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Signaling Private Choices

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For many applications of signaling, senders rather than nature choose unobserved features such as their private choices of quality, capacity, investment, contract or price, along with other actions that are (partially) observed by receivers. Despite the large number of different applications, these games have not been studied in any systematic way. We identify and study a general class of such games, which we call "endogenous signaling games". These games normally suffer from a plethora of equilibria. To focus on reasonable equilibria, we propose to solve such games by requiring that the solution be invariant to a particular reordering of the senders' moves. For a class of single-sender monotone endogenous signaling games, we show that the sender's private choice can still have some commitment value even though it is not observed and that the sender's signals must be exaggerated in equilibrium. Applications to loss-leader pricing, costly announcements, limit pricing, advertising, corporate financing, and private contracting are given.

1. INTRODUCTION

The classic treatment of signaling games assumes that a sender's unobserved features (or types) are exogenously determined by nature so that the game is one of incomplete information. While this makes sense for some applications (for instance, when a player's type is their innate ability), for other applications it may be more reasonable that the sender's unobserved features be endogenously determined (i.e. chosen by the sender). Examples include setting a low price to signal a firm's unobserved investment in cost-reducing technology (and so likely production costs), advertising to signal a firm's unobserved investment in quality control (and so likely product quality), and an entrepreneur's proposal of financing terms to signal its private choice of effort (and so likely project quality). In some other applications it makes no sense at all for nature to determine senders' unobserved features. Consider the problem of multi-product firms choosing the level of prices, as well as which prices to advertise. Clearly it makes no sense to think of nature determining the level of firms' unadvertised prices. Other examples where this kind of signaling naturally arises include unobserved capacity choices, private contracts, or a country's secret military tactics. Surprisingly, this class of games does not appear to have been studied formally, at least in any systematic way. We attempt to fill this gap.

We consider a class of imperfect-information games in which some of senders' actions are not observed by receivers but can be signaled to receivers. We call this class of games, "endogenous signaling games" to reflect that in these signaling games the unobserved features are endogenously determined by the senders. The class of games we define is quite general and includes, for example, games in which there are multiple senders and receivers, where senders can determine which choices are observed and which are unobserved (as arises when firms choose which prices to advertise), and where different receivers may have different information on senders' actions (as arises when a manufacturer sells to competing retailers through private contracts).

We propose a way to solve endogenous signaling games. To motivate our approach, consider a simple game in which a sender chooses an observed price and an unobserved price for a service it offers, and a receiver then chooses whether to subscribe to the service or not. If the sender chooses the observed price before the unobserved price, then subgame perfection would pin down the receiver's belief about the unobserved price, which should be optimally chosen given the observed price and the receiver's equilibrium strategy. If instead the unobserved price is chosen first or at the same time as the observed price, subgame perfection does not pin down the receiver's belief about the unobserved price for off-equilibrium levels of the observed price. In this case, there will generally be a multitude of subgame perfect equilibria or sequential equilibria due to different specifications of the receiver's off-equilibrium beliefs. Nevertheless, provided the sender makes its choices of unobserved and observed prices without gaining any new payoff-relevant information in between, the order in which it actually makes these choices should not matter. This is because, even if the unobserved price is set before or at the same time as the observed price, the sender should have in mind the observed price it wants to set. For this reason, a rational sender would make the same choices for the two prices irrespective of which price is actually set first, or indeed if the two prices are set at the same time. The receiver, reasoning in this way, is therefore able to form a belief about the choice of the unobserved price by noting that it should be chosen optimally given the choice of the observed price and the receiver's equilibrium strategy. That is, to form its beliefs about the unobserved price, the receiver can treat the observed price as if it was set first.

Based on this principle, in order to pin down receivers' beliefs in endogenous signaling games in which unobserved actions are chosen first or at the same time as observed actions, we use a hypothetical reordered game or games in which observed actions are modeled as though they are chosen before unobserved actions such that the reordered game or games share the same reduced normal form as the original game. We explain how the receivers' beliefs and strategies in the equilibria of the reordered games can be taken to narrow down the equilibria of the original game. In most of the examples we present, this narrowing of equilibria leads to a unique equilibrium outcome. We call this procedure *Reordering Invariance*, an equilibrium that satisfies it, an *RI-equilibrium*, and the associated beliefs *RI-beliefs.*

Requiring *Reordering Invariance* can eliminate unreasonable beliefs. As one such example, consider a game where a sender (e.g. a firm) chooses unobserved and observed effort at the same time, and a receiver (e.g. a client) then decides whether to award the contract to the firm. Passive beliefs in which the client believes that the unobserved effort is always equal to some equilibrium level regardless of the observed effort would not be reasonable if the observed effort is so low that the client would never want to award the contract—in this case, the firm is better off not engaging in any unobserved effort and the client should realize this. Instead, *Reordering Invariance* implies the client should work out what unobserved effort would be optimal for the firm given the level of observed effort *and* given the client's own equilibrium strategy.

We recognize that similar notions of invariance are present in existing solution concepts. For example, a much stronger version of invariance is used in strategic stability (Kohlberg and Mertens, 1986), and is also a feature of proper equilibrium (Myerson, 1978). As a result, the set of proper equilibrium outcomes is a subset of the set of *RI-equilibrium* outcomes, with the inclusion being strict for some games. Despite this, we view the utility of applying *Reordering Invariance* to endogenous signaling games as being threefold. First, *Reordering Invariance* is much weaker than the concept of invariance used in the literature, which requires invariance to any transformation of the extensive-form game that leaves the reduced normal form unchanged, rather than the particular reordering we propose, making *Reordering Invariance* easier to verify and interpret. Second, *Reordering Invariance* requires no other aspects to verify or interpret. For example, proper equilibrium involves limits of sequences of vanishingly small "trembles" of a particular kind. The analyst may have little intrinsic interest in whether an equilibrium satisfies these requirements, much less with any difficulties that come with actually verifying them, at least not in endogenous signaling games. Third, the burden of applying such refinements may be significantly increased in games with uncountably infinite actions (Simon and Stinchcombe, 1995), which comprise a host of important endogenous signaling games. *Reordering Invariance* adapts straightforwardly to such games.[1]

We show how to apply *Reordering Invariance* to a wide variety of endogenous signaling games, many of which are not traditionally thought of as games involving signaling. As we detail in Section 2, researchers have studied games like these over the last several decades. However, they have done so seemingly in ignorance of the common class of games they have been solving, and often based on *ad-hoc* or intuitive arguments for why they pin down beliefs in a certain way.

In a simple class of single-sender games in which a sender chooses unobserved actions before (or at the same time as) observed actions and then a receiver chooses some actions, we show an *RI-equilibrium* outcome yields the best payoff to the sender among all the equilibria of the original game. This reflects that in the reordered game, the sender can choose the observed action to select the subgame which generates the best payoff to the sender from all the equilibria of the proper subgames of the reordered game, including those corresponding to all the equilibria of the original game. To the extent that it is natural to focus on an equilibrium that yields the best payoff to the first mover, this provides an additional reason to focus on *RI-equilibrium.*

Adding a monotone structure to the sender's and receivers' payoffs, we obtain some additional properties. In these games, which we call "monotone endogenous signaling games", the sender can potentially influence the receivers' choices to its benefit through its signal. Compared to the equilibrium where no signaling arises because the receiver holds fixed beliefs about the sender's unobserved actions (i.e. the receiver has passive beliefs), in the equilibrium satisfying *Reordering Invariance* the sender chooses "higher" levels of both the unobserved and observed actions, and obtains a higher payoff. This shows that a sender can still benefit indirectly from its "investment" through its effect on the receivers' beliefs and therefore choices, even though its investment choice is unobserved, provided it can be signaled through some observed actions such as prices or quantities.

Another implication of the monotone endogenous signaling games we study is that a sender will exaggerate its choice of signal in equilibrium, compared to the equivalent choice of signal in a full-information game. This is somewhat similar to the well-known property of separating equilibria in classical signaling games, in which a high-type sender has to exaggerate its signal to ensure the low-type sender does not want to mimic its choice of signal. In our setting, such "signal exaggeration" arises in equilibrium to ensure

1. The connection between *Reordering Invariance* and existing equilibrium concepts is further discussed in Section 5.

that the sender itself does not have any incentive to further manipulate beliefs by choosing a different signal.

The rest of the paper proceeds as follows. Section 2 discusses the related literature. A simple illustrative example is provided in Section 3. Section 4 derives some implications for a class of endogenous signaling games with a single sender. These implications are illustrated with several applications in Section 4.2. Section 5 generalizes the class of endogenous signaling games, explains how *Reordering Invariance* can be applied in some more complicated settings, and provides the linkages with other approaches to narrowing down the equilibria such as requiring equilibria be proper and using "wary beliefs". Section 6 briefly concludes.

2. RELATED LITERATURE

As mentioned in the introduction, we do not know of any systematic analysis of the class of games we consider. In this section we briefly note studies of specific applications that belong to our class. Since the application to private contracting and the use of wary beliefs in the existing literature (McAfee and Schwartz, 1994; Rey and Vergé, 2004) will be discussed in detail in Section 5.4, we will not discuss it further here.

A well-established literature has studied how firms use their price to signal their choice of quality. Prominent early examples include Klein and Leffler (1981), Wolinsky (1983), Farrell (1986), Riordan (1986), Rogerson (1988) and Bester (1998). In contrast to classical signaling models, in this literature, equilibria have generally been pinned down by a seemingly *ad-hoc* approach to how expectations are formed for off-equilibrium price offers. For example, several of these studies assume that although prices and quality are set jointly (at one move), consumers interpret prices that are different from equilibrium levels as "an indication of high quality if the seller has no incentive to disappoint this expectation" (Bester, p. 834).

Some other specific applications of endogenous signaling games that precede our paper include (i) Dana (2001), who studies firms that compete by choosing unobserved inventory and observed prices, (ii) Rao and Syam (2001), who study supermarkets that compete by choosing which of two different goods to advertise the price of at the same time as determining the two prices, and (iii) Bueno de Mesquita and Stephenson (2007), who study a policymaking agent that chooses both observed and unobserved effort (to improve the quality of regulation) in the face of an overseer that can veto new regulation. Each of these studies has its own justification for pinning down receivers' off-equilibrium beliefs in a particular way.

Some more recent work has explicitly applied our *Reordering Invariance* approach to pin down the equilibria in the endogenous signaling games they consider. In monetary economics, Rocheteau (2011) uses *Reordering Invariance* to study the issue of counterfeiting, in which buyers can first choose whether to counterfeit an asset which acts as a medium of exchange and then choose what price to offer when paying with this asset. In a similar way, Li *et al.* (2012) applies *Reordering Invariance* in their study of the liquidity of assets in trade, in which agents choose a portfolio of genuine and fraudulent assets for trade, and the terms of trade. See also Lester *et al.* (2012). In our own work, In and Wright (2014), we use *Reordering Invariance* to solve for the equilibrium in a setting where multi-product firms set prices of substitute goods, and not all prices are observed by consumers. Rhodes (2015) analyzes a richer environment in which consumers search for prices at multi-product firms and want to buy multiple products, and also refers

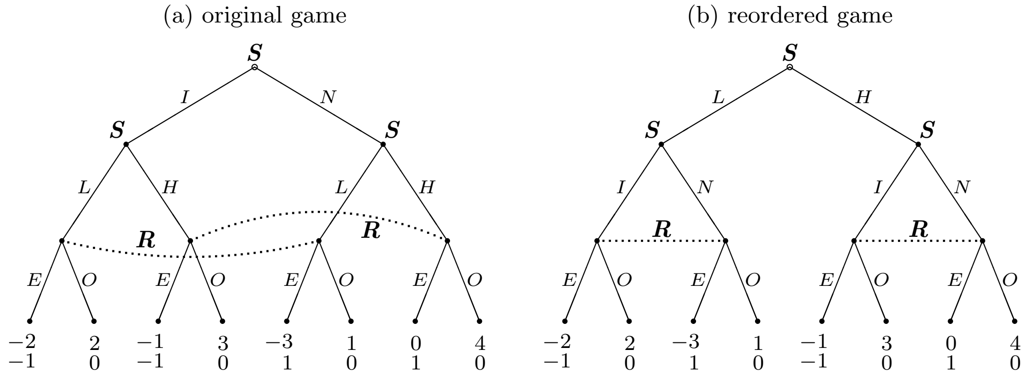

FIGURE 1
Illustration of original and reordered games

to *Reordering Invariance* to justify why he focuses on a reordered game in which the advertised price is set first.

3. ENTRY DETERRENCE WITH UNOBSERVABLE INVESTMENT

As an example of an endogenous signaling game, consider the following simple game. There are two players, one sender (S) and one receiver (R). The sender first chooses an unobserved action followed by an observed action.[2] Subsequently, the receiver chooses an action. Panel (a) of Figure 1 illustrates the game. Note that this example would become a classical signaling game if the first move was made by nature rather than by the sender.

In the figure, the sender, an incumbent firm, first chooses whether to invest in some cost-reducing project (I) or not (N). Following this, the incumbent chooses a low price (L) or high price (H). Observing the incumbent's choice of price but not whether it invested in the project, the receiver, a potential rival, chooses whether to enter (E) or stay out (O). The particular payoffs given are meant to capture the key features of an entry deterrence game with unobservable investment such that (i) the incumbent always prefers the rival to stay out; (ii) the rival prefers to stay out (enter) if it thinks the incumbent has (not) invested in the project; and (iii) the incumbent prefers (not) to invest in the project if it intends to choose a low (high) price.

For this simple game, there are many different subgame-perfect equilibria, which due to the lack of proper subgames also coincide with the set of Nash equilibria. After a tedious process, one can find a continuum of subgame-perfect equilibria as follows[3]:

$$(I, L, pL + (1-p)H; O, qE + (1-q)O)_{p\in[0,1],\, q\in[\frac{1}{2},1]}$$
$$(N, pL + (1-p)H, H; qE + (1-q)O, E)_{p\in[0,1],\, q\in[\frac{1}{2},1]}.$$

2. We could have equally started from a situation where the sender chooses the unobserved action together with the observed action at a single move. In this case, the reordered game remains the same as that specified below, and the equilibrium outcome that we obtain from *Reordering Invariance* is equivalent to that specified below. We could have also started with a situation where the sender chooses the observed action first, but in this case *Reordering Invariance* is trivial and has no effect.

3. Note that there are also a continuum of sequential (perfect Bayesian) equilibria as follows: $(I, L, L; O, qE + (1-q)O)_{q\in[\frac{3}{4},1]}$, $(I, L, pL + (1-p)H; O, \frac{3}{4}E + \frac{1}{4}O)_{p\in(0,1)}$, $(I, L, H; O, qE + (1-q)O)_{q\in[\frac{1}{2},\frac{3}{4}]}$, $(N, L, H; qE + (1-q)O, E)_{q\in[\frac{1}{2},\frac{3}{4}]}$, $(N, pL + (1-p)H, H; \frac{3}{4}E + \frac{1}{4}O, E)_{p\in(0,1)}$, and $(N, H, H; qE + (1-q)O, E)_{q\in[\frac{3}{4},1]}$.

The multiplicity problem also remains in a game where the incumbent makes the investment and price choices together at a single move.

The subgame-perfect equilibria where the rival stays out with a positive probability upon observing the incumbent's choice of the high price are unreasonable because the incumbent's choice of investing is dominated by its choice of not investing in case it intends to choose the high price. The subgame-perfect equilibria where the rival enters with a positive probability upon observing the incumbent's choice of the low price are unreasonable because the incumbent's choice of not investing is dominated by its choice of investing in case it intends to choose the low price. Note all passive-beliefs equilibria belong to this set of unreasonable subgame-perfect equilibria since they involve the rival entering with probability one regardless of whether the price is high or low. In this illustrative example, only subgame-perfect equilibria $(I, L, pL+(1-p)H; O, E)_{p\in[0,1]}$ are free from these problems, all of which support the same outcome (I, L, O), in which the incumbent invests in the cost-reducing project, sets a low price and the rival stays out.

Now consider the extensive-form game in panel (b) of Figure 1, where we reordered the incumbent's investment and price choices. This game, which we call the "reordered game", shares the same reduced normal form (up to the relabeling of the strategies) as the original game. Surprisingly, even allowing the players to randomize over their actions, we find a unique subgame-perfect equilibrium of this game $(L, I, N; O, E)$.[4] This reflects the fact that the reordering creates proper subgames. The incumbent chooses the low price, invests following the choice of the low price, and does not invest following the choice of the high price; the rival stays out upon observing the low price, and enters upon observing the high price.

If the rival's strategy in the original game is required to be the same as that in the reordered game, the resulting subgame-perfect equilibria are $(I, L, pL + (1 - p)H; O, E)_{p\in[0,1]}$. This corresponds to the set of equilibria that we argued above were reasonable. All these equilibria share the same outcome (up to the reordering of the incumbent's chosen actions) as the subgame-perfect equilibrium in the reordered game: the incumbent chooses to invest and sets the low price; the rival stays out. It can also be checked that the incumbent obtains its best payoff at this outcome, among all the subgame-perfect equilibria of the original game.

4. CANONICAL ENDOGENOUS SIGNALING GAMES

We first consider a simple class of games in which there is a single sender and a single receiver,[5] as in the example of Section 3. This enables us to define a monotone structure on payoffs. The timing of moves is as follows:

1 In stage 1a, the sender chooses $t \in T$.
2 In stage 1b, the sender chooses $a \in A$.
3 In stage 2, having observed the sender's choice of a, but not t, the receiver chooses $b \in B$.

The moves in stages 1a and 1b constitute the signaling stage and the moves in stage 2 constitute the reaction stage. The analysis in this section continues to apply even if the

4. Indeed, the unique sequential equilibrium is the strategy profile $(L, I, N; O, E)$, together with a system of beliefs, including the rival's beliefs such that it believes the incumbent has invested in the cost-reducing project upon observing the low price, and not invested upon observing the high price.

5. In case there are multiple receivers that share the same information regarding the sender's actions, the results in this section continue to hold.

sender chooses t and a jointly at one move in stage 1. The payoffs, when the outcome is (t, a, b), are denoted $u_S(t, a, b)$ and $u_R(t, a, b)$ to each player respectively. We call this class of games "canonical endogenous signaling games", denote the class by Γ_S^1, and its typical element by G_S^1. Section 5 defines a more general class of endogenous signaling games.

One example of a game in Γ_S^1, applied to entry deterrence with unobservable investment, was illustrated in Section 3. Other examples abound. In the case of product-quality signaling, t could be a firm's unobserved investment in quality, a the firm's price and advertising, and b the buyer's decision to purchase or not. In a corporate finance setting, t could be the entrepreneur's private effort to determine the likely project quality, a the entrepreneur's offer of financing contract, and b the investor's decision to invest or not. These and other examples are discussed in Section 4.2.

For any $G_S^1 \in \Gamma_S^1$, we consider the following hypothetical extensive-form game, where the timing of the sender's moves is now reversed as follows:

1 In stage 1a, the sender chooses $a \in A$.
2 In stage 1b, the sender chooses $t \in T$.
3 In stage 2, having observed the sender's choice of a, but not t, the receiver chooses $b \in B$.

The payoffs are the same for each player as in the original game if the outcome is the same up to the reordering of the sender's chosen actions. Abusing notation slightly, we use the same notation u_S and u_R for the payoff functions of the reordered game as well. We denote this game by G_r^1.

We can now apply *Reordering Invariance* to the subgame-perfect equilibrium concept for the class of canonical endogenous signaling games Γ_S^1.

Definition 1 (*RI-equilibrium*). A subgame-perfect equilibrium of a game in Γ_S^1 satisfies *Reordering Invariance* if the receiver's strategy is also a part of a subgame-perfect equilibrium in the reordered game. We call such a subgame-perfect equilibrium an *RI-equilibrium.* ||

Applying *Reordering Invariance* has a strong decision-theoretic justification. Given that each receiver's strategies and beliefs at each of its information sets are the same across the two games, a sender's optimal choice of actions at sequential moves should not depend on the order of its moves if the sender does not gain any new payoff-relevant information between its moves. Indeed, as one of the requirements for strategic stability (Kohlberg and Mertens, 1986), invariance was proposed, which states that a solution of a game should also be a solution of any equivalent game (i.e. having the same reduced normal form). Instead of considering all equivalent variants, we only consider particular reordered variants of the original game since doing so addresses the primary source of indeterminacy in canonical endogenous signaling games. Specifically, for games in Γ_S^1, reordering produces proper subgames, which explains why our definition of *RI-equilibrium* used in this section is based on the subgame-perfect equilibrium concept.[6]

6. In the more general class of games in Section 5, reordering may not create proper subgames, but as will be shown, a similar type of reordering still works by "disentangling" the receivers' information sets. This explains why the definition of *RI-equilibrium* used in Section 5 is based on the sequential equilibrium concept. Moreover, even if reordering creates proper subgames, it may not lead to a unique equilibrium outcome. In this case, one may still apply additional requirements in the reordered games, such as requiring that strategies be undominated or survive the iterative elimination of weakly-dominated strategies.

Note that the subgame-perfect equilibrium in the reordered game induces a Nash equilibrium in each proper subgame of the reordered game. In the Nash equilibrium, the receiver chooses its best response to its (correct) belief about the sender's choice of action. We call these beliefs *RI-beliefs*.[7]

A practical advantage of applying *Reordering Invariance* in endogenous signaling games in which senders choose unobserved actions before (or at the same time as) observed actions is that it is often relatively easy to apply. Solving for the equilibria of the original game can be cumbersome for these games, unless *ad-hoc* belief functions (such as passive beliefs) are adopted. This point was illustrated with the example in Section 3. A more dramatic illustration of the point is given in Section 4.2.1.

In some applications, when there are multiple equilibria and the first mover can choose its action to signal its intention to play a particular equilibrium strategy, it is natural to focus on the equilibrium that yields the best payoff to the first mover. The following proposition shows that in canonical endogenous signaling games, where there is a single sender, such an equilibrium outcome is always an *RI-equilibrium* outcome.[8]

Proposition 1 (Optimality). *Consider a game $G^1_S \in \Gamma^1_S$. Suppose that the set of RI-equilibria is nonempty and the set of their payoffs to the sender admits a maximal element. Then <u>at least</u> <u>one</u> RI-equilibrium yields the best payoff to the sender among all subgame-perfect equilibria. Furthermore, any subgame-perfect equilibrium that is not outcome-equivalent to an RI-equilibrium yields a strictly lower payoff to the sender than any RI-equilibrium.*

The proof of the proposition is based on the fact that the payoffs in a subgame-perfect equilibrium of G^1_S correspond to Nash equilibrium payoffs in some of the proper subgames of its reordered game. In case the sender's payoff in a subgame-perfect equilibrium is greater than or equal to that in some inferior *RI-equilibrium*, this allows us to construct a new *RI-equilibrium* from the subgame-perfect equilibrium and the inferior *RI-equilibrium*, which is outcome-equivalent to the subgame-perfect equilibrium. In the following corollary, we provide a sufficient condition such that any *RI-equilibrium* yields the best payoff to the sender. Under the sufficient condition, if the receiver chooses its strategy in the game G^1_S in accordance with *Reordering Invariance*, the best payoff is always available to the sender, making an *RI-equilibrium* with a lower payoff to the sender a contradiction.[9]

Corollary 1 (Complete optimality). *Consider a game $G^1_S \in \Gamma^1_S$ and its reordered game G^1_r. Suppose that the set of RI-equilibria is nonempty and the set of their payoffs to the sender admits a maximal element. Consider proper subgames (in G^1_r) which contain Nash equilibria in which the sender gets the same payoff as the best payoff to the sender among all subgame-perfect equilibria in G^1_S. Suppose that for at least one such proper subgame, all Nash equilibria yield the same payoff to the sender. Then <u>all</u> RI-equilibria yield this best payoff to the sender.*

7. Note that there is a one-to-one correspondence between the receiver's information sets in the original game and those in the reordered game. Therefore, we can use *RI-beliefs* to refer to the receiver's particular beliefs at the (non-singleton) information sets in the original game as well.

8. Obviously this need not be the case when there are multiple senders, for example, if the senders are competitors.

9. Formal proofs of Proposition 1 and Corollary 1 are given in an Online Appendix.

4.1. *Monotone endogenous signaling games*

Thus far, we have not put any particular structure on the players' payoffs. However, typically our interest in these games stems from the fact that the sender can influence the receiver's choices to its benefit through its own choice of signals. In this section we explore a particular structure of payoffs which gives rise to this feature and explore some of its economic implications.

Specifically, we consider a setting which has a monotone structure, such that (i) the sender has an incentive to induce the receiver to choose a higher level of action; (ii) in order that the receiver prefers to choose a higher level of action, the sender must convince the receiver that it has chosen a higher level of the unobserved action; and (iii) the sender wants to choose a higher equilibrium level of the unobserved action when it intends to choose a higher level of the observed action.[10]

In order to make our assumptions on the monotone structure, we assume first that the action sets T, A, and B are linearly ordered sets, such as $\{\text{Enter}, \text{Not enter}\}$ with an order Enter $<$ Not enter and $\{1, 2, 3\}$ with an order $1 < 2 < 3$. We also assume that there exists a unique pure-strategy subgame-perfect equilibrium $(a^*, t^*(\cdot); b^*(\cdot))$ in the reordered game G_r^1 of a game $G_S^1 \in \Gamma_S^1$ such that[11]

$$\begin{aligned} a^* &\in \arg\max\nolimits_{a \in A} u_S(t^*(a), a, b^*(a)), \\ t^*(a) &\in \arg\max\nolimits_{t \in T} u_S(t, a, b^*(a)) \quad \forall a \in A, \quad \text{and} \\ b^*(a) &\in \arg\max\nolimits_{b \in B} u_R(t^*(a), a, b) \quad \forall a \in A. \end{aligned}$$

This implies that the outcome (t^*, a^*, b^*) is an *RI-equilibrium* outcome in G_S^1.

Corresponding to the informal conditions (i)-(iii) above, suppose (i) the sender's payoff function u_S is strictly increasing in b, (ii) $\arg\max_{b \in B} u_R(t, a, b)$ is unique and strictly increasing in t for all $a \in A$, and (iii) the function $t^*(\cdot)$ is strictly increasing in a. Since the term "increasing" is based on particular linear orders over the sets, this setting actually captures eight different monotone structures, corresponding to changing "increasing" to "decreasing" anywhere in the conditions (i)-(iii), or equivalently reversing the linear orders over the sets T, A, and B. These conditions are sufficient but not necessary to obtain our results. The functions need not be increasing globally and in the strict sense as stated above but we assume these stronger properties to simplify the proofs.[12]

We consider these conditions on the reordered game as the primitives of the problem of characterizing properties of the equilibria of the original game. This takes advantage of the fact that analyzing the reordered game is often straightforward. We call original games satisfying the conditions (i)-(iii) on their corresponding reordered games "monotone

10. Cho and Sobel (1990, pp. 391-392) identified three main conditions (their A1′, A3, and A4) to give classical signaling games a certain monotone structure. Our first two conditions correspond to their A1′ and A3. Our third condition is distinct from their A4, which guarantees that the sender of higher type is more willing to send a higher signal. Instead, in our setting what matters is that the sender wants to choose a higher "type" (i.e. unobserved feature) in the equilibrium when it intends to choose a higher signal. This reflects the difference in game structure (i.e. who chooses the "type") rather than any difference in payoff structure. It explains why sometimes we obtain qualitatively different results, as shown in Section 4.2.

11. When we use the same letter to denote both a function and a dependent variable of the function, we distinguish the function from the dependent variable by adding $(\cdot)$ after the letter, as in $t(\cdot)$. For brevity, hereafter we suppress the arguments of $t(\cdot)$, $a(\cdot)$, and $b(\cdot)$ at the equilibrium outcomes, so that $t^* \equiv t^*(a^*)$, $b^* \equiv b^*(a^*)$, etc.

12. Nevertheless, the following three links are necessary for our results: (i) the sender's payoff must depend on b, (ii) $\arg\max_{b \in B} u_R(t, a, b)$ must depend on t, and (iii) $t^*(\cdot)$ cannot be a constant function.

endogenous signaling games", and denote this class of games by Γ_S^+, and its typical element G_S^+.

Let $(\tilde{t}, \tilde{a}(\cdot); \tilde{b}(\cdot))$ be a pure-strategy *RI-equilibrium* of G_S^+ which is outcome-equivalent to and shares the same receiver's strategy with the unique pure-strategy subgame-perfect equilibrium of the reordered game, $(a^*, t^*(\cdot); b^*(\cdot))$.

4.1.1. Commitment effect. The first property we explore is whether the sender's private choice can still have some commitment value in the *RI-equilibrium* even though it is not observed. This is possible if the sender's choice of observed action a can signal its choice of unobserved action t. The benchmark for determining whether the sender's private choice has some commitment value is an equilibrium sustained by the receiver's passive beliefs. In this case, the receiver ignores any observed actions by the sender when forming its beliefs about the unobserved actions.

Consider a pure-strategy passive-belief equilibrium of G_S^+, $(t^{pa}, a^{pa}(\cdot); b^{pa}(\cdot))$, which satisfies

$$\begin{aligned} t^{pa} &\in \arg\max\nolimits_{t \in T} u_S(t, a^{pa}(t), b^{pa}(a^{pa}(t))), \\ a^{pa}(t) &\in \arg\max\nolimits_{a \in A} u_S(t, a, b^{pa}(a)) \quad \forall t \in T, \quad \text{and} \\ b^{pa}(a) &\in \arg\max\nolimits_{b \in B} u_R(t^{pa}, a, b) \quad \forall a \in A. \end{aligned}$$

Note that the receiver's belief about t is passive and fixed at t^{pa} in this equilibrium.

For games in Γ_S^+, we prove in the appendix that the sender's choices of both observed and unobserved actions at the *RI-equilibrium* outcome are (weakly) higher than those in a passive-belief benchmark.

Proposition 2 (Commitment effect). *For any game $G_S^+ \in \Gamma_S^+$, $\tilde{t} \geq t^{pa}$, $\tilde{a} \geq a^{pa}$, and $u_S(\tilde{t}, \tilde{a}, \tilde{b}) \geq u_S(t^{pa}, a^{pa}, b^{pa})$.*

Starting from a passive-belief equilibrium outcome, an increase in a has three effects on the sender's payoff: it may (1) directly change the sender's payoff (direct effect), (2) directly affect the receiver's choice of action b and so the sender's payoff (strategic effect), and (3) indirectly affect the receiver's choice of action b through the receiver's belief about the sender's choice of t and so the sender's payoff (signaling-commitment effect). The sender's choice of a in the passive-belief equilibrium outcome is "optimal" with respect to the sum of the first two effects, given that the third effect is zero by the definition of passive beliefs, which means the first-order effect of the sum of the direct and strategic effects is zero in differentiable cases. On the other hand, the third effect is positive when the receiver holds *RI-beliefs*.

When $\tilde{t} > t^{pa}$ and $u_S(\tilde{t}, \tilde{a}, \tilde{b}) > u_S(t^{pa}, a^{pa}, b^{pa})$ because of a positive signaling-commitment effect, we say there is a "commitment effect". In this case, even though the sender's private choice is unobserved, it is still chosen as though it is observed, to some extent. That is, the sender's private choice may still have some commitment value in terms of influencing the receiver's choices to its benefit (relative to the outcome under passive beliefs). We discuss several applications in which the commitment effect arises in Section 4.2.

Under the class of games with monotone structure considered in this section, Proposition 2 shows that the relevant *RI-equilibrium* outcomes differ in general from the corresponding passive-belief equilibrium outcomes. The applications in Sections 4.2 and 5.4 further illustrate the differences between the outcomes under *RI-beliefs* and passive beliefs. Given that in our view one should focus on *RI-equilibria* in such games, these findings can be viewed as a critique of using passive beliefs in endogenous signaling games in which the signaling-commitment effect is non-zero.

4.1.2. Signal exaggeration. Another interesting property of monotone endogenous signaling games is what we refer to as "signal exaggeration", in which the sender's signal is exaggerated in equilibrium, compared to the equivalent choice of signal in a full-information game.

To formalize what we mean by signal exaggeration, we define an observable benchmark and compare the sender's choice in the *RI-equilibrium* outcome with that in the benchmark. Consider a variant of the game $G_S^+ \in \Gamma_S^+$ where the action t is now observed by the receiver. We denote it by G^{ob}. Consider an equilibrium of G^{ob}, $(t^{ob}, a^{ob}(\cdot); b^{ob}(\cdot,\cdot))$, where

$$t^{ob} \in \arg\max_{t\in T} u_S(t, a^{ob}(t), b^{ob}(t, a^{ob}(t))),$$
$$a^{ob}(t) \in \arg\max_{a\in A} u_S(t, a, b^{ob}(t, a)) \quad \forall t \in T, \quad \text{and}$$
$$b^{ob}(t, a) \in \arg\max_{b\in B} u_R(t, a, b) \quad \forall (t, a) \in T \times A.$$

For games in Γ_S^+, we prove in the appendix that the sender would choose a (weakly) higher observed action at the *RI-equilibrium* outcome than it would choose if the unobserved action was actually observed by the receiver.

Proposition 3 (Signal exaggeration). *For any game $G_S^+ \in \Gamma_S^+$, $\tilde{a}(\tilde{t}) \geq a^{ob}(\tilde{t})$.*

Starting from the *RI-equilibrium* outcome, the sum of the direct and strategic effects (defined earlier) is negative. This is because the sum of all three effects (direct, strategic, and signaling-commitment effects) should be zero at the *RI-equilibrium* outcome in differentiable cases and because the signaling-commitment effect is positive from the monotone structure of the games. An implication is that the sender would prefer to choose a lower action a when $\tilde{t}$ is observed, compared to when it chooses a to manipulate the receiver's belief (as it does at the *RI-equilibrium* outcome).

Signal exaggeration arises when a multi-product firm sets a low advertised price to signal to consumers that it has set low prices for its other substitute products that are not advertised. Starting from its normal multi-product monopoly prices, a small decrease in the monopolist's observed price does not give rise to any first-order loss to the firm. However, since it lowers consumers' beliefs about the level of its other unobserved prices (for substitute goods), it can attract more consumers to shop at the firm, and so it does have a first-order benefit to the firm. Thus, in equilibrium, the firm's announced price must be exaggerated (in the downward direction) to ensure that it does not have any incentive to further manipulate beliefs, i.e. so consumers are not fooled.

In a separate paper we have applied this logic to the pricing of upgrades, where the price of a firm's basic version of some good is advertised but the upgraded version is not (see In and Wright, 2014). Applying *Reordering Invariance*, we find that the advertised price (i.e. the price of the basic version) is set below cost, consistent with loss-leader pricing. Rhodes (2015) finds a similar effect in a more general multi-product setting. Thus, loss-leader pricing can be viewed as an implication of signal exaggeration.

4.2. *Applications*

In this section we illustrate the above results using several different applications, contrasting some of the implications with those from classical signaling games. For each application, the full model and corresponding analysis is provided in the Online Appendix.

4.2.1. Costly announcements and inflation. In monotone endogenous signaling games, the sender would naturally want to reveal its actual choice of its unobserved feature to the receiver, thereby avoiding the need to engage in costly signal exaggeration. This may not be possible if its actual choice is not verifiable. However, provided there is some cost to lying, the sender's announcement can still work as an inflated signal of its actual choice. A costly announcement game is therefore a somewhat generic example of a monotone endogenous signaling game.

Consider the following simple game. In stage 1a, a sender chooses its private effort t which is unobserved by a receiver. In stage 1b, the sender makes an announcement a on its level of effort. In stage 2, the receiver reacts by choosing some action b. To capture this situation, we adopt the standard quadratic payoff specification of cheap-talk games as in Crawford and Sobel (1982) (the first term in (4.1)), and modify it to introduce a cost to lying as in Kartik *et al.* (2007) (the second term in (4.1)) and so that the sender cares about its choice of effort (the third term in (4.1)). The payoffs are

$$u_S(t,a,b) = -(b-(t+\theta))^2 - \kappa(a-t)^2 - \lambda(t-\hat{t}\,)^2 \tag{4.1}$$

for the sender and $u_R(t,a,b) = -(b-t)^2$ for the receiver, with $\kappa > 0$ and $\lambda > 0$.

For example, this specification captures a government that privately instructs a (non-independent) central bank to achieve a target level of inflation t while publicly announcing a target level a, where $\hat{t}$ represents the government's first-best level of inflation. The choice of b captures the private sector's expectation of the level of inflation. The parameter θ captures the government's bias: here $\theta < 0$ captures the government's incentive to pursue higher inflation rates than are expected. Another application of the model is to false advertising.

This simple game highlights the difficulty of finding all the equilibria of the original game. Focusing only on pure-strategy equilibria, and assuming linearity (not to mention differentiability) of the receiver's equilibrium strategy, i.e. $\bar{b}(a) = c_0 + c_1 a$, there are a continuum of subgame perfect equilibria in the original game. In contrast, solving for the subgame perfect equilibrium of the reordered game is trivial, and leads to the unique *RI-equilibrium* outcome of the original game $\tilde{t} = \tilde{b} = \hat{t} - \theta/(\lambda+\kappa)$ and $\tilde{a} = \hat{t}$.

Consistent with the commitment effect, if $\theta > 0$, then t and a will both be strictly higher in the *RI-equilibrium* than in the passive-beliefs equilibrium. Even though the sender's effort is unobserved, it is still able to obtain a commitment benefit in the *RI-equilibrium*. This is not an obvious result, given that the sender chooses its announcement after (or equivalently, at the same time as) fixing its level of effort, so that in general the receiver need not react to the announcement at all (i.e. passive beliefs) or in any particular way.

Consistent with signal exaggeration, we also find the announcement is inflated: $\tilde{a} = \hat{t} > a^{ob}(\tilde{t}) = \tilde{t} = \hat{t} - \theta/(\lambda+\kappa)$ if $\theta > 0$. Although the sender still chooses less effort than its first-best level $\hat{t}$, due to signal exaggeration it turns out that its announced effort level a is exactly equal to its ideal level of effort $\hat{t}$. This surprising result is specific to the quadratic payoff specification (in general it may announce a somewhat higher or lower level of effort compared to $\hat{t}$).

4.2.2. Limit pricing and business strategy. The commitment effect of Proposition 2 implies that much of the large literature studying business strategy in which firms commit to observed "investments" so as to benefit from strategic effects can be redone using our framework even when investments are not observed, provided they can be signaled through related observed actions, such as an incumbent's pricing or output choices. Such applications are a cross between the literature studying business strategy in which firms' endogenous (but observed) "investments" have commitment value and the classical models of limit pricing and signaling in oligopolies where firms' unobserved (but exogenously determined) costs are signaled through the product market.

Consider, for example, limit pricing, in which an incumbent monopolist sells at a price below its normal monopoly price to make entry appear unattractive. Traditional limit pricing models (e.g. Milgrom and Roberts, 1982) assume that the incumbent's marginal cost of production is determined exogenously by nature. Suppose instead that the incumbent's marginal cost of production is determined endogenously by its unobserved investment in cost-reducing R&D in the first stage. If the R&D is successful, it lowers the incumbent's production costs to the level of an otherwise more efficient potential entrant's production costs. The more the incumbent invests, the greater the probability of success. Initially, the incumbent chooses how much to invest and announces its price. After the outcome of its R&D efforts are realized, the incumbent produces output to meet demand at its announced price. Upon observing this price but not the incumbent's investment or the success of its R&D, the potential entrant faces a one-off opportunity to enter the market. If it enters, the rival learns the outcome of the incumbent's R&D effort, and the firms compete in prices. If there is no entry, the incumbent sets it price in the final stage to maximize its monopoly profit taking into account the outcome of its R&D efforts. In the Online Appendix, we characterize the *RI-equilibrium* outcome of this game.

In the specification of payoffs we use, the incumbent prices below its original marginal cost in the first stage if it prefers to invest to deter entry. Thus, since the cost-reducing R&D may not be successful, this implies one element of predatory pricing holds (pricing below marginal cost is possible). This below-cost pricing is an extreme case of signal exaggeration. Starting from its normal monopoly price, a small decrease in the monopolist's price does not give rise to any first-order loss. It, however, raises the rival's beliefs about the level of cost-reducing R&D the monopolist has made, since a lower pre-entry price means the incumbent will face higher demand prior to the rival's entry, and so it has a higher payoff from investing in R&D to lower its costs. In equilibrium, the incumbent's pre-entry price is exaggerated (in the downward direction) to the point that the incumbent wants to invest at a level that would deter entry. This below-cost price signals to the potential entrant, that the incumbent has invested a lot in cost-reducing R&D and so it stays out.

Note the mechanism of limit pricing here is quite distinct from that in the separating equilibrium of a traditional limit pricing model, in which a low-cost incumbent sets a "low" price in order to prevent a high-cost incumbent mimicking it. Unlike traditional models, here limit pricing has effects beyond just the pricing of firms. Consistent with the commitment effect, we find that the incumbent will invest as much in cost-reducing R&D when it engages in limit pricing as in the case its investment is observable (and so can directly deter entry), although this happens over a smaller range of parameter values compared to the case in which investment is observable.

4.2.3. Quality choice, burning money and advertising. The previous limit pricing application illustrates the differences that can arise when the sender's unobserved features are modeled as chosen by the sender rather than by nature. A key difference arises from the fact that the sender makes an additional choice in our setting (the choice of its unobserved action), meaning there are additional ways it can deviate, which must be unprofitable in any equilibrium. In addition, what matters for equilibria in monotone endogenous signaling games is that the sender wants to choose a higher "type" (i.e. unobserved feature) when it intends to choose a higher signal, whereas what matters for separating equilibria in classical signaling games is that the sender of higher type is more willing to send a higher signal. This distinction can sometimes lead to qualitatively very different results.

For example, consider the standard result of exogenous-quality settings in which a high quality firm may choose a positive level of wasteful advertising (together with a high price) to make it unprofitable for a low quality firm to mimic it (Milgrom and Roberts, 1986). In contrast, when quality is chosen by the firm, "burning money" will never arise as an *RI-equilibrium* outcome since such advertising is treated like a sunk cost, which has no bearing on the firm's optimal choice of its unobserved quality even in a setting with repeat purchases. As we show in the Online Appendix, advertising can play a "signaling" role in an endogenous-quality setting once it has a demand-expanding effect.

4.2.4. Corporate finance and costly collateral pledging. Another setting in which the qualitative results depend on who determines the unobserved feature (nature or the sender) is the signaling model of costly collateral pledging. In the classical signaling model (Tirole, 2006, pp.251-254), an improvement in the high type borrower's prospects would increase the equilibrium level of collateral pledged to offset the fact that mimicking by a low type would become more profitable. In contrast, in an identical model except that the borrower's quality is now determined by its prior efforts, we find that an improvement in the prospects for a borrower that has chosen high effort decreases the *RI-equilibrium* level of collateral pledged. This reflects that the need to engage in costly signaling to incentivize effort is less important when the sender anyway prefers to put in more effort. Thus, the comparative statics of the *RI-equilibrium* in our setting may be easier to reconcile with the empirical evidence (Coco, 2000, p. 191), which suggests borrowers post less collateral when their prospects improve.

5. ENDOGENOUS SIGNALING GAMES

In this section we show how *Reordering Invariance* applies to a broader class of endogenous signaling games. We also discuss how it relates to other approaches, including proper equilibrium and wary beliefs.

5.1. *Class of games*

The approach of using reordered games to narrow down the equilibria as in Section 4 can potentially be applied to a broad class of games where endogenous signaling may arise — a class of games where reordering the moves of any player would not change the reduced normal form of the game. A simple requirement for games to be in this class is that no player obtains more refined information on other players' actions or nature's random choices between its own moves that we need to reorder. In order that signaling can arise, we add the requirement that some actions or combinations of actions of a player are partially observed by some other players. Even with these requirements, reordering may not ensure the information sets of the receivers are "disentangled" in a way that enables the consistency requirement of the sequential equilibrium concept to have any bite. To ensure this we also require that the private information that a player possesses and therefore can potentially signal to another player is only with respect to its own choice of actions. In what follows, we define a class of endogenous signaling games that satisfies all three requirements.

Definition 2 (Endogenous signaling games). Let Γ denote the class of finite extensive-form games with perfect recall. A game in Γ is called an *endogenous signaling game* if it satisfies the following properties[13]:

1 There are two stages: the *signaling stage* followed by the *reaction stage*.
2 In each stage, player(s) move without observing each other's actions.
3 For at least one player who moves in the signaling stage and one player who moves in the reaction stage, some of the first player's actions (or combinations of actions) are partially observed by the second player. For all such cases, the first player is called a *sender* and the second player is called a *receiver*. Receivers move only in the reaction stage.
4 Each player's information on another player's previous actions is independent of its information on a third player's previous actions.

We denote the collection of endogenous signaling games by Γ_S. ||

A sender's action (or combination of actions if the sender has more than one information set) is partially observed by a receiver if the receiver knows at its particular information set that some actions (including the action) but not all actions are possibly taken at the sender's information set. Obviously, signaling cannot arise if all actions of a player are either observed or unobserved by other players. If all actions are observed there would be no information to signal, whereas if all actions are unobserved there would be no way to signal any information. We require receivers move only in the reaction stage for expositional simplicity. The property in item 4 above is a mild property that holds in all applications we have considered, including all games that fit within the class in Section 4, multiple-sender games discussed below, and the games in Section 5.4. In the Online Appendix we formally define this property.

5.2. *RI-equilibrium*

5.2.1. Reordering in simple cases. In case all actions available at each of senders' first information sets are unobserved and all actions available at each of senders' succeeding information sets are observed by any receiver, the reordering is very simple. In this case, we just need to reorder the senders' moves such that all observed actions are chosen before all unobserved actions, while maintaining that senders do not observe each others' actions in this reordering. In the example in Section 3, where the sender's actions L and H are observed and I and N are unobserved, the reordering involves placing the choice between L and H before the choice between I and N (see panel (b) of Figure 1). In the game in panel (a) of Figure 2, which is a multiple-sender version of the previous game

13. We can trivially allow for random choices by nature provided the information on nature's random choices is the same across all players.

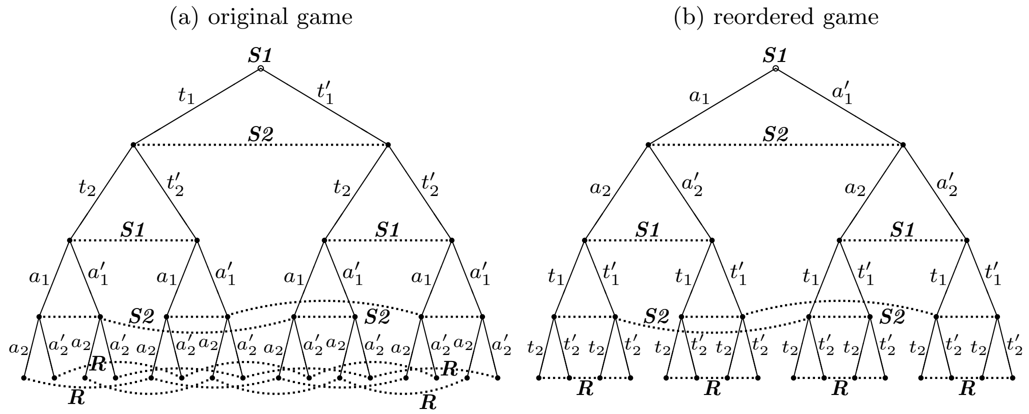

FIGURE 2
Multiple senders

and where the senders' actions a_1, a_1', a_2, a_2' are observed and t_1, t_1', t_2, t_2' are unobserved, the reordering means that choices are made in the sequence of a_1/a_1', a_2/a_2', t_1/t_1', t_2/t_2' (see panel (b) of Figure 2).

5.2.2. Reordering in general. To handle more complicated cases, for example where some of the senders' actions are partially observed by receivers or the observability of a sender's first action depends on its choice of the subsequent action, we provide a general algorithm in the Online Appendix along with a guide to reordering. Henceforth, we refer to the reordering based on the algorithm as "reordering", and games obtained by the reordering as "reordered games". An example of the former case is a modified version of the game in panel (a) of Figure 1, in which the sender makes the two choices jointly at one move. Applying the algorithm, we would obtain the same reordered game as in panel (b) of Figure 1. An example of the latter case is a game in which competing supermarkets (senders) choose the prices of all their goods, as well as which prices to advertise, without observing each other's choices. In this case, the advertising decision will determine which price(s) will be observed by the buyers (receivers) at the time they decide which supermarket to go to. The game in panel (a) of Figure 3, where the nine unlabeled information sets belong to the receiver (R), has a similar structure. If sender i ($i = 1, 2$) chooses the action r_i ("reveal") then its chosen action at the preceding information set becomes observed by the receiver whereas if it chooses the action n_i ("not reveal") then its chosen action at the preceding information set becomes unobserved. Applying the algorithm, we would obtain the reordered game as in panel (b) of Figure 3.

5.2.3. *RI-equilibrium.* We use sequential equilibrium (Kreps and Wilson, 1982) as our basic equilibrium concept for the class of endogenous signaling games Γ_S. It is natural to use a more refined equilibrium concept for games with a richer structure. A sequential equilibrium is a pair consisting of a system of beliefs and a strategy profile satisfying sequential rationality and consistency. Following convention, we often refer to only the strategy profile, omitting the system of beliefs, as a sequential equilibrium.

As was noted for the example in Section 3, typically there are a multitude of equilibria in games in Γ if players' beliefs are not pinned down at non-singleton information sets.

(a) original game (b) reordered game

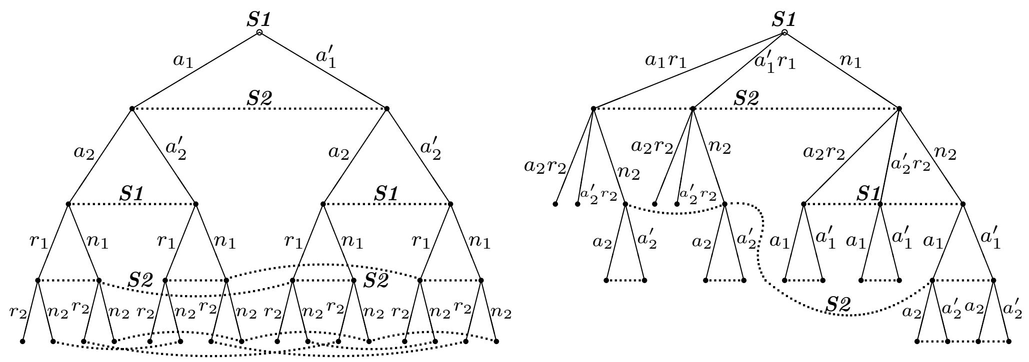

Figure 3
Endogenized observability

Definition 3 (Definiteness). We call an extensive-form game without particular payoff functions a “game structure”. Consider an extensive-form game in Γ and its game structure. We say that a non-singleton information set is *definite*[14] in the game structure if for any strategy profile, there is a unique belief at the information set such that the consistency requirement of sequential equilibrium is satisfied in any game with the game structure, and *indefinite* otherwise. ‖

In the Online Appendix, we provide a necessary and sufficient condition on the game structure for a non-singleton information set to be definite, which is used in the proof of Proposition 4 below.

For games in Γ_S, our proposed algorithm involves constructing the reordered game for each group of receivers with the same information on the senders' actions, which we call *associated receivers.* The reordering only uses a combination of the coalescing of moves and the interchange of moves (see Thompson, 1952 for the definition of these two transformations), and therefore does not affect the reduced normal form. The reordering places “more observable” actions before “less observable” actions, and by doing so makes the relevant non-singleton information sets definite. This property of reordered games is summarized as a proposition below. As illustrated in Figures 2 and 3, the reordering “disentangles” the receivers' information sets while leaving the senders' decision problems essentially unchanged.

Proposition 4 (Disentanglement). *Consider any game in Γ_S. Suppose that if the associated receivers' information on the senders' actions changes as the game progresses, the way it changes is independent of the associated receivers' choice of actions. Then, in any of its reordered games, all non-singleton information sets of the associated receivers are definite.*

Proposition 4, which is proven in the Online Appendix, explains why it is easier to solve for the sequential equilibria of the reordered games than those of the original game.

14. What is definite is not the information set itself but rather the beliefs conditional on the information set for any given behavior strategy profile (provided the consistency requirement is satisfied), but we use this terminology for the sake of brevity.

This is a generalization of the fact that reordering creates proper subgames for canonical endogenous signaling games. In the richer class of endogenous signaling games Γ_S, the reordering may not create proper subgames (for example, see panel (b) of Figures 2 and 3), yet requiring the equilibria to be invariant under the reorderings can still help narrow down the equilibria of the original game. The condition in Proposition 4 is a sufficient condition to obtain the definiteness. We explain how it can be relaxed in the Online Appendix. Now we are ready to apply the *Reordering-Invariance* requirement to the sequential equilibrium concept to use for this more general class of games Γ_S.

Definition 4 (*RI-equilibrium*). A sequential equilibrium of a game in Γ_S satisfies *Reordering Invariance* if each receiver's strategy and beliefs at all of its non-singleton information sets are also a part of a sequential equilibrium in every reordered game of the original game. We call such equilibrium an *RI-equilibrium* and the receivers' beliefs *RI-beliefs*. ||

A natural question that arises is how we find all the *RI-equilibria* for a game in Γ_S. Note that the set of terminal nodes in each of the reordered games correspond one-to-one to the set of terminal nodes in the original game. For example, the terminal node resulting from the terminal history (I, H, E) in the original game corresponds to the terminal node resulting from the terminal history (H, I, E) in the reordered game in Figure 1. We say outcomes in the original game and reordered games are *equivalent* if they exhibit the same probability distributions over the set of terminal nodes when the one-to-one correspondence is considered. Since the games in Γ_S are finite, it is trivial to show the equivalence of the set of *RI-equilibrium* outcomes and the intersection of the sets of sequential equilibrium outcomes in the reordered games.

Remark 1 (Outcome equivalence). *A. For any game in Γ_S, if an outcome is induced by a sequential equilibrium in each of its reordered games and the sequential equilibria contain the same receivers' strategies and beliefs, then there exists an RI-equilibrium in the original game with the equivalent outcome and the same receivers' strategies and beliefs. B. For any game in Γ_S, if an outcome is induced by an RI-equilibrium, then there exists a sequential equilibrium in each of its reordered games with the equivalent outcome and the same receivers' strategies and beliefs.*

Since it is easier to solve for the sequential equilibria of the reordered games than those of the original game, this outcome-equivalence result provides a shortcut to find all the *RI-equilibrium* outcomes for games in Γ_S. We just need to solve the reordered games for the sequential equilibria and reorder/relabel the actions of the sequential equilibrium outcomes to fit the original game.

5.3. *Relationship with proper equilibrium*

Outcome equivalence, when combined with existing theorems in the literature, provides us with some additional results. Consider a proper equilibrium in the reduced normal-form game of a game in Γ_S, which exists by Myerson's theorem (Myerson, 1978) because the reduced normal-form game is finite. From proposition 0 in Kohlberg and Mertens (1986) we know that in each of the reordered games there exists a behavior strategy profile which is equivalent[15] to the proper equilibrium and forms a sequential equilibrium.

15. "Equivalent" is in the sense of Kuhn's theorem (Kuhn, 1953).

By the outcome-equivalence result, we can construct an *RI-equilibrium* in the original game, which has the same outcome (and the same receivers' strategies and beliefs) as the sequential equilibria in the reordered games. Therefore, we have the following results.

Remark 2 (Existence of *RI-equilibrium*). *For any game in Γ_S, there exists an RI-equilibrium.*

Remark 3 (Relationship with proper equilibrium). *For any game in Γ_S, the set of proper equilibrium outcomes is a subset of the set of RI-equilibrium outcomes.*

It can be shown that the inclusion is strict for some games in Γ_S. This is because a proper equilibrium outcome is invariant for any transformation of the extensive-form game that leaves the reduced normal form unchanged, whereas an *RI-equilibrium* outcome is required to be invariant only for particular reordering transformations of the extensive form. Sometimes, considering only *RI-equilibria* that are proper may be too restrictive. To illustrate this, in the Online Appendix we provide an example of a canonical endogenous signaling game which is also in the broader class Γ_S. In the example, there is an *RI-equilibrium* which is not a proper equilibrium, yet it is arguably the most reasonable equilibrium—it has a strong forward-induction justification and it survives the iterative elimination of weakly-dominated strategies (IEWDS) for any order of elimination in the corresponding reduced normal form. In contrast, the proper equilibria do not survive IEWDS for any order of elimination in the example.

An implication of Remark 3 is that if there is a unique *RI-equilibrium* outcome in a game in Γ_S, it has to be the proper equilibrium outcome. In such a case, *Reordering Invariance* provides a simpler way to find the proper equilibrium outcome than the usual methods. For example, we know that in the example in Section 3, the unique proper equilibrium outcome is (I, L, O) from the fact that it is the unique *RI-equilibrium* outcome.

5.4. *Relationship with wary beliefs*

In this section, we show how the approach of using wary beliefs as defined in the literature on vertical contracting relates to applying *Reordering Invariance*. This also illustrates how to handle the case where receivers have different information on senders' actions and thus more than one reordering is required to find the *RI-equilibria*.

Consider first the classic opportunism problem of vertical contracting. Consider a game with one sender (an upstream supplier) and two receivers (downstream retailers). The upstream supplier chooses contracts $a_1 \in A_1$ and $a_2 \in A_2$ jointly at one move. Then downstream retailers, R_1 and R_2, decide whether to accept contracts and choose prices (or quantities) $b_i \in B_i$ ($i \in \{1, 2\}$) simultaneously, having partially observed the supplier's contracts. Specifically, retailer 1 only observes contract a_1 and retailer 2 only observes contract a_2.

There are two reordered games. In reordered game 1, the supplier chooses $a_1 \in A_1$ for retailer 1 in stage 1a, and $a_2 \in A_2$ for retailer 2 in stage 1b. In stage 2, the retailers choose $b_i \in B_i$ ($i \in \{1, 2\}$) simultaneously. In reordered game 2, stage 1a and 1b are reversed, so the supplier chooses $a_2 \in A_2$ for retailer 2 first and then $a_1 \in A_1$ for retailer 1. With these two reordered games, we can find *RI-equilibria* as follows. We solve both reordered games simultaneously assuming that each retailer holds the same beliefs and chooses the same strategy across the two reordered games.

Formally, the strategy profile $(\tilde{a}_1, \tilde{a}_2; \tilde{b}_1(\cdot); \tilde{b}_2(\cdot))$ is a pure-strategy *RI-equilibrium* if for each reordering i ($i \in \{1,2\}$ with $j \neq i$)

$$\tilde{a}_i \in \arg\max_{a_i \in A_i} u_S(a_i, \tilde{a}_j(a_i), \tilde{b}_i(a_i), \tilde{b}_j(\tilde{a}_j(a_i))), \quad (5.2)$$

$$\tilde{a}_j(a_i) \in \arg\max_{a_j \in A_j} u_S(a_i, a_j, \tilde{b}_i(a_i), \tilde{b}_j(a_j)) \quad \forall a_i \in A_i, \quad (5.3)$$

$$\tilde{b}_1(a_1) \in \arg\max_{b_1 \in B_1} u_{R_1}(a_1, \tilde{a}_2(a_1), b_1, \tilde{b}_2(\tilde{a}_2(a_1))) \quad \forall a_1 \in A_1, \quad (5.4)$$

$$\tilde{b}_2(a_2) \in \arg\max_{b_2 \in B_2} u_{R_2}(\tilde{a}_1(a_2), a_2, \tilde{b}_1(\tilde{a}_1(a_2)), b_2) \quad \forall a_2 \in A_2. \quad (5.5)$$

Note that reordering i by itself does not pin down retailer j's beliefs $\tilde{a}_i(a_j)$, which are used in (5.5) for reordering 1 and in (5.4) for reordering 2. By combining the equilibrium conditions from both reordered games, we can pin down both retailers' beliefs in the *RI-equilibrium.* Note also that by combining (5.2) and (5.3) for $i \in \{1,2\}$, it is straightforward to show that (5.2) for $i \in \{1,2\}$ can be replaced by

$$(\tilde{a}_1, \tilde{a}_2) \in \arg\max_{a_1 \in A_1, a_2 \in A_2} u_S(a_1, a_2, \tilde{b}_1(a_1), \tilde{b}_2(a_2)). \quad (5.6)$$

Then (5.3) for $i \in \{1,2\}$ together with (5.4)-(5.6) define the *RI-equilibrium* in the original game.

How does the approach above compare to the approach taken in the literature on vertical contracting? A key issue in this literature is how downstream firms form beliefs about the suppliers' offers to rivals in response to unexpected (off-equilibrium) offers they receive. McAfee and Schwartz (1994) proposed three different types of beliefs, one of which was wary beliefs. Wary beliefs require that "each firm thinks that others received offers that are the monopolist's optimal choices given the offer it made to that firm". However, the exact meaning of "optimal choices" is not clear without additional conditions. McAfee and Schwartz (p.222) added the precise conditions, written using our notation, as follows: when R_i receives a contract a_i from the set of all contracts that a retailer would accept, it believes that (a) the supplier expects it to accept this contract; (b) the supplier offers R_j ($j \neq i$) the contract $a_j^*(a_i)$ that is the best for the supplier, given that R_i accepts a_i, among all contracts acceptable to R_j; and (c) R_j reasons in the same way. In case a_i is not from the set of contracts that a retailer would accept, then McAfee and Schwartz note that condition (b) becomes: the supplier offers R_j ($j \neq i$) the contract $a_j^*(a_i)$ that is the best for the supplier, given that R_i does not accept a_i, among all contracts acceptable to R_j.[16] Rey and Vergé (2004) argue wary beliefs are more plausible than passive beliefs in the context of the opportunism problem of vertical contracting.

In the vertical contracting games considered by McAfee and Schwartz (1994) and Rey and Vergé (2004), wary beliefs turn out to be just a special case of *RI-beliefs*. Specifically, replacing a_i with the two-part tariff (f_i, w_i) and replacing b_i with the retailer's decision (I_i, q_i) of whether to participate and a quantity (or price), the conditions for an equilibrium with wary beliefs characterized by Rey and Vergé coincide with (5.3)-(5.6) above. Given that only acceptable contracts need to be considered for this particular game, conditions (a) and (c) in the definition of wary beliefs are actually redundant since they are implicit in the definition of a sequential equilibrium. The fact that wary beliefs is an example of *RI-beliefs* in this particular vertical contracting game provides a strong

16. Note that there is a circularity in the definition of wary beliefs. On the one hand, the set of contracts that is acceptable to R_i depends on determining $a_j^*(a_i)$. On the other hand, $a_j^*(a_i)$ depends on determining the set of such contracts since the supplier's optimal contract to R_j for a given a_i depends on whether R_i accepts the contract a_i or not.

game-theoretic justification for the focus by Rey and Vergé on wary beliefs rather than passive beliefs in this game.

The opportunism problem of vertical contracting described above is just one of many examples of private contracting where *Reordering Invariance* could be applied. Segal (1999) considers a number of applications where there are multiple agents (receivers) each of which might receive a private offer from the principal (the sender) that could fit within our class of endogenous signaling games. These include applications to exclusive dealing (in case the exclusive deal is private and includes the terms of the offer), selling a patent (or other indivisible asset) where there are externalities across buyers and terms are private, and takeovers involving private offers to existing shareholders. In each of these cases, it may no longer always be optimal to make offers that are accepted by all agents, meaning the focus on acceptable contracts in the definition of wary beliefs may be problematic. More generally, receivers' choices may not involve binary decisions which implies that the contract acceptance part of the definition of wary beliefs would make no sense at all. An example is the announcement game in Section 4.2.

The interpretation of wary beliefs can also be unclear when there are multiple senders who interact, such as games with competing suppliers, each of which chooses actions that are partially observed by buyers. The interpretation of "the contract that is best for the supplier" is not clear from the existing definition of wary beliefs, given what is best for a supplier will depend on its expectation about what other suppliers will do, and how receivers will react. For example, Hagiu and Halaburda (2014) consider competing platforms that have to attract users and developers, where the utility of users depends on the number of developers that sign up (and vice-versa). In their setting, while developers know the prices charged on each side, users only observe their own prices. Hagiu and Halaburda characterize an equilibrium, which they note is an *adaptation* of wary beliefs to their setting. Requiring *Reordering Invariance* makes it clear how to proceed for such games, and indeed all endogenous signaling games.

6. CONCLUDING REMARKS

There are many interesting applications of endogenous signaling games, and we had space to only consider a few. Here we conclude by mentioning two further applications worthy of further exploration.

A common assumption in the literature on add-on pricing, such as Ellison (2005), is that firms set unobserved add-on prices after setting the advertised prices of their base goods. But if all prices are actually set at the same time, as is more reasonable in many settings, the results could differ from the existing literature since firms would not be able to condition on each others' advertised prices when setting unobserved add-on prices. Provided each seller's best add-on price depends on the price of its base good, say because a lower price for the base good selects consumers who are more sensitive to add-on prices or alternatively because some consumers are fully informed of both sets of prices, then we have an endogenous signaling game to which *Reordering Invariance* could fruitfully be applied.

Existing price-signaling models in which firms choose quality require some mechanism for a high price to signal that a firm has chosen high quality such as the possibility of a repeat purchase. In a search context, in which consumers observe prices (e.g. through advertising) but quality only after a search, and then decide whether to complete the purchase or continue searching, such mechanisms are not needed anymore. Consumers will continue searching if the seller has not chosen a sufficiently high quality relative to the price it has set. Future work could explore such search settings using *Reordering Invariance*.

APPENDIX

Proof of Proposition 2. If $(\tilde{t}, \tilde{a}, \tilde{b}) = (t^{pa}, a^{pa}, b^{pa})$ then the inequalities are trivially satisfied. Therefore, we assume $(\tilde{t}, \tilde{a}, \tilde{b}) \neq (t^{pa}, a^{pa}, b^{pa})$.

Suppose $\tilde{a} = a^{pa}$ but $(\tilde{t}, \tilde{b}) \neq (t^{pa}, b^{pa})$. Then both $(\tilde{t}, \tilde{b})$ and (t^{pa}, b^{pa}) are Nash equilibria in the subgame following $\tilde{a}$. The sender's payoff is strictly greater at $(\tilde{t}, \tilde{b})$ than at (t^{pa}, b^{pa}) since there would be more than one subgame-perfect equilibrium in the reordered game otherwise. Therefore, the two inequalities are satisfied: $\tilde{a} \geq a^{pa}$ and $u_S(\tilde{t}, \tilde{a}, \tilde{b}) \geq u_S(t^{pa}, a^{pa}, b^{pa})$. In the following, we show $\tilde{t} \geq t^{pa}$. Suppose not ($\tilde{t} < t^{pa}$). Since $\arg\max_{b \in B} u_R(t, a, b)$ is unique and strictly increasing in t for all $a \in A$, this implies $\tilde{b} < b^{pa}$. For (t^{pa}, b^{pa}) to be a Nash equilibrium in the subgame, it should be satisfied that $u_S(t^{pa}, \tilde{a}, b^{pa}) \geq u_S(\tilde{t}, \tilde{a}, b^{pa})$. From $\tilde{b} < b^{pa}$, we also have $u_S(\tilde{t}, \tilde{a}, b^{pa}) > u_S(\tilde{t}, \tilde{a}, \tilde{b})$ since u_S is strictly increasing in b. Combining these two inequalities on the sender's payoffs, we obtain $u_S(t^{pa}, \tilde{a}, b^{pa}) > u_S(\tilde{t}, \tilde{a}, \tilde{b})$, which contradicts the fact that the sender's payoff is strictly greater at $(\tilde{t}, \tilde{b})$ than at (t^{pa}, b^{pa}) in the subgame.

Now we assume $\tilde{a} \neq a^{pa}$. Since $\arg\max_{b \in B} u_R(t, a, b)$ is strictly increasing in t for all $a \in A$, if a is such that

$$\left\{ \begin{array}{c} t^*(a) < t^{pa} \\ t^*(a) = t^{pa} \\ t^*(a) > t^{pa} \end{array} \right\} \text{ then } \left\{ \begin{array}{c} \arg\max_{b \in B} u_R(t^*(a), a, b) < \arg\max_{b \in B} u_R(t^{pa}, a, b) \\ \arg\max_{b \in B} u_R(t^*(a), a, b) = \arg\max_{b \in B} u_R(t^{pa}, a, b) \\ \arg\max_{b \in B} u_R(t^*(a), a, b) > \arg\max_{b \in B} u_R(t^{pa}, a, b) \end{array} \right\}.$$

Since $\tilde{b}(a) = b^*(a)$, it follows that $\tilde{b}(a) < b^{pa}(a)$, $\tilde{b}(a) = b^{pa}(a)$, and $\tilde{b}(a) > b^{pa}(a)$ respectively. Since u_S is strictly increasing in b, this implies that

$$\text{if } a \text{ is such that } \left\{ \begin{array}{c} t^*(a) < t^{pa} \\ t^*(a) = t^{pa} \\ t^*(a) > t^{pa} \end{array} \right\} \text{ then } \left\{ \begin{array}{c} u_S(t, a, \tilde{b}(a)) < u_S(t, a, b^{pa}(a)) \\ u_S(t, a, \tilde{b}(a)) = u_S(t, a, b^{pa}(a)) \\ u_S(t, a, \tilde{b}(a)) > u_S(t, a, b^{pa}(a)) \end{array} \right\} \quad \forall t \in T.$$

Note $t^*(a^{pa}) = t^{pa}$ since (t^{pa}, b^{pa}) is the unique Nash equilibrium in the subgame following a^{pa} in the reordered game. This is because the strategy profile (t^{pa}, b^{pa}) is a Nash equilibrium in the subgame, and furthermore, it is the unique Nash equilibrium in the subgame since there exists a unique subgame-perfect equilibrium in the reordered game and the subgame is off the equilibrium path. Since $t^*(\cdot)$ is strictly increasing in a,

$$\text{if } \left\{ \begin{array}{c} a < a^{pa} \\ a = a^{pa} \\ a > a^{pa} \end{array} \right\} \text{ then } \left\{ \begin{array}{c} u_S(t, a, \tilde{b}(a)) < u_S(t, a, b^{pa}(a)) \\ u_S(t, a, \tilde{b}(a)) = u_S(t, a, b^{pa}(a)) \\ u_S(t, a, \tilde{b}(a)) > u_S(t, a, b^{pa}(a)) \end{array} \right\} \quad \forall t \in T.$$

Note that $u_S(\tilde{t}, \tilde{a}, \tilde{b}) = \max_{(t, a(\cdot)) \in T \times A^T} u_S(t, a(t), \tilde{b}(a(t)))$ and $u_S(t^{pa}, a^{pa}, b^{pa}) = \max_{(t, a(\cdot)) \in T \times A^T} u_S(t, a(t), b^{pa}(a(t)))$. Therefore, $u_S(\tilde{t}, \tilde{a}, \tilde{b}) \geq u_S(t^{pa}, a^{pa}, b^{pa})$ and $\tilde{a} \geq a^{pa}$. Since $t^*(\cdot)$ is strictly increasing in a, $\tilde{t} = t^*(\tilde{a}) \geq t^*(a^{pa}) = t^{pa}$. ||

Proof of Proposition 3. Note that $\tilde{b}(a) = b^*(a) \in \arg\max_{b \in B} u_R(t^*(a), a, b)$ for all $a \in A$, and $b^{ob}(t, a) \in \arg\max_{b \in B} u_R(t, a, b)$ for all $(t, a) \in T \times A$. Since $\arg\max_{b \in B} u_R(t, a, b)$ is strictly increasing in t for all $a \in A$ and u_S is strictly increasing in b, following the same argument as in the proof of Proposition 2, we obtain the following statements:

$$\text{If } a \text{ is such that } \left\{ \begin{array}{c} t^*(a) < t \\ t^*(a) = t \\ t^*(a) > t \end{array} \right\} \text{ then } \left\{ \begin{array}{c} u_S(t, a, \tilde{b}(a)) < u_S(t, a, b^{ob}(t, a)) \\ u_S(t, a, \tilde{b}(a)) = u_S(t, a, b^{ob}(t, a)) \\ u_S(t, a, \tilde{b}(a)) > u_S(t, a, b^{ob}(t, a)) \end{array} \right\} \quad \forall t \in T.$$

Substituting $\tilde{t}$ for t in the above statements, and noting that $t^*(\tilde{a}(\tilde{t})) = \tilde{t}$ and the fact that $t^*(\cdot)$ is strictly increasing in a, we obtain the following statements:

$$\text{If } \left\{ \begin{array}{c} a < \tilde{a}(\tilde{t}) \\ a = \tilde{a}(\tilde{t}) \\ a > \tilde{a}(\tilde{t}) \end{array} \right\} \text{ then } \left\{ \begin{array}{c} u_S(\tilde{t}, a, \tilde{b}(a)) < u_S(\tilde{t}, a, b^{ob}(\tilde{t}, a)) \\ u_S(\tilde{t}, a, \tilde{b}(a)) = u_S(\tilde{t}, a, b^{ob}(\tilde{t}, a)) \\ u_S(\tilde{t}, a, \tilde{b}(a)) > u_S(\tilde{t}, a, b^{ob}(\tilde{t}, a)) \end{array} \right\}.$$

Note that $u_S(\tilde{t}, \tilde{a}(\tilde{t}), \tilde{b}(\tilde{a}(\tilde{t}))) = \max_{a \in A} u_S(\tilde{t}, a, \tilde{b}(a))$ and $u_S(\tilde{t}, a^{ob}(\tilde{t}), b^{ob}(\tilde{t}, a^{ob}(\tilde{t}))) = \max_{a \in A} u_S(\tilde{t}, a, b^{ob}(\tilde{t}, a))$. Therefore, $\tilde{a}(\tilde{t}) \geq a^{ob}(\tilde{t})$. ||

Supplementary Data

Supplementary data are available at Review of Economic Studies online.

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