Securitization and Lending Competition*

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Abstract

We study the effects of securitization on interbank lending competition. An applicant’s observable features are seen by a remote bank, while her true credit quality is known only to a local bank. Without securitization, the remote bank does not compete because of a winner’s curse. With securitization, in contrast, ignorance is bliss: the less a bank knows about its loans, the less of a lemons problem it faces in selling them. This enables the remote bank to compete successfully in the lending market. Consistent with the empirical evidence, remote and securitized loans default more than observationally equivalent local and unsecuritized loans, respectively.

JEL: D82, G14, G21.

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1 Introduction

Securitization of conventional home mortgages began in 1970 with the founding of the Federal Home Loan Mortgage Corporation. The proportion of mortgages held in market-based instruments rose steadily from 20% in 1980 to 68% in 2008. Earlier evidence indicates that securitization rose from 1975 to 1980 as well (Jaffee and Rosen [30, Table 2]).

Remote lending has also grown. Petersen and Rajan [42, Figures I and II] find an upward trend in distances between small firms and their lenders that began in about 1978 or 1979 and continued through the end of their data in 1992. The mean borrower-lender distance in a sample of small business loans studied by De Young, Glennon, and Nigro [19, pp. 125-6] rose from 5.9 miles in 1984 to 21.5 miles in 2001. Remote lending of residential mortgages also rose from 1992 to 2007 (Loutskina and Strahan [36, p. 1477]).

We show that securitization can lead to remote lending even when remote banks have an informational disadvantage vis-a-vis local banks. We assume that an applicant’s soft information is known only to a local bank while her hard information is known also to a remote bank. Without securitization, any profitable loan offer of the remote bank would be outbid by the local bank. Anticipating this, the remote bank cedes the entire market to the local bank. Under securitization, the remote bank’s ignorance has an advantage: investors will not suspect it of choosing only its bad loans to sell. This enables the remote bank to compete successfully for applicants with strong enough observables.

Our model yields several empirical predictions that are confirmed by recent research.

\[1\] A detailed history of securitization appears in Hill [29].

\[2\] The source is unpublished data underlying Figure 3 in Shin [48].

\[3\] This phenomenon was first studied theoretically by Hauswald and Marquez [28], Rajan [43], and Sharpe [47]. They show that if banks must hold their loans to maturity, then banks with superior information (or, in Hauswald and Marquez [28], a lower cost of gathering information) about loan applicants will have a competitive advantage in lending because of a winner’s curse.

\[4\] In a prior empirical paper, Loutskina and Strahan [36] suggest that banks may have an incentive to lend remotely in order to avoid private information at the time of securitization. They do not model this phenomenon theoretically.
Securitization stimulates lending in general and remote lending in particular. Securitized loans have higher conditional default rates than unsecuritized loans. Remote lenders securitize a higher proportion of their loans. Remote borrowers have stronger observables than local borrowers and pay lower interest rates, but have higher conditional default rates.

Without securitization, lending is limited as the local bank cannot sell its loans. By lifting this limitation, securitization expands lending, which raises welfare. But it also encourages entry by the remote bank, which makes worse lending decisions as it lacks soft information. Despite this, securitization cannot lower welfare. Moreover, securitization raises welfare if, with securitization, either bank lends.

An intuition is as follows. The remote bank cannot be harmed by securitization as it can always choose not to lend. In practice, it ensures positive profits by lending only to agents with strong observables, thus ensuring that its proportion of low-quality borrowers will be small. Hence, securitization helps the remote bank when it lends and does not harm it otherwise.

As for the local bank and the agents, securitization affects them in two ways. It lowers the interest rate the local bank can charge because of competition from the remote bank. This is a pure transfer with no welfare effects. It also allows the local bank to resell some of its loans. This cannot harm the local bank, which can always choose not to securitize. We show, moreover, that the local bank will profitably securitize a portion of its portfolio if, under securitization, it lends at all. Finally, investors are assumed to be fully rational and competitive, so their payoffs are identically zero: they are unaffected by securitization. This completes the intuition.

Our finding that securitization does not harm investors conflicts with the popular narrative in which securitization enabled sophisticated finance professionals to profit by foisting toxic securities on unsuspecting, naive security buyers. However, the available scientific evidence does not support this narrative. Cheng, Raina, and Xiong [10] find that midlevel

\[\text{The evidence for this consists mainly of selected emails from before the crash, as well as testimony from afterwards (e.g., Financial Crisis Inquiry Commission [21, pp. 3-24.]).}\]
managers in securitized finance systematically overinvested in their own private housing in the years leading up to the crash. Ma [37] finds that the chief executive officers of banks that lent more aggressively during the boom had a greater tendency to increase their own holdings of their banks’ stock which, during the subsequent crash, fell more than the stock of less aggressive lenders.

The evidence for optimism among securitizers and lenders has two possible interpretations. First, it may be that beliefs were correct on average and the crash resulted from an unusually bad shock. In our model, a bad shock will lead many projects to fail. Investors who bought the banks’ securities will lose money. Banks will be harmed by low prices for their loans and by poorly performing loans that remain on their books. In this way, the widespread losses experienced during the 2008-9 crash are consistent with correct \textit{ex ante} beliefs combined with an unusually low realization of the macro shock.

A second - and perhaps more likely - interpretation is that there was a housing bubble in the early 2000s that led to unrealistic expectations of continued house price appreciation among market participants. Our model is consistent with this theory if we assume that the players’ prior beliefs are incorrect. Our results then imply that securitization raised expected social welfare under these incorrect beliefs. It may well have lowered welfare under correct beliefs. Unfortunately, it is not clear how to distinguish between the bursting of a bubble and a particularly bad shock.

This paper contributes to the literature on security issuance under asymmetric information. In Leland and Pyle [35], an issuer sells a security to a continuum of risk-neutral, uninformed investors. Before choosing how much to sell, the issuer sees private information about her security’s value. In equilibrium, she varies the amount that she sells in order to signal her information to investors. This is very costly for her, as in equilibrium she must

\footnote{In particular, the house price return forecasts in some analysts’ reports in 2005 and 2006 were much higher than the long-run historical average, although they were in line with the lofty experience of the prior few years (Foote, Gerardi, and Willen [22, p. 18]). Similarly, homebuyers’ expectations of long-run house price appreciation were unusually high, relative to mortgage rates, at the height of the boom, and have fallen sharply since then (Case, Shiller, and Thompson [9]).}
sell less of her security precisely when the gains from trade are higher.\textsuperscript{7} DeMarzo and Duffie [16] show that these costs give an issuer an incentive to design a security whose payout is insensitive to her private information.\textsuperscript{8}

A central insight of our paper is that an issuer can accomplish the same goal by acquiring assets about which she has little private information. In particular, a bank may lend to a remote loan applicant about whom it knows only hard information such as the credit score and loan-to-value ratio. Since the bank lacks soft information, it can securitize this loan without costly signaling. This gives remote banks an advantage over local banks that can more than offset the remote banks’ poorer screening ability at the lending stage.

In our base model, banks issue equity securities. We also consider an extension in which each bank instead designs a monotone security that is secured by its loans. The local bank can lessen its lemons problem by choosing standard debt, which is less informationally sensitive than equity. In contrast, security design does not help the remote bank, which does not face a lemons problem. By selectively helping the local bank, security design lessens the extent of remote lending but does not eliminate it.

The rest of the paper is as follows. Our main model is presented in section 2. Section 3 analyzes a base case without securitization; the full model is solved in section 4. Section 5 studies the welfare effects of securitization, while section 6 discusses the model’s empirical implications. Three extensions are studied in section 7. Concluding comments appear in section 8.

\textsuperscript{7}Similarly, Myers and Majluf [40] show that the lemons problem can prevent a privately informed firm from raising a fixed amount of capital to fund a worthwhile project.

\textsuperscript{8}This incentive exists also when it is the security buyers who have market power (Biais and Mariotti [5]) or private information about the security’s value (Axelson [2]; Dang, Gorton, and Holmström [12]), and when the amount of capital to be raised is fixed (Myers and Majluf [40]; Nachman and Noe [41]).
2 The Main Model

There is a single region that contains a unit measure of agents. Each agent is endowed with a project that requires one unit of capital and pays a fixed gross return of \( \rho > 1 \) if it succeeds and zero otherwise. An agent has no capital of her own, so in order to implement her project she must borrow a unit of capital from a bank. There are two banks: a local bank \( L \) and a remote bank \( R \). There is also a continuum of uninformed, deep-pocket investors. All participants are risk-neutral and fully rational.

There are three periods. Lending competition occurs in period 1. First, the remote bank publicly announces whether it is willing to lend to the agents and, if so, at what gross interest rate \( r \). We assume \( r \) is not higher than the gross project return \( \rho \), since an agent cannot pay more than \( \rho \). If the remote bank declines to lend, we let \( r \) equal the gross project return \( \rho \). With this convention, \( r \) is now the maximum interest rate that the agents are willing to pay the local bank. For each agent, the local bank can then announce an offer of its own. If it does so, it will offer the agent’s willingness to pay \( r \) and the agent will agree. The measure of loans made by each bank is commonly observed.

The success probability of an agent’s project is the product of two independent random variables: the agent’s idiosyncratic type \( \theta \) and a common, region-specific macroeconomic shock \( \zeta \), both of which lie in \( (0, 1) \). Project outcomes, conditional on these success probabilities, are mutually independent. The unconditional mean of \( \zeta \) is denoted \( \overline{\zeta} \). While the shock \( \zeta \) is realized in period 3, a signal of it will be seen by the local bank in period 2.

An agent’s type \( \theta \) is seen only by the local bank. It represents soft information about the agent’s creditworthiness and project quality. The remote bank and investors see only the agents’ hard information, which is summarized by a parameter \( \overline{\theta} \in (0, 1) \) that we call the

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9 The case of multiple local and remote banks is studied in section 7.3.

10 Regulation C (enacted in 1989) of the U.S. Home Mortgage Disclosure Act requires lenders to report the amount of each mortgage loan as well as other data such as loan type (conventional loan, FHA loan, VA loan, etc.).

11 That is, a type \( \theta \) agent’s project succeeds with probability \( \theta \zeta \) regardless of the outcomes of the other projects in the region.
agents’ credit score. We treat the credit score $\bar{\theta}$ as an exogenous parameter to be varied. One interpretation is that $\bar{\theta}$ is the realization of a random variable, and that our analysis is contingent on this realization.\textsuperscript{12}

Conditional on the credit score $\bar{\theta}$, the agents’ types $\theta$ have a commonly known, increasing distribution function $G_{\bar{\theta}}$, which has a continuous density and support equal to $[0, 1]$. Higher credit scores $\bar{\theta}$ are good news about the agents’ types, in the following sense.

**Increasing Conditional Expectation (ICE).** The expectation $E_{\bar{\theta}}[\theta|\theta \leq c]$ of $\theta$ conditional on $\theta \leq c$ is increasing in the credit score $\bar{\theta}$ for any constant $c > 0$. As $\bar{\theta}$ goes to zero and one, this expectation converges to zero and $c$, respectively.

By ICE, the agents’ mean type $E_{\bar{\theta}}[\theta]$ is also increasing in the credit score $\bar{\theta}$.\textsuperscript{13} Henceforth we reparametrize the credit score, if necessary, so that it equals this mean type: $\bar{\theta} = E_{\bar{\theta}}[\theta]$. We also restrict to parameters for which the \textit{ex ante} expected return of a random agent’s project exceeds the cost of funding that project:

$$\rho \bar{\theta} \zeta > 1. \tag{1}$$

If the local bank could be certain to sell all its loans to investors, it would not care about the types of its borrowers. To rule this out, we assume that there is an infinitesimal chance that the securitization market will be disrupted, forcing each bank to hold its loans to maturity.\textsuperscript{14} Under these beliefs, a threshold strategy must be optimal for the local bank: it will offer loans to the set of agents whose types $\theta$ exceeds some cutoff $\theta_1$ that will, in general, depend on the agents’ willingness to pay $r$. Since the distribution function $G_{\bar{\theta}}$ is increasing,

\textsuperscript{12}In practice, loan applicants with different credit scores coexist. A model with this feature is studied in the working paper version of this paper (Frankel and Jin [23]). The essential results are analogous to those of the present model.

\textsuperscript{13}This follows directly from ICE with $c = 1$.

\textsuperscript{14}For instance, the crisis of 2008-9 caused such a disruption in the markets for subprime/Alt-A and jumbo mortgage loans (Keys \textit{et al} [33, Figs. 1 and 2]).
investors can infer the local bank’s lending threshold $\theta_1$ from the measure $1 - G_\pi(\theta_1)$ of its loans which, as noted above, is commonly observed.

In period 2, the local bank sees a private signal $t \in [0,1]$, $t \sim \Psi$ of the macroeconomic shock $\zeta$.\textsuperscript{15} Arbitrarily low signals can occur: for any $t_0 > 0$, the probability $\Psi(t_0)$ that the signal is at most $t_0$ is positive. Let $\Phi(\zeta|t)$ denote the distribution of the shock conditional on the signal. We assume that $\Psi$ and $\Phi$ are continuously differentiable in their arguments and have no atoms.\textsuperscript{16} Moreover, higher signals are good news:

**First Order Stochastic Dominance (FOSD).** For any $\zeta_0 \in (0,1)$, the probability $\Phi(\zeta_0|t)$ that the shock does not exceed the cutoff $\zeta_0$ is decreasing in the signal $t$.

We also assume that the expectation of the shock $\zeta$, conditional on the signal $t$ taking its minimum value of zero, is strictly positive: $E[\zeta|t = 0] > 0$. Intuitively, even if the local bank sees the lowest signal, it cannot be sure that all projects will fail.

After the local bank sees its signal $t$, each bank simultaneously selects a proportion of its loans to securitize.\textsuperscript{17,18} These proportions are commonly observed.\textsuperscript{19} Since the remote bank knows nothing about its borrowers’ types, it must securitize each loan with the same probability $q_R \in [0,1]$. The measure of loans that the remote bank securitizes is thus $q_R G_\pi(\theta_1)$. As for the local bank, it sees the type $\theta$ of each of its loans while investors do not. Thus, it will securitize its lowest quality loans:

\textsuperscript{15}A model with no macroeconomic signal is discussed in section 7.1.

\textsuperscript{16}A distribution $F$ on $[0,1]$ is atomless if $F$ is continuous and $F(0) = 0$.

\textsuperscript{17}We restrict here to equity securities; an extension to general monotone securities appears in section 7.2.

\textsuperscript{18}The assumption of simultaneous securitization is without loss of generality. Why? In equilibrium, the remote bank will realize its full gains from trade with investors. This is its theoretical maximum securitization profit. Hence, it cannot benefit from delay. Moreover, the issuance choice of the remote bank is uninformative and thus does not affect the outcome of the signalling game played between the local bank and investors. Thus, delaying would not help the local bank either.

\textsuperscript{19}Under the SEC’s Regulation AB (enacted in 2005), issuers of mortgage backed securities are required to report a "mortgage loan schedule" that lists, for each loan, information such as the loan amount, interest rate, loan-to-value ratio, loan purpose, and property type (Wang [50, p. 47]).
Proposition 1  Suppose the local bank has two loans of types \( \theta' > \theta'' \). For any signal \( t \), if the bank securitizes the type \( \theta' \) loan, then it must also securitize the type \( \theta'' \) loan.

Proof. Suppose not. On seeing the signal \( t \), let the local bank now secretly securitize the type \( \theta'' \) loan instead of the type \( \theta' \) loan. As this deviation cannot be detected, its only effect is to lower the bank’s expected payment to the holders of its security from \( r \theta' E[\zeta | t] \) to \( r \theta'' E[\zeta | t] \): the bank is better off. Hence, its original strategy is not optimal. ■

Let \( \bar{q}_L \) be the proportion of its loans that the local bank securitizes. As this proportion is commonly observed, investors can infer that the local bank has securitized the set of local loans whose types \( \theta \) lie in \([\theta_1, \theta_2]\), where \( \theta_2 \) is the local bank’s securitization threshold and is given implicitly by \( \bar{q}_L = \frac{G(\theta_2) - G(\theta_1)}{1 - G(\theta_1)} \).

In period 3, the macroeconomic shock \( \zeta \) is realized. Each borrower’s project then succeeds with probability \( \theta \zeta \) and fails with probability \( 1 - \theta \zeta \). If her project succeeds, an agent pays the interest rate \( r \) to the bank that financed it; her payoff is thus \( \rho - r \). If her project fails or was not funded, she pays nothing and her payoff is zero. By the law of large numbers, if the remote bank lent in period 1, then in period 3 it receives aggregate loan repayments of

\[
Y_R = \int_{\theta=0}^{\theta_1} [r \theta \zeta] \, dG_{\theta} (\theta)
\]

from its borrowers and pays \( q_R Y_R \) to its security holders. As for the local bank, in period 3 it receives aggregate loan repayments of

\[
Y_L = \int_{\theta=\theta_1}^{1} [r \theta \zeta] \, dG_{\theta} (\theta)
\]

from its borrowers and pays \( \int_{\theta=\theta_1}^{\theta_2} [r \theta \zeta] \, dG_{\theta} (\theta) \) to its security holders. The latter quantity

\[20\text{This equation has a unique solution as } G_{\theta} (\theta_2) \text{ is increasing in } \theta_2. \text{ (As previously noted, investors can infer the local bank’s lending threshold } \theta_1 \text{ from the measure of its loans.)} \]
can be written as $q_L Y_L$ where

$$q_L = \int_{\theta_1}^{\theta_2} \theta dG_\theta (\theta) \int_{\theta_1}^{1} \theta dG_\theta (\theta) \in [0, 1]$$

is the proportion of its aggregate loan repayments that the local bank must pay to investors. Since there is an increasing, one to one relationship between the securitization threshold $\theta_2$ and the quantity $q_L$, we may assume that the local bank chooses $q_L$ rather than $\theta_2$.

Of the two quantity choices $q_R$ and $q_L$, only the latter can convey information about the local bank’s signal $t$. Let

$$p_i (q_L) = E [Y_i | q_L]$$

denote investors’ posterior expected value of the loan portfolio of bank $i = R, L$ given the quantity $q_L$.\footnote{This pricing function is endogenous; it depends on the local bank’s equilibrium issuance strategy.} Since they are competitive and risk-neutral, investors pay bank $i$ a total of $E [q_i Y_i | q_L] = q_i p_i (q_L)$ for its security.

We now specify the payoffs of the banks and investors. While periods 1 and 2 occur at the same point of real time, there is a unit of delay between periods 2 and 3. The banks are liquidity constrained: the discount factor of investors, which we normalize to one, exceeds the discount factor of the banks, which is denoted $\delta \in (0, 1)$. This assumption, common in the prior literature, is thought to capture the typical reason cited for why banks sell loans: the availability of attractive alternative investments together with the existence of regulatory capital ratios (e.g., Gorton and Haubrich [26]).

Both banks have the same unitary cost of capital. A bank’s cost of lent funds is thus the measure of its loans: $C_L = 1 - G_\theta (\theta_1)$ for the local bank and $C_R = G_\theta (\theta_1)$ for the remote bank (if it lent in period 1). Bank $i$’s direct lending profits are just its discounted loan repayments $\delta Y_i$ less its cost $C_i$ of lent funds. Its securitization profits are the payment $q_i p_i (q_L)$ from its security buyers in period 2, less its discounted repayment $\delta q_i Y_i$ to the same
buyers in period 3. Its realized payoff $\Pi_i$ is the sum of these two types of profits:

$$\Pi_i = \underbrace{\delta Y_i - C_i}_{\text{direct lending profits}} + \underbrace{q_i (p_i (q_L) - \delta Y_i)}_{\text{securitization profits}}. \quad (6)$$

The payoff of bank $i$’s security buyers equals their payment $q_i Y_i$ from the bank in period 3 less the amount $q_i p_i (q_L)$ they pay in period 2. The joint surplus of bank $i$ and its security buyers is thus $\delta Y_i - C_i + (1 - \delta) q_i Y_i$. It is increasing in the bank’s period-3 payment $q_i Y_i$ to its security buyers as the bank discounts this payment while investors do not.

## 3 Competition without Securitization

We first analyze a base case without securitization: each bank must hold all of its loans to maturity. If a bank lends, at an interest rate $r$, to an agent of type $\theta$, its expected profit is $\delta r i \overline{\zeta} - 1$: the discounted interest payment $\delta r$ times the ex ante probability $i \overline{\zeta}$ of project success, less the unitary cost of capital.

In the base case, only the local bank lends and it extracts the full surplus. This is due to the winner’s curse. Suppose the remote bank offers $r$. In the absence of securitization, the two banks have common values: the profit from lending to an agent is simply her discounted expected repayment less the banks’ common cost of capital. Thus, the local bank will slightly underbid the remote bank on its profitable offers and not compete for its unprofitable ones. As a result, only unprofitable agents will accept the remote bank’s offer. Knowing this, the remote bank will not make any offer. But then the local bank can charge an agent the maximum possible interest rate of $\rho$. It will do so if and only if the resulting discounted expected repayment, $\delta \rho i \overline{\zeta}$, exceeds the banks’ unitary cost of capital. We have proved the following result.

**Theorem 1** Without the option of securitization, only the local bank lends. The agents’ payoffs are zero: the gross interest rate on each loan equals the gross project return $\rho$. An agent of type $\theta$ is financed if and only if her project’s discounted expected gross return, $\delta \rho i \overline{\zeta}$,
exceeds the unitary cost of capital.

Without securitization, an agent gets a loan if and only if her discounted expected project return exceeds the banks’ common cost of capital. Hence, the allocation of loans is efficient: one agent is funded while another is not if and only if the first agent’s project has a higher expected return than the second’s. This efficiency property will not hold with securitization: a bank may prefer not to lend to a creditworthy agent whom it knows well, since the agent’s loan is harder to securitize.\footnote{A full welfare analysis appears in section 5.}

Our conclusion that all lending is local and the loan allocation is efficient relies on our assumption that the remote bank makes the first offer, followed by the local bank. A similar order of offers is used by Dell’Ariccia, Friedman, and Marquez \cite{dell2011credit} and Dell’Ariccia and Marquez \cite{dell2011credit2}. Others have instead assumed simultaneous offers. They generally find that the uninformed bank plays a mixed strategy and sometimes wins, while earning zero expected profits.\footnote{See, in particular, Rajan \cite[pp. 1380-81]{rajan2009bank} and von Thadden \cite[pp. 17-18]{von2009credit}.}
The extension of our model to this case might be an interesting topic for future research.

4 Competition with Securitization

We now permit securitization. Suppose first that the remote bank offers an interest rate \( r \) while the local bank makes no offers: its lending threshold \( \theta_1 \) is at least one. Then all agents will accept the remote bank’s offer. As there is symmetric information and positive gains from trade between the remote bank and investors, this bank will sell all of its loans: its securitization proportion \( q_R \) will equal one. By (2) and (5), investors assign the value \( p_R = E[Y_R] = r \bar{\delta} \bar{C} \) to the remote bank’s loan portfolio. Hence, by (2) and (6), the remote bank’s expected securitization profits \( E[q_R (p_R - \delta Y_R)] \) equal the gains from trade, \((1 - \delta) r \bar{\delta} \bar{C}\): the difference in discount rates times the unconditional expected payout of the portfolio.
Now assume instead that the local bank makes some loans: its lending threshold $\theta_1$ is less than one. Bank $L$’s expected securitization profit from selling the quantity $q_L$, conditional on its signal $t$, is
\[
\pi_L (q_L, t) = q_L \cdot (p_L (q_L) - \delta E [Y_L | t]) ,
\] (7)
which equals the revenue it gets now, $q_L p_L (q_L)$, less its discounted expected future payment to investors, $\delta q_L E [Y_L | t]$.

Assume bank $R$ also lends, and sells the quantity $q_R$. In equilibrium, bank $R$ knows the quantity $q_L (t)$ that bank $L$ sells as a function of the signal $t$. By analogy to (7), bank $R$’s expected securitization profits, conditional on $t$, are simply $q_R \cdot (p_R (q_L (t)) - \delta E [Y_R | t])$. Bank $R$’s expected securitization profits when it chooses $q_R$ are just the integral of this expression over all signals $t$:
\[
\pi_R (q_R) = \int_{t=0}^{1} q_R \cdot (p_R (q_L (t)) - \delta E [Y_R | t]) d\Psi (t) .
\] (8)

We use the following definition of equilibrium in this subgame.

**Definition 1** A Bayesian Nash equilibrium of the subgame that begins in period 2, if the local bank made some loans in period 1, is a measurable quantity function $q_L (t)$ chosen by the local bank, a quantity $q_R$ chosen by the remote bank, and a pair $p_L (q_L)$, $p_R (q_L)$ of measurable price functions chosen by investors such that:

1. Best Response: $q_L (t) \in \arg \max_{q \in [0,1]} \pi_L (q, t)$ and $q_R \in \arg \max_{q \in [0,1]} \pi_R (q)$, almost surely;

2. Bayesian Updating: for $i = R, L$, $p_i (q_L (t)) = E [Y_i | q_L (t)]$ almost surely.

The equilibrium is separating if the following condition also holds.

3. Separation: $p_L (q_L (t)) = E [Y_L | t]$ almost surely.

We restrict to separating equilibria, which satisfy all three conditions. This restriction uniquely determines the banks’ behavior and profits. By (2), (8), condition 2 of Definition
1, and the law of iterative expectations, the remote bank’s securitization profits are

\[ \pi_R (q_R) = q_R \left( 1 - \delta \right) E [Y_R] = q_R \left( 1 - \delta \right) \left( \int_{\theta = 0}^{\theta_1} [r \theta \zeta] \ dG_{\theta} (\theta) \right). \]

As the right hand side is proportional to the quantity \( q_R \), the remote bank securitizes all its loans as before: \( q_R = 1 \). This proves part 1 of the following result. It also implies that the remote bank’s optimal securitization decision is invariant to the behavior of the local bank, which therefore acts as a single issuer. The single issuer problem was previously analyzed by DeMarzo and Duffie [16, Proposition 2]. Their results imply that the local bank issues the quantity \( q_L (t) = \left( \frac{E[Y_L | t=0]}{E[Y_L | t]} \right)^{\frac{1}{1-\delta}} \) and the investors’ price function is \( p_L (q_L) = \frac{E[Y_L | t=0]}{(q_L)^{1-\delta}} \).

Using (3) to simplify the quantity function \( q_L (t) \), we obtain part 2 below. The local bank’s securitization profits, which appear in part 3, follow immediately using equations (3) and (7).

**Proposition 2** The subgame that begins in period 2, if the local bank lent in period 1, has a unique separating Bayesian Nash equilibrium, with the following properties.

1. The remote bank securitizes all its loans in period 2: \( q_R = 1 \). Its expected securitization profits are

\[ (1 - \delta) \left( \int_{\theta = 0}^{\theta_1} [r \theta \zeta] \ dG_{\theta} (\theta) \right). \]

This equals the difference \( 1 - \delta \) in discount factors times the unconditional expected gross return \( E [Y_R] \) of the remote bank’s loans.

2. Conditional on its signal, the local bank sells the quantity \( q_L (t) = \left( \frac{E[Y_L | t=0]}{E[Y_L | t]} \right)^{\frac{1}{1-\delta}} \). The investors’ price function is given by \( p_L (q_L) = \frac{E[Y_L | t=0]}{(q_L)^{1-\delta}} \).

3. The local bank’s expected securitization profits, conditional on its signal, are

\[ \pi_L (q_L (t), t) = (1 - \delta) \left( \int_{\theta = \theta_1}^{\theta_1} [r \theta \delta] \ dG_{\theta} (\theta) \right) \left( \frac{E[\zeta | t=0]}{E[\zeta | t]^{\delta}} \right)^{\frac{1}{1-\delta}}. \]
This equals the difference $1 - \delta$ in discount factors times the value of trade $p_L q_L$ between the local bank and investors for the given signal $t$.

By part 1, the remote bank always sells its entire portfolio and thus realizes its full potential gains from trade. The local bank does so only when $t = 0$: when its signal is the lowest possible (part 2). As its signal $t$ rises, the local bank sells less in order to signal higher quality. Indeed, its quantity $q_L$ falls so fast that its securitization profits are decreasing in $t$ (part 3): it sells less, and profits less, precisely when the potential gains from trade are larger. In this sense, the signaling outcome is quite inefficient (first noted by DeMarzo and Duffie [16]).

We now compute the banks’ payoffs in the full game as functions of their actions in period 1: before the local bank sees its signal $t$. The local bank’s direct lending profits are

$$
\int_{\theta = \theta_1}^{1} (\delta r \theta \zeta - 1) \, dG_\theta (\theta) \quad \text{the integral, over all types } \theta \text{ to whom it lends, of the discounted expected gross loan return } \delta r \theta \zeta \text{ minus the unitary cost of capital.}
$$

The expectation over signals $t$ of the local bank’s securitization profits in part 3 of Proposition 2 may be written

$$
\int_{\theta = \theta_1}^{1} (1 - \delta) [r \theta] \Lambda dG_\theta (\theta)
$$

where we define the parameter

$$
\Lambda = E \left[ \left( \frac{E [\zeta | t = 0]}{E [\zeta | t]} \right)^{1-\delta} \right]
$$

which lies in $(0, E [\zeta | t = 0])$. (The outer expectation in (10) is over signals $t$.) An increase in

---

24 The assumption that the local bank sees a signal $t$ after lending is just one way to create a cost advantage for the remote bank in the securitization market; section 7.1 presents another approach. If the model is interpreted literally, the local bank might avoid its costly signalling problem by contracting to sell its entire portfolio of loans before learning its signal $t$. Such contracts, which we rule out, are probably infeasible in practice. Investors generally do not know when a bank receives private information about its existing loans. Hence, the offer of such a contract by the bank would likely signal to investors that the bank has already received bad news.

25 The parameter $\Lambda$ is positive since $E [\zeta | t = 0] > 0$. The conditional expectation $E [\zeta | t]$ can be written...
the conditional expectation $E[\zeta|t]$ of the shock relative to its lowest possible value $E[\zeta|t=0]$ may be interpreted as a rise in the informativeness of the signal $t$. Hence, the parameter $\Lambda$ - and thus the local bank’s securitization profits - are lower when the local bank is more informed about local economic conditions. Intuitively, having more information worsens the lemons problem the bank faces at the securitization stage.

Bank $L$’s payoff $\Pi_L$ is the sum of its expected direct lending and securitization profits:

$$
\Pi_L(\theta_1|r, \bar{\theta}, \mu) = \int_{\theta_1}^{1} (r\theta - 1) dG_{\bar{\theta}}(\theta)
$$

(11)

where we will refer to

$$
\mu = (1 - \delta) \Lambda + \delta \tilde{\zeta}
$$

(12)
as the local profitability parameter. By (10) and FOSD, $\mu$ lies in $(\Lambda, \tilde{\zeta})$. By (11), the local bank’s optimal lending threshold is

$$
\theta_1 = (r\mu)^{-1}.
$$

(13)

This is the type $\theta$ for whom $r\theta - 1$ - the sum of the local bank’s expected lending revenue $\delta r \theta \tilde{\zeta}$ and securitization revenue $(1 - \delta) [r\theta] \Lambda$ - equals the unitary cost of capital.

If the remote bank lends, it sells all of its loans. Its payoff from offering an interest rate $r \leq \rho$, given the agents’ credit score $\bar{\theta}$, is just its expected direct lending profits $\int_{\theta_1}^{\theta} (r\theta - 1) dG_{\bar{\theta}}(\theta)$ plus its expected securitization profits from part 1 of Proposition 2. Substituting for $\theta_1$ using (13), the remote bank’s payoff is

$$
\Pi_R(r|\bar{\theta}, \mu) = \int_{0}^{(r\mu)^{-1}} (r\theta - 1) dG_{\bar{\theta}}(\theta).
$$

(14)

If the remote bank competes, it will charge an interest rate $r$ that maximizes $\Pi_R(r|\bar{\theta}, \mu)$.\[\int_{z=0}^{1} z d[\Phi(z|t) - 1] as d1 = 0.\] Integrating by parts, it equals $\int_{z=0}^{1} (1 - \Phi(z|t)) dz$ which, by FOSD, is increasing in $t$. Hence, $\Lambda$ is also bounded above by $\left(\frac{E[\zeta|t=0]}{E[\zeta|t=0]}\right)^{\frac{1}{t}} = E[\zeta|t=0]$.\[\text{16}\]
Since the integral is at most \( r \hat{\theta} \zeta - 1 \), the remote bank will not choose an interest rate below \( (\hat{\theta} \zeta)^{-1} \). And since \( \Pi_R \) is continuous on \( r \in \left[ (\hat{\theta} \zeta)^{-1}, \rho \right] \), an optimal interest rate exists in this interval.\(^{26}\)

Let \( \Pi^*_R (\theta, \mu) \) equal the remote bank’s payoff \( \Pi_R (r|\theta, \mu) \) at an optimal interest rate \( r \), and let

\[
\theta^* = \inf \{ \theta : \Pi^*_R (\theta, \mu) \geq 0 \}
\]

(15) denote the greatest lower bound on the set of credit scores for which the remote bank can profitably lend. The remote bank will lend if and only if the agents’ credit score \( \theta \) lies in \( (\theta^*, 1) \), which is nonempty as \( \theta^* \) is less than one:

\begin{enumerate}
\item The threshold \( \theta^* \) lies in \( (0, 1) \) and is nondecreasing in the local profitability parameter \( \mu \).

\item If \( \theta < \theta^* \), the remote bank does not compete and agents with types \( \theta \) below \( (\rho\mu)^{-1} \) do not borrow. Agents with types above \( (\rho\mu)^{-1} \) borrow from the local bank at an interest rate \( r \) equal to the gross project return \( \rho \). On seeing its signal \( t \), the local bank securitizes its loans to all types \( \theta \in [(\rho\mu)^{-1}, \theta_2] \), where its securitization threshold \( \theta_2 \), which is decreasing in \( t \), is determined implicitly by (4) using \( \theta_1 = (\rho\mu)^{-1} \) and \( q_L = \left( \frac{E[\zeta|t=0]}{E[\zeta|t]} \right)^{\frac{1}{1-\delta}} \).

\item If \( \theta > \theta^* \), all agents borrow. The remote bank offers an interest rate \( r \) in the nonempty interval \( [(\theta^* \zeta)^{-1}, \rho] \) that maximizes \( \Pi_R (r|\theta, \mu) \). If an agent’s type \( \theta \) exceeds \( (r\mu)^{-1} \), she borrows from the local bank; else she borrows from the remote bank. In either case, she pays the interest rate \( r \). The remote bank securitizes all of its loans. On seeing its signal \( t \) of the macroeconomic shock \( \zeta \), the local bank securitizes its loans to all types \( \theta \in [(r\mu)^{-1}, \theta_2] \), where its securitization threshold \( \theta_2 \), which is decreasing in \( t \), is determined implicitly by (4) using \( \theta_1 = (r\mu)^{-1} \) and \( q_L = \left( \frac{E[\zeta|t=0]}{E[\zeta|t]} \right)^{\frac{1}{1-\delta}} \).
\end{enumerate}

\(^{26}\)The interval is nonempty by (1). The remote bank’s payoff at its optimal interest rate may be negative, in which case it will not lend. This motivates equation (15), which follows.
Proof. Appendix. ■

In equilibrium, the remote bank follows a threshold policy: it competes if and only if the agents’ credit score \( \theta \) lies in the nonempty interval \((\theta^*, 1)\). Moreover, \( \theta^* \) is nondecreasing in the local profitability parameter \( \mu \). Intuitively, given the interest rate \( r \), the remote bank lends to those agents whose types \( \theta \) lie below the local bank’s lending threshold \((r \mu)^{-1}\). This threshold does not depend on the credit score \( \theta \): as the local bank knows an agent’s actual type \( \theta \), it does not care additionally about her credit score. Hence, ICE implies that the remote borrowers’ mean type \( E_{\theta}[\theta|\theta \leq (r \mu)^{-1}] \) is increasing in the credit score \( \theta \). It is also clearly nonincreasing in the local profitability parameter \( \mu \). So the remote bank’s profits from offering any given interest rate \( r \) are increasing in \( \theta \) and nonincreasing in \( \mu \). Thus, bank \( R \)’s profits from its best offer \( r \) must also be increasing in \( \theta \) and nonincreasing in \( \mu \): the remote bank must use a credit score threshold, which is nondecreasing in \( \mu \).

Theorem 2 is illustrated in Figure 1. The Figure assumes that without securitization, the local bank lends \((\delta \rho \zeta)^{-1} < 1\) and that the remote bank’s optimal interest rate \( r \) is less than the gross project return \( \rho \) and does not vary with the credit score \( \theta \geq \theta^* \). An agent’s type \( \theta \), which only the local bank sees, appears on the vertical axis. The credit score \( \theta \), which all see, is depicted on the horizontal axis. Without securitization, the local bank lends to agents whose types \( \theta \) exceed the threshold \((\delta \rho \zeta)^{-1}\) by Theorem 1: only agents in areas \( A_0 \) and \( A_3 \) are funded.

Securitization extends funding to those in areas \( A_1 \), \( A_4 \), and \( A_5 \). Why? First suppose the credit score \( \theta \) lies below \( \theta^* \). Only the local bank lends as before, but the ability to securitize some loans entices the bank to lower its lending threshold to \((\rho \mu)^{-1}\): it extends funding to agents in area \( A_1 \).\(^{27}\) If instead the credit score \( \theta \) exceeds \( \theta^* \), bank \( R \) offers some optimal interest rate \( r \leq \rho \). (The Figure assumes \( r < \rho \).) The local bank extends funding to those in area \( A_4 \), while the remote bank funds agents in area \( A_5 \).

\(^{27}\)Its threshold is lower since, by (12), \( \mu > \delta \zeta \).
Figure 1: Illustration of Theorems 1 and 2. Figure assumes (a) local bank lends without securitization ($[\delta \rho \zeta]^{-1} < 1$) and (b) remote bank’s optimal interest rate $r$ is less than gross project return $\rho$ and independent of credit score $\overline{\theta}$. Without securitization, local bank lends to agents in areas $A_0$ and $A_3$. With securitization, local bank lends to agents in areas $A_0$ and $A_1$ while remote bank lends to those in areas $A_3$, $A_4$, and $A_5$. 
4.1 Bidding on One Another’s Securities

Our model assumes that one bank cannot bid on the other’s security. Clearly, only the local bank might profit from doing so since only it has an informational advantage (via its signal $t$) that offsets its greater impatience vis-a-vis investors. When the remote bank lends, it sells its entire portfolio to investors for a price equal to the portfolio’s expected payout $r\zeta \int_{\theta=0}^{\theta_1} \theta dG_\theta(\theta)$. The local bank’s valuation of this portfolio is at most $\delta r E[\zeta | t = 1] \int_{\theta=0}^{\theta_1} \theta dG_\theta(\theta)$. Hence, in the above equilibrium, the local bank can never profit from bidding on the remote bank’s portfolio if

$$\delta < \frac{\zeta}{E[\zeta | t = 1]].}$$

(16)

This condition states that the bank’s degree of patience $\delta$ is less than the maximum informational advantage that it gets from its private signal $t$ of the macroeconomic shock $\zeta$. If (16) holds, then the above outcome remains an equilibrium if banks can bid on each others’ securities.

5 Welfare

The remote bank cannot screen on an agent’s type. Hence, when it lends, some of its borrowers will have types that are close to zero. Under symmetric information, these agents would not be financed. This is an efficiency cost of securitization. On the other hand, securitization allows a welfare-enhancing exchange between patient investors and impatient banks. We now show that the cost never exceeds the benefit: securitization cannot lower welfare. And if, with securitization, either bank lends, then it raises welfare for generic parameters.

Formally, we analyze social welfare as follows. Let $U$ denote \textit{ex ante} agent welfare: the integral of the agents’ expected payoff $(\rho - r) \theta \zeta$ over all types $\theta$ that receive loans. Let $\Pi_L$ and $\Pi_R$ denote the \textit{ex ante} payoffs of the local and remote bank, which are given in
equations (11) and (14).

Our model is nonstandard as the banks are less patient than the investors and agents. So in constructing the welfare function we consider two alternative weighting schemes. In scheme A, we simply sum the players’ payoffs: \( SW_{\text{scheme A}} = U + \Pi_L + \Pi_R \). As the investors’ payoff is identically zero, it is omitted.

Scheme A puts unit weight on the income of all players in all periods except the banks’ period-3 income, which is given the weight \( \delta < 1 \). Thus, scheme A favors changes (such as a decline in the interest rate) that transfer income from the banks to the agents in period 3. However, our motivation for the bank’s lower discount factor is that the bank faces capital requirements that prevent it from making profitable investments in periods 1 and 2. Thus, instead of underweighting the banks’ period-3 income, it may be more reasonable to overweight their income in the earlier periods. This is accomplished with the following welfare function: \( SW_{\text{scheme B}} = U + (\Pi_L + \Pi_R) / \delta \). Scheme B puts unit weight on the income of all players in all periods except the banks’ income in periods 1 and 2, which receives the weight \( \omega = 1 / \delta \). The weight \( \omega \) captures the expected gross return of the banks’ profitable alternative investments and can take on any value in \((1, \infty)\).

Under either scheme, securitization raises welfare as long as it leads to some lending:

**Theorem 3** Securitization does not lower welfare. And if, under securitization, either bank lends, then securitization generically raises welfare. These claims hold for both weighting schemes.

**Proof.** There are two cases.

1. Under securitization, neither bank lends. Then the local bank’s profit \( \rho \mu \zeta \) from lending to the highest type \( (\theta = 1) \) must be nonpositive. But then without securitization, the local bank does not lend either: its profit \( \delta \rho \zeta \) from lending to the highest type must be negative by (12). Thus, social welfare is identically zero both with and without securitization.

2. Under securitization, some bank lends. There are two subcases.
(a) The remote bank does not lend under securitization. Then it gets zero: it is unaffected by securitization. The interest rate $r$ remains equal to the gross project return $\rho$ by part 2 of Theorem 2. So the agents are also unaffected by securitization. As for the local bank, there are two possibilities. In the first, it does not lend without securitization. With securitization, it lends by hypothesis, so its payoff is generically positive: securitization raises welfare. In the second, the local bank lends without securitization. Securitization then changes its profit on a loan to a type $\theta$ agent from $\delta \rho \theta \zeta - 1$ to $\rho \theta \mu - 1$, which is higher by (12) and since $\Lambda > 0$. So securitization raises welfare here as well.

(b) The remote bank lends under securitization. Then for generic parameters, securitization must raise its payoff $\Pi_R$, which is zero without securitization. It thus suffices to show that securitization cannot lower the remainder of the social welfare function: $U + \Pi_L$ under scheme A and $U + \Pi_L/\delta$ under scheme B. We will refer to this remainder as the "partial surplus". An outline is as follows; a rigorous proof appears in Appendix A.

When the remote bank lends, securitization affects the agents and the local bank in three distinct ways. First, agents to whom the local bank does not lend can now borrow from the remote bank. Second, the interest rate $r$ falls from $\rho$ to some $r_0 \leq \rho$ that is chosen by the remote bank. Third, the local bank gains access to the securitization market. Consider the following thought experiment, in which these steps occur sequentially:

i. The remote bank first offers loans to all agents at the interest rate $\rho$. As the interest rate is, by assumption, held constant at $\rho$, the agents’ payoffs are still zero. And the local bank is not affected since the agents’ willingness to pay remains at $\rho$. In particular, it still lends to the set of agents whose types $\theta$ exceed $\left(\delta \rho \zeta\right)^{-1}$. Hence, this step has no effect on the partial surplus.

ii. The remote bank then gradually lowers its interest rate from $\rho$ to $r_0$. This has three effects. First, it raises the income $\rho - r$ that a remote borrower
gets if her project succeeds. This raises the partial surplus (from which, crucially, the remote bank’s payoff is omitted). Second, period-3 income is transferred to local borrowers from the remote bank. (As we are not yet permitting the local bank to securitize, the decline in $r$ cannot affect its securitization profits.) This raises the partial surplus under scheme A and leaves it unchanged under scheme B. Third, the local bank gradually raises its lending threshold $\theta_1 = \left(\frac{\delta r}{\zeta}\right)^{-1}$ as $r$ falls. This leaves the local bank’s payoff unchanged by the envelope theorem. It does not affect the agents either: those who are dropped by the local bank simply borrow from the remote bank at the prevailing interest rate $r$. Hence, it does not affect the partial surplus.

iii. Finally, the local bank is permitted to securitize some or all of its loans. This has no effect on the agents, who are all still funded at the interest rate $r_0$. And it cannot harm the local bank, which can always choose not to securitize. So it cannot lower the partial surplus.

We conclude that securitization cannot lower the partial surplus.

Another question pertains to the efficient allocation of funds across agents. Without securitization, the local bank lends to those agents whose expected returns $\rho \theta \zeta$ exceed the fixed threshold $1/\delta$ (Theorem 1). Hence, the allocation of loans across agents is efficient: if one agent gets a loan while another does not, the former agent’s project must have a higher expected return. While securitization expands lending, the expansion is biased towards agents with higher credit scores. In particular, some agents in area A5 (Figure 1), all of whom are funded, have lower types $\theta$ than some agents in area A2, none of whom are funded. Thus, the allocation of loans across agents is now inefficient: while securitization raises welfare, an omniscient planner could reallocate loans across agents so as to obtain a further welfare improvement.
6 Empirical Implications

Several features of the recent securitization episode in the U.S. are consistent with our model. For example, Keys et al [33] find that the share of loans with low or no documentation dramatically increased as securitization expanded. This mirrors our prediction that securitization favors screening based on hard information such as credit scores rather than the soft information that may be produced, e.g., from an analysis of loan documentation.

Some other predictions that find empirical support are as follows.

1. **Securitization Stimulates Lending.** As in Shin [48], securitization leads to expanded lending by connecting liquid investors with loan applicants. In Figure 1, areas $A_1$, $A_4$, and $A_5$ are added. There is considerable evidence that the securitization boom in the 2000s led to expanded lending (Demyanyk and Van Hemert [18]; Krainer and Laderman [34]; Mian and Sufi [38]).

2. **Securitization Favors Remote Lenders, who Securitize More.** The introduction of securitization enables the remote bank to compete for some loan applicants. In addition, the remote bank sells all of its loans while the local bank retains a portion of its loan portfolio. Loutskina and Strahan [36] find that as securitization rose, the market share of concentrated lenders - those which originate at least 75% of their mortgages in one metropolitan statistical area (MSA) - fell from 20% to 4% from 1992 to 2007. Moreover, concentrated lenders retain a higher proportion of their loans. Finally, when they expand to new MSA’s, these lenders are more likely to sell their remote loans than those made in their core MSA’s.

3. **Remote Borrowers have Stronger Observables and Higher Conditional Default Rates.** By Theorem 2, all agents with strong observables can borrow remotely. In contrast, agents with weak observables can borrow only locally, and only if their soft information is strong enough. Agarwal and Hauswald [1] find that applicants with strong observables tend to apply online for loans, while in-person applicants tend to be those with weaker observables but positive estimates of the bank’s soft information.
about them. Now consider an agent whose credit score $\bar{\theta}$ exceeds the remote bank’s lending threshold $\bar{\theta}^*$. She borrows remotely (locally) if her default probability, based on her type $\theta$, is high (low) enough. Thus, remote loans have higher conditional default rates. Indeed, Agarwal and Hauswald [1] find that online loans default more than observationally equivalent in-person loans, while De Young, Glennon, and Nigro [19] find that banks that lend remotely have higher default rates. Loutskina and Strahan [36, p. 1456] find that concentrated lenders (defined above) have lower loan losses despite lending to applicants who are riskier in terms of loan to value ratios.

4. **Securitized Loans have Higher Conditional Default Rates.** In our model, among agents with credit scores above the remote bank’s lending threshold, high (low) types get local (remote) loans, which are partially (wholly) securitized. Thus, securitized loans have higher conditional default rates. Krainer and Laderman [34] find that controlling for observables, privately securitized loans default at a higher rate than retained loans, while Elul [20] finds that securitized loans perform worse than observationally similar unsecuritized loans. Rajan, Seru, and Vig [45] find that conditional default rates rose between 1997-2000 and 2001-6 with the rise of securitization; similarly, Demyanyk and Van Hemert [18] find that conditional and unconditional default rates rose from 2001 to 2007. Our model also predicts a discontinuity at the remote bank’s lending threshold: as agents with credit scores slightly above this threshold qualify for remote loans, they have discretely higher securitization and default rates than agents whose scores lie slightly below the threshold. Keys, Seru, and Vig [32] and Keys *et al* [31] find that loans just above the 620 FICO credit score threshold are much more likely to be securitized and to default than loans of borrowers with credit scores just slightly below this threshold.

5. **Securitization Lets Borrowers with Strong Observables Get Cheap Remote Loans.** By Theorem 2, agents with weak observables pay the maximum interest rate to their local bank if they borrow, while agents with strong observables pay a
generally lower rate that results from competition between the remote and local bank. This has two implications. First, the securitization boom in the 2000s should have strengthened the (negative) relation between borrower observables and interest rates. Rajan, Seru, and Vig [45] find that borrower credit scores and LTV ratios explain just 9% of interest rate variation among loans originated in 1997-2000 but 46% of this variation among loans originated in 2006. Second, remote loans should carry lower interest rates. Agarwal and Hauswald [1] find that internet loans carry lower interest rates than in-person loans, while Degryse and Ongena [13] and Mistrulli and Casolaro [39] find that interest rates decrease with the distance between small firms and their lenders.

7 Extensions

We now consider several variations of the basic model. These are a model with no macroeconomic signal; a model with security design; and a model with multiple local and remote banks.

7.1 No Macroeconomic Signal

A model with no macroeconomic signal is equivalent to the special case of our model in which the conditional expected value of the shock, \( E[\zeta|t] \), is independent of the signal \( t \). This implies \( \Lambda = \mu = \zeta \) by (10) and (12). Substituting \( \zeta \) for \( \mu \) in (14), one finds that the remote bank loses money on every type \( \theta \) to which it lends except the highest type \( \theta = \left( r\zeta \right)^{-1} \), on which it breaks even: the remote bank will not lend. Intuitively, there is now symmetric information between the local bank and investors at the securitization stage. As each bank can reap its full gains from trade with investors, the lending game has common values. Since the uninformed remote bank bids first, it must lose from competing as in the case without securitization (section 3).

One way to reintroduce a lemons problem is to modify the model so that only the local
bank knows the distribution of types \( \theta \) in its loan portfolio. A simple model with this property is one with a *single* agent whose type \( \theta \) is known only to the local bank. If the local bank lends, investors know only that the agent’s type \( \theta \) does not lie below the local bank’s lending threshold. They do not know the precise value of \( \theta \). Hence, the local bank may still retain part of the loan in order to signal that \( \theta \) is high. As the remote bank does not need to signal, it may still be able to compete with the local bank.

We analyze such a model in our online appendix (Frankel and Jin [24]). Remote lending can still occur. However, unlike our main model, there are now multiple separating equilibria. Intuitively, with a single agent the market does not observe the local bank’s lending threshold. Hence, if the local bank deviates (e.g., by selling a higher than expected proportion of its loan), the market may conclude that this threshold, and the agent’s type, is zero: the loan has no chance of being repaid. Such punishing beliefs can prevent the local bank from securitizing more than an arbitrary proportion (including zero) of its loans. This permits multiple equilibria with different such arbitrary proportions.\(^{28}\)

### 7.2 Ex Post Security Design

We now modify the main model of section 2 to permit *ex post* security design, in which each bank can design a general monotone security after the local bank sees its signal.\(^{29}\) Our qualitative results remain intact. However, security design permits the local bank to signal its information more efficiently. This strengthens the local bank’s position at the lending stage and thus makes remote lending less likely.

The changes to the model begin in period 2 after the local bank sees its signal \( t \). Rather than choosing a subset of its loans to sell, each bank \( i = R, L \) announces a function \( F_i \)

---

\(^{28}\)We also show that there is a unique equilibrium that survives the D1 refinement of Banks and Sobel [4]. There is no remote lending in this equilibrium. However, it is hard to justify the strong restrictions on beliefs that D1 imposes. We discuss this issue further in the online appendix; see also Fudenberg and Tirole [25, p. 460].

\(^{29}\)The working paper version of this paper (Frankel and Jin [23]) instead considers *ex ante* security design: each bank designs its security, sees its signal, and decides how many shares of its security to sell. The results are essentially the same in the two cases.
that specifies the payment \( F_i (Y_i) \in [0, Y_i] \) that investors will receive for any given gross return \( Y_i \) of bank \( i \)'s loan portfolio.\(^{30}\) We restrict to monotone securities, for which both the security payout \( F_i (Y_i) \) and the portion \( Y_i - F_i (Y_i) \) of its portfolio return that bank \( i \) keeps are nondecreasing in the portfolio return \( Y_i \).\(^{31}\)

We make the following technical assumptions. First, the signal density \( \Psi' (t) = \frac{d\Psi (t)}{dt} \) is bounded and Lipschitz continuous:

**Lipschitz-\( \Psi \).** There are constants \( k_0, k_1 \in (0, \infty) \) such that for all signals \( t, t' \in [0, 1] \),

\[
\Psi' (t) \leq k_0 \text{ and } |\Psi' (t) - \Psi' (t')| \leq k_1 |t - t'|.
\]

Moreover, the conditional distribution \( \Phi \) of the shock \( \zeta \) given the signal \( t \) is Lipschitz continuous and has some minimum sensitivity to its arguments:

**Lipschitz-\( \Phi \).** There are constants \( k_2, k_3 \in (0, \infty) \) such that for all \( \zeta, t \in [0, 1] \),

\[
\frac{\partial \Phi (\zeta|t)}{\partial \zeta} \in (k_2, k_3) \text{ and } \frac{\partial \Phi (\zeta|t)}{\partial t} \in [k_2 \zeta (1 - \zeta), k_3]. \tag{17}
\]

An intuition for (18) is as follows. A higher signal \( t \) is good news about the shock, so \( \Phi (\zeta|t) \) is decreasing in \( t \). Thus, the absolute sensitivity of \( \Phi \) to \( t \) is represented by the nonnegative

\[^{30}\text{We assume that a bank must securitize all of its loans and sell its security in its entirety. This is without loss of generality. Why? Suppose instead that the remote bank securitizes each loan with probability } q_R \text{ and writes a fixed security } F_R \text{ on the result, and then sells a proportion } \tilde{q}_R \text{ of this security to investors. The payout to investors is thus } \tilde{q}_RF_R (q_RY_R). \text{ But this is equivalent to securitizing all loans for sure and selling in its entirety a security with payout } \tilde{F}_R (Y_R) = \tilde{q}_RF_R (q_RY_R). \text{ Likewise, suppose the local bank securitizes all loans with types } \theta \in [\theta_1, \theta_2] \text{ and, given its signal } t, \text{ writes a security } F_L^t \text{ on the result and sells a proportion } \tilde{q}_L \text{ of this security. The payout to investors, given } t, \text{ is thus } \tilde{q}_LF_L^t (q_LY_L) \text{ where } q_L \text{ is determined by } \theta_1 \text{ and } \theta_2 \text{ via (4). But this is equivalent to securitizing all loans for sure and, on seeing the signal } t, \text{ selling (in its entirety) a security with payout } \tilde{F}_L^t (Y_L) = \tilde{q}_LF_L^t (q_LY_L). \tag{18}\]

\[^{31}\text{The former can be justified by assuming that the bank can hide debts. Thus, if } F_i \text{ were decreasing, bank } i \text{ could borrow money to inflate } Y_i, \text{ pay investors the lower payout, and then return the loan. The second assumption follows from free disposal. See DeMarzo, Frankel, and Jin [17]. The effect of relaxing monotonicity is not known for the case of ex post security design. With ex ante design, standard debt is no longer optimal if monotonicity is dropped for a class of conditional distribution functions } \Phi \text{ (Nachman and Noe [41, n. 3]).}
\]
quantity \( -\frac{\partial \Phi(\zeta | t)}{\partial t} \). As \( \Phi(0, t) \) and \( \Phi(1, t) \) equal zero and one, respectively, for any \( t \), we cannot require that \( \frac{\partial \Phi(\zeta | t)}{\partial t} \) be sensitive to the signal \( t \) for all shocks \( \zeta \). However, we can require that as \( \zeta \) moves away from zero (one), this sensitivity rises at least linearly in \( \zeta \) (respectively, in \( 1 - \zeta \)). The factor \( \zeta (1 - \zeta) \) ensures this property as it is approximately equal to \( \zeta \) in a neighborhood of \( \zeta = 0 \) and to \( 1 - \zeta \) in a neighborhood of \( \zeta = 1 \).

We also assume that the partial derivatives of the conditional distribution function \( \Phi \) are Lipschitz continuous in the signal \( t \):

**Lipschitz Partial Derivatives (LPD).** There is a \( k_4 \in (0, \infty) \) such that for all \( \zeta, t', t'' \in [0, 1] \),

\[
\max \left\{ \left| \frac{\partial \Phi(\zeta | t')}{\partial \zeta} - \frac{\partial \Phi(\zeta | t'')}{\partial \zeta} \right|, \left| \frac{\partial \Phi(\zeta | t)}{\partial t} \right|_{t=t'} - \left| \frac{\partial \Phi(\zeta | t)}{\partial t} \right|_{t=t''} \right\} < k_4 |t' - t''|.
\]

Finally, we assume that \( \Phi \) satisfies the following strengthening of First Order Stochastic Dominance:

**Hazard Rate Ordering (HRO).** For all \( t' > t'' \), \( \frac{1 - \Phi(\zeta | t')}{1 - \Phi(\zeta | t'')} \) is increasing in \( \zeta \in [0, 1] \).

HRO is weaker than the monotone likelihood ratio property, which is commonly assumed in signaling games (DeMarzo, Frankel, and Jin [17]).

By (2) and (3), the realized value \( Y_i \) of bank \( i \)'s loans may be written \( y_i^{\theta_1} \zeta \) where \( y_i^{\theta_1} = \int_{\theta=0}^{\theta_1} r \theta dG_\theta(\theta) \) and

\[
y_i^{\theta_1} = \int_{\theta=\theta_1}^{1} r \theta dG_\theta(\theta) \tag{19}
\]

are common knowledge at the securitization stage. The realized payout to investors of bank \( i \) in period 3 is thus \( F_i(y_i^{\theta_1} \zeta) \).

We first consider the remote bank’s security design problem. Let \( F_L^t \) equal the security designed by the local bank when its signal is \( t \). Let \( E[f(\zeta) | F_L^t] \) denote the expectation of a function \( f(\zeta) \) given what the local bank’s security design \( F_L^t \) reveals about the signal \( t \). \(^{32}\) Let \( p_R(F_R, F_L^t) \) denote the price \( E[F_R(y_R^{\theta_1} \zeta) | F_L^t] \) that investors offer for the remote bank’s security \( F_R \) when the local bank announces the security \( F_L^t \). The unconditional

\[^{32}\text{In particular, in a separating equilibrium } F_L^t \text{ reveals } t \text{ so } E[f(\zeta) | F_L^t] \text{ equals } E[f(\zeta) | t].\]
expected price of the remote bank’s security is just $\int_{t=0}^{1} p_R (F_R, F^L_1) d\Psi (t)$ which, by the law of iterated expectations, equals the unconditional expected payout $E [F_R (y^{\theta_1}_R, \zeta)]$ to investors. From this we subtract the discounted unconditional expected payout to investors $\delta E [F_R (y^{\theta_1}_R, \zeta)]$ to obtain the remote bank’s unconditional expected securitization profits $\pi_R (F_R) = (1 - \delta) E [F_R (y^{\theta_1}_R, \zeta)]$. To maximize this, bank $R$ simply sets $F_R (y^{\theta_1}_R, \zeta)$ equal to its maximum value, $y^{\theta_1}_R$: the bank sells a 100% equity stake in all the loans that it made.

Turning now to the local bank, assume it made some loans: its lending threshold $\theta_1$ is less than one. Given its security $F^L_1$ and signal $t$, the local bank’s expected securitization profits $\pi_L (F^L_1, t)$ equal the price $p_L (F^L_1)$ of its security less the discounted conditional expected payout to investors $\delta E [F^L_1 (y^{\theta_1}_L, \zeta) \mid t]$. Define the function

$$f (m, t) = - \frac{1}{1 - \delta} \frac{\partial E \min \{m, \zeta\} \mid t}{\partial m} \left( \frac{\delta \Phi (\zeta \mid t)}{\partial m} \right) = \frac{1}{1 - \delta} \int_{\zeta = 0}^{m} \frac{\partial \Phi (\zeta \mid t)}{\partial m} d\zeta$$

Consider the following initial value problem:

**Initial Value Problem (IVP).** The differential equation $\frac{dm}{dt} = f (m, t)$ with $m \in [0, 1]$ → $\mathbb{R}$, together with the initial value $m (0) = 1$.

**Proposition 3** Assume Lipschitz-$\Psi$, Lipschitz-$\Phi$, LPD, and HRO.

1. There exists a unique solution $m$ to IVP, which is strictly positive and decreasing in $t$.

2. There is an equilibrium in which, for each signal $t$, the local bank issues standard debt with face value $y^{\theta_1}_L m (t)$. This security has the payout $y^{\theta_1}_L \min \{\zeta, m (t)\}$. In this equilibrium, the local bank’s unconditional expected securitization profit equals the expected gains from trade $(1 - \delta) y^{\theta_1}_L E [\min \{\zeta, m (t)\}]$ from the issuer’s security, where this expectation is taken with respect to both $t$ and $\zeta$.

**Proof.** See DeMarzo, Frankel, and Jin [17].

While there may be other signaling equilibria, there are good reasons to focus on this one. First, if the signal $t$ and shock $\zeta$ come from discrete distributions, there is a unique
equilibrium that satisfies the Intuitive Criterion of Cho and Kreps [11]. Moreover, this unique equilibrium converges to the equilibrium of Proposition 3 as the gaps between signals and shocks shrink to zero.\footnote{These two results appear in DeMarzo, Frankel, and Jin [17].} Finally, the Intuitive Criterion has found experimental support in the work of Brandts and Holt [6] and Camerer and Weigelt [8].

Define

\[ \widehat{\Lambda} = E \{ \min \{ \zeta, m(t) \} \} . \]  

(21)

**Proposition 4** \( \widehat{\Lambda} \) lies in \((0, \zeta)\).

**Proof.** Appendix. \( \blacksquare \)

The local bank’s payoff \( \widehat{\Pi}_L (\theta_1, r) \) from choosing the lending threshold \( \theta_1 \) consists of the discounted expected portfolio return \( \delta y_{\theta_1} \zeta \), plus its expected securitization profits \( (1 - \delta) y_{\theta_1} \widehat{\Lambda} \) (by part 2 of Proposition 3), less the cost of loaned funds \( 1 - G_\pi (\theta_1) \). By equation (19) this payoff is just \( \Pi_L (\theta_1 | r, \bar{\theta}, \widehat{\mu}) \) where \( \Pi_L \) is defined in (11) and

\[ \widehat{\mu} = (1 - \delta) \widehat{\Lambda} + \delta \zeta, \]  

(22)

which lies in \( (\widehat{\Lambda}, \zeta) \) by Proposition 4. By (11), the local bank chooses the lending threshold \( [r \widehat{\mu}]^{-1} \) and so the remote bank’s payoff in the game is just \( \Pi_R (r | \bar{\theta}, \widehat{\mu}) \) where \( \Pi_R \) is defined in (14). Thus, our analysis of lending competition in the main model applies unchanged to this version except that \( \mu \) is replaced by \( \widehat{\mu} \) and \( \Lambda \) by \( \widehat{\Lambda} \). In light of proposition 3, this implies parts 2 and 3 of the following result. Part 4 states that security design makes securitization more profitable for the local bank: \( \widehat{\mu} \) exceeds \( \mu \).\footnote{As noted in section 1, it does so by letting the local bank signal its private information more efficiently.} Finally, let

\[ \widehat{\theta}^* = \inf \{ \bar{\theta} : \Pi^*_R (\bar{\theta}, \widehat{\mu}) \geq 0 \} \]  

(23)

denote the greatest lower bound on the set of credit scores for which the remote bank can
profitably lend.\textsuperscript{35} Part 5 states that security design weakly raises the remote bank’s lending threshold: $\hat{\theta}^* \geq \overline{\theta}^*$. Intuitively, by allowing the local bank to signal more efficiently, security design shrinks the remote bank’s advantage at the securitization stage, thus making remote lending less likely. However, remote lending still occurs for credit scores $\overline{\theta}$ in the interval $(\hat{\theta}^*, 1)$, which is nonempty by part 1.

**Theorem 4** Assume Lipschitz-$\Phi$, Lipschitz-$\Psi$, LPD, and HRO. Also assume that, in the security design subgame, the local bank plays the equilibrium described in Proposition 3. Then the following properties hold for generic parameters.

1. The remote lending threshold $\hat{\theta}^*$ lies in $(0, 1)$ and is nondecreasing in the local profitability parameter $\hat{\mu}$.

2. If $\overline{\theta} < \hat{\theta}^*$, the remote bank does not compete and agents with types $\theta$ below $\theta_1 = (\rho\hat{\mu})^{-1}$ do not borrow. Agents with types above $\theta_1$ borrow from the local bank at an interest rate $r$ equal to the gross project return $\rho$. On seeing its signal $t$, the local bank issues standard debt with face value $y_{Lm}(t)$ where the function $m$ is decreasing in $t$ and is the unique solution to IVP.

3. If $\overline{\theta} > \hat{\theta}^*$, all agents borrow. The remote bank offers an interest rate $r$ in the nonempty interval $[(\overline{\theta} \overline{\zeta})^{-1}, \rho]$ that maximizes $\Pi_R (r|\overline{\theta}, \hat{\mu})$. If an agent’s type $\theta$ exceeds $(r\hat{\mu})^{-1}$, she borrows from the local bank; else she borrows from the remote bank. In either case, she pays the interest rate $r$. The remote bank issues a 100% equity stake in its loans. The local bank’s securitization behavior is as in part 2 of this theorem, except that the local bank’s lending threshold $\theta_1$ now equals $(r\hat{\mu})^{-1}$ rather than $((\rho\hat{\mu})^{-1})$.

4. The local profitability parameter $\hat{\mu}$ exceeds the analogous parameter $\mu$ in the main model.

5. The remote lending threshold $\hat{\theta}^*$ with security design is at least as high as the remote lending threshold $\overline{\theta}^*$ in the main model.

\textsuperscript{35}The function $\Pi_R (\overline{\theta}, \hat{\mu})$, defined in section 4, is the remote bank’s payoff $\Pi_R (r|\overline{\theta}, \hat{\mu})$ at an optimal interest rate $r$ when the local profitability parameter is $\hat{\mu}$. 

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7.3 Multiple Local and Remote Banks

We now modify our main model (section 2) to incorporate multiple local and remote banks. While greater competition does lead to lower interest rates, it does not alter the set of projects that are financed. Hence, the welfare results of section 5 are robust to this change.

There is now also a continuum of remote banks \( i \in [0, 1] \) and local banks \( j \in [0, 1] \). The remote banks first make simultaneous offers. Let \( r^i \) be the offer of remote bank \( i \); if this bank makes no offer, let \( r^i = \infty \). We assume that if any remote bank makes an offer, the minimum such offer \( r \) exists.\(^{36}\) If no remote bank makes an offer, let \( r = \rho \).

The local banks then make simultaneous offers. Let \( r_{\theta j} \) be the offer that a type \( \theta \) agent gets from local bank \( j \); if no such offer is made, let \( r_{\theta j} = \infty \). For each type \( \theta \), we assume that the minimum \( r_\theta = \min \{ r_{\theta j} : j \in [0, 1] \} \) of the local banks’ bids exists. If \( r_\theta \leq r \) (respectively, \( r_\theta > r \)), we assume that a type \( \theta \) agent borrows from the local (remote) bank with the lowest offer; if more than one local (remote) bank made this offer, she flips a coin to choose among them.

We first analyze a base case without securitization: each bank must hold every loan to maturity. If a bank lends, at a gross interest rate \( r \), to a type \( \theta \) agent, its expected profit is \( \delta r \theta \zeta - 1 \): the discounted interest payment \( \delta r \) times the probability \( \theta \zeta \) of project success, less the unitary cost of capital. The proof of the following result, which follows that of Theorem 1, is omitted.

**Theorem 5** Without the option of securitization, only the local banks lend. A type \( \theta \) agent is financed if and only if her project’s discounted expected gross return, \( \delta \rho \theta \zeta \), exceeds the unitary cost of capital. Such an agent pays the interest rate \( [\delta \theta \zeta]^{-1} \) and receives positive profits, while her lender’s profits are zero.

\(^{36}\)For instance, this rules out the following profile of offers: \( r^i = 1 + i \) for \( i > 0 \) and \( r^i = \infty \) for \( i = 0 \).
A comparison with Theorem 1 shows that introducing more banks does not affect the set of projects that are funded. It merely transfers rents from the banks to the agents.

We now turn to the effects of securitization. We assume the local banks belong to a local cooperative that pools and securitizes their loans.\textsuperscript{37} After the lending stage, the local cooperative sees the macroeconomic signal $t$ and selects a subset of its loans to securitize, with the goal of maximizing its securitization profits. The local cooperative’s profits are divided among the local banks in a manner to be described below.

To facilitate comparison with our main model, we restrict attention to equilibria in which each local bank $j$ offers a loan to an agent if and only if the agent’s type $\theta$ is not less than some threshold $\theta_1^j \in \mathbb{R}_+$. As the local banks win all ties with the remote banks, the cooperative’s portfolio must then consist of all types $\theta \geq \theta_1 = \min_j \theta_1^j$ (which we assume exists and may depend on $r$).\textsuperscript{38} If any remote bank competes, each remote bank that bids $r$ receives a representative sample of the agents whose types $\theta$ are less than $\theta_1$, while remote banks that bid higher than $r$ attract no borrowers. Investors observe the measure of loans in each portfolio, and thus can infer the threshold $\theta_1$.

The securitization stage is equivalent to that of our main model, with the cooperative playing the role of the local bank. Hence, if any remote banks compete, the sum of the expected payoffs of the remote banks that offer the minimum bid $r$ equals the payoff $\Pi_R (r | \tilde{\theta}, \mu)$ of the remote bank in our main model (equation (14)). Each remote bank that bids $r$ gets an equal share of this payoff since they all make an equal proportion of remote loans.

As for the cooperative, the proportion of its loan repayments that it sells is $q_L(t) = \left( \frac{E[\zeta | t = 0]}{E[\zeta | t]} \right)^{\frac{k-\delta}{\delta}}$ by part 2 of Proposition 2. The unconditional expected payout to investors that derives from the securitization of a loan to a type $\theta$ agent is thus $E \left[ r_\theta E \left[ \zeta | t \right] q_L(t) \right]$, which equals $r_\theta \theta \Lambda$ by equation (10). Since the investors are competitive and risk-neutral,

\textsuperscript{37}By assigning the issuance decision to a cooperative, we sidestep the technical issues that arise in signalling games when multiple senders see the same signal (see Bagwell and Ramey [3]). As the local banks are small, fixed costs of issuance would create an incentive to issue their loans through a cooperative. As information among them is symmetric, bargaining costs would be minimal. However, an explicit model of the bargaining process that would lead to such an agreement is outside the scope of this paper.

\textsuperscript{38}If $\theta_1$ is not less than one, the local cooperative’s portfolio is empty.
the securitization of the marginal borrower \( \theta = \theta_1 \) raises the cooperative’s securitization revenue by \( r_{\theta_1} \Lambda \). By gradually lowering the marginal type from one (its upper bound) to zero, one finds that securitizing each type \( \theta \) increases the cooperative’s securitization revenue by \( r_{\theta} \Lambda \). We assume that for each type \( \theta \geq \theta_1 \), the cooperative pays this marginal revenue to the local bank that lent to agent \( \theta \). Later, when project returns are realized, the expected gross return of this loan is \( r_{\theta} \Lambda \), of which the expected amount \( r_{\theta} \Lambda \) is paid to investors. The remainder (whose expectation is \( r_{\theta} [\zeta - \Lambda] \)) is paid to bank \( j \), which discounts this payment at the rate \( \delta \). In this way, the cooperative passes all revenues on to its member banks. Local bank \( j \)’s total expected discounted profit from lending to a type \( \theta \) agent is thus 

\[
R_j = \sum_{\theta=\theta_1}^{\theta_1} (r_{\theta} \mu - 1) dG_\theta(\theta),
\]

which is identical to the local bank’s profit function in our main model (equation (11)) except that the interest rate \( r \) in that equation is replaced by the interest rate \( r_{\theta} \) that here is offered to agent \( \theta \).

In equilibrium, the local banks bid the interest rate \( r_{\theta} \) of any type \( \theta \geq \theta_1 \) down to the point where their expected profit \( r_{\theta} \mu - 1 \) from lending to this type is zero. Hence, all local banks offer the interest rate \( r_{\theta} = (\theta \mu)^{-1} \) to type \( \theta \) as long as this rate does not exceed the agent’s willingness to pay \( r \). Otherwise, they do not compete for agent \( \theta \). The lending threshold \( \theta_1 \) must therefore satisfy \( (\theta_1 \mu)^{-1} = r \) or, equivalently, \( \theta_1 = (r \mu)^{-1} \): the local banks have the same lending threshold as in our main model (equation (13)). And as noted, each remote bank that bids \( r \leq \rho \) receives profits that are proportional to \( \Pi_R(r|\theta, \mu) \) as in the main model. Hence, for generic parameters, if there is any \( r \) for which \( \Pi_R(r|\theta, \mu) \) is positive - if \( \Pi_R^*(\theta, \mu) > 0 \) - the remote banks all bid the lowest such \( r \); else they do not compete.

This implies the following modification of Theorem 2. The threshold \( \overline{\theta}^* \) is defined in (15) and is identical to that of the main model. Hence, part 1 of this result follows from part 1 of Theorem 2.

**Theorem 6**

1. The threshold \( \overline{\theta}^* \) lies in \((0, 1)\) and is nondecreasing in the local profitability parameter \( \mu \). It is identical to the remote bank’s lending threshold \( \overline{\theta}^* \) in the main model.

2. If \( \overline{\theta} < \overline{\theta}^* \), the remote banks do not compete and agents with types \( \theta \) below \((\rho \mu)^{-1} \) do not
borrow. Agents with types above \((\rho \mu)^{-1}\) borrow from a local bank at an interest rate \(r\) equal to \((\theta \mu)^{-1}\). On seeing its signal \(t\), the local cooperative securitizes all local loans to types \(\theta \in [(\rho \mu)^{-1}, \theta_2]\), where its securitization threshold \(\theta_2\) is determined implicitly by (4) using \(\theta_1 = (\rho \mu)^{-1}\) and \(q_L = \left(\frac{E[\zeta|t=0]}{E[\zeta|t]}\right)^{\frac{1}{1-\rho}}\).

3. If \(\bar{\theta} > \bar{\theta}^*\), all agents borrow. The remote banks offer the lowest interest rate \(r\) in the nonempty interval \([(\bar{\theta} \zeta)^{-1}, \rho]\) for which \(\Pi_R (r|\theta, \mu)\), defined in (14), is nonnegative. If an agent’s type \(\theta\) exceeds \((r \mu)^{-1}\), she borrows from a local bank at the interest rate \((\theta \mu)^{-1}\); else she borrows from a remote bank at the interest rate \(r\). The remote banks securitize all of their loans. On seeing its signal \(t\), the local cooperative securitizes all local loans to types \(\theta \in [(r \mu)^{-1}, \theta_2]\), where its securitization threshold \(\theta_2\) is determined implicitly by (4) using \(\theta_1 = (r \mu)^{-1}\) and \(q_L = \left(\frac{E[\zeta|t=0]}{E[\zeta|t]}\right)^{\frac{1}{1-\rho}}\).

A comparison of Theorems 1 and 2 with Theorems 5 and 6 shows that whether or not banks can securitize, the introduction of multiple remote and local banks merely lowers the interest rates paid by borrowers. It does not alter the effects of securitization on the set of projects that are financed or securitized. Thus, the welfare implications of securitization, discussed in section 5, remain the same.

8 Concluding Comments

We treat securitization as an exogenous innovation that encourages remote lending. If instead securitization were initially possible and an exogenous barrier to remote lending were then lifted, our model would also predict a simultaneous increase in both remote lending and securitization.\(^{39}\) In practice, legal barriers to interstate banking fell gradually starting in Maine in 1978 and ending with the federal government’s passage of the Interstate Banking

\(^{39}\)Since we assume banks lack private information about their remote loans and have a lower discount factor than investors, banks securitize all of their remote loans. Since - in our model - they securitize only some of their local loans, removing a barrier to remote lending would raise the proportion of loans that are securitized.
and Branching Efficiency Act of 1994, which abolished all remaining restrictions (Loutskina and Strahan [36, pp. 1451-2]). Since securitization was invented earlier, these barriers may have fallen partly in response to pressure from large banks who were eager to increase their securitization profits. Alternatively, their fall may have been due to an exogenous change in regulatory philosophy. This might be an interesting topic for future empirical research.

Our model follows the literature in assuming that banks securitize because they are less patient than investors. However, if banks are risk-averse, they might instead securitize in order to reduce their exposure to local macroeconomic shocks. This benefit would likely be smaller for the local bank for two reasons: asymmetric information prevents it from selling its entire portfolio, and the price of its security varies with its signal which creates risk. Hence, securitization should still favor remote lending. It should also favor applicants whose project outcomes are more correlated with local macroeconomic shocks, such as real estate developers: a risk averse bank would be especially hesitant to lend to such applicants if it could not securitize. This is an interesting question for future research.

Finally, securitization in our model worsens screening by allowing the entry of uninformed remote lenders. Another theory is moral hazard: securitization weakens a lender’s incentive to screen when doing so is costly (Rajan, Seru, and Vig [44]). The relative importance of these two theories is an interesting open question.

A Proofs

Proof of Theorem 2. The remote bank’s profit per borrower from offering the interest rate \( r \) is \( \phi_R (r|\bar{\theta}, \mu) = r \xi E_{\bar{\theta}} [\theta|\theta \leq (r\mu)^{-1}] - 1 \): its total profits \( \Pi_R (r|\bar{\theta}, \mu) \) divided by its measure of borrowers \( G_{\bar{\theta}}((r\mu)^{-1}) \). It is increasing in the mean type \( E_{\bar{\theta}} [\theta|\theta \leq (r\mu)^{-1}] \) of the remote bank’s borrowers which, in turn, is increasing in the credit score \( \bar{\theta} \) by ICE and

\[ \text{This assumes that investors are either less risk averse or better able to construct diversified portfolios.} \]

\[ \text{Some policy responses are analyzed in Bubb and Kaufman [7] and Hartman-Glaser, Piskorski, and Tchistyi [27].} \]
rate and the parameter the remote bank lends. Let remains only to show rigorously that securitization cannot lower the partial surplus when Proof of Theorem 3, continued. The first part of this proof appears in section 5. It remains only to show rigorously that securitization cannot lower the partial surplus when the remote bank lends. Let \( \omega \) equal either one or \( 1/\delta \), depending on whether the chosen scheme is A or B, respectively. Let \( U (r, z) = \int_{\theta=z}^{\theta=1} (\rho - r) \theta \zeta dG_\theta (\theta) \) denote the collective payoff \( U \) of the agents when the interest rate is \( r \) and types \( \theta \geq z \) are financed. Let \( \Omega_L (r, \sigma) = \int_{\theta=(r \sigma-1)}^{\theta=1} (r \sigma \theta - 1) dG_\theta (\theta) \) denote the local bank’s payoff where \( r \) is the interest rate and the parameter \( \sigma \) equals \( \delta \zeta \) without securitization and \( \mu \) with securitization. Without securitization, the agents get zero, so the partial surplus \( PS_{\omega}^{\text{nosec}} \) is \( \omega \Pi_L (\rho, \delta \zeta) \). With securitization, the partial surplus \( PS_{\omega}^{\text{sec}} \) is \( U (r_0, 0) + \omega \Pi_L (r_0, \mu) \) where \( r_0 \) is the interest rate offered by the remote bank. We decompose the change in the partial surplus that results

\[ \omega \Pi_L (\rho, \delta \zeta) \]

\[ \omega \Pi_L (r_0, \mu) \]

\[ U (r_0, 0) \]

\[ \omega \Pi_L (r_0, \mu) \]

\[ U (r_0, 0) + \omega \Pi_L (r_0, \mu) \]

\[ \omega \Pi_L (r_0, \mu) \]

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\[ U (r_0, 0) + \omega \Pi_L (r_0, \mu) \]

\[ \omega \Pi_L (r_0, \mu) \]

\[ U (r_0, 0) + \omega \Pi_L (r_0, \mu) \]

\[ \omega \Pi_L (r_0, \mu) \]

\[ U (r_0, 0) + \omega \Pi_L (r_0, \mu) \]
from securitization as \(P_{\omega}^{\text{sec}} - P_{\omega}^{\text{nosec}} = A + B + C\) where

\[
A = U(\rho, 0) - U\left(\rho, (\delta \rho \zeta)^{-1}\right) = 0
\]

is the effect of the remote bank’s extending loans to all agents at the interest rate \(\rho\),

\[
B = -\int_{r=r_0}^{\rho} \frac{\partial}{\partial r} \left(U(r, 0) + \omega \Pi_L (r, \delta \zeta)\right) dr
\]

\[
= \int_{r=r_0}^{\rho} \left[ \int_{\theta=0}^{1} \theta \zeta dG_\Theta (\theta) - \omega \int_{\theta=(\rho \delta \zeta)^{-1}}^{1} \delta \theta \zeta dG_\Theta (\theta) \right] dr
\]

\[
= \int_{r=r_0}^{\rho} \left[ \int_{\theta=0}^{(\rho \delta \zeta)^{-1}} \theta \zeta dG_\Theta (\theta) + (1 - \delta \omega) \int_{\theta=(\rho \delta \zeta)^{-1}}^{1} \delta \theta \zeta dG_\Theta (\theta) \right] dr \geq 0
\]

is the effect of gradually lowering the interest rate from \(\rho\) to \(r_0\), and

\[
C = \omega \int_{\sigma=\delta \zeta}^{\mu} \frac{\partial}{\partial \sigma} \Pi_L (r_0, \sigma) d\sigma = \omega \int_{\sigma=\delta \zeta}^{\mu} \left[ \int_{\theta=(r_0 \sigma)^{-1}}^{1} r_0 \theta dG_\Theta (\theta) \right] d\sigma \geq 0
\]

is the effect of giving the local bank access to the securitization market. Hence \(P_{\omega}^{\text{sec}} \geq P_{\omega}^{\text{nosec}}\) as claimed. Q.E.D.

**Proof of Proposition 4.** Since \(\Psi\) has no atoms, \(\Psi(t) \leq k_0 t\) by Lipschitz-\(\Psi\). Let \(t_0 = (2k_0)^{-1}\), so \(\Psi(t_0) \leq 1/2\), whence \(t_0 \in (0, 1)\). Let \(m_0 = m(t_0)\), which lies in \((0, 1)\) by part 1 of Proposition 3. By Lipschitz-\(\Phi\),

\[
\zeta - \hat{\lambda} = \int_{t=0}^{1} \int_{\zeta=0}^{1} \max \{0, \zeta - m(t)\} d\Phi(\zeta|t) d\Psi(t)
\]

\[
\geq \int_{t=t_0}^{1} \left( \int_{\zeta=\frac{1+m_0}{2}}^{1} (\zeta - m_0) d\Phi(\zeta|t) \right) d\Psi(t) \geq \frac{k_2}{2} \left( \frac{1-m_0}{2} \right)^2 > 0,
\]

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so \( \hat{\Lambda} < \zeta \). Similarly,

\[
\hat{\Lambda} = \int_{t=0}^{1} \int_{\zeta=0}^{1} \min \{ \zeta, m(t) \} \, d\Phi(\zeta|t) \, d\Psi(t) \\
\geq \int_{t=0}^{1} \int_{\zeta=m_0}^{1} m_0 d\Phi(\zeta|t) \, d\Psi(t) \geq k_2 m_0 (1 - m_0) \Psi(t_0)
\]

which is positive since, by assumption, \( \Psi(t_0) > 0 \) (see p. 8). Q.E.D.

**Proof of Theorem 4:** The proof of part 1 is just the proof of part 1 of Theorem 2, with \((\mu, \theta^*)\) replaced by \((\hat{\mu}, \hat{\theta}^*)\). Parts 2 and 3 are proved in the text. For part 4, by (12) and (22) it suffices to show that \( \hat{\Lambda} > \Lambda \). Define \( v(m(t), t) = E[\min \{ \zeta, m(t) \}] | t \), so \( \hat{\Lambda} = E[v(m(t), t)] \) by the law of iterative expectations. By (10), it suffices to show that when \( t \geq 0 \), \( v(m(t), t)^{1-\delta} E[\zeta | t]^\delta \geq E[\zeta | t = 0] \).\(^4\) The case \( t = 0 \) holds since \( m(0) = 1 > \zeta \). As for \( t > 0 \), \( E[\zeta | t = 0] \) is independent of \( t \) and \( m(t) \) is decreasing in \( t \). Hence, it suffices to show that when \( m(t) < 1 \),

\[
0 < \frac{\frac{d}{dt} \left( v(m(t), t)^{1-\delta} E[\zeta | t]^\delta \right)}{v(m(t), t)^{1-\delta} E[\zeta | t]^\delta} = (1 - \delta) \frac{\frac{d}{dt} v(m(t), t)}{v(m(t), t)} + \delta \frac{\frac{d}{dt} E[\zeta | t]}{E[\zeta | t]}, \quad (24)
\]

By (20) and IVP, \( \frac{d m(t)}{dt} = -\frac{1}{1-\delta} \frac{v_2(m(t), t)}{v_1(m(t), t)} \), so \( \frac{d}{dt} [v(m(t), t)] = -\frac{\delta}{1-\delta} v_2(m(t), t) \), whence (24) can be rewritten as

\[
0 < \frac{\frac{d}{dt} E[\zeta | t]}{E[\zeta | t]} = \frac{v_2(m(t), t)}{v(m(t), t)}. \quad (25)
\]

Let \( c \in [0, 1] \). Integrating by parts, \( v(c, t) = c - \int_{\zeta=0}^{c} \Phi(\zeta | t) \, d\zeta \), so (25) must hold if, for \( c < 1 \),

\[
0 < \frac{d^2}{dt^2 \, dc} \ln \left( \int_{\zeta=0}^{c} [1 - \Phi(\zeta | t)] \, d\zeta \right) = \frac{d}{dt} \left[ \frac{1 - \Phi(c | t)}{\int_{\zeta=0}^{c} [1 - \Phi(\zeta | t)] \, d\zeta} \right]. \quad (26)
\]

By HRO, for all \( \zeta' > \zeta'' \), \( 1 - \Phi(\zeta'' | t) \) is decreasing in \( t \). Integrating over \( \zeta'' < \zeta' \), \( \int_{\zeta=0}^{c} [1 - \Phi(\zeta | t)] \, d\zeta \) is decreasing in \( t \), which implies (26): \( \hat{\Lambda} \) exceeds \( \Lambda \). Part 5 follows from part 1 of Theorem

\(^4\)The notation "\( f_0(t) \geq f_1(t) \) when \( t \geq 0 \)" means that \( f_0(t) \) exceeds (equals) \( f_1(t) \) when \( t \) exceeds (equals) zero.
2 since the definition of $\hat{\theta}$ (equation (23)) is identical to that of $\tilde{\theta}$ (equation (15)) except that $\mu$ is replaced by the higher $\tilde{\mu}$. Q.E.D.

References


Securitization and Lending Competition:

Online Appendix*

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1 Introduction

This document is the online appendix to Frankel and Jin [5], which we first summarize. There are two banks, one local and one remote. There is a continuum of loan applicants who have different credit qualities, which are known only to the local bank. Finally, there is a continuum of risk-neutral investors. The remote bank first makes loan offers, followed by the local bank. Applicants then choose which, if any, offer to accept. The local bank then observes a macroeconomic shock that affects loan repayment probabilities. Finally, each bank chooses a proportion of their loans to offer for sale.

While the local bank has an informational advantage at the lending stage, the macroeconomic shock creates a lemons problem that forces it to retain some of its loans in order to signal their quality. As the remote bank faces no such limitation, its profits from securitizing a given loan portfolio are greater. Hence, remote lending can occur.

In Frankel and Jin [5], investors can infer the distribution of credit qualities in each bank’s loan portfolio by observing the measure of loans that each bank makes. Hence, if there were no subsequent macroeconomic shock, there would be no lemons problem for the local bank. Given its informational advantage, all lending would be local.

In this appendix we consider a different approach to creating a lemons problem: aggregate uncertainty about the distribution of credit qualities. We study the simplest case, in which there is a single applicant whose credit quality is unknown.

We first show that there are multiple separating equilibria. In some the remote bank lends while in others it does not. Intuitively, with a single agent investors cannot infer the local bank’s lending standards with certainty. Hence, if the local bank deviates (e.g., by selling a higher than expected proportion of its loan), investors may conclude that the borrower has no possibility of repayment. Threatened with such pessimistic beliefs, the local bank can be prevented from securitizing more than an arbitrary proportion of its loans. This permits multiple equilibria with different such arbitrary proportions. When this proportion is low, the remote bank’s easier access to the securitization market outweighs the informational disadvantage it faces at the lending stage: remote lending can occur.
When faced with multiple equilibria, researchers often look to equilibrium refinements to sharpen their predictions. In games with a continuum of signals and actions such as ours, one needs a strong selection criterion such as the D1 refinement of Banks and Sobel [2]. We apply D1 and show that a unique equilibrium survives, in which all lending is local. Intuitively, if the local bank sells an unexpectedly high proportion of its loan, D1 forces investors to believe that the loan is of the lowest quality \( \theta_1 \) to which the local bank lends. (A loan’s quality is just its repayment probability.) Hence, the local bank can securitize its entire loan to an applicant of quality \( \theta \) that is slightly below \( \theta_1 \) and receive the price \( r\theta_1 \), where \( r \) is the interest rate. For the local bank not to want to deviate in this way, \( r\theta_1 \) must equal the common cost of capital. If the local bank competes, it attracts the applicant only if her quality is below \( \theta_1 \). Hence, the most investors will pay for the remote bank’s loan is \( rE[\theta|\theta < \theta_1] \), which is less than the cost of capital \( r\theta_1 \): the remote bank will not lend.

A problem with this result is that the restriction on off-equilibrium beliefs that D1 imposes is somewhat arbitrary. Roughly speaking, D1 states that following a deviation, investors believe with certainty that the loan is of the type \( \theta \) for which the local bank most benefits from the deviation. However, the local bank may benefit from deviating for other loan types as well. D1 prevents investors from assigning any positive probability to these other loan types. A discussion appears in section 4.1.

2 The Model

There is a single agent with a project that succeeds with probability \( \theta \). The random variable \( \theta \) has the distribution function \( G_{\bar{\theta}} \), which is indexed by the mean value \( \bar{\theta} = E[\theta] \) as in Frankel and Jin [5]. The agent’s project has a gross return of \( \rho > 1 \) if it succeeds and zero if it fails. It requires a unit of capital that the agent must borrow. There is a local bank and a remote bank. The local bank knows the agent’s success probability \( \theta \) while the remote bank knows only its distribution \( G_{\bar{\theta}} \).

The remote bank first chooses whether to make an offer and, if so, at which interest rate
r. Seeing $r$, the local bank then decides whether or not to make an offer of its own. The applicant then chooses which, if any, offer to accept. (The applicant can accept at most one offer.) Let $r$ equal the gross return $\rho$ if the remote bank refrained from making an offer. With this convention, the agent is willing to pay the local bank the interest rate $r$ but no more. Hence, if the local bank does make an offer, it will offer $r$ and the agent will accept.

Assume an offer is accepted. With probability $a \in (0, 1)$, the lending bank can then offer any proportion of its loan for sale to a continuum of uninformed, risk-neutral investors with deep pockets. With complementary probability $1 - a$, the bank must hold the loan to maturity.\(^1\) The bank’s discount factor is $\delta$ while the investors have a unit discount factor. In order to obtain an analytic solution for the interest rate, we focus on the case in which the distribution of types $\theta$ is not too concave, and its concavity is nondecreasing in $\theta$:

**No Cream Skimming (NCS).** For all $\theta$ in the interior of the support of $G_\theta$, $\theta G_\theta^u(\theta) / G_\theta^L(\theta)$ is greater than $-1$ and is weakly increasing in $\theta$.

We first establish a monotonicity property that must hold in any pure strategy equilibrium. Fix an interest rate $r$ offered by the remote bank. The local bank’s payoff from lending without securitization is the discounted expected gross loan return $\delta \theta r$ less the unitary cost of capital. If, in addition, the securitization market functions (probability $a$) and the bank sells a proportion $Q$ at the price $P$, the bank gets an additional payoff of $Q [P - \delta \theta r]$. Hence, the local bank’s total payoff from lending is

$$U^r(\theta, Q, P) = \delta \theta r - 1 + aQ [P - \delta \theta r] = -1 + \delta \theta r [1 - aQ] + aQP$$

which is increasing in the repayment probability $\theta$. Let $P^r(Q)$ be the equilibrium price function. For all $\theta \in [0, 1]$, let

$$Q^r(\theta) = \arg \max_{Q \in [0, 1]} U^r(\theta, Q, P^r(Q))$$

\(^1\)In Frankel and Jin [5] there is no constant $a$ as we focus on the limit as $a \to 1$. In this appendix, we must fix $a < 1$ in order to apply D1. As the results do not place conditions on $a$ other than $a < 1$, they also hold in the limit as $a \to 1$. 

4
be the local bank’s optimal quantity and let

$$
\Pi(\theta) = \max_{Q \in [0,1]} U^r(\theta, Q, P^r(Q))
$$

be its maximum expected profits if it lends to an applicant of type $\theta$. Equations (2) and (3) actually define $Q^r$ and $\Pi$ on the whole real line, although for $\theta \notin [0,1]$ they have no economic interpretation.

**Proposition 1** Fix $r > 0$. The function $Q^r$ is nonincreasing. The function $\Pi$ is continuous and increasing and satisfies $\lim_{\theta \to -\infty} \Pi(\theta) = \infty$ and $\lim_{\theta \to -\infty} \Pi(\theta) = -\infty$.

An immediate consequence is that the local bank follows a threshold lending rule:

**Corollary 2** Fix $r > 0$. There is a unique threshold $\theta^r_1 \in \mathbb{R}$ that satisfies $\Pi(\theta^r_1) = 0$. The local bank lends if and only if the repayment probability $\theta$ is at least $\theta^r_1$.

### 3 Separating Equilibria with Remote Lending

We now show that for any $q \in [0,1)$, there are equilibria in which the local bank is limited to securitizing a proportion $q$ of any loan it makes. Moreover, by increasing the remote bank’s advantage at the securitization stage, this partial securitization property allows remote lending to occur.

**Theorem 3** Fix any $q \in [0,1)$.

1. In the subgame following the remote bank’s offer of any interest rate $r$, there is an equilibrium in which the local bank never securitizes a proportion greater than $q$ of any loan it makes.

2. Assume $a > \frac{e^{1-\delta}}{1-\delta}$ and No Cream Skimming. There is a cutoff $\mu \in (0,1)$, which is nondecreasing in $q$, such that if the expected type $\bar{\theta}$ is at least $\mu$, then the full game has

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2 We assume that the local bank lends if it is indifferent.
an equilibrium in which only the remote bank competes and offers an interest rate $r^*$ that deters the local bank from lending except to the highest type $\theta = 1$, of whose loan the local bank securitizes a proportion $q$.

Intuitively, the investors’ pessimistic beliefs limit the local bank to a securitization proportion $q < 1$ whenever it does lend. These beliefs raise the deterring rate $r^*$, at which the local bank just willing to lend to the highest type. Able to collect the higher interest rate $r^*$ without losing its best applicants to cream-skimming, the remote bank will lend if the expected type $\bar{\theta}$ is high enough.

4 Equilibria that Survive D1

We now consider equilibria that survive the D1 refinement of Banks and Sobel [2]. By protecting the local bank from pessimistic beliefs, D1 strengthens the local bank’s competitive position to such an extent that the remote bank cannot compete.

**Theorem 4** Assume the refinement D1.

1. In the subgame following the remote bank’s offer of any interest rate $r$:

   (a) If the local bank sometimes lends (if $\theta^*_1 \leq 1$) then the signalling game has a unique equilibrium that survives D1. Investors respond to the quantity $Q$ with the price $P^r(Q) = \frac{r^\gamma}{Q^{1-\delta}}$ and for all $\theta \in [\theta^*_1, 1]$, the local bank sells the quantity $Q^r(\theta) = \left(\frac{\theta^*}{\bar{\theta}}\right)^{1-\delta}$ and earns total profits $\Pi(\theta) = \delta r \theta - 1 + r (1 - \delta) \left(\frac{\theta^*}{\bar{\theta}}\right)^{1-\delta}$. If $\theta = \theta^*_1$, the local bank’s profits are $\Pi(\theta^*_1) = r \theta^*_1 - 1$, which equals zero and exceeds the remote bank’s profits from offering $r$.

   (b) If the local bank never lends (if $\theta^*_1 > 1$), $r$ must be low enough that the the remote bank’s profits from offering $r$ are less than $\theta - 1$ which is negative.

2. As the remote bank’s profits in both cases are negative, it will not compete.
The intuition runs roughly as follows. First suppose that the interest rate $r$ is high enough that the local bank sometimes lends: $\theta_1^r \leq 1$ (part 1a). If the local bank then securitizes its entire loan, D1 forces investors to conclude that the applicant’s success probability $\theta$ equals the local bank’s lending threshold $\theta_1^r$. Hence, the local bank can reap the entire gains from trade $(1 - \delta) r\theta_1^r$ if it sells the whole loan. The local bank’s profits from such a loan are thus $r\theta_1^r - 1$: the sum of its securitization profits $(1 - \delta) r\theta_1^r$ and its discounted gross expected loan return $\delta r\theta_1^r$, less its unit cost of capital. As the local bank is indifferent between lending and not, its profits $r\theta_1^r - 1$ must be zero. But this leaves the remote bank with applicants $\theta < \theta_1^r$, for whom the joint payoff of the bank and investors, $r\theta - 1$, is negative. Since the investors must break even, the remote bank loses money. Accordingly, the remote bank will not make such an offer.

Now suppose the interest rate $r$ is so low that the local bank never lends: $\theta_1^r > 1$ (part 1b). If the local bank unexpectedly issues a loan, by D1 investors believe that the loan type $\theta$ equals one as this type maximizes the profitability of the local bank’s deviation. They thus assign the price $r$ to the loan. As this exceeds the local bank’s gross return $\delta r\theta$ from retaining the loan, it sells the entire loan, getting a profit of $r - 1$. This cannot be positive since, by assumption, the remote bank never lends: $r \leq 1$. Hence, the remote bank’s profits $r\bar{\theta} - 1$ are at most $\bar{\theta} - 1$, which is negative: the remote bank will not make such an offer either. Since the remote bank does not have a profitable offer, it will not compete.

4.1 D1: Discussion

As Fudenberg and Tirole [6] note, the beliefs restrictions that D1 imposes are strong and somewhat arbitrary. In our model, D1 seems to force investors to hold very specific and arbitrary beliefs about what a deviating local bank must think. In particular, for each loan type $\theta$ and securitization quantity $Q$, let $\underline{P}_Q^\theta$ be the minimum price that investors could assign to the loan that would make the local bank willing to deviate in the given way. D1

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3 This intuition assumes the securitization market functions: $a = 1$. The proof does not assume this.
states that investors believe that if the local bank lends and sells $Q$, investors conclude with certainty that the type $\theta$ is the type $\theta^*$ that yields the lowest such minimum price $P_{Q\theta}^\theta$. But for investors to believe this, they must think that the local bank expects $P$ to equal the price $P_{Q\theta}^{\theta^*}$ at which it is willing to deviate only if its loan type is $\theta^*$. For if the local bank expects the price $P$ possibly to exceed $P_{Q\theta}^{\theta^*}$, it would want to deviate when its loan type exceeds $\theta^*$ as well. It is not clear why investors should think the local bank has these very specific beliefs.

A Proofs

Proof of Theorem 3: Fix a constant $q \in [0,1)$. We seek a separating equilibrium in which $Q^r (\theta_1^r) = q$ for any interest rate $r > 0$ for which $\theta_1^r \in [0,1]$. Let $R$ be the set of such interest rates. We may assume w.l.o.g. that for any $r \in R$, if the local bank deviates to a quantity above $q$ then investors conclude that the loan has no possibility of repayment.\footnote{By Proposition 1, the local bank never securitizes more than $q$ in equilibrium.} This belief indeed prevents such a deviation since the resulting price $P$ is zero.

Fix any $r \in R$. The price $P$ that results from the quantity $q$ is $\theta_1^r r$ and the local bank’s payoff $U^r (\theta_1^r, q, \theta_1^r r)$ at $\theta = \theta_1^r$ is zero by Corollary 2, so $\theta_1^r = [aq + \delta (1 - aq)]^{-1} r^{-1}$ by (1). Now consider the price function $P^r (Q) = \frac{r \theta_1^r q^{1-\delta}}{Q^{1-\delta}}$ and the quantity function $Q^r (\theta) = q \left( \frac{\theta_1^r}{\theta} \right)^{1-\delta}$. The first order condition for an optimal quantity $Q$, evaluated using the given pricing function, is

$$0 = \frac{d}{dQ} U^r (\theta, Q, P^r (Q)) = \frac{d}{dQ} \left[ \delta \theta r - 1 + aQ [P^r (Q) - \delta \theta r] \right]$$

$$= a \left[ P^r (Q) - \delta \theta r + Q \frac{dP^r (Q)}{dQ} \right] = a \delta r \left[ \frac{\theta_1^r q^{1-\delta}}{Q^{1-\delta}} - \theta \right]$$

which, at $Q = Q^r (\theta)$, is zero as $\frac{\theta_1^r q^{1-\delta}}{Q^r (\theta)^{1-\delta}} = \theta$. Moreover, for any $\theta$, $U^r (\theta, Q, P^r (Q))$ is globally concave in $Q$ as

$$\frac{d^2}{dQ^2} U^r (\theta, Q, P^r (Q)) = \frac{d^2}{dQ^2} [QP^r (Q)] = \frac{d^2}{dQ^2} \left[ r \theta_1^r q^{1-\delta} Q^\delta \right]$$
which is negative as \( \delta \in (0, 1) \). Finally, the pricing function reflects Bayesian updating since

\[
P^r (Q_r (\theta)) = \frac{r \theta^r_q q^{1-\delta}}{Q^r (\theta)^{1-\delta}} = \frac{r \theta^r_q q^{1-\delta}}{q \left( \frac{\theta^r_q}{\theta} \right)^{1-\delta}} = \theta r.
\]

The local bank’s equilibrium payoff to lending to any type \( \theta \in [\theta^r_1, 1] \) is

\[
U^r (\theta) = U^r (\theta, Q_r (\theta), P^r (Q_r (\theta))) = \delta \theta r - 1 + aQ^r (\theta) [P^r (Q_r (\theta)) - \delta \theta r]
\]

\[
= \delta \theta r - 1 + aq \left( \frac{\theta^r_q}{\theta} \right)^{1-\delta} \theta [1 - \delta] = \delta \theta r - 1 + aqr (1 - \delta) \left( \frac{\theta^r_q}{\theta^r_q} \right)^{1-\delta}.
\]

We have produced a separating equilibrium in which the local bank sells up to the fixed proportion \( q \), independent of \( r \). We now turn to the remote bank’s behavior: which \( r \) will it choose, and are its lending profits positive at this \( r \)? If so, there can be remote lending. If the remote bank offers \( r \), it lends if and only if \( r < \theta^r_1 \) where

\[
\theta^r_1 = b r^{-1}
\]

and \( b = [aq + \delta (1 - aq)]^{-1} \). Moreover, it securitizes the whole loan, getting the loan’s expected value \( rE [\theta | \theta < \theta^r_1] \). Let

\[
r^* = b
\]

be the deterring rate: if \( r = r^* \), the local bank’s threshold \( \theta^r_1 = 1 \) so it is just deterred from competing. As the securitization market exists with probability \( a \), the remote bank’s expected payoff from offering \( r \) is \([a + \delta (1 - a)] rE [\theta | \theta < \theta^r_1] - 1\). Let

**Lemma 5** Under No Cream Skimming, if the remote bank competes it will offer the rate \( \min \{ \rho, r^* \} \) and the local bank will not lend.

**Proof of Lemma 5:** As the securitization market exists with probability \( a \), the remote bank’s expected payoff from offering \( r \) is

\[
[a + \delta (1 - a)] rE [\theta | \theta < \theta^r_1] - 1 = c^{-1} \int_{\theta=0}^{br^{-1}} (r \theta - c) dG_\pi (\theta) \overset{d}{=} c^{-1} I (r)
\]

where \( c = [a + \delta (1 - a)]^{-1} \). Suppose bank \( R \) does compete. Then it chooses an interest rate \( r \leq \rho \) to maximize \( I (r) \) and, moreover, \( I (r) > 0 \) at this optimal \( r \). If \( r < \min \{ \rho, r^* \} \),
then by (4) \( \theta_1' \) exceeds one so \( I'(r) = \int_{\theta=0}^{1} r \theta dG_{\theta}(\theta) > 0 \): bank \( R \)'s optimal \( r \) is at least \( \min \{ \rho, r^* \} \). It remains to show that its optimal \( r \) is at most \( \min \{ \rho, r^* \} \). If \( r^* \geq \rho \), we are done since \( \min \{ \rho, r^* \} = \rho \) and, by assumption, bank \( R \) competes (chooses \( r \leq \rho \)). If \( r^* < \rho \), consider any \( r > r^* \). We will show, using No Cream Skimming, that if \( I(r) > 0 \) then \( I'(r) < 0 \): no such \( r \) can be optimal for bank \( R \). By (4) and (5), \( \theta_1' = br^{-1} = r^*/r < 1 \), so

\[
I(r) = \int_{\theta=0}^{r^*/r} (r \theta - c) dG_{\theta}(\theta).
\]

With the change of variables \( s = r \theta \), \( I(r) = \frac{1}{r} \int_{s=0}^{r^*} (s - c) G'_{\theta}(\frac{s}{r}) ds \). Thus,

\[
I'(r) = -\frac{1}{r^2} \left( \int_{s=0}^{r^*} (s - c) \left[ \frac{G'_{\theta}(\frac{s}{r}) + G''_{\theta}(\frac{s}{r}) \frac{s}{r}}{G'_{\theta}(\frac{s}{r})} \right] G'_{\theta}(\frac{s}{r}) ds \right).
\]

Changing variables back,

\[
I'(r) = -\frac{1}{r} \left( \int_{\theta=0}^{r^*/r} (r \nu - c) \left[ \frac{G'_{\theta}(\nu) + G''_{\theta}(\nu) \nu}{G'_{\theta}(\nu)} \right] dG_{\theta}(\nu) \right).
\]

For any functions \( \varphi_0(\theta) \) and \( \varphi_1(\theta) \), let \( E^*(\varphi_0) \) and \( Cov^*(\varphi_0, \varphi_1) \) denote the expectation of \( \varphi_0 \) and covariance of \( \varphi_0 \) and \( \varphi_1 \), both conditioned on \( \theta \) lying in \( [0, r^*/r] \). Then

\[
I'(r) = -\frac{1}{r} E^*(xy) G_{\theta}(r^*/r), \text{ where } x(\theta) = r \theta - c \text{ and } y(\theta) = \frac{G_x'(\theta) + G_y''(\theta) \theta}{G_y' G_{\theta}(\theta)}.
\]

By definition of covariance, \( E^*(xy) = Cov^*(x, y) + E^*(x) E^*(y) \). By assumption \( E^*(x) = \frac{I(r)}{G_{\theta}(r^*/r)} > 0 \). By No Cream Skimming, \( y(\nu) \) is positive and nondecreasing in \( \nu \), so \( E^*(y) > 0 \) and \( Cov^*(x, y) \geq 0 \). Thus \( E^*(xy) > 0 \), so \( I'(r) < 0 \) as claimed. Q.E.D. Lemma 5

Thus, if the local bank competes, its profits are \( \Pi_R(r) = [a + \delta (1 - a)] \min \{ \rho, r^* \} \bar{\theta} - 1 \) by Lemma 5. It will compete if this is positive: if the unconditional expected success probability \( \bar{\theta} \) exceeds the threshold

\[
[a + \delta (1 - a)]^{-1} = [a + \delta (1 - a)]^{-1} \min \{ \rho, [aq + \delta (1 - aq)]^{-1} \}^{-1} = \max \left\{ \frac{1}{\rho [a + \delta (1 - a)]}, \frac{aq + \delta (1 - aq)}{a + \delta (1 - a)} \right\}.
\]

The numerator in the second ratio is increasing in \( q \) as \( \delta < 1 \); the denominator in the second ratio equals the numerator evaluated at \( q = 1 \). Hence, the second ratio is less than one.
for any \( q < 1 \). The first ratio is less than one as long as \( a > \frac{\rho^{-1-\delta}}{1-\delta} \). Thus, as long as this condition on \( a \) holds, there exist distributions of success probabilities \( \theta \) that prompt the remote bank to compete. Q.E.D.

**Proof of Lemma 6:** Assume \( \theta' > \theta'' \geq \theta_1' \); let \( Q' = Q^r (\theta') \), \( Q'' = Q^r (\theta'') \), \( P' = P^r (Q') \), and \( P'' = P^r (Q'') \). Then incentive compatibility requires that \( \Pi (\theta') = U^r (\theta', Q', P') \geq U^r (\theta', Q'', P'') \) and \( \Pi (\theta'') = U^r (\theta'', Q'', P'') \geq U^r (\theta'', Q', P') \). Hence,

\[
U^r (\theta', Q', P') - U^r (\theta'', Q', P') \geq U^r (\theta', Q'', P'') - U^r (\theta'', Q'', P'')
\]

By (1), \( (\theta' - \theta'') (Q'' - Q') \geq 0 \) so \( Q'' \geq Q' \) as claimed. Moreover, by (1), \( \Pi (\theta') - \Pi (\theta'') \) is bounded above by \( U^r (\theta', Q'', P'') - U^r (\theta'', Q'', P'') = \delta (\theta' - \theta'') r [1 - aQ''] \) and below by \( U^r (\theta', Q', P') - U^r (\theta'', Q', P') = \delta (\theta' - \theta'') r [1 - aQ'] \). Both bounds are positive and at most \( \delta r |\theta' - \theta''| \), so \( \Pi () \) is continuous and increasing in \( \theta \). Finally as \( Q' \in [0, 1] \),

\[
\frac{\Pi (\theta') - \Pi (\theta'')}{\theta' - \theta''} \geq \delta r (1 - a) > 0 \text{ which implies the two limiting properties. Q.E.D.}
\]

**Proof of Theorem 4:** Part 1. Fix an equilibrium. Let \( R \) be the set of interest rates \( r \) for which the local bank ever lends: for which \( \theta_1' \leq 1 \). For any \( r \in R \), let \( q^r \) be the maximum the local bank ever securitizes conditional on lending: \( q^r = \max_{\theta \geq \theta_1'} Q (\theta) \). We first show that D1 rules out equilibria in which the local bank never securitizes conditional on lending.

**Lemma 6** In any equilibrium that survives D1, for any \( r \in R \), \( q^r > 0 \).

**Proof of Lemma 6:** W.l.o.g. we can assume any positive quantity \( Q \) leads to the belief that \( \theta = 0 \), so \( P = 0 \). In this equilibrium, \( \delta \theta_1' r = 1 \), so \( \theta_1' = 1/\delta r \). Fix a quantity \( Q > 0 \). For each price \( P \), and each type \( \theta \), let \( \pi (\theta, P) \) be the local bank’s set of optimal probabilities of lending and then selling \( Q \), vs. sticking to the equilibrium, if its type is \( \theta \) and the price it anticipates from this deviation is \( P \). There are two cases.

1. If \( \theta \geq \theta_1' = 1/\delta r \), the local bank lends in equilibrium, getting \( \delta \theta r - 1 \). So its relative payoff from this deviation is

\[
U^r (\theta, Q, P) - (\delta \theta r - 1) = aQ [P - \delta \theta r].
\]
This is nonnegative iff $\theta \leq \frac{P}{\delta r} = \gamma_P$, which is possible only if $P \geq 1$. Intuitively, we know that $\delta \theta r \geq 1$, as the local bank is willing to lend without securitizing. But for it to be willing to securitize any of the loan, it must be that the price $P$ that it gets is not less than the discounted expected return $\delta \theta r$ of the loan. Hence, $P \geq 1$. For $P > 1$,

$$\pi(\theta, P) = \begin{cases} 
1 & \text{if } \frac{1}{\delta r} \leq \theta < \gamma_P \\
[0, 1] & \text{if } \theta = \gamma_P \\
0 & \text{if } \theta > \gamma_P
\end{cases}$$

If $P = 1$, $\overline{\mu}(\theta, P) = \begin{cases} 
[0, 1] & \text{if } \theta = \gamma_P = \frac{1}{\delta r}; \text{ if instead } P < 1, \text{ then } \overline{\mu}(\theta, P) = 0 \text{ for all } \theta \geq \theta^*_1.
\end{cases}$

2. If $\theta \leq \theta^*_1 = 1/\delta r$, the local bank is willing not to lend in equilibrium. So its relative payoff from this deviation is

$$U^r(\theta, Q, P) = -1 + \delta \theta r [1 - aQ] + aQP.$$ 

This is nonnegative iff $\theta \leq \frac{1-aQP}{\delta r[1-aQ]} = \lambda^Q_P$, which is possible only if $\lambda^Q_P \leq \frac{1}{\delta r} = \theta^*_1$ or, equivalently, if $1 - aQP \leq 1 - aQ$ or, equivalently, if $P \geq 1$ (the same condition as in case 1). In this case, if $P > 1$,

$$\pi(\theta, P) = \begin{cases} 
1 & \text{if } \lambda^Q_P < \theta \leq \frac{1}{\delta r} \\
[0, 1] & \text{if } \theta = \lambda^Q_P \\
0 & \text{if } \theta < \lambda^Q_P
\end{cases}$$

while if $P = 1$,

$$\overline{\mu}(\theta, P) = \begin{cases} 
[0, 1] & \text{if } \theta = \lambda^Q_P = \frac{1}{\delta r}. \text{ Finally, if } P < 1, \text{ then } \overline{\mu}(\theta, P) = 0 \text{ for all } \theta < \theta^*_1.
\end{cases}$$

Collecting cases 1 and 2, if $P > 1$, then $\lambda^Q_P < \frac{1}{\delta r} < \gamma_P$ so

$$\overline{\mu}(\theta, P) = \begin{cases} 
1 & \theta \in \left(\frac{1-aQP}{\delta r[1-aQ]}, \frac{P}{\delta r}\right) \triangleq \left(\lambda^Q_P, \gamma_P\right) \\
[0, 1] & \text{if } \theta \in \left\{\lambda^Q_P, \gamma_P\right\} \\
0 & \text{otherwise}
\end{cases}$$
while if \( P = 1 \), then \( \lambda_P^Q = \frac{1}{\delta r} = \gamma_P \) so \( \overline{\mu}(\theta, P) = \begin{cases} [0, 1] & \text{if } \theta = \frac{1}{\delta r}, \text{ and if } P < 1, \text{ then} \overline{\mu}(\theta, P) = 0 \text{ for all } \theta: \text{ no bank wants to deviate.} \end{cases} \)

Finally, D1 states that if each price \( P \) that makes type \( \theta \) (say) weakly prefer to deviate to \( Q \) also makes type \( \theta' \) strictly prefer to deviate to \( Q \), then on seeing \( Q \) the investors believe that the local bank’s type is \( \theta' \). Hence, they believe that \( \theta = \min \{1, 1/\delta r\} \), regardless of \( Q \), and thus assign the price \( P = r\theta = \min \{r, 1/\delta\} \). As \( r \in R \), consider \( \theta = \theta_1^r = 1/\delta r \leq 1 \). If the local bank sells a proportion \( Q \) of the loan, it gains \( Q [P - \delta r\theta] = Q [1/\delta - 1] > 0 \). Hence, \( q^r > 0 \) as claimed. Q.E.D.

Lemma 6
We now turn to equilibria of the securitization subgame in which \( q^r \in (0, 1] \). We show that D1 implies that \( q^r = 1 \) and that these equilibria have the DeMarzo-Duffie form.

**Lemma 7** Consider an equilibrium that survives D1. Suppose that for some given \( r \in R, q^r \in (0, 1] \). Then \( q^r = 1, P^r (Q) = \frac{r\theta^r}{Q^1}, \text{ and for all } \theta \in [\theta_1^r, 1], Q^r (\theta) = \left( \frac{\theta_1^r}{\theta} \right)^{\frac{1}{1-r}} \text{ and } \Pi (\theta) = \delta r \theta - 1 + r (1 - \delta) \left( \frac{\theta_1^r}{\theta} \right)^{\frac{1}{1-r}}. \)

**Proof of Lemma 7:** By Claim 1, \( Q^r (\theta_1^r) = q^r \). By Corollary 2, \( \Pi (\theta_1^r) = 0 \). The local bank’s signal has two components: the decision to lend, and the quantity \( Q \) to securitize. Let \( 1_{\text{lend}} \) equal one if the local bank lends and zero otherwise. The local bank’s realized profit from its choice given the repayment probability \( \theta \) and the anticipated security price \( P \) is

\[
\widehat{U}^r (\theta, (1_{\text{lend}}, Q), P) = U^r (\theta, Q, P) * 1_{\text{lend}}
\]

where \( U \) is defined in (1). Hence, the analogue to the Spence-Mirrlees sorting condition (Fudenberg and Tirole [6, ch. 11]) in our model is

\[
- \frac{\partial U^r}{\partial Q} = \frac{\partial U^r}{\partial P} = \begin{cases} - \frac{\partial [P - \delta \theta r]}{\partial Q} = \frac{\delta \theta r - P}{Q} & \text{which is increasing in } \theta \text{ if } 1_{\text{lend}} = 1 \\ \text{undefined} & \text{if } 1_{\text{lend}} = 0 \end{cases}
\]

Lemma 11.2 of Fudenberg and Tirole [6] can now be adapted to our model as follows.
Lemma 8 Let $\theta'' > \theta' \geq \theta_1$. Also assume that when $\theta = \theta''$, the local bank lends and sells $Q'$ with positive probability in equilibrium. Then D1 implies that $\mu(\theta' | Q'') = 0$ for all $Q'' < Q'$.

Proof of Lemma 8: Fix an equilibrium $(\sigma_1^* (1_{\text{lend}}, Q|\theta), \sigma_2^* (P|1_{\text{lend}}, Q))$ such that (a) type $\theta''$ lends and sells $Q'$ with positive probability ($\sigma_1^* (1, Q'|\theta'') > 0$) and (b) type $\theta'$ sometimes lends (there is a $Q$ for which $\sigma_1^* (1, Q'|\theta') > 0$). Let $P^* ((1_{\text{lend}}, Q))$ denote the investors’ equilibrium price in response to the local bank’s action $(1_{\text{lend}}, Q)$. For each $Q'' < Q'$ and every $\theta$ for which the local bank sometimes lends, let $\widehat{P}(\theta)$ in the set $\Pi = [0, r]$ of optimal prices for any type $\theta$ satisfy $U^r \left( \theta, (1, Q''), \widehat{P}(\theta) \right) = U^r \left( \theta, (1, Q''), \widehat{P}(\theta'') \right)$ (the equilibrium payoff of the local bank when the type is $\theta$). If no such action $\widehat{P}(\theta)$ exists, let $\widehat{P}(\theta) = \infty$. We claim that $\widehat{P}(\theta') > \widehat{P}(\theta'')$. For assume not: $\widehat{P}(\theta') \leq \widehat{P}(\theta'')$. By definition of $\widehat{P}(\theta)$, type $\theta''$ is indifferent between the action pair $A = ((1, Q'), P^* ((1, Q')))$ and $B = ((1, Q''), \widehat{P}(\theta''))$. By Spence-Mirrlees, conditional on $1_{\text{lend}} = 1$, the indifference curve in $(Q, P)$ space of type $\theta'$ is steeper than that of type $\theta''$ at any point. But since $\widehat{P}(\theta') \leq \widehat{P}(\theta'')$, the indifference curve of type $\theta''$ lies above the indifference curve of type $\theta'$ at $Q = Q''$. Thus, these two indifference curves cannot intersect at any $Q < Q''$. Therefore, type $\theta'$ strictly prefers $((1, Q'), P^* ((1, Q'))) to his equilibrium strategy - and thus will deviate from the assumed equilibrium. This is a contradiction. (I.e., it’s not an equilibrium after all.) We have shown that $\widehat{P}(\theta') \geq \widehat{P}(\theta'')$. Thus, the set $\left[ \widehat{P}(\theta'), r \right]$ of prices $P$ that make type $\theta'$ willing to sell the quantity $Q''$ is strictly contained in the set $\left( \widehat{P}(\theta''), r \right]$ of prices that make type $\theta''$ strictly prefer to sell the quantity $Q''$. Hence, by D1, on seeing the local bank lend and then sell $Q''$, investors must assign probability zero to the type being $\theta'$. Q.E.D.
results from $Q^*$ is $r\bar{\theta}$. Since there is some pooling at $Q^*, \bar{\theta} < \theta^*$. Suppose type $\theta^*$ deviates to the quantity $Q^* - \iota$ for any $\iota \in (0, Q^*)$. By the lemma, the investors’ posterior over $\theta$ when they see the quantity $Q^* - \iota$ is at least $\theta^*$. Hence, by switching from $Q^*$ to $Q^* - \iota$ the local bank can raise the price by at least $\alpha = r [\theta^* - \bar{\theta}] > 0$, independent of $\iota$. Its change in its profits is at least

$$U^r (\theta^*, Q^* - \iota, r\bar{\theta} + \alpha) - U^r (\theta^*, Q^*, r\bar{\theta})$$

$$= \delta \theta^* r [1 - a (Q^* - \iota)] + a (Q^* - \iota) (r\bar{\theta} + \alpha) - (\delta \theta^* r [1 - aQ^*] + aQ^* r\bar{\theta})$$

$$= \delta \theta^* r \alpha + a [-\iota r\bar{\theta} + Q^* \alpha - \iota \alpha]$$

which must be positive if $\iota < \frac{Q^* \alpha}{\alpha + r \bar{\theta}} \in \mathbb{R}_{++}$. Q.E.D.

**Lemma 9**

**Lemma 10** The function $Q^r$ is continuous.

**Proof of Lemma 10:** Suppose it is not. By Claim 1, its range must have a gap $G \subset (0, 1)$ of the form $(a, b]$ or $[a, b)$. Let $G$ be maximal in the sense that for all $\iota > 0$, some quantities in $(a - \iota, a]$ and in $[b, b + \iota)$ are offered in equilibrium. If the gap is $(a, b]$, let $\theta'$ be the type that sells the quantity $b$ in equilibrium; if $G = [a, b)$, let $\theta'$ sell $Q = a$. Since $G \subset (0, 1)$, $\theta' > 0$, so type $\theta'$ is unique by Lemma 9. Since the support of the types $\theta$ is connected and by Claim 1, for any $\varepsilon > 0$ there is an $\iota > 0$ such that (a) any type $\theta$ that sells any quantity $Q_0$ in $(a - \iota, a]$ must lie in the interval $[\theta', \theta' + \varepsilon/2]$ and (b) any type $\theta'$ that sells any quantity $Q$ in $[b, b + \iota)$ must lie in the interval $(\theta' - \varepsilon/2, \theta']$. Now pick a type $\theta$ that sells a quantity $Q_0$ in $(a - \iota, a]$. Suppose $\theta$ deviates to the quantity $Q_1 = b - \varepsilon$. Let the price that follows the quantity $Q_i$ be $P_i$ for $i = 0, 1$. Since there is no pooling except at $Q = 0$, $P_0 = \theta r$. The payoff from this deviation is thus

$$U^r (\theta, Q_1, P_1) - U^r (\theta, Q_0, P_0) = a [Q_1 (P_1 - P_0) + \theta r (Q_1 - Q_0) (1 - \delta)]$$

$$\geq a [-\varepsilon r + \theta r (b - a - \varepsilon) (1 - \delta)]$$

as $Q_1 (P_1 - P_0) \geq -\varepsilon r$ (since $Q_1 \leq 1$ and the investors’ posterior over $\theta$ falls by at most $\varepsilon$), while $Q_1 - Q_0 \geq b - a - \varepsilon$. Hence, the deviation is profitable as long as $\varepsilon \in \left(0, \frac{\theta r (b - a) (1 - \delta)}{r + \theta r [1 - \delta]} \right)$. Q.E.D.
Lemma 11 \( Q^r (1) > 0 \).

Proof of Lemma 11: Suppose not. Let \( \theta \) be the lowest type \( \theta \) for which \( Q^r (\theta) = 0 \). This type exists by Lemma 10 and exceeds \( \theta^r_1 \) as \( Q^r (\theta^r_1) = q^r > 0 \). Let the price that results from the quantity 0 be \( P_0 \). By Lemma 10, for any \( \varepsilon > 0 \) there is an \( \lambda > 0 \) such that if the local bank (of type \( \theta \)) deviates to some \( Q \in (0, \lambda) \), the investors believe that \( \theta \in (\theta - \varepsilon, \theta) \). Let the price that results from this deviation be \( P_1 \). The local bank’s payoff changes by

\[
U^r (\theta, Q_1, P_1) - U^r (\theta, 0, P_0) = [\delta \theta r [1 - aQ_1] + aQ_1 P_1] - \delta \theta \theta r
= aQ_1 (P_1 - \delta \theta r) \geq aQ_1 r ((1 - \delta) \theta - \varepsilon)
\]

which is positive if \( \varepsilon \in (0, (1 - \delta) \theta) \). Hence, the local bank will deviate, a contradiction. Q.E.D.

Lemma 11

By Lemma 9, only the type \( \theta^r_1 \) sells the quantity \( q^r > 0 \). Hence \( P^r (q^r) = r \theta^r_1 \), so

\[
0 = U^r (\theta^r_1, q^r, P) = \delta \theta^r_1 r - 1 + aq^r [P - \delta \theta^r_1 r] = \theta^r_1 (\delta r + aq^r r [1 - \delta]) - 1
\]

and thus

\[
\theta^r_1 = r^{-1} (\delta + aq^r [1 - \delta])^{-1}.
\]

By Lemmas 9, 10, and 11, the function \( Q^r : [\theta^r_1, 1] \rightarrow [0, 1] \) is continuous and strictly decreasing, and its range is a subinterval of \((0,1)\). Hence, it has a continuous, strictly decreasing inverse \( \Psi \). For all \( \theta \in [\theta^r_1, 1] \), on seeing \( Q = Q^r (\theta) \) investors know the type is \( \Psi (Q) \) and thus assign the price \( P^r (Q) = r \Psi (Q) \). Let \( q^r > Q_1 > Q_0 > Q^r (1) \). Hence, there are \( \theta_0, \theta_1 \in (\theta^r_1, 1) \), \( \theta_0 > \theta_1 \), such that, for \( i = 0, 1 \), \( Q_i = Q^r (\theta_i) \). Let \( P_i = r \theta_i = P^r (Q_i) \). By incentive compatibility,

\[
0 \geq U^r (\theta_0, Q_1, P_1) - U^r (\theta_0, Q_0, P_0) = a (Q_1 (P_1 - P_0) + (1 - \delta) P_0 [Q_1 - Q_0])
\]

and

\[
0 \geq U^r (\theta_1, Q_0, P_0) - U^r (\theta_1, Q_1, P_1) = a (Q_0 (P_0 - P_1) + (1 - \delta) P_1 [Q_0 - Q_1])
\]
Rearranging, \( P_1 - P_0 \leq - (1 - \delta) \frac{P_0}{Q_1} [Q_1 - Q_0] \) and \( P_1 - P_0 \geq - (1 - \delta) \frac{P_0}{Q_0} [Q_1 - Q_0] \). Hence,

\[
(1 - \delta) \left[ \frac{P_0}{Q_0} - \frac{P_1}{Q_0} \right] \leq \frac{P_1 - P_0}{Q_1 - Q_0} \leq - (1 - \delta) \frac{P_0}{Q_0} \left[ Q_1 - Q_0 \right] \leq (1 - \delta) \left[ \frac{P_0}{Q_0} - \frac{P_0}{Q_1} \right].
\]

Since \( P^r() \) is continuous and \( Q^r(1) > 0 \), this implies that as \( |Q_1 - Q_0| \) shrinks to zero, so does \( \frac{P_1 - P_0}{Q_1 - Q_0} = \left[ - (1 - \delta) \frac{P_0}{Q_0} \right] \). Hence, \( P^r() \) is differentiable at all points \( Q \in (Q^r(1), q^r) \) and satisfies

\[
\frac{dP^r(Q)}{dQ} = - \frac{P^r(Q) - \delta \theta r}{Q} = - (1 - \delta) \frac{P^r(Q)}{Q}.
\]

(7)

Since the function \( \frac{1 - \delta}{Q} \) is continuous in \( Q \in [Q^r(1), q^r] \), (7) has a unique solution on this interval (see, e.g., Apostol [1, ch. 8.3]). It must be \( P^r(Q) = \frac{r \theta^r(q^r)^{1-\delta}}{Q^{1-\delta}} \), as this function satisfies (7) as well as the boundary condition \( P^r(q^r) = r \theta^r \). Inverting this function, \( Q^r(\theta) = q^r \left( \frac{\theta^r}{\theta} \right)^{\frac{1}{1-\delta}} \). Hence,

\[
P^r(Q^r(\theta)) = \frac{r \theta^r(q^r)^{1-\delta}}{Q^r(\theta)^{1-\delta}} = \frac{r \theta^r(q^r)^{1-\delta}}{\left( q^r \left( \frac{\theta^r}{\theta} \right)^{\frac{1}{1-\delta}} \right)^{1-\delta}} = \theta r
\]

as required for separation. This permits us to compute the equilibrium payoffs of all types \( \theta \in [\theta^r_1, 1] \):

\[
U^r(\theta) = U^r(\theta, Q^r(\theta), P^r(Q^r(\theta))) = \delta \theta r - 1 + aQ^r(\theta) [P^r(Q^r(\theta)) - \delta \theta r]
\]

\[
= \delta \theta r - 1 + aq^r \left( \frac{\theta^r}{\theta} \right)^{\frac{1}{1-\delta}} \theta r [1 - \delta]
\]

\[
= \delta \theta r - 1 + aq^r r (1 - \delta) \left( \frac{\theta^r}{\theta^r} \right)^{\frac{1}{1-\delta}}.
\]

We can now check D1. We first compute the payoff change of each type \( \theta \in [0, 1] \) from (lending and) deviating to each quantity \( Q \in (q^r, 1) \) when each price \( P \) is anticipated. For \( \theta \leq \theta^r_1 \), this payoff change is

\[
U^r(\theta, Q, P) = \delta \theta r - 1 + aQ [P - \delta \theta r]
\]

which is positive if and only if \( P > \frac{1 - (1 - aQ) \delta \theta r}{aQ} \). As the right hand side of this inequality is decreasing in \( \theta \), the inequality holds for the widest range of prices \( P \) when \( \theta \) takes its
maximum value of $\theta_i^r$. Hence, investors cannot believe that $\theta < \theta_i^r$. For $\theta$ in $[\theta_i^r, 1]$ the payoff change from the deviation is

$$U^r(\theta, Q, P) - U^r(\theta) = \delta \theta r - 1 + a Q [P - \delta \theta r] - \left[ \delta \theta r - 1 + a q^r r (1 - \delta) \left( \frac{\theta_i^r}{\theta^r} \right)^{1-\delta} \right]$$

$$= a \left[ Q [P - \delta \theta r] - q^r r (1 - \delta) \left( \frac{\theta_i^r}{\theta^r} \right)^{1-\delta} \right]$$

which is positive if and only if $P > r \left[ \delta \theta + (1 - \delta) \frac{q^r}{Q} \left( \frac{\theta_i^r}{\theta^r} \right)^{1-\delta} \right]$. The right hand side of this inequality is of the form $a \theta + b \theta^{-\frac{\delta}{1-\delta}}$ where $a = r \delta$ and $b = r (1 - \delta) \frac{q^r}{Q} \theta_i^r$. The first derivative is $a - \frac{\delta b}{1-\delta} \theta^{-\frac{2\delta}{1-\delta}}$ and second derivative is $\frac{\delta b}{(1-\delta)^2} \theta^{-\frac{2-2\delta}{1-\delta}} > 0$. Hence, the function is strictly convex and has a unique minimum, which is

$$\theta(Q) = \left( \frac{\delta b}{1-\delta a} \right)^{1-\delta} = \left( \frac{\delta (1 - \delta) \frac{q^r}{Q} \left( \theta_i^r \right)^{1-\delta}}{1-\delta} \right) \left( \frac{q^r}{Q} \right)^{1-\delta} \theta_i^r,$$

which in turn is strictly less than $\theta_i^r$. Hence, the function is increasing in $\theta \geq \theta_i^r$. Thus, D1 implies that investors cannot believe that $\theta > \theta_i^r$. We have shown that following a deviation to $Q > q^r$, investors must believe that $\theta = \theta_i^r$. This means that type $\theta = \theta_i^r$, which gets zero in equilibrium, gets

$$U^r(\theta_i^r, 1, r\theta_i^r) = \delta \theta_i^r r - 1 + a [\theta_i^r r - \delta \theta_i^r r] = \theta_i^r r [\delta + a (1 - \delta)] - 1$$

$$= \frac{\delta + a (1 - \delta)}{\delta + a q^r (1 - \delta)} - 1$$

(by (6)) which exceeds zero unless $q^r = 1$. Thus, the only equilibrium that survives D1 is

$$P^r(Q) = \frac{r \theta_i^r}{Q^{1-\delta}}, \quad Q^r(\theta) = \left( \frac{\theta_i^r}{\theta} \right)^{\frac{1}{1-\delta}}.$$

The local bank’s total profits $\Pi(\theta)$ equal the discounted gross loan return $\delta r \theta$ less the unitary cost of capital plus its securitization profits:

$$\Pi(\theta) = \delta r \theta - 1 + [P^r(Q^r(\theta)) - \delta r \theta] Q^r(\theta) = \delta r \theta - 1 + r (1 - \delta) \left( \frac{\theta_i^r}{\theta} \right)^{1-\delta}$$

as claimed. Q.E.D. Lemma 7
Lemmas 6 and 7 together imply part 1 of the theorem. We now turn to part 2. First, the remote bank will not offer an interest rate \( r \in R \) (for which \( \theta_1^r \leq 1 \)). For suppose it does. By Lemma 7, if \( \theta = \theta_1^r \), the local bank gets total profits \( 0 = \Pi(\theta) = r\theta_1^r - 1 \) and hence \( \theta_1^r = 1/r \). The remote bank lends only if \( \theta < \theta_1^r \). In this case, since there is symmetric information between the remote bank and investors, the remote bank sells its loan and gets the price \( rE[\theta|\theta < 1/r] < 1 \). Since its cost of capital is one, it loses money on the loan. Hence it will not offer such an interest rate, as claimed.

Finally, the remote bank will not offer an interest rate \( r \notin R \) (for which \( \theta_1^r > 1 \)) either. For suppose it does. D1 states that if each price \( P \) that makes type \( \theta \) (say) weakly prefer to deviate to \((1,\text{ lend},Q) = (1,Q)\) also makes type \( \theta^r \) strictly prefer to deviate to \((1,Q)\), then on seeing \((1,Q)\) the investors believe that the local bank’s type is \( \theta^r \). Now suppose that \( \theta_1^r > 1 \) and the local bank unexpectedly chooses action \((1,Q)\): it lends and offers a proportion \( Q \) for sale. Let it anticipate the price \( P \). Its payoff from this deviation is \( U^r(\theta,Q,P) = -1 + \delta r [1 - aQ] + aQP \) which is strictly increasing in \( \theta \) for any given \( Q \) and \( P \) as \( a < 1 \). Thus, by D1, investors must believe that \( \theta = 1 \), regardless of \( Q \). They thus assign the price \( r \). This is more than the local bank can get by retaining the loan, so its best such deviation sets \( Q = 1 \). Hence, its deviation profits are \( U^r(\theta,1,r) = r[\delta(1 - a) + a] - 1 \). The cutoff \( \theta_1^r \) sets this to zero: \( \theta_1^r = \frac{r^{-1} - a}{\delta(1 - a)} \) which, by assumption, exceeds one: \( r < \frac{a + (1 - a)\delta}{(1 - a)} \). Thus, the remote bank’s profits from offering \( r \) are

\[
 r\bar{\theta}(a + (1 - a)\delta) - 1 < \bar{\theta} - 1 < 0 :
\]

the remote bank will not offer such an interest rate \( r \) either. This proves part 2. Q.E.D.

References


