_{np,t}$ are by construction uncorrelated to the fiscal policy shock. Thus, we use $\tilde{e}_{np,t}$ as instruments to estimate the contemporaneous elasticities in the fiscal rule described in Equation (8). To implement the simple fiscal rule, we set to zero all elasticities other than ψ_{gpd}^{p}. Thus, given the estimate of M_0 implied by the proxy, we can compute ψ_{gpd}^{p} by inverting the analytical expression presented in Equation (11).

In our application, the proxy $m_{p,t}$ is the Mertens and Ravn (2011) series of unanticipated tax shocks, a subset of the Romer and Romer (2010) tax shocks identified by studying narrative records of tax policy decisions. By contrast, we do not have a proxy for unanticipated government spending shocks, as the narrative literature—most prominently Ramey (2011)—identifies only anticipated shocks.

As shown by the green diamond in Figure 1, the proxy SVAR approach estimates an impact tax multiplier of about 1, the largest attainable multiplier within the range of possible multipliers. Our framework allows us to identify the tax rule that accounts for the large impact tax multiplier, while generating the covariance of tax revenue and output observed in the data. This tax rule features a systematic component that is the largest of all identification strategies, described by an elasticity ψ_{gdp}^{tr} of about 3.5.

3.3 Dynamic Fiscal Multipliers under the General Fiscal Rules

We now turn to the description of the results for the full estimation of the model. We compare results under the simple rules and under general rules. To characterize the general rules, we refer to the contemporaneous elasticities that do not appear in the simple tax rule using the following notation: ψ_{π}^{tr} (inflation), ψ_{r}^{tr} (interest rate), and ψ_{g}^{tr} (government spending). The contemporaneous elasticities in the unrestricted government spending rule are defined accordingly.

For the BP approach, we follow the implementation of Perotti (2005) and deviate from the simple rule by setting ψ_{π}^{tr} and ψ_{π}^{g} to 1.25 and -0.5, respectively. A positive elasticity of taxes with respect to inflation is consistent with the U.S. tax system prior to 1985, as the personal income tax schedule did not adjust for inflation, giving rise to so-called bracket creep; a negative elasticity of (real) government spending with respect to inflation is plausible if (nominal) government spending is not fully indexed to inflation. For the penalty function approach, in addition to the business cycle shock—we identify a monetary policy shock and an inflation shock prior to the identification of the fiscal shocks. In particular, the monetary policy shock is an innovation that maximizes the positive response of the interest rate and the negative response to inflation over a one-quarter horizon, restrictions that rule out the price puzzle; the inflation shock is identified assuming that inflation does not respond contemporaneously to the fiscal variables. Controlling for monetary policy and inflation shocks results in fiscal rules that feature a contemporaneous systematic response to interest rate and inflation. For the proxy SVAR approach, we make use of all contemporaneous elasticities implied by the identification strategy. Finally, we separately identify tax and spending shocks by imposing that government spending does not respond contemporaneously to taxes.

Table 1 reports the median and the 68% credible sets of the parameters of the general rules imposed by the BP approach—the first column—and implied by the penalty function and proxy SVAR approaches, the second and third columns. The median estimates of the systematic response of taxes to output are virtually identical to the estimates obtained under the simple fiscal rules reported in Section 3.2. All approaches identify a positive systematic response of taxes to inflation. This response is particularly strong under the proxy SVAR approach, with a median ψ_{π}^{tr} of 2.4. Hence, controlling for bracket creep in the specification of the tax rule could be important for the identification of tax shocks and for the estimation of the associated tax multipliers. The penalty function is the only approach that implies a negative systematic component of tax and spending policies to contemporaneous changes in the interest rate. These elasticities could capture

Table 1: Contemporaneous Elasticities in the Fiscal Policy Rules
(General Rules)

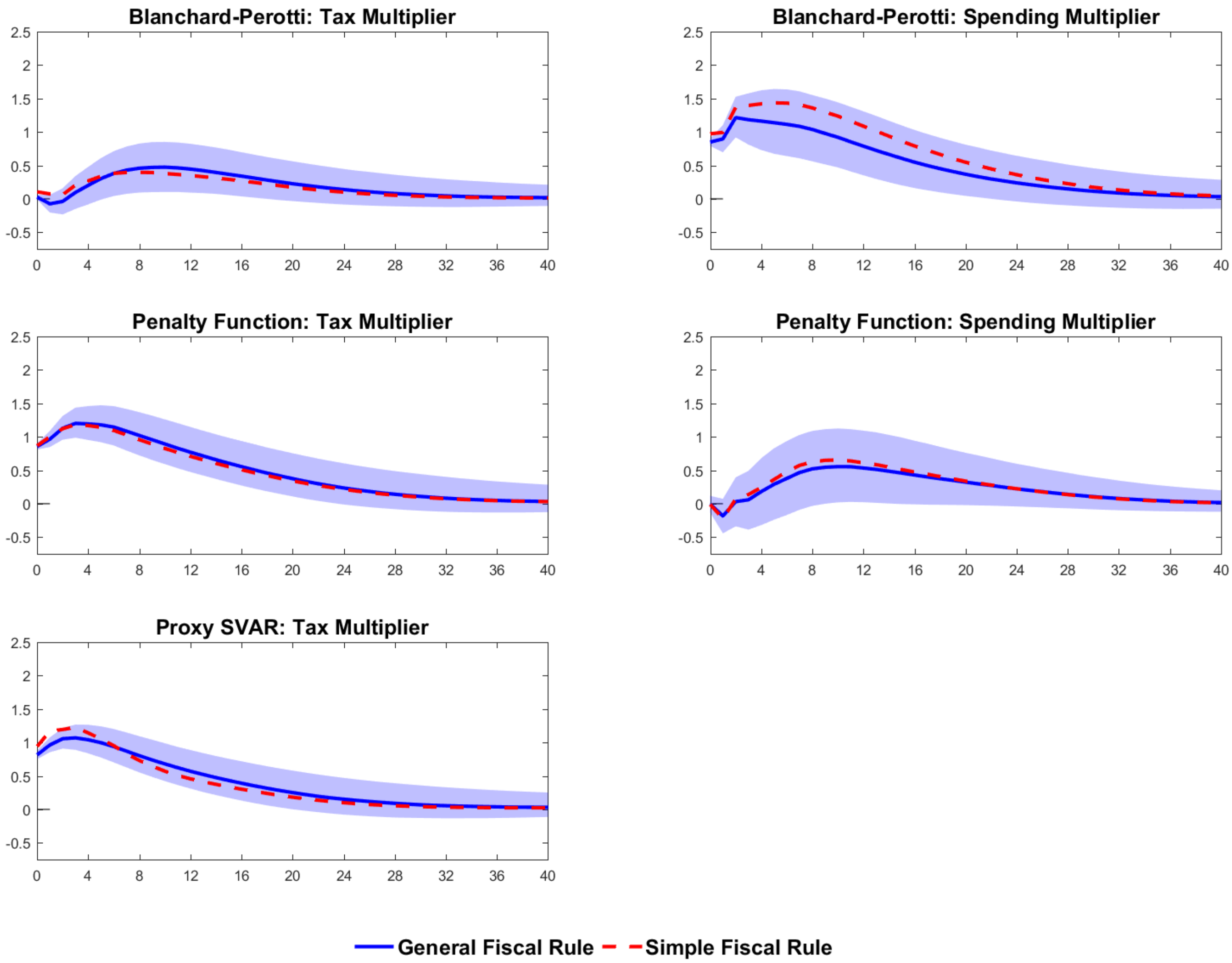
	Blanchard-Perotti	Penalty Function	Proxy SVAR
(A.) *Tax Rule*			
$\psi^{tr}_{0,gdp}$	1.70	3.24	3.58
		[3.04 3.45]	[3.22 3.98]
$\psi^{tr}_{0,\pi}$	1.25	0.48	2.41
		[0.23 0.74]	[1.95 2.91]
$\psi^{tr}_{0,r}$	0.00	-0.42	-0.01
		[-0.65 -0.20]	[-0.42 0.39]
$\psi^{tr}_{0,g}$	-0.14	0.01	-0.29
	[-0.17 -0.10]	[-0.15 0.18]	[-0.50 -0.09]
(B.) *Government Spending Rule*			
$\psi^{g}_{0,gdp}$	0.00	0.55	
		[0.44 0.66]	
$\psi^{g}_{0,\pi}$	-0.50	0.41	
		[0.30 0.52]	
$\psi^{g}_{0,r}$	0.00	-0.36	
		[-0.45 -0.26]	
$\psi^{g}_{0,tr}$	0.00	0.00	

Note: The entries in the table denote the posterior median estimates of the contemporaneous elasticities that characterize the systematic component in the tax rule (panel A) and in the government spending rule (panel B) identified by the Blanchard and Perotti (2002) approach, the penalty function approach, and the proxy SVAR approach. The 68 percent credible sets from the posterior distributions are reported in brackets. See the main text for details.

the automatic response of fiscal variables to changes in interest sensitive taxable income, such as corporate profits. Finally, the BP and proxy SVAR approaches imply an economically meaningful contemporaneous response of taxes to government spending.

Figure 2 plots the median dynamic multipliers under the general fiscal rules (blue solid lines), the associated 68 percent credible sets (blue shaded area), and the median multipliers under the simple rules (red dashed line). Two results emerge from this figure. First, differences in dynamic multipliers across identification schemes are consistent with differences in impact multipliers. For the first year after the shock, dynamic tax multipliers are close to zero under the BP approach, while they are close to one under the penalty function and proxy SVAR approaches. Conversely, dynamic spending multipliers are about one under the BP approach, while they are below one under the penalty function approach. Second, for all approaches, the median multipliers estimated under the simple fiscal rules are within the credible sets

Figure 2: Dynamic Fiscal Multipliers
(Comparison over Identification Schemes)



Note: The lines in each panel depict dynamic tax and spending multipliers. The underlying fiscal shocks are calculated using general fiscal rules (blue dashed) and simple fiscal rules (red dashed) identified under the Blanchard and Perotti (2002) approach (top row), the penalty function approach (middle row), and the proxy SVAR approach (bottom row). Shaded bands denote the 68 percent pointwise credible sets calculated under the general fiscal rules. See text for details.

associated with the general fiscal rules. Thus, we confirm that the analysis of identification under the simple fiscal rules provides the key insights to understand the different estimates of fiscal multipliers found in the literature.

3.4 Summary of Findings

Summing up, in this section we characterized how the specification of the systematic component of tax and spending policies—and in particular of the coefficients that describe the systematic response of taxes and spending to contemporaneous movements in output—determine the sign and size of of fiscal multipliers. Moreover, we showed how three influential identification schemes widely used in the literature impose, either explicitly or implicitly, different restrictions on the systematic component of fiscal policy. We found that such differences account for the wide range of fiscal multiplier estimates reported in the literature.

4 Proxy SVAR Identification with Non-Fiscal Proxies

In this section we propose a new identification strategy to discipline the systematic component of tax and spending policies. The identification is based on the proxy SVAR methodology described in Section 3. The novelty is that, instead of using fiscal proxies to identify the fiscal shocks, we use non-fiscal proxies taken from prominent papers in the literature to directly estimate the coefficients that characterize the contemporaneous elasticities that characterize the systematic component in the fiscal rules. The basic idea is that the non-fiscal shocks move fiscal and non-fiscal variables simultaneously and for reasons unrelated to discretionary changes in fiscal policy. Thus, the relative strength of their responses allows to make inference about the coefficients in the fiscal rules.

Let $m_{np,t}$ be a $n-k \times 1$ vector of proxy variables for the non-policy shocks $e_{np,t}$ that we assume satisfies:

$$\mathbb{E}[m_{np,t}e'_{np,t}] = \Gamma, \tag{15}$$

$$\mathbb{E}[m_{np,t}e'_{p,t}] = 0_{n-k\times k}, \tag{16}$$

where Γ is an $n-k \times n-k$ non-singular matrix. The first condition requires that the proxies are contemporaneously correlated with the non-policy shocks and the second condition requires that they are uncorrelated with the fiscal shocks. If the proxies satisfy these conditions, we can directly estimate ψ_0 in Equation (8) by instrumenting $u_{np,t}$ with $m_{np,t}$.[14] With the estimates of the elasticities at hand, we identify the fiscal shocks using the fiscal rules described in Equation (7). As in Section 3.3, we jointly identify tax

[14]Note that we do not require the proxies to be orthogonal to each other: to be valid instruments for $u_{np,t}$ they only need to satisfy Equations (15) and (16), and Γ is not required to be diagonal.

and spending shocks by imposing that government spending does not respond contemporaneously to taxes.

4.1 The Proxies

The 5-equation VAR described in Section 3 has three non-policy shocks—to output, inflation, and the interest rate—which we proxy using the following three variables: m_{tfp}, the Fernald (2012) technology series, which measures total factor productivity adjusted for changes in factor utilization; m_{oil}, the Hamilton (2003) oil shocks based on a nonlinear transformation of the nominal price of crude oil; m_r, the Romer and Romer (2004) monetary policy shocks (quarterly sums of their monthly variable), extended through 2006 by Barakchian and Crowe (2013). While m_{tfp} and m_{oil} are available from 1950 to 2006, m_r is available starting in 1969. For this reason, and following standard practice in the literature (Stock and Watson, 2012; Gertler and Karadi, 2015), we estimate the reduced-form VAR from 1950 to 2006, and regress the residuals on the proxies based on sample availability. Thus, we use m_{tfp} and m_{oil}—the proxies available for the full sample—as proxies for the non-policy shocks associated with the output and inflation equations; we use m_r as proxy for the monetary policy shock.[15]

We provide evidence that the three proxies satisfy the relevance and exogeneity conditions stated in equations (15) and (16) by running a battery of predictive regressions. First, to assess the relevance of our proxies, we follow Gertler and Karadi (2015) and regress the non-policy VAR residuals $u_{np,t}$ on the proxies. Results are tabulated in panel A of Table 2. The first two columns report results for the regression of $u_{gdp,t}$ and $u_{\pi,t}$ on the productivity and oil shocks m_{tfp} and m_{oil}, while the third column reports results for the regression of the interest rate residual $u_{r,t}$ on m_r. The F statistic is larger than ten—the threshold to detect weak instruments recommended by Stock et al. (2002)—for the output and interest rate residuals. The F statistic is only 2.5 for the regression of the inflation residual. Nonetheless, the response of inflation to an oil shock is significant at the 5% level.

To assess the exogeneity of the proxies, we regress them on the narrative measure of tax shocks constructed by Mertens and Ravn (2011), m_{tax}, and on the narrative measure of expected exogenous changes in military spending constructed by Ramey (2011), m_g. Results are tabulated in panel B of Table 2. The F tests suggest that we cannot reject that all regression coefficients are zero, indicating that the proxies are orthogonal to contemporaneous movements in narrative measures of tax and government spending

[15] Coibion and Gorodnichenko (2012) use similar instruments to identify the parameters of a Taylor rule in a single-equation instrumental variable regression.

Table 2: PREDICTABILITY REGRESSIONS

(A.) *Relevance of Non-Fiscal Proxies*			
	u_{gdp}	u_{π}	u_r
m_{tfp} (Utilization-Adjusted Productivity)	0.10	0.00	
	[0.01]	[0.01]	
m_{oil} (Oil Shocks)	-0.01	0.01	
	[0.01]	[0.01]	
m_r (Monetary Policy Shocks)			0.79
			[0.07]
F-statistic	26.27	2.46	120.52

(B.) *Exogeneity of Non-Fiscal Proxies*			
	m_{tfp}	m_{oil}	m_r
m_{tax} (Narrative Tax Shocks)	-2.06	0.93	-0.04
	[1.53]	[2.22]	[0.31]
m_g (Military Spending Shocks)	-0.23	0.38	0.04
	[0.21]	[0.31]	[0.04]
F-statistic	1.44	0.85	0.54

NOTE: Panel A: The dependent variable in each specification is the VAR residual u_j, for $j = gpd, \pi, r$, calculated at the OLS estimates of (Φ, Σ). u_{gdp} and u_{π} are regressed on m_{tfp} and m_{oil} on the sample 1950–2006. u_r is regressed on m_r on the sample 1969–2006; Panel B: The dependent variable in each specification is the level of the specified non-fiscal proxy. The regression of m_r on the fiscal shocks is estimated on the sample 1969–2006. Each regression includes a constant. Standard errors are reported in brackets. See the text for details.

shocks.[16] Therefore, one may reasonably expect that the exogeneity condition is satisfied for our proxies.

4.2 Fiscal Rules and Multipliers

Panel A of Table 3 reports the median and the 68% credible set of the systematic component of tax policy estimated with all non-fiscal proxies (first column), and with only m_{tfp} under the assumption of a simple fiscal rule (second column). Under the general rule, the median estimate of the contemporaneous response of taxes to output is 2.2, with the 68% credible set ranging from 2.0 to 2.4. Under the simple rule, the median estimate for this elasticity is slightly larger at 2.4 and lies just outside the 68% credible set estimated for the general rule. Under the general rule, the median estimate of the contemporaneous response of taxes to inflation and the interest rate is about 1.1 and 0.6, respectively.

Panel B of Table 3 reports the estimated coefficients of the systematic component of government

[16] In Section 5 we show that results are robust to including m_{tax} and m_g in the estimation of the reduced-form VAR model, and thus controlling for lags of m_{tax} and m_g has a modest impact on results.

Table 3: Contemporaneous Elasticities in the Fiscal Policy Rules
(Proxy SVAR Identification with Non-Fiscal Proxies)

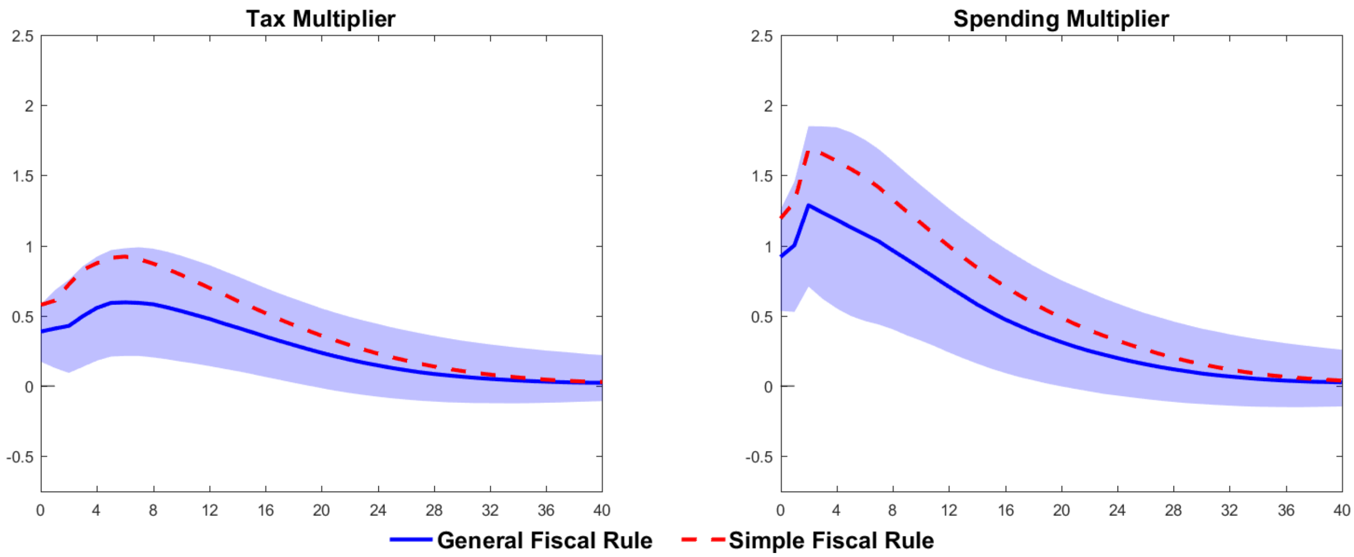
	General Rule	Simple Rule
(A.) *Tax Rule*		
$\psi^{tr}_{0,gdp}$	2.18	2.43
	[1.96 2.41]	[2.21 2.66]
$\psi^{tr}_{0,\pi}$	1.06	
	[0.09 2.10]	
$\psi^{tr}_{0,r}$	0.56	
	[0.39 0.73]	
$\psi^{tr}_{0,g}$	-0.23	
	[-0.46 -0.02]	
(B.) *Government Spending Rule*		
$\psi^{g}_{0,gdp}$	-0.13	-0.15
	[-0.28 0.01]	[-0.27 -0.03]
$\psi^{g}_{0,\pi}$	-0.75	
	[-1.62 -0.08]	
$\psi^{g}_{0,r}$	0.01	
	[-0.09 0.13]	
$\psi^{g}_{0,tr}$	0.00	

Note: The entries in the table denote the posterior median estimates of the contemporaneous elasticities in the tax rule (panel A) and in the government spending rule (panel B) identified by the proxy SVAR. The first column reports estimates for the general rules estimated using three non-fiscal proxies, while the second column for the simple fiscal rule identified using only Fernald (2012) capacity adjusted productivity (m_{tfp}). The 68 percent credible sets from the posterior distributions are reported in brackets. See the main text for details.

spending policy. The median estimate of the contemporaneous response of government spending to output is -0.13 for the general rule and -0.15 for the simple rule. Under the general rule, the median estimate of the contemporaneous response to inflation is -0.8, similar to the calibration used by Perotti (2005). A negative elasticity reflects the fact that nominal government spending is not fully indexed to inflation in the U.S., implying that real government spending can fall in response to an increase in inflation. Finally, the median estimate of the contemporaneous response of government spending to the interest rate is close to zero, which is consistent with our definition of the spending aggregate which excludes government interest payments.

Overall, we find a systematic response of taxes to output that lies within the range spanned by the three identification approaches studied in Section 3.2, slightly tilted towards the elasticity used in the BP approach. We find a negative estimate of the systematic response of government spending to output that

Figure 3: Dynamic Fiscal Multipliers
(Simple vs. General Fiscal Rules - Proxy SVAR with Non-Fiscal Proxies)



Note: The lines in each panel depict dynamic tax and spending multipliers. The underlying fiscal shocks are computed using the general fiscal rules (blue solid line) and the simple fiscal rules (red shaded line) identified using the proxy SVAR identification with non-fiscal proxies. Shaded bands denote the 68 percent pointwise credible sets calculated under the general fiscal rules. See text for details.

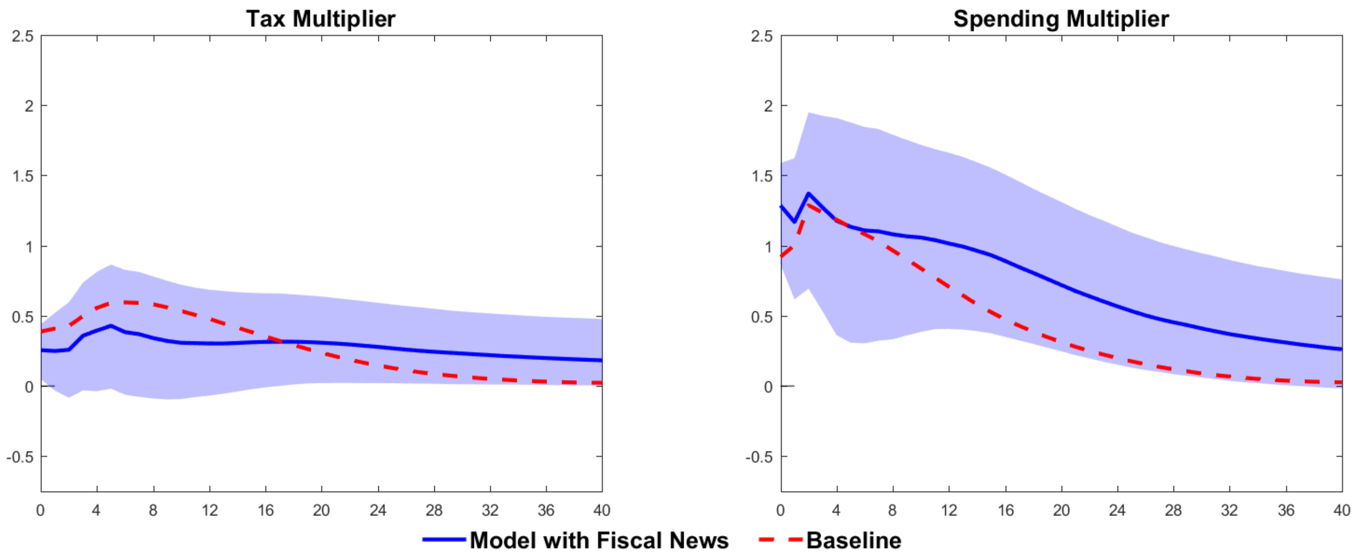
lies outside the range spanned by the existing identification schemes, although—at a median estimate of -0.13—it is close to the value of zero used in the BP approach.

Figure 3 plots the tax and spending multipliers under the general fiscal rules (the blue lines) and under the simple fiscal rule (the red dashed line) based on the estimation of the rules reported in Table 3. Under the general rules, the impact tax multiplier ranges between 0.2 and 0.6, with a median of 0.4. The median tax multiplier peaks at 0.6 seven quarters after the shock. The median tax multiplier under the simple rule is within the 68 percent credible set estimated under the general rules.

The impact spending multiplier ranges between 0.5 and 1.3 under the general rule, with a median of about 0.9. The median spending multiplier peaks at 1.3 about 3 quarters after the shock. The median multiplier under the simple rule is about 1.2 on impact, and it peaks at 1.7 about three quarters after the shock. The difference is accounted for by the large systematic response of government spending to inflation under the general rule. Yet, since the posterior distributions have a wide support, the median multiplier under the simple rule is within the 68 percent credible set estimated under the general rules.

All told, in this section we have provided novel estimates of the endogenous component of fiscal policy. This approach leads to the conclusion that short-run spending multipliers are larger than tax multipliers,

Figure 4: Dynamic Fiscal Multipliers
(Fiscal News Data in the VAR)



Note: The lines in each panel depict dynamic tax and spending multipliers based on the baseline reduced-form VAR specification (red dashed line) and on an alternative specification that includes data on fiscal news (blue solid line). The underlying fiscal shocks are computed using the general fiscal rules identified under the proxy SVAR identification with non-fiscal proxies. Shaded bands denote the 68 percent pointwise credible sets based on the alternative specification of the reduced-form model.

with the former around 1 and the latter around 0.5.

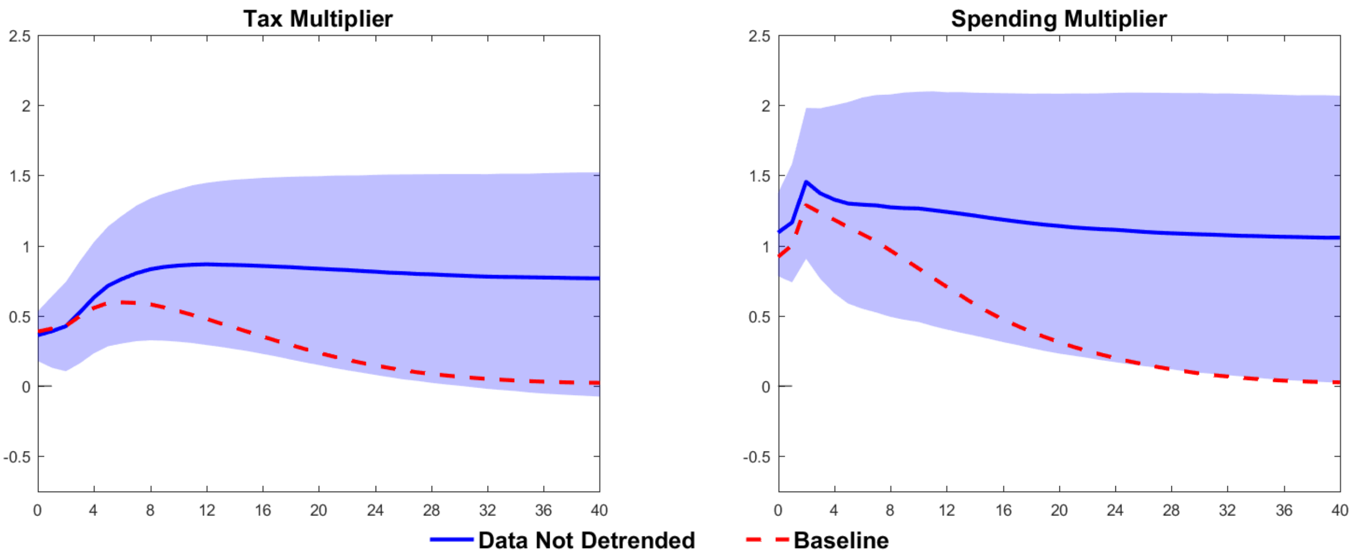
5 Robustness

In this section we investigate the robustness of the fiscal multipliers estimated under general fiscal rules using the proxy SVAR identification proposed in Section 4 with respect to four issues: fiscal foresight, the modeling of trends in the data, the ordering of policy variables, and the definition of fiscal multipliers.

Fiscal Foresight. Leeper et al. (2013) argue that SVARs that do not account for fiscal foresight—the reaction of economic agents to anticipated fiscal shocks—might obtain biased estimates of fiscal multipliers. To explore the importance of fiscal foresight within our framework, we estimate a larger system that includes measures of fiscal news. In particular, we add to the baseline model the measure of news about military spending constructed by Ramey (2011) and the measure of news about taxes constructed by Leeper et al. (2012). In the presence of fiscal foresight, the inclusion of fiscal news affects the estimation of the VAR reduced-form residuals to the extent that the variation in these residuals is predictable.

Figure 4 plots the median tax and spending multipliers identified using the model that includes fiscal

Figure 5: Dynamic Fiscal Multipliers
(Alternative Specification Of Trends)



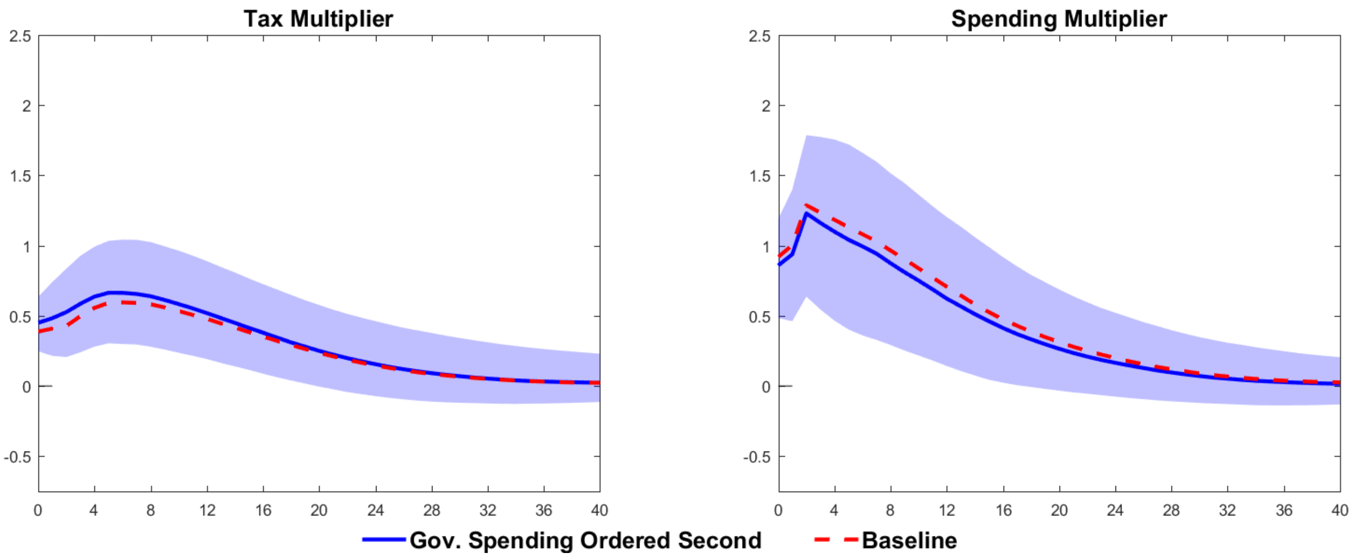
Note: The lines in each panel depict dynamic tax and spending multipliers based on the baseline reduced-form VAR specification (red dashed line) and on an alternative specification that uses non-detrended data (blue solid line). The underlying fiscal shocks are computed using the general fiscal rules identified under the proxy SVAR identification with non-fiscal proxies. Shaded bands denote the 68 percent pointwise credible sets based on the alternative specification of the reduced-form model.

news, and compares them against those identified using the baseline specification.[17] The inclusion of fiscal news lowers the path for the tax multiplier for up to four years after the shock, while it increases the path for the spending multiplier starting six quarters after the shock. Yet, the median estimates of the fiscal multipliers under the baseline specification of the model lie within the 68 percent credible set associated with the model that includes news.

Specification of Trends in the Data. Since the literature offers little guidance on the specification of the long-run properties of the VAR, we compare fiscal multipliers for the baseline specification, where we linearly detrended some variables, to a specification where data are not detrended. In the latter case, the reduced-form model is non-stationary, with the largest eigenvalue being 1. Figure 5 shows that in the non-stationary model, tax shocks have permanent effects, while spending shocks have temporary effects—albeit much more persistent than in the baseline model specification. While the estimates of long-run multipliers strongly depend on the econometrician's choices regarding the specification of the reduced-form VAR model, short-run multipliers—which are the focus of our analysis—are robust to these choices.

[17]Note that—due to the availability of data on tax news—the model with news is estimated on data starting in 1955 instead of 1950. The exclusion of the Korean War from the sample partly accounts for the differences in multipliers.

Figure 6: Dynamic Fiscal Multipliers
(Alternative Ordering of Fiscal Variables)



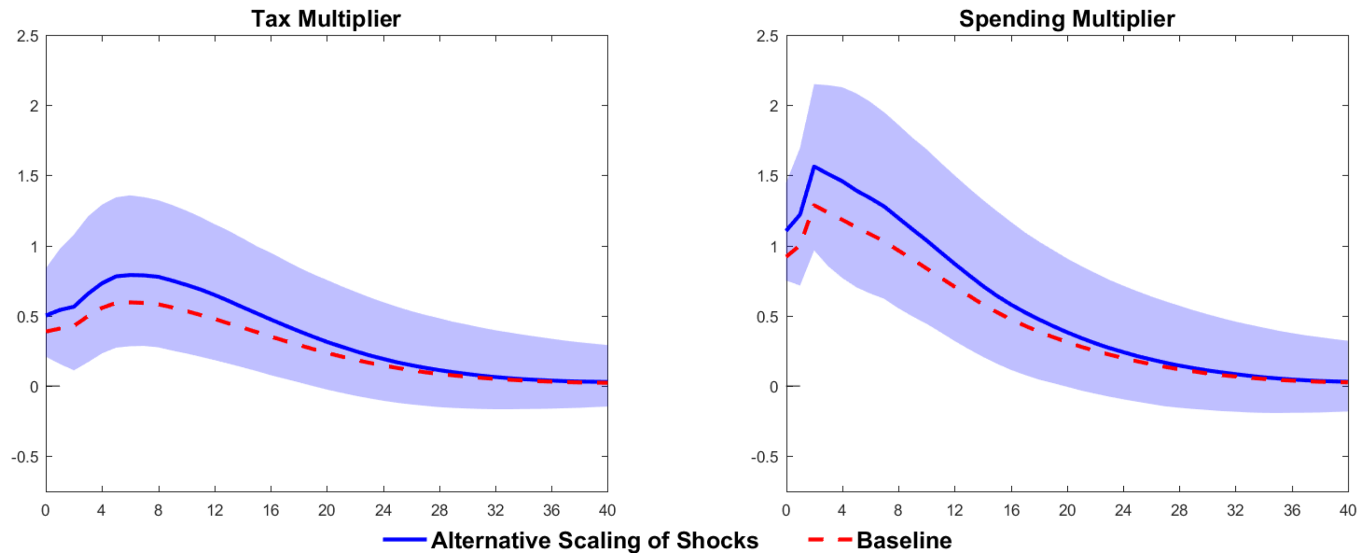
Note: The lines in each panel depict dynamic tax and spending multipliers based on the baseline ordering of fiscal variables within the policy block (red dashed line), and on the alternative assumption that orders government spending second, so that it can contemporaneously respond to tax revenue (blue solid line). The underlying fiscal shocks are computed using the general fiscal rules identified under the proxy SVAR identification with non-fiscal proxies. Shaded bands denote the 68 percent pointwise credible sets based on the alternative ordering within the policy block.

Ordering of Policy Variables. As explained in Section 2, when we include two policy instruments in the VAR we need to make an additional identification assumption on how the fiscal instruments interact with each other. Figure 6 compares fiscal multipliers estimated under the baseline ordering—in which government spending is ordered before taxes—and under the alternative ordering—in which government spending is ordered after taxes. Under the alternative ordering, the median tax multiplier is higher and the spending multiplier lower than under the baseline ordering, while remaining well within the bounds of the credible sets.

Definition of Fiscal Multiplier. We compare two definitions of fiscal multipliers used in the literature. Our benchmark definition follows Blanchard and Perotti (2002): the fiscal multiplier is the dollar response of output to a shock of size 1 dollar.[18] The alternative definition—followed for instance by Mountford and Uhlig (2009), Barro and Redlick (2011), and Mertens and Ravn (2013)—defines the fiscal multiplier as the dollar increase in output to an effective change *in the fiscal variable* of 1 dollar. If the

[18] The scaling adopted by BP is explained on page 1342 and can be noted from Table III, page 1344 in the original article, where the impact response of tax revenue to a tax shock is 0.74 instead of 1.

Figure 7: Dynamic Fiscal Multipliers
(Alternative Scaling of Fiscal Multiplier)



Note: The lines in each panel depict dynamic tax and spending multipliers based on the baseline definition of fiscal multipliers (red dashed line) and on the alternative definition described in the text (blue solid line). The underlying fiscal shocks are computed using the general fiscal rules identified under the proxy SVAR identification with non-fiscal proxies. Shaded bands denote the 68 percent pointwise credible sets based on the alternative definition of multipliers.

policy variable does not respond contemporaneously to any variable in the system, the two definitions are equivalent. Except for this special case, the response of policy to its own shock depends on the size of the systematic component, and thus it differs from the size of the shock.

To illustrate this point, Figure 7 plots multipliers under the two alternative definitions. The definition of multiplier matters both for taxes and spending, although the median estimates are similar. Multipliers are larger under the alternative definition because in our proxy SVAR identification, fiscal policy systematically responds to economic activity: a fiscal shock of size 1\$ induces a contemporaneous response of the associated fiscal variable of less than \$1. Thus, under the alternative definition, to induce a 1\$ effective change in the fiscal variable, the shock needs to be larger than 1\$. However, the figure shows that the relative size of spending versus tax multipliers is robust to alternative definitions of the multiplier.

6 Conclusions

This paper derives an analytical framework to compare fiscal multipliers implied by commonly used identification schemes. To this end, we first characterize how the specification of the systematic component of tax and spending policies determines the identification of fiscal shocks and, by implication, the sign and size of of fiscal multipliers. We then show how existing identification schemes impose, either explicitly or implicitly, different restrictions on the systematic component of fiscal policy. We find that differences in fiscal multiplier estimates documented in the literature can be accounted for by different assumptions on the systematic response of tax and spending policies to output.

We provide new estimates of the systematic component of tax and spending policies based on a proxy SVAR identified using non-fiscal proxies. We find a positive and large systematic response of taxes to output, and a small but negative systematic response of government spending to output. The implied spending multipliers tend to be larger than the tax multipliers. Our estimation results suggest that the automatic stabilization embedded in U.S. tax and spending policies might be different than commonly assumed in the public finance literature. The dispersion in the estimates of the systematic component of fiscal policies documented in this paper, together with their importance to pin down fiscal multipliers, suggests that more research should be devoted to estimate the cyclical properties of U.S. fiscal policy.

The framework developed in this paper can be applied to study identification in models that feature time variation in coefficients. The use of such models can help unveil whether the transmission of policy shocks has changed over time and whether it depends on the state of the economy (see for instance Primiceri, 2005; Auerbach and Gorodnichenko, 2012).

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Appendices

A Analytical Derivations in Section 2

Let us consider the partition $u_t = [u'_{p,t}, u'_{np,t}]'$ where $u_{p,t}$ is a $k \times 1$ vector of policy variables, and $u_{np,t}$ is $n-k \times 1$ vector of non-policy variables. Without loss of generality, we order gdp_t first in the non-policy block. We apply a similar partition to $e_t = [e'_{p,t}, e'_{np,t}]'$, where $e_{p,t}$ is a $k \times 1$ vector of policy shocks, and $e_{np,t}$ is a $n-k \times 1$ vector of non-policy shocks. Using this partition, we can write the relation between reduced-form residuals and structural shocks as follows:[19]

$$
\begin{aligned}
u'_{p,t} &= u'_{np,t}\psi^p + e'_{p,t}B_p, && (17)\\
u'_{np,t} &= u'_{p,t}\xi + e'_{np,t}B_{np}, && (18)
\end{aligned}
$$

where B_p is a $k \times k$ matrix, B_{np} is a $n-k \times n-k$ matrix, and ψ^p and ξ are $(n-k) \times k$ and $k \times n-k$ matrices of structural parameters. We can use Equation (17) to write the policy shocks $e_{p,t}$ in terms of the reduced-form residuals u_t and the $(n-1) \times 1$ vector ψ^p:

$$e'_{p,t}B_p = u'_{p,t} - u'_{np,t}\psi^p. \tag{19}$$

Thus, we can express the vector of impact IRFs to the policy shocks $e'_{p,t}B_p$ as function of ψ^p and u_t by running a projection of u_t on $e'_{p,t}B_p$:

$$L_0(\psi^p, u_t)_{\cdot,p} = \left[(u'_{p,t} - u'_{np}\psi^p)(u'_{p,t} - u'_{np,t}\psi^p)'\right]^{-1}(u'_{p,t} - u'_{np,t}\psi^p)u'_t. \tag{20}$$

For an arbitrary u_t, Equation (20) characterizes the response of all variables to the fiscal policy shocks as function of the matrix ψ^p. Using simple linear algebra, we can express analytically $L_0(\psi^p, u_t)_{\cdot,p}$ in terms of ψ^p and Σ. For instance, if we set $k = 1$ and $n = 3$, Equation (20) can be written as:

$$L_0(\psi^p, \Sigma)_{j,p} = \frac{\sigma_{p,j} - \psi^p_{gdp}\sigma_{gdp,j} - \psi^p_{\pi}\sigma_{\pi,j}}{\sigma^2_p + (\psi^p_{gdp}\sigma_{gdp})^2 - 2\psi^p_{gdp}\sigma_{p,gdp} + (\psi^p_{\pi}\sigma_{\pi})^2 + 2\psi^p_{gdp}\psi^p_{\pi}\sigma_{gdp,\pi} - 2\psi^p_{\pi}\sigma_{p,\pi}}, \tag{21}$$

[19] Mertens and Ravn (2013) also use this representation —often referred to as the AB representation of a SVAR (Lütkepohl, 2005)— to derive some analytical results.

where ψ^p_{gdp} and ψ^p_π are the element of ψ^p that describe the contemporaneous response of policy to output and the other non-policy variable that—for exposition purposes—we assume to be inflation (π_t). When $j = gdp$, under the simple fiscal rule described in Equation (10), Equation (21) collapses to Equation (11).

If $k > 1$ as in our empirical application, we need to jointly identify multiple policy shocks. The additional complication is that—as we see from Equation (17)—$e'_{p,t}B_p$ are orthogonal to movements in the non-policy variables but are not orthogonal to each other because their covariance matrix $B_pB'_p$ is in general not diagonal. Hence, to identify the fiscal shocks of interest we need to impose restrictions on B_p. Results in the paper are based on the assumption that B_p reflects a Cholesky ordering where either g_t does not respond contemporaneously to tr_t (baseline specification) or vice versa (as explored in Section 5).[20]

Finally, in Equation (9) we showed how to recover the contemporaneous elasticities ψ_0 from an arbitrary $e_{p,t}$, which is needed to calculate the restrictions on the systematic component of fiscal policy implied by identification schemes that do not directly place restrictions on ψ_0. The two-step procedure described in Section 2 applies also if $k > 1$, as the mapping between the impulse responses to the policy shocks $e_{p,t}$ and the vector ψ^p is still unique. To see this, let us assume that we identify a fiscal policy shock with our preferred identification strategy. We can estimate ξ in Equation (18) by instrumenting $u_{np,t}$ with $e_{np,t}$. For a given estimate of ξ, we use Equation (18) to isolate movements in non-policy variables unrelated to the policy shock:

$$\tilde{e}'_{np,t} = e'_{np,t}B_{np} \equiv u'_{np,t} - u'_{p,t}\hat{\xi}. \tag{22}$$

$\tilde{e}_{np,t}$ is a set of instruments that we use to estimate ψ^p in Equation (17). The resulting system of instrumental variable regressions is just identified, as we have $n-k$ instruments in $\tilde{e}_{np,t}$ to estimate the $n-k$ elasticities in ψ^p.

Proof of Proposition 1

First, we prove existence of a global minimum and maximum of $M_0(\psi^p_{gdp}, \Sigma)$. Note that M_0 belongs to the family of rational functions, which are continuous and differentiable. So in order to find the global extrema of M_0 we have to investigate its first and second derivatives. Equating the first derivative to zero,

[20] Mertens and Ravn (2013) use a similar strategy to identify shocks to labor income and capital income taxes.

we obtain two points that satisfy the necessary conditions for an extremum of M_0:

$$\psi_{gdp}^{p,min} = \frac{\rho_{p,gdp} + \sigma_p\sqrt{1-\rho_{p,gdp}^2}}{\sigma_{gdp}},\ \psi_{gdp}^{p,max} = \frac{\rho_{p,gdp} - \sigma_p\sqrt{1-\rho_{p,gdp}^2}}{\sigma_{gdp}},$$

where $\rho_{p,gdp}$ denotes the correlation coefficient. It can be immediately seen that $\psi_{gdp}^{p,min} > \psi_{gdp}^{p,max}$.

The sufficient condition for extremum is checked by deriving the second derivatives of M_0 and evaluating it at $\psi_{gdp}^{p,min}$ and $\psi_{gdp}^{p,max}$:

$$M_0''\left(\psi_{gdp}^{p,min}, \Sigma\right) = \frac{\sigma_{gdp}^3\sqrt{1-\rho_{p,gdp}^2}}{2\sigma_p^3} > 0,\ M_0''\left(\psi_{gdp}^{p,max}, \Sigma\right) = -\frac{\sigma_{gdp}^3\sqrt{1-\rho_{p,gdp}^2}}{2\sigma_p^3} < 0,$$

provided that $|\rho_{p,gdp}| < 1$.

The global minimum and maximum of M_0 are

$$M_0\left(\psi_{gdp}^{p,min}, \Sigma\right) = -\frac{\sigma_{gdp}}{2\sigma_p\sqrt{1-\rho_{p,gdp}^2}} < 0,\ M_0\left(\psi_{gdp}^{p,max}, \Sigma\right) = \frac{\sigma_{gdp}}{2\sigma_p\sqrt{1-\rho_{p,gdp}^2}} > 0.$$

The second statement can be proved by studying the sign of $M_0'\left(\psi_{gdp}^{p}; \Sigma_u\right)$. The third statement can be proved using the definition of M_0:

$$M_0\left(\psi_{gdp}^{p}, \Sigma\right) = 0 \Longleftrightarrow \sigma_{p,gdp} - \psi_{gdp}^{p}\sigma_{gdp}^2 = 0 \Longleftrightarrow \psi_{gdp}^{p,chol} = \frac{\sigma_{p,gdp}}{\sigma_{gdp}^2}.$$

B The Penalty Function Approach in a Bivariate Model

In this section we study the penalty function approach (PFA) to identification applied to a bivariate model in output and taxes. To characterize analytically this approach, we describe the relationship between the reduced-form and the structural parameters by $L_0 = PQ$, where L_0 is the matrix of impact IRFs, P is the lower-triangular Cholesky factor of Σ, and Q is an orthogonal matrix (Rubio-Ramírez et al., 2010). We use the analytical expression for P and a Givens matrix representation for Q (see Golub and van Loan, 1996) to express L_0 as:

$$L_0(\theta,\Sigma) = \begin{bmatrix} \sigma_{gdp}\cos\theta & -\sigma_{gdp}\sin\theta \\ \sigma_{tr}\cos(\theta-\varphi_{gdp,tr}) & -\sigma_{tr}\sin(\theta-\varphi_{gdp,tr}) \end{bmatrix},$$

where $\theta \in [-\pi,\pi]$ is a rotation angle and $\varphi_{gdp,tr} \equiv \arccos(\rho_{gdp,tr})$ is the trigonometric representation of the correlation coefficient of the residuals u_t. By varying the rotation angle θ, we can explore all possible configurations of structural models that are consistent with the reduced-form covariance matrix Σ.

As in Mountford and Uhlig (2009), we identify the business cycle shock first and the tax shock second. In a bivariate model the identification of the business cycle shock is sufficient to identify the entire system, including the tax shock. We identify the business cycle shock by searching for an innovation that maximizes the *positive* response of output and tax revenue over a one-quarter horizon. The business cycle shock is the solution to the following constrained maximization problem:

$$\max_{\theta} \Omega^{PFA}(\theta,\Sigma) \equiv \frac{L_{0,11}(\theta,\Sigma)}{\sigma_{gdp}} + \frac{L_{0,21}(\theta,\Sigma)}{\sigma_{tr}}, \tag{23}$$

subject to

$$L_{0,11}(\theta,\Sigma) > 0, L_{0,21}(\theta,\Sigma) > 0. \tag{24}$$

The following proposition characterizes analytically the solution to the constrained optimization problem described by Equations (23) and (24). Moreover, the proposition characterizes two key properties of the solution: (i) the business cycle shock maximizes the covariance between $u_{gdp,t}$ and $u_{tr,t}$, which explains the selection of a large systematic component; (ii) the impact tax multiplier is always positive, and consequently the penalty function approach is not agnostic about the sign of the multiplier.

Proposition 2. *The constrained optimization problem defined by equations (23) and (24) has a unique global maximum with*

$$\theta^{PFA} = \frac{\varphi_{gdp,tr}}{2} \Longleftrightarrow \psi_{gdp}^{tr,PFA} = \frac{\sigma_{tr}}{\sigma_{gdp}}. \tag{25}$$

The impact tax multiplier, i.e. the response of output to a negative one-unit tax shock, is

$$-\frac{L_{0,12}(\theta^{PFA},\Sigma)}{\omega_{tr}} = \frac{\sigma_{gdp}}{2\sigma_{tr}}, \tag{26}$$

which is positive for any Σ. Finally, for $0 < \rho_{gdp,tr} < 1$, θ^{PFA} maximizes

$$\frac{L_{0,11}(\theta, \Sigma) L_{0,21}(\theta, \Sigma)}{\sigma_{gdp,tr}}, \tag{27}$$

the covariance of the one-step-ahead forecast errors explained by the business cycle shock.

Proof. First, we prove that the maximum is unique and global. The sign restrictions in Equation (24) restrict the set of admissible rotation angles as follows:

$$\begin{aligned} L_{0,11} &\geq 0 \Leftrightarrow -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \\ L_{0,21} &\geq 0 \Leftrightarrow -\frac{\pi}{2} + \varphi_{gdp,tr} \leq \theta \leq \frac{\pi}{2} + \varphi_{gdp,tr}. \end{aligned}$$

Consequently, the set of admissible θ is given by

$$S \equiv \left\{ \theta \in [-\pi, \pi] : -\frac{\pi}{2} + \varphi_{gdp,tr} \leq \theta \leq \frac{\pi}{2} \right\}. \tag{28}$$

This set is non-empty for all admissible values of the error correlation coefficient as $-\frac{\pi}{2} + \varphi_{gdp,tr} < \frac{\pi}{2}$ for all $\varphi_{gdp,tr} \in (0, \pi)$. Thus, the domain of Ω^{PFA} is a closed and bounded interval on $\mathbb{R}$.

The objective function Ω^{PFA} in Equation (23) is continuous and concave on S. The cosine function is continuous for all real numbers. To show that Ω^{PFA} is concave, it is sufficient to prove that $\cos(\theta)$ and $\cos(\theta - \varphi_{gdp,tr})$ are both concave on S (see Simon and Blume (1994), Theorem 21.8, page 519). Using the second-derivative test, it is straightforward to show that $\cos(\theta)$ is concave on the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $\cos(\theta - \varphi_{gdp,tr})$ is concave on the interval $[-\frac{\pi}{2} + \varphi_{gdp,tr}, \frac{\pi}{2} + \varphi_{gdp,tr}]$. Since S is a subset of both intervals for all $\varphi_{gdp,tr} \in (0, \pi)$, by Weierstrass's theorem, Ω^{PFA} achieves its global maximum on its domain.

To establish uniqueness we need to show that Ω^{PFA} is strictly concave on its entire domain. Note, first, that $\cos(\theta)$ and $\cos(\theta - \varphi_{gdp,tr})$ have continuous derivatives with respect to θ of every order, implying that Ω^{PFA} is also infinitely continuously differentiable. Concavity implies that $\frac{d^2 \cos(\theta)}{d\theta^2} = -\cos(\theta) \leq 0$ on S and $\frac{d^2 \cos(\theta - \varphi_{gdp,tr})}{d\theta^2} = -\cos(\theta - \varphi_{gdp,tr}) \leq 0$ on S for all $\varphi_{gdp,tr} \in (0, \pi)$. Furthermore, $\frac{d^2 \cos(\tilde{\theta})}{d\theta^2} = \frac{d^2 \cos(\tilde{\theta} - \varphi_{gdp,tr})}{d\theta^2} = 0$ for identical angle of rotation $\tilde{\theta} \in S$ if and only if $\varphi_{gdp,tr} = \pi$ (perfect negative correlation), which is ruled out by assumption. Thus, the objective function Ω^{PFA} is strictly concave on S for all $\varphi_{gpd,tr} \in (0, \pi)$, ensuring uniqueness of its global maximum.

Second, we prove that θ^{PFA} is the maximizer of Ω^{PFA} on S. The first-order condition is

$$\frac{d\Omega^{PFA}}{d\theta} = -\sin\theta - \sin(\theta - \varphi_{gdp,tr}) \stackrel{!}{=} 0,$$

which yields $\theta^{PFA} = \frac{\varphi_{gdp,tr}}{2}$ as its unique solution. The second-order condition is

$$\frac{d^2\Omega^{PFA}}{d\theta^2} = -\cos\theta - \cos(\theta - \varphi_{gdp,tr}) \stackrel{!}{<} 0,$$

which evaluated at $\theta = \theta^{PFA}$ yields

$$\left.\frac{d^2\Omega^{PFA}}{d\theta^2}\right|_{\theta=\theta^{PFA}} = -\cos\frac{\varphi_{gdp,tr}}{2} - \cos(-\frac{\varphi_{gdp,tr}}{2}) \equiv -2\cos\frac{\varphi_{gdp,tr}}{2} < 0 \text{ for all } \varphi_{gdp,tr} \in (0,\pi).$$

Third, θ^{PFA} is equivalent to the systematic component $\psi_{gdp}^{tr,PFA} \equiv \frac{L_{0,21}}{L_{0,11}} = \frac{\sigma_{tr} cos(-\frac{\varphi_{gdp,tr}}{2})}{\sigma_{gdp} cos(\frac{\varphi_{gdp,tr}}{2})} = \frac{\sigma_{tr}}{\sigma_{gdp}}$. Substituting $\psi_{gdp}^{tr,PFA}$ into Equation (11)—and accounting for the fact that we express the multiplier in terms of a tax *cut*—yields the multiplier stated in Proposition 2, which is positive for any Σ.

Finally, we prove that θ^{PFA} maximizes the one-step-ahead forecast error covariance explained by the business cylce shock:

$$\widetilde{\Omega} \equiv \frac{L_{0,11}L_{0,21}}{\sigma_{gdp,tr}} = \frac{\cos\theta\cos(\theta - \varphi_{gdp,tr})}{\rho_{gdp,tr}},$$

which is to be maximized with respect to θ. The first-order condition is

$$\frac{d\widetilde{\Omega}}{d\theta} = -(\sin\theta\cos(\theta - \varphi_{gdp,tr}) + \cos\theta\sin(\theta - \varphi_{gdp,tr})) = -\sin(2\theta - \varphi_{gdp,tr}) \stackrel{!}{=} 0,$$

which has $\theta^{PFA}(\varphi_{gdp,tr}) = \frac{\varphi_{gdp,tr}}{2}$ as its unique solution. The second derivative of $\widetilde{\Omega}$, evaluated at $\theta = \theta^{PFA}(\varphi_{gdp,tr})$, is negative

$$\frac{d^2\widetilde{\Omega}}{d\theta^2} = -\frac{2\cos(2\theta - \varphi_{gdp,tr})}{\rho_{gdp,tr}} = -\frac{2}{\rho_{gdp,tr}} < 0 \text{ for all } \rho_{gdp,tr} > 0.$$

□