Credit Markets, Limited Commitment, and
Government Debt*

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Abstract

A dynamic model with credit under limited commitment is constructed, in which limited memory can weaken the effects of punishment for default. This creates an endogenous role for government debt in credit markets, and the economy can be non-Ricardian. Default can occur in equilibrium, and government debt essentially plays a role as collateral and thus improves borrowers’ incentives. The provision of government debt acts to discourage default, whether default occurs in equilibrium or not.

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1 Introduction

Why does government debt matter? There is a rich and varied literature which delivers a number of different answers to that question. For example, government debt can be part of a mechanism for effecting intergenerational transfers; it can permit the relaxation of credit constraints for private sector economic agents; it can provide a medium for self-insurance in the context of incomplete insurance markets; and it can permit the smoothing of tax distortions over time. In this paper, we explore a novel mechanism by which government debt might act to improve economic welfare. In particular, with limited commitment and imperfect information flows, government debt can play an important role in disciplining credit market behavior. This role arises in contexts where default in credit markets is only a threat and, alternatively, where default occurs in equilibrium. Our model gives some insight into the role of government debt as collateral, and has implications for financial crisis phenomena.

Diamond (1965) shows how government debt, supported by taxation, can effect appropriate intergenerational transfers and support efficient capital accumulation. Woodford (1990) and Kiyotaki and Moore (2008) study the role of government debt in relaxing private credit and liquidity constraints in incomplete markets environments, while Aiyagari and McGrattan (1998) examine some quantitative issues in a related economic environment which also includes tax distortions. Holmstrom and Tirole (1998) explore related issues to Woodford (1990), but in a model with explicit limited commitment.

Woodford (1990) and Holmstrom and Tirole (1998) both study environments in which the government is endowed with an advantage in credit markets. In particular, these researchers assume that, while private economic agents are credit constrained either for exogenous reasons (as in Woodford 1990) or because of limited commitment (as in Holmstrom and Tirole 1998), there is no problem for the government in getting the same private economic agents to pay their taxes. While it may be true in reality that governments have some advantages over private sector agents in collecting on debts, our analysis begins by leveling the playing field. We assume that the government has no special advantage in collecting on debts owed to it, relative to private sector creditors. This will help to make clear our departure from the existing literature.

Monetary theorists have explored some of the consequences of the symmetric treatment of default on public and private liabilities. A key idea in modern monetary economics, emphasized by Kocherlakota (1998), is that “money is memory,” in that limited recordkeeping is the key friction that explains a role for public liquidity. Indeed, Sanches and Williamson (2010) show, in part using some ideas from Andolfatto (2011), how an economy with no memory, monetary exchange, and an optimal monetary policy achieves the same equilibrium allocation as an economy with perfect memory and private credit, in which money is not valued. An important element in that argument is limited commitment, which restricts private credit, and also limits the government’s ability to tax private economic agents. Gu and Wright (2011), and Gu, Mattesini and Wright (2013) explore issues of money and credit in related models. The flow of infor-
information – i.e. the extent of “memory” – will be key to our results, so in this sense we are using insights from the monetary literature to help understand the role played by public liquidity in a broader sense. However, an important point to note at the outset is that the role we have in mind for public debt cannot be filled by an anonymously-traded asset such as currency.

The basic model builds on Lagos and Wright (2005) and Rocheteau and Wright (2005), using some elements of the money-credit economy in Sanches and Williamson (2010). In the model, exchange can be carried out using credit, subject to limited commitment. To support credit in equilibrium, borrowers must face the threat of punishment if they default. But punishment is potentially limited due to the inability of some lenders in decentralized exchange to observe past defaults by would-be borrowers. These limitations on punishment will play a critical role in credit market dysfunction in the model, and provide the novel role for government debt that we are interested in.

In the model, we start by considering equilibria with global punishments. In such equilibria, all would-be borrowers are punished if anyone defaults on their debts. In this case, punishment is not limited, and we show that government debt is neutral. Ricardian equivalence holds, as the government is assumed to have no advantage over private sector lenders in collecting on its debts. In the model, when a borrower defaults on his or her private debts, he or she also defaults on his or her tax liabilities.

More realistically, we analyze what happens in equilibria in which punishments are individual-specific – confined to the borrower who defaults. In these instances, the economy is non-Ricardian. First, in symmetric equilibria in which all would-be borrowers are treated the same, if a borrower were to default, in the absence of government debt, then he or she would on occasion be able to borrow from a lender unaware that default had taken place. But in this context, government debt acts to make default more costly, as a defaulter must acquire government debt in order to trade. Thus, incentive constraints are relaxed, and the volume of exchange and economic welfare increase. Effectively, government debt is good collateral, as the government never defaults (though there is potential for default on tax liabilities), and creditors can readily identify the government’s liabilities, while they sometimes have difficulty distinguishing among private debt instruments.

Second, we study asymmetric equilibria with individual-specific punishments in which some borrowers default in equilibrium while others do not. Borrowers in the model are intrinsically identical, but those borrowers who default have no reputation to lose from doing so, while those who repay their debts do it because there is sufficient loss from defaulting. Typically, in models with limited commitment, for example Kehoe and Levine (1993), Kocherlakota (1996), or Sanches and Williamson (2010), there is only potential default, with credit supported by the threat of off-equilibrium punishments. This is problematic if we want to explain regularities in real-world default behavior. In our model, asymmetric equilibria display credit market dysfunction that corresponds to features of financial crises. In particular, in such equilibria there is an endogenous breakdown in credit relationships, with self-fulfilling default behavior.
In asymmetric equilibria, there are instances in which a lender faces an adverse selection problem— he or she cannot tell the difference between a would-be borrower who has defaulted in the past, and one who has not. Such lenders will then charge borrowers a default premium. Then, if would-be borrowers can exchange government debt rather than engage in credit contracts, this can eliminate the adverse selection problem, though it need not eliminate default, as agents can still default on their tax liabilities. This can be interpreted as a role for government debt as collateral. Private lenders who have no access to credit histories can require that government debt be posted to secure credit contracts, and this generates more exchange, even if incentive constraints are not binding in the absence of government debt. The beneficial effects of government debt as collateral work through the distribution of the losses from default, using tax collection and the issue of government debt. Thus, government debt can act to eliminate default by ruling out equilibria with a large number of defaulting borrowers.

The paper proceeds as follows. The model is constructed in the second section. In the third section, the properties of equilibria with global punishments are studied. Then symmetric equilibria with individual punishments, and asymmetric equilibria with individual punishments, respectively, are examined in Sections 4 and 5. Finally, Section 6 concludes.

2 The Baseline Model of Private Credit

The baseline model we build on is a version of Lagos and Wright (2005), or Rocheteau and Wright (2005). Most often, models of this type are used to address issues in monetary economics, but more recently they have also proven useful in the study of credit economies with limited commitment, for example in Sanches and Williamson (2010) or Gu and Wright (2011). Some of our ideas will indeed make use of key results from the monetary economics literature, and those ideas can have a monetary interpretation, but our attention will be focused on the role of government liquidity, in the context of credit market frictions, rather than on government-provided money, narrowly defined.

Time is indexed by \( t = 1, 2, 3, \ldots \), and each period consists of two subperiods, in which trade occurs, respectively, in a centralized market (\( CM \)) and a decentralized market (\( DM \)). There is a continuum of agents with mass 2, half of whom are buyers, with the other half being sellers. Each buyer has preferences given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t [ -H_t + u(x_t) ]
\]

where \( H_t \) is labor supply minus consumption during the \( CM \), \( x_t \) is consumption in the \( DM \), and \( 0 < \beta < 1 \). Assume that \( u(\cdot) \) is strictly concave, strictly increasing, and twice continuously differentiable with \( u(0) = 0, u'(0) = \infty \),

\[ 2 \]
$u'(\infty) = 0$, and $-\frac{xu''(x)}{u'(x)} < 1$. A seller has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t),$$

where $X_t$ is consumption in the $CM$, and $h_t$ is labor supply in the $DM$. Buyers can produce only in the $CM$, and sellers produce only in the $DM$. When productive, an agent has access to a technology which permits the production of one unit of the perishable consumption good for each unit of labor input.

During the $DM$, each buyer is randomly matched with a seller. A fraction $\rho$ of $DM$ meetings are limited-information meetings, in which the seller does not have access to the buyer’s history. Even though there is limited information in this sense, the interaction between the buyer and seller in the meeting will be publicly recorded. The remaining fraction $1 - \rho$ of $DM$ meetings are full-information meetings, in which the seller has access to the public record and the interaction between buyer and seller is recorded. Thus, the key assumption about “memory” (see Kocherlakota 1998) in the model is that agents engaged in exchange may sometimes not have access to the public record, but all information that could possibly be useful for any agent living in this world always resides in the public record. Note that the public record includes information on whether meetings in the $DM$ were limited information or full information meetings. Credit histories are perfect, but a would-be lender may not have access to credit histories.

Another key credit friction, in addition to imperfect recordkeeping, is limited commitment (Kehoe-Levine 1993, Kocherlakota 1996, Sanches-Williamson 2010), in that economic agents in the model cannot be forced to work. Thus, a private debt will be repaid only if it is in the debtor’s interest to do so.

In a $DM$ meeting between a buyer and a seller, the buyer makes a take-it-or-leave it offer to the seller. In this baseline credit model, a take-it-or-leave-it offer will be a credit contract, involving goods produced by the seller and given to the buyer in the current $DM$, in exchange for a promise by the buyer to supply goods to the seller in the next $CM$. The nature of that contract will depend on the information available to the seller – whether the meeting is limited-information or full-information – and what the buyer stands to lose if he or she should default on the credit contract.

For readers who are unfamiliar with the Lagos-Wright (2005) structure, in this credit market context, the nature of heterogeneity among agents provides us with a simple motive for intertemporal exchange and credit contracts. Random matching in the $DM$ is helpful, as this permits the coexistence of credit arrangements with poor information and with good information about credit histories, respectively. This will play an important role in the analysis. Finally, quasilinear preferences for buyers eliminates wealth effects, and makes the $CM$ a period when debts are settled and the problem restarts. This gives some elements of decisionmaking a two-period structure, while maintaining an infinite horizon – the latter being critical for supporting the credit arrangements. Linear preferences for sellers, combined with take-it-or-leave-it offers by buyers in
the $DM$ imply that behavior by sellers is trivial, simplifying our analysis by allowing us to focus on the behavior of buyers.

As a visual aid, Figure 1 shows the sequence of activities during a period in the model. The figure includes transactions involving government debt, which we will cover in the next section.

2.1 Symmetric Stationary Equilibria with Global Punishments

We will first analyze equilibria that are symmetric and stationary, in that each buyer and each seller receive the same allocation, and consume the same amount in each period. Typically, in models with limited commitment, credit is supported by the threat of punishment for default. This punishment never occurs in equilibrium, but agents have equilibrium beliefs about how that punishment occurs – off equilibrium. As a benchmark, we start by considering equilibria supported by off-equilibrium-path global punishments, in which all economic agents are punished for the bad behavior of any one agent. This is a valid equilibrium to be considering, within the economic environment at hand, though global punishments have unrealistic features. In later sections we will consider symmetric equilibria with more realistic individual punishments, along with asymmetric equilibria in which some economic agents default in equilibrium. Note that there are many equilibria, and we confine attention to ones that are particularly interesting for the issues we want to address.

In the equilibria under consideration, because the buyer will face global punishment in the event of default, and because default enters the public record whether a buyer is defaulting on a limited-information or full-information contract, the contracts will be the same in all $DM$ meetings. Therefore, in any $DM$ meeting, the buyer makes a take-it-or-leave-it offer $(x, \frac{x}{\beta})$, where $x$ is the quantity of goods the seller produces in the $DM$ for the buyer, and $\frac{x}{\beta}$ is the quantity of goods the buyer produces for the seller in the next $CM$. This offer makes the seller indifferent to accepting. Then, letting $v$ denote the continuation value (constant for all $t$) for a buyer at the end of the $CM$, and $\hat{v}$ the punishment continuation value, $v$ is determined by

$$v = \max_x [u(x) - x + \beta v]$$  \hspace{1cm} (3)

subject to

$$x \leq \beta (v - \hat{v}).$$  \hspace{1cm} (4)

Inequality (4) is an incentive constraint which states that, given limited commitment, the buyer must have the incentive to repay the loan during the $CM$ rather than face punishment by the market, represented by the continuation value $\hat{v}$.

Since no one can be forced to work, the worst possible punishment is $\hat{v} = 0$, i.e. perpetual autarky. Here $\hat{v} = 0$ is accomplished with off-equilibrium global
punishments. If any buyer defaults then this triggers global autarky. Note that
global autarky is also an equilibrium, since if \( \hat{v} = 0 \) then \( v = x = 0 \) solves the
problem (3) subject to (4). Thus, if any individual defaults, this is observed by
everyone at the beginning of the CM. On the off-equilibrium path triggered by
a default, each seller strictly prefers not to trade, as the belief that any buyer
who receives a loan will default is self-fulfilling on the off-equilibrium path.

2.1.1 Incentive Constraint Does Not Bind

To construct equilibria, first suppose that the incentive constraint (4) does not
bind, which implies, from (3), that \( x = x^* \), where \( x^* \) solves

\[
u'(x^*) = 1.
\]

Then, from (3), we have \( v = \frac{u(x^*) - x^*}{1 - \beta} \), and checking the incentive constraint (4)
with \( \hat{v} = 0 \), this equilibrium exists if and only if

\[
\beta \geq \frac{x^*}{u(x^*)},
\]

i.e. buyers have to be sufficiently patient for this equilibrium to exist, in that
they need to suffer sufficiently from the off-equilibrium punishment.

2.1.2 Incentive Constraint Binds

Next, suppose that the incentive constraint (4) binds. Then, \( x = \beta v \), and from
(3), \( x \) solves

\[
x = \beta u(x).
\]

Equation (6) has two solutions, one with \( x = 0 \), and one with \( x > 0 \). This first
equilibrium always exists, and is inefficient, while the equilibrium with \( x > 0 \) is
efficient if it exists, and exists if and only if (checking that \( x < x^* \))

\[
\beta < \frac{x^*}{u(x^*)}.
\]

Thus, from (5) and (7), two equilibria exist, one with \( x = 0 \) and one with
\( x > 0 \), and in the latter either the incentive constraint binds or it does not. Note
that, in the equilibrium where \( x > 0 \), efficient trade is supported in spite of the
fact that the seller does not observe the buyer’s history in a limited-information
meeting during the DM. If a buyer defaults on any loan contract, whether the
loan was received in a limited-information or full-information meeting, this will
trigger global autarky, so that no loans are made on the off-equilibrium path.

3  Government Debt

A key element in our model will be the role played by government liquidity, and
we want to ask what role this liquidity plays in the context of the credit frictions
that exist in this environment. To emphasize our key points, we will assume that the government cannot force people to work (limited commitment), and thus is no better at collecting on its debts – in the government’s case the tax liabilities of private-sector agents – than are private creditors.

The government taxes buyers lump-sum in the $CM$, and issues one-period government bonds in the $CM$, each of which is a claim to one unit of consumption in the $CM$ of the next period. In the $CM$, agents first meet in a centralized location, where debts from the previous $DM$ are settled and taxes are paid to the government. Then, in the latter part of the $CM$, anonymity holds. Government bonds are sold on a Walrasian market, where the government also makes payoffs on the government bonds issued in the previous period. Thus, at the end of the $CM$, the government does not know the identities of the buyers of its new bond issues, nor of the economic agents redeeming maturing bonds. Again, consult Figure 1 for a time line of transactions during a period.

Suppose that the government issues $B$ units of government bonds each period in the $CM$. Each bond sells, in the stationary equilibrium we consider, at the price $q$. Further, each buyer incurs a tax $\tau$ during the $CM$ to pay the net interest on the government’s debt. Then, the continuation value $v$ is determined by

$$v = \max_{l, b, b'} \{-qb + u(l + \beta b - \beta b') - l + \beta b' - \beta \tau + \beta v\}$$

subject to

$$l + \beta \tau - \beta b' \leq \beta (v - \hat{v}) ,$$

$$b' \leq b$$

where $l$ is the quantity borrowed by the buyer during the $DM$, $b$ denotes the quantity of bonds acquired by the buyer in the $CM$, and $b'$ is the quantity of bonds that are not sold by the buyer in the subsequent $DM$, but are held to be redeemed in the next $CM$. Note that, in the incentive constraint (9), the left-hand side represents how much the buyer has to work to pay off his or her debts, where these debts include tax liabilities.

The key issue to address in the setup of problem (8) subject to (9) and (10), is the specification of the off-equilibrium punishment the buyer faces if he or she defaults. With global punishments, this is quite straightforward. If the government observes a default – either on tax liabilities or private debt – it will issue no debt in the current $CM$, and in every $CM$ in the future, and it will not make payments on the outstanding government debt. A default by any individual then will trigger global autarky, which we know, from our analysis in the previous section, is an equilibrium in which only private credit is available. Thus, $\hat{v} = 0$.

In equilibrium, the demand for government debt is equal to the supply,

$$b = B,$$

and the government budget constraint holds, or

$$\tau = B(1 - q).$$
3.0.3 Incentive Constraint Does Not Bind

First, if the constraint (9) does not bind, then in equilibrium \( l + \beta b - \beta b' = x^* \), and \( q = \beta \), so from (8), (11), and (12), we have

\[
v = \frac{u(x^*) - x^* - \beta B(1 - \beta)}{1 - \beta},
\]

and the incentive constraint (9) in equilibrium then reduces to (5), which is the same condition we obtained without government debt, and the equilibrium allocation is identical, so \( B \) is irrelevant if (5) holds.

3.0.4 Incentive Constraint Binds

Next, consider the case where the incentive constraint (9) binds. Then, from (8)-(12), the quantity of consumption in the \( DM \), \( x \), solves (6), and

\[
q = \beta u'(x).
\]

If the incentive constraint (9) binds, then from (8)-(12) and (14), we can write (8) as

\[
x = \beta u(x),
\]

and so an equilibrium with \( x > 0 \) and a binding incentive constraint exists if and only if (7) holds. There also always exists an equilibrium with \( x = 0 \). Thus, in this case \( B \) is irrelevant, just as when the incentive constraint does not bind.

Therefore, in these equilibria with global punishments, the issue of government debt accomplishes nothing. With global punishments, the economy is Ricardian. In spite of the limited commitment friction, government debt is irrelevant as the government is no better at collecting on its debts than are private sector lenders. If the incentive constraint binds in the absence of government debt, then issuing government debt does not relax the incentive constraint, as taxation is required to support tradeable government debt, and buyers can default on their tax liabilities to the same extent they can default on their private debts. It should be clear why Ricardian equivalence holds in this case, as the government is doing nothing that a private sector agent could not do. It issues promises to pay, and the fact that these promises are issued in the \( CM \) rather than the \( DM \) does not make any difference. Those promises are kept as the government is able to collect taxes to pay off the government debt because of a threat of global autarky faced by defaulting taxpayers – the same implicit threat faced by economic agents in the private credit economy.

4 Symmetric Equilibria with Individual Punishment

Studying equilibria with global punishments, as in the previous section, serves as a useful baseline, but global punishments are obviously unrealistic. In this
section, we construct equilibria with individual punishments, under which a
default triggers retribution directed only against the individual defaulter, off
equilibrium. As with global punishment equilibria, for now we confine attention
to symmetric equilibria.

4.1 Private Credit and No Government Debt

Just as in the previous section, \( v \) is determined by (3) subject to (4). But, with
individual punishments, \( \hat{v} \) will be different from the case with global punish-
ments. If a buyer chooses to default, then off-equilibrium he or she will not
receive a loan on meeting a seller in a full-information meeting in the \( DM \).
Off-equilibrium, sellers believe that a buyer who has defaulted will not receive a
loan from any seller in a limited information meeting in the \( DM \). Thus, a buyer
who has defaulted has nothing to lose from defaulting in future periods, and
thus continues to do so. Therefore, if a seller knows that a buyer has defaulted,
then he or she strictly prefers not to lend.

If a buyer who has defaulted were in a limited-information meeting in the
\( DM \) during any period after default occurs, the seller does not know that this
individual buyer defaulted. Let \((x, \frac{\xi}{\beta})\) denote the offer that each seller receives
in equilibrium from a buyer in limited information \( DM \) trades. This is the offer
of consumption for the buyer \( x \) in the \( DM \) in return for a payment \( \frac{\xi}{\beta} \) in the next
\( CM \). The seller is just indifferent to accepting this offer, knowing that the offer
is optimal for the buyer among offers which do not give the buyer the incentive
to default. We assume that the seller believes that any offer other than \((x, \frac{\xi}{\beta})\)
comes from a buyer who has defaulted in the past. Thus, off equilibrium, any
buyer who has defaulted will make the offer \((x, \frac{\xi}{\beta})\) so as to get the same loan \( x \)
as other buyers receive in equilibrium. Thus, the continuation utility if default
occurs is

\[
\hat{v} = \frac{\rho u(x)}{1 - \beta}.
\]  

(16)

4.1.1 Incentive Constraint Does not Bind

If there is efficient exchange in all \( DM \) meetings, with \( x = x^* \), then from (3)
and (16) we obtain

\[
v - \hat{v} = \frac{(1 - \rho)u(x^*) - x^*}{1 - \beta},
\]

and checking the incentive constraint (4), this equilibrium exists if and only if

\[
\beta \geq \frac{x^*}{(1 - \rho)u(x^*)}.
\]  

(17)

4.1.2 Incentive Constraint Binds

Now, suppose that the incentive constraint (4) binds. Then from (3), (4), and
(16), we can solve for the quantity of goods \( x \) consumed by the buyer in the
DM, i.e. $x$ solves
\[ x = \beta (1 - \rho) u(x), \tag{18} \]
and we require that the solution satisfy $x < x^*$, for the incentive constraint to bind. There always exists an equilibrium with $x = 0$, and an equilibrium with $x > 0$ exists if and only if
\[ \beta < \frac{x^*}{(1 - \rho) u(x^*)}. \tag{19} \]

### 4.1.3 Individual Punishments vs. Global Punishments

Not surprisingly, with weaker punishments relative to the global punishment case, the quantity of exchange and welfare are in general reduced. Let the welfare measure be the sum across agents of period utilities, so that welfare is denoted by
\[ W = u(x) - x. \tag{20} \]
Then, welfare is increasing in $x$ for $x \leq x^*$. Let $x_G$ denote the quantity of goods exchanged in DM meetings with global punishments, and $x_I$ the quantity with individual punishments. Then, from (5), (6), (7), (17), (18), and (19),

1. If $\beta \geq \frac{x^*}{(1 - \rho) u(x^*)}$, then $x_G = x_I = x^*$ and welfare is the same whether there are global or individual punishments.
2. If $\frac{x^*}{u(x^*)} \leq \beta < \frac{x^*}{(1 - \rho) u(x^*)}$, then $x_G = x^* > x_I$, and welfare is higher with global punishments.
3. If $\beta < \frac{x^*}{u(x^*)}$, then $x_I < x_G < x^*$, and welfare is higher with global punishments.\(^1\)

Thus, with individual punishments – which are weaker than global punishments – the incentive constraint for buyers is tighter, in general. As a result, less exchange is supported in the DM, and welfare is lower.

### 4.2 Equilibria with Government Debt

To focus on whether government debt can improve matters, we will construct equilibria in which the government issues just enough debt to completely crowd out private credit. This will make our analysis more straightforward, and will also allow us to make fairly strong statements, as it turns out that these equilibria have nice welfare properties.

In the equilibria we construct, in which there is no private lending in the DM, and all government debt is exchanged by buyers in the DM, the buyer’s

\[ ^1 \text{When the incentive constraint binds with individual punishments, } x_I \text{ is determined by } x_I = \beta (1 - \rho) u(x_I), \text{ whereas when the incentive constraint binds with global punishments } x_G \text{ is determined by } x_G = \beta u(x_G). \text{ Since } \rho < 1, \text{ therefore } x_G > x_I. \]
consumption in the $DM$ is given by $x = \beta b = \beta B$. Then, from (9) and (12), we can write the incentive constraint for a buyer as

$$B(1 - q) \leq v - \hat{v}.$$  

(21)

But what is $\hat{v}$, the continuation value if the buyer chooses (out of equilibrium) to default on his or her tax liabilities? Without government debt, sellers in full information $DM$ meetings have an incentive to deny credit to buyers who have defaulted as they believe, correctly, that these buyers will continue to default. With government debt, sellers do not have a similar incentive to punish those who default on tax liabilities, so the government must provide this incentive. We then need to make additional assumptions about the government’s ability to punish tax cheats. As well, we will be consistent, in assuming symmetry with respect to private and public sector punishment abilities.

So, assume that there is a technology which permits an agent to post government debt as collateral, which means a creditor – or the government – has the right to seize the collateral in the event of default. Seizure can only occur at the beginning of the $CM$ when an individual buyer can be identified. Further, in this environment, note that the outright exchange of government debt for goods in the $DM$ is equivalent to an arrangement where government debt is posted as collateral in a credit arrangement in the $DM$, with the threat that the collateral is seized if default occurs in the following $CM$. So we also assume that, if there is an outright exchange of government debt for goods in the $DM$, then it is possible for the government to seize this government debt from the seller at the beginning of the next $CM$. If a buyer acquires government debt in the current $CM$, and then redeems it in the next $CM$, the debt cannot be confiscated by anyone in the interim.

If a buyer defaults on his or her taxes then, off equilibrium, sellers in full information meetings in the $DM$ strictly prefer not to trade with a buyer who has defaulted. Why? In one case, if the buyer were to offer the seller government debt in exchange for goods in the $DM$, then the government would confiscate the government bonds from the seller when he or she arrives in the $CM$. In the other case, if the defaulting buyer were to offer the seller a credit contract, collateralized with government debt, in the $DM$, then the government would confiscate the collateral in the next $CM$, and the buyer would default on the loan. Thus, in either case, the seller anticipates getting nothing in return for the goods he or she produces, and so strictly prefers not to trade. Similarly, off equilibrium, a defaulting buyer and a seller might consider engaging in an unsecured credit contract in a full-information meeting in the $DM$. But in that case, the seller believes that no one will offer the buyer credit in the future, off equilibrium, so the seller understands that the buyer will default. Thus, a seller strictly prefers not to make a loan to a buyer, off equilibrium, if he or she knows the buyer has defaulted on his or her tax liabilities.

These assumptions about the seizure of assets and collateral seem natural and reasonable in light of what is feasible in reality. Secured private credit arrangements are ubiquitous, and government debt is usually considered to be
good collateral. Further, the government typically collects from tax defaulters through the seizure of assets, and is willing to punish third parties, for example under laws that hinder money-laundering. Moreover, under the US Bankruptcy Code any payment made by a bankrupt debtor, in the 90 days prior to the bankruptcy filing, must be paid back to the bankruptcy court.

In instances in which a buyer who has defaulted is matched in a limited information meeting in the DM, the seller will trade with the buyer if and only if the buyer behaves in the same way as buyers who have not defaulted. If a seller were to trade with a buyer who had defaulted and who behaved differently, this would be detected by the government, with the same punishment as applied for full information meetings. Thus, it is optimal for a seller not to trade with an agent who can be identified as having defaulted. Therefore, in order to engage in any trades in the DM, a buyer who has defaulted must arrive in the DM with government bonds, so

\[ v = \max \left( 0, -qB + \rho u(\beta B) + (1 - \rho)\beta B \right) \frac{1}{1 - \beta}, \tag{22} \]

and a defaulting buyer thus chooses autarky, or to pool with non-defaulting buyers, depending on which yields higher utility. For non-defaulting buyers, the presence of a single defaulter is irrelevant, off equilibrium.

In equilibrium, from (8), (11), and (12), the continuation value for a buyer is

\[ v = -qB + u(\beta B) - \beta B(1 - q) + \beta v, \tag{23} \]

and optimization by buyers implies that the bond price \( q \) is determined by

\[ q = \beta u'(\beta B). \tag{24} \]

The government’s problem is:

\[ \max_B [u(\beta B) - \beta B] \tag{25} \]

subject to (21), (23), (22), and (24), i.e. the government chooses the quantity of government bonds \( B \) to maximize welfare – the surplus from trade in the DM – which is in general determined in equilibrium from the solution \((\hat{v}, v, q)\) to (23), (22), and (24), satisfying the incentive constraint (21).

### 4.2.1 Incentive Constraint Does Not Bind

If the choice of \( B \) were unconstrained for the government, from (25) it would choose \( x = \beta B = x^* \). Then, from (24), \( q = \beta \), and from (23) and (22) we get

\[ v = \frac{-x^* + u(x^*) - x^*(1 - \beta)}{1 - \beta}, \]

2See http://www.justice.gov/opa/pr/2013/August/13-tax-975.html for an example of agreements aiming at the freeze of bank accounts linked to tax evasion and prosecution of banks failing to cooperate with the prosecution of tax evaders. See Chapter V, Articles 51–59 of the United Nation Convention against Corruption for an example of recovery of asset linked to illicit cross-border transfers.

3Unless the creditor proves that they are preferential transfers. See the U.S. Bankruptcy Code, Section 547.
\[ \hat{v} = \frac{\rho [u(x^*) - x^*]}{1 - \beta}. \]

Checking the incentive constraint (21), this is the solution to the government’s problem if and only if
\[ \beta \geq \frac{x^*}{(1 - \rho)u(x^*) + \rho x^*}. \] (26)

Note that, if (26) holds, and the government sets \( B = \frac{x^*}{\beta} \), then efficient exchange in the DM guarantees that no buyer in the DM would want to make an offer to a seller involving private credit. Each buyer acquires \( \frac{x}{\beta} \) units of government debt in the CM at a price \( q = \beta \), exchanges all of this debt for a surplus-maximizing quantity of goods in the DM, and then pays his or her taxes in the subsequent CM.

4.2.2 Incentive Constraint Binds and \( \hat{v} > 0 \)

If the solution to the government’s problem is not \( B = \frac{x^*}{\beta} \), then the incentive constraint must bind. If the incentive constraint binds, we need to consider two possibilities. First, it may be the case that \( \hat{v} > 0 \), so that a defaulting buyer prefers to mimic the equilibrium behavior of other buyers. Second, we could have \( \hat{v} = 0 \), in which case a defaulting buyer prefers autarky. In this subsection, we consider the first case.

From (9) we have \( q = \beta u'(x) \) where \( x \) is the quantity of goods exchanged in DM meetings. Then, (21)-(23) imply that \( x \) solves
\[ x [1 - \beta u'(x)] = \beta (1 - \rho) [u(x) - x]. \] (27)

Proposition 1 For \( x \in (0, \infty) \), equation (27) has a unique solution \( x_E \).

Proof. Rewrite equation (27) as
\[ 1 - \beta u'(x) = \beta (1 - \rho) \left[ \frac{u(x)}{x} - 1 \right]. \] (28)

The right-hand side of (28) is monotonically decreasing in \( x \), since \( u(\cdot) \) is strictly concave. The right-hand side of (28) tends to \( \infty \) as \( x \to 0 \), and to \( 0 \) as \( x \to \infty \). The left-hand side of (28) is monotonically increasing in \( x \). The left-hand side of (28) tends to \( -\infty \) as \( x \to 0 \) and to \( 1 \) as \( x \to \infty \). Therefore, by the intermediate value theorem, and given monotonicity, there exists a unique \( x_E \in (0, \infty) \) that solves (28) and (27).

An equilibrium of this type must involve a solution \( x_E \) to (27), and we have shown that a unique solution always exists. However, for this to be an equilibrium of the type we are looking for, \( x_E \) must satisfy two other properties. First, the incentive constraint binds if and only if \( x_E < x^* \). Second, it must be the case that \( \hat{v} > 0 \) in equilibrium, or \( \phi(x_E) > 0 \), where
\[ \phi(x) \equiv -xu'(x) + \rho u(x) + (1 - \rho)x. \] (29)
and let $\tilde{x}$ denote the solution to

$$
\phi(\tilde{x}) = 0.
$$

**Proposition 2** Assume that $-\frac{xu''(x)}{u'(x)}$ is constant. If $-\frac{xu''(x)}{u'(x)} \geq 1 - \rho$, then

$$
\beta < \frac{x^*}{(1 - \rho)u(x^*) + \rho x^*}
$$

is necessary and sufficient for existence of an asymmetric equilibrium with individual punishments and a binding incentive constraint with $\hat{\nu} > 0$. If $-\frac{xu''(x)}{u'(x)} < 1 - \rho$, then (30) and

$$
\beta > \frac{\tilde{x}}{(1 - \rho)[u(\tilde{x}) - \tilde{x}] + \tilde{x}u'(\tilde{x})}
$$

are necessary and sufficient for existence, where $\tilde{x}$ solves $\phi(\tilde{x}) = 0$.

**Proof.** First determine necessary and sufficient conditions for a binding incentive constraint, i.e. $x_E < x^*$. If we rewrite (27) as (28), then the right-hand side of (28) is monotonically decreasing and the left-hand side of (28) is monotonically increasing. The previous proposition shows that the solution $x_E$ is unique, so it follows that $x_E < x^*$ if and only if (30) holds. Second, we want to find necessary and sufficient conditions for $x_E > 0$. Differentiating (29), we obtain

$$
\phi'(x) = u'(x)[-1 + \rho + \eta] + 1 - \rho
$$

Therefore, if $\eta \geq 1 - \rho$, then $\phi'(x) > 0$ for $0 \leq x \leq x^*$, and since $\phi(0) = 0$ therefore $\phi(x) > 0$ for $x \in [0, x^*]$. Thus, if $\eta \geq 1 - \rho$ then (30) is necessary and sufficient for existence of the equilibrium. Alternatively, suppose $\eta < 1 - \rho$, then $\phi'(0) = -\infty$, $\phi''(x) > 0$ for $x \in [0, x^*]$, $\phi(0) = 0$, $\phi(x^*) = \rho[u(x^*) - x^*] > 0$, so there exists a unique $\tilde{x} \in (0, x^*)$ which solves $\phi(\tilde{x}) = 0$. Further, $\phi(x) > 0$ for $x \in (\tilde{x}, x^*)$ and $\phi(x) \leq 0$ for $x \in (0, \tilde{x})$. It is then necessary and sufficient for $\phi(x_E) > 0$ that $x_E \in (\tilde{x}, x^*)$. Then, once more using (28), $x_E \in (\tilde{x}, x^*)$ if and only if (30) and (31) hold. $\blacksquare$

### 4.2.3 Incentive Constraint Binds and $\hat{\nu} = 0$

Next, we consider the case where the incentive constraint binds in the government’s problem, and a buyer who defaults (off equilibrium) chooses autarky. In this case, from (21)-(24), the quantity of goods traded in DM meetings, $x$, solves (15).

**Proposition 3** Assume that $-\frac{xu''(x)}{u'(x)}$ is constant. A necessary condition for existence of a symmetric equilibrium with individual punishments and a binding incentive constraint with $\hat{\nu} = 0$ is (7). If $-\frac{xu''(x)}{u'(x)} \leq 1 - \rho$, then this equilibrium does not exist. If $-\frac{xu''(x)}{u'(x)} > 1 - \rho$, then (7) and

$$
\beta \leq \frac{\tilde{x}}{u(\tilde{x})}
$$


15
are necessary and sufficient conditions for existence.

Proof. Checking that \( x < x^* \), from (15) a necessary condition for the equilibrium to exist is (7). As well, \( \bar{v} = 0 \) requires that a defaulting buyer not wish to mimic equilibrium behavior, which requires \( \phi(x) \leq 0 \). Therefore, this equilibrium does not exist if \( -\frac{xu''(x)}{w'(x)} \geq 1 - \rho \), as this implies \( \phi(x) > 0 \) for \( x \in (0, x^*) \).

If \( -\frac{xu''(x)}{w'(x)} < 1 - \rho \), then \( \phi(x) \leq 0 \) for \( x \in [0, \bar{x}] \), and so from (15), a necessary and sufficient condition for existence in this case is (33).

This, then, serves to characterize symmetric equilibria with individual punishments, government debt, and binding incentive constraints. In the next subsection, we examine how government debt affects the equilibrium allocation, relative to equilibria with private credit only.

4.2.4 Effects of Government Debt with Individual Punishments

Government debt clearly matters in equilibria with individual punishments, as the equilibrium allocation with government debt is in general different from the one without government debt. Thus, Ricardian equivalence does not hold in general, in spite of the fact that the government is no better than private sector agents at collecting on debts. The government’s advantage arises from the recognizability of its liabilities, and the fact that the government has no incentive to default, with the circulation of government debt limited only by the government’s ability to tax.

We want to compare welfare with and without government debt, using our measure of welfare (20), which implies that welfare is increasing in the quantity of goods \( x \) exchanged in each meeting in the DM, for \( x \leq x^* \), which will always hold in equilibrium. The following proposition demonstrates that government debt at worst has no effect on welfare and at best increases it by increasing the volume of exchange in the DM. Let \( x_N \) denote the quantity of goods exchanged in each meeting in the DM in the absence of government debt, and \( x_D \) the quantity exchanged when there is government debt.

Proposition 4 (i) If \( \beta \geq \frac{x^*}{(1-\rho)u(x^*)} \), then \( x_D = x_N = x^* \), and welfare is the same in the equilibrium without government debt and the one with government debt. (ii) If \( \beta \leq \frac{x^*}{(1-\rho)u(x^*)+\rho x^*} \), then \( x^* = x_D > x_N \), and welfare is greater with government debt than without. (iii) If \( -\frac{xu''(x)}{w'(x)} \) is constant, if \( -\frac{xu''(x)}{w'(x)} \geq 1 - \rho \) or if \( -\frac{xu''(x)}{w'(x)} < 1 - \rho \) and \( \beta > \frac{2}{(1-\rho)(u(x)-\bar{x})+2w'(x)} \), and \( \beta < \frac{x^*}{(1-\rho)u(x^*)+\rho x^*} \), then \( x_N < x_D < x^* \). (iv) If \( -\frac{xu''(x)}{w'(x)} \) is constant, if \( -\frac{xu''(x)}{w'(x)} > 1 - \rho \) and \( \beta \leq \frac{x^*}{u(x)} \), then \( x_N < x_D < x^* \).

Proof. (i) From (17) and (26), equilibria with a nonbinding incentive constraint exist if \( \beta \geq \frac{x^*}{(1-\rho)u(x^*)} \), both with and without government debt. (ii) From (26), the incentive constraint does not bind with government debt if \( \beta < \frac{x^*}{(1-\rho)u(x^*)+\rho x^*} \), so \( x_D = x^* \). However, since (19) holds, the incentive constraint
beneﬁts without government debt, so \( x_N < x^* \). (iii) From (19), (30), and (31), the incentive constraint binds in both equilibria, so \( x_N < x^* \) and \( x_D < x^* \). Further, we can write equation (18) as

\[
x[1 - \beta(1 - \rho)] = \beta(1 - \rho)[u(x) - x].
\] (34)

Then, since the right-hand sides of equations (34) and (27) are identical, and \( u'(x) > 1 - \rho \) for \( x < x^* \), therefore \( x_D > x_N \). (iv) From (19) and (33), the incentive constraint binds in both equilibria, so \( x_N < x^* \) and \( x_D < x^* \). In the equilibrium with no government debt, \( x \) is determined by (18), and in the equilibrium with government debt, \( x \) is determined by (15). It is immediate from (18) and (15) that \( x_N < x_D \). ■

Note that, since

\[
-\ddot{x}u'(\ddot{x}) + \rho u(\ddot{x}) + (1 - \rho)\ddot{x} = 0
\]
determines \( \ddot{x} \), therefore

\[
\frac{\ddot{x}}{u(\ddot{x})} = \frac{\ddot{x}}{(1 - \rho)[u(\ddot{x}) - \ddot{x}] + \ddot{x}u'(
\ddot{x})},
\]
so the right-hand sides of inequalities (31) and (33) are equal. As a result, the above proposition exhausts the parameter space. With government debt, the equilibrium is unique among the class of equilibria we are examining in this section, and an equilibrium always exists. The same is the case without government debt.

Figure 2 shows how the parameter space is subdivided, with the parameter \( \rho \) on the horizontal axis, and \( \beta \) on the vertical axis. In region 1, part (i) of the above proposition applies. In this case, the incentive constraint does not bind with or without government debt, and there is efﬁcient exchange in either case, or \( x_N = x_D = x^* \). In region 2, part (ii) of the above proposition applies, in which case the incentive constraint binds without government debt, but does not bind with government debt, or \( x_N < x_D = x^* \). In region 2, the introduction of government debt is welfare-improving, as it increases exchange in the \( DM \). In region 3, part (iii) of the above proposition applies, with incentive constraints binding in equilibrium with or without government debt. With government debt, defaulting buyers choose to mimic equilibrium behavior, or \( \ddot{v} > 0 \). In region 3, exchange is greater in the \( DM \) with government debt, and welfare is higher. Finally, in region 4 of the parameter space, in Figure 3, part (iv) of the above proposition applies. In this case, the incentive constraint binds with or without government debt, but there is more exchange in the \( DM \), and higher welfare, with government debt. In the equilibrium with government debt in region 4, defaulting buyers choose autarky, or \( \ddot{v} = 0 \).

[Figure 2 here.]

We can conclude from the above four propositions that introducing government debt in symmetric equilibria with individual punishments is welfare
improving as, in general, it increases the quantity of exchange in decentralized meetings. The introduction of government debt acts to relax incentive constraints, as it tends to make default—in this case on tax liabilities rather than private debt—less desirable. At best, the introduction of government debt can make it so costly to mimic equilibrium behavior for a defaulting buyer, that a buyer will choose autarky if default occurs, off-equilibrium. In the absence of government debt, a buyer who defaults can get something for nothing. By posing as a buyer who has not defaulted, a defaulting buyer can receive a loan with probability $\rho$. However, when government debt is traded, a defaulting buyer has to work to acquire the government debt in order to pose as a buyer who has not defaulted. It is critical for these results that government debt is distinguishable as such. Even in limited information meetings in the DM, the seller recognizes government debt, even though the debt of all private buyers in these circumstances is indistinguishable.

Some readers may be puzzled as to why the welfare-improving role of government debt is not generated from a liquidity premium on government debt that implies positive transfers from the government, which of course all buyers are willing to accept. For example, if the incentive constraint is tight, then $x$ is small in equilibrium, so from (14) $q$ is high, and if $q > 1$, then from (12) the tax is negative. Then, there would be no potential default problem. But from our analysis above, it is straightforward to show that $q < 1$ in the equilibria with government debt we consider. To see why $q < 1$ given the optimal provision of government debt, if $q \geq 1$ at the optimum, then $x < x^*$. But this implies that the government could increase the quantity of government debt, increase welfare, and the incentive constraint of buyers would still hold. Thus, $q \geq 1$ cannot be optimal.

As discussed above, we can interpret exchange as occurring in the government debt economy by way of outright exchange of government debt for goods in the DM, or as a credit arrangement with government debt used as collateral. With the collateral contract, the buyer can sell the government debt in the next CM in order to repay the debt. Effectively, the government supplies a safe asset, supported by taxation, that is used to enhance credit market activity. With a ready supply of government debt, the collateralized credit arrangements arise endogenously, and they increase welfare by promoting more exchange. This role for government debt as collateral in the credit market is consistent with empirical observations (see Garbade 2006), in that activity in the repurchase agreement market appears to have grown partly in response to growth in the quantity of government debt outstanding in the United States.

In may not be obvious why the provision of government debt is not bad for incentives in this context, since it potentially allows for anonymous exchange. For example, in Aiyagari and Williamson (2000), the exchange of government liabilities (fiat money, in their case) provides an alternative to participation in a credit system, and the better this system of government-liability exchange works, the worse are incentives in the credit system. In the Aiyagari-Williamson (2000) model, government liabilities are effectively a substitute for credit, while in this model (at least in the symmetric equilibria we have studied thus far)
government liabilities are complementary to credit, as government debt essentially serves as collateral. This is important, as the non-anonymous nature of government debt is what contributes to its role in improving welfare. In this sense, the role of government liquidity here is different from what we find in typical monetary models.

We have made the case that government debt can improve on the allocation achieved in an economy with private credit only. But are there more sophisticated private credit arrangements that could achieve the same outcome as considered above with government debt? For example, perhaps we could look for private credit arrangements that mimic the desirable properties of the arrangement where government debt is traded. In particular, suppose that buyers issue debt in the CM, that this debt is sold to other buyers, traded in the DM, and then redeemed in the next CM. Further, suppose that agent A has defaulted on his or her own debt in the past, and agent A meets a seller in the DM who accepts the debt of agent B from agent A in exchange for goods. Then, assume that agent B can and will confiscate his or her debt from the seller at the beginning of the next CM so as to punish the seller. Why would agent B carry out the punishment? Because he or she anticipates being treated as a defaulter if he or she does not do so.

The problem with this arrangement is that it does no better than the private credit arrangement we considered above, in which buyers issue their own debt in DM exchange. Off equilibrium, in the arrangement proposed in the previous paragraph, if a buyer defaults he or she can always obtain consumption in limited information DM meetings, by trading his or her own debt. From the seller’s point of view, such debt is indistinguishable in such a meeting from the debt of any other buyer, as the seller cannot identify the buyer. Thus, in contrast to government debt, private debt does not impose costs on defaulters. With government debt, a defaulting buyer has to acquire government debt off equilibrium in order to consume. Why? Because government debt is identifiably a liability of the government, whereas private debt cannot be associated with a particular buyer in limited information exchange in the DM.

5 Asymmetric Equilibria with Individual Punishment and Equilibrium Default

We will now consider equilibria where agents behave asymmetrically, with some buyers defaulting in equilibrium. These are equilibria where a fraction $\alpha$ of buyers (the good buyers) never defaults, but a fraction $1 - \alpha$ (bad buyers) will default on their debts if anyone chooses to lend to them. This is interesting as buyers are fundamentally identical, but in an asymmetric equilibrium identical economic agents are treated differently. It will in general be possible to support equilibria with differing values for $\alpha$. Indeed, a special case is equilibria with $\alpha = 1$, which we have already considered, and such equilibria can coexist with other equilibria with $\alpha < 1$. Thus, note that $\alpha$ is endogenous but, as we hope
to make clear, it is indeterminate. In an asymmetric equilibrium, good buyers never default because they would be punished for default by being treated in the same way as bad buyers, thus losing access to exchange in the DM under full information. Bad buyers always default as they have nothing to lose – sellers who know their type will not lend to them.

5.1 Private Credit and No Government Debt

In the equilibria we construct, bad buyers will always default on a loan received from a seller in the DM. Therefore, if a bad buyer meets a seller in a full-information meeting, the buyer will not receive a loan. Similarly, since a good buyer does not default in equilibrium, the good buyer will receive a loan in a full-information meeting, as the seller knows the buyer’s type. Since the buyer makes a take-it-or-leave it offer in a full-information meeting, the offer of a good buyer will take the form of a loan quantity $x_F$, which is the quantity of consumption goods produced by the seller for the buyer in the DM, coupled with a repayment $\frac{x_F}{\alpha^2}$ by the buyer in the next CM.

The situation is potentially more complicated if a good buyer or bad buyer meets a seller in a limited-information meeting. Given our assumption that the buyer makes a take-it-or-leave-it offer in the DM, a limited information meeting involves signalling under private information. Our candidate equilibrium is a pooling equilibrium in DM limited information meetings, in which all buyers (good and bad) make an offer $x_L$ to the seller in a limited-information meeting in the DM, and promise a repayment $X_L = \frac{x_L}{\alpha^2}$ in the next CM. To completely specify the signalling game, we need to add beliefs for the seller, including those for off-equilibrium offers by buyers. If the equilibrium offer is $(\tilde{x}_L, \tilde{X}_L)$ assume that sellers in limited information meetings believe that an offer with $x_L \leq \tilde{x}_L$ is made by a good buyer with probability $\alpha$. However, sellers believe that an offer $x_L > \tilde{x}_L$ comes from a bad buyer with certainty. Thus, in equilibrium, the seller is indifferent to accepting an offer from a buyer given the seller’s beliefs, which are correct in equilibrium. The offer made by a good buyer is a best response to the seller, given the seller’s beliefs, and a bad buyer clearly optimizes by making the same offer as a good buyer, as posited.

There are equilibrium refinements that could eliminate this equilibrium from consideration. A good discussion of this is in the appendix of Rocheteau (2008), for a related model. For example, if we use the intuitive criterion (Cho and Kreps 1987), then the good buyer could deviate from the equilibrium contract by offering to accept slightly less than $\tilde{x}_L$ in exchange for a payment $X_L = \frac{x_L}{\alpha^2}$ to the seller in the next CM, which the seller and the good buyer both strictly prefer to the equilibrium contract. Further, the bad buyer strictly prefers the equilibrium contract to the deviation, so the proposed equilibrium is ruled out by the intuitive criterion. But, using the notion of undefeated equilibrium (Mailath et al. 1993), which is arguably deeper than the intuitive criterion, we ask more of a deviation. In this model, any equilibrium must be a pooling equilibrium for limited information exchange, as the seller would never want to trade with a buyer who he or she identifies as bad. Thus, under undefeated equilibrium,
deviations involve cases where, in this model, the seller believes that an off-equilibrium offer of \(x_L \leq \hat{x}_L\) comes from a good buyer with probability \(\alpha\). By construction of the pooling equilibrium we consider (which gives the seller zero expected surplus), good and bad buyers must be worse off for any deviation. Thus, the equilibrium we focus on is undefeated.

Therefore, in the absence of government debt, the continuation value \(v\) for a good buyer is given by

\[
v = \max_{x_L, x_F} \left\{ \rho u(x_L) + (1 - \rho)u(x_F) - \rho \frac{x_L}{\alpha} - (1 - \rho)x_F + \beta \hat{v} \right\}
\]  

subject to

\[
\frac{x_L}{\alpha} \leq \beta (v - \hat{v}),
\]

\[
x_F \leq \beta (v - \hat{v})
\]

Note that, in the pooling equilibrium, limited-information loans carry a default premium. Constraints (36) and (37) are the incentive constraints that must hold for a good buyer following a limited-information meeting and a full-information meeting, respectively. Here, \(\hat{v}\) is not only the off-equilibrium continuation utility the good buyer receives if he or she defaults, but the continuation utility of a bad buyer, who always defaults.

A bad buyer consumes the same quantity as a good buyer in a limited information meeting, and consumes zero in a full-information meeting in the DM, so

\[
\hat{v} = \frac{\rho u(x_L)}{1 - \beta}.
\]

If neither incentive constraint, (36) or (37), binds, then \(x_L = \hat{x}\) and \(x_F = x^*\), where \(\hat{x}\) solves

\[
u'(\hat{x}) = \frac{1}{\alpha}.
\]

Clearly \(\hat{x} < x^*\) for \(\alpha < 1\), so a smaller quantity of goods is exchanged in limited information meetings, even if incentive constraints do not bind. Further, since \(-\frac{xu'(x)}{u(x)} < 1\), \(\frac{\hat{x}}{\alpha} < x^*\) for \(\alpha < 1\), which implies that, in equilibrium, incentive constraint (37) is always tighter than (36). Thus, the only cases we need to consider, in the absence of government debt, are equilibria where neither incentive constraint binds, where constraint (37) binds and (36) does not, and where both incentive constraints bind. We will consider each of these three cases in turn.

5.1.1 Neither Incentive Constraint Binds

If (36) and (37) do not bind, then from (35) - (38) we get

\[
v - \hat{v} = \frac{(1 - \rho)u(x^*) - \rho \frac{\hat{x}}{\alpha} - (1 - \rho)x^*}{1 - \beta}.
\]
To check incentive constraints, it is sufficient to check (37), as this is always the tighter constraint. This tells us that this equilibrium exists if and only if

\[ \beta \geq \frac{x^*}{(1 - \rho)u(x^*) + \rho (x^* - \frac{\bar{x}}{\alpha})} \] (40)

5.1.2 Limited Information Incentive Constraint Does Not Bind, Full-Information Incentive Constraint Does

Next, we analyze the case where (36) does not bind, but (37) does. Then, from (35) - (38) we get

\[ v = \rho u(\hat{x}) + (1 - \rho)u[\beta(v - \hat{v})] - \frac{\rho}{\alpha} + \beta\rho v + \beta(1 - \rho)v, \] (41)

\[ \hat{v} = \rho u(\hat{x}) + \beta\hat{v}, \] (42)

and then (41), (42), and (37) with equality gives

\[ \beta(1 - \rho)u(x^F) - x^F + \rho \beta \left( x^F - \frac{\bar{x}}{\alpha} \right) = 0, \] (43)

which solves for \( x^F \). There are potentially two solutions to (43), one solution with \( x^F < \frac{\bar{x}}{\alpha} \), and one with \( x^F > \frac{\bar{x}}{\alpha} \). Only the latter can be an equilibrium as, if \( x^F < \frac{\bar{x}}{\alpha} \), then incentive constraint (36) must bind, but we are attempting to construct an equilibrium in which this constraint does not bind. For an equilibrium, we first require that there be a solution to (43). A necessary and sufficient condition for that is

\[ \beta \geq \frac{\frac{\bar{x}}{\alpha}}{(1 - \rho)u \left( \frac{\bar{x}}{\alpha} \right)}. \] (44)

As well, we require that the solution satisfy \( x^F < x^* \), so that the full-information incentive constraint binds or, from (43),

\[ \beta < \frac{x^*}{(1 - \rho)u(x^*) + \rho (x^* - \frac{\bar{x}}{\alpha})}. \] (45)

Then, (44) and (45) are necessary and sufficient conditions for this equilibrium to exist.

5.1.3 Both Incentive Constraints Bind

If (36) and (37) bind, then \( x_L = \alpha x_F \), and from (35) - (38), \( x_F \) solves

\[ x_F = \beta(1 - \rho)u(x_F). \] (46)

To determine necessary and sufficient conditions for existence, it is sufficient to check that the solution to (46) implies that the incentive constraint (36) binds,
since (37) is always the tighter constraint. Thus, we require that $x_F < \frac{x}{\alpha}$, which from (46) gives

$$\beta < \frac{\frac{x}{\alpha}}{(1 - \rho)u \left( \frac{x}{\alpha} \right)}.$$  

(47)

Then, inequality (47) is necessary and sufficient for the existence of this equilibrium.

5.1.4 Asymmetric Equilibria vs. Symmetric Equilibria

In an asymmetric equilibrium, if we measure welfare as the sum of expected utilities across agents, then we need to take account of bad buyers as well as good ones, which gives us the welfare measure

$$W = \rho \left( u(x_L) - x_L \right) + \alpha(1 - \rho) \left( u(x_F) - x_F \right),$$

(48)

where $x_L$ and $x_F$ denote, respectively, consumption in limited information and full information meetings in the DM, and $W$ is strictly increasing in $x_L$ and $x_F$ for $x_L \leq \frac{x}{\alpha}$ and $x_F < x^*$. A symmetric equilibrium is just a special case where $\alpha = 1$ and $x_L = x_F$.

If we compare asymmetric and symmetric equilibria, for given parameter values, what are the key differences that would cause welfare, as measured in (48), to differ? First, bad borrowers in the asymmetric equilibrium do not consume in full information meetings in the DM, which lowers welfare in the asymmetric relative to the symmetric equilibrium. That is, given $x_L$ and $x_F$, lower $\alpha$ in (48) causes $W$ to decrease. Second, given the presence of defaulting borrowers in equilibrium, there is an adverse selection problem in limited information meetings, which tends to make $x_L$ lower in asymmetric equilibria. For these two reasons, we would expect welfare to be lower in asymmetric than in symmetric equilibria, in general. However, given the complications of general equilibrium, particularly the effects of bad borrowers on the incentive constraints of good borrowers, in some circumstances it is difficult to show this. We can make some precise statements, though.

To that end, note first that

$$\frac{x^*}{(1 - \rho)u(x^*)} + \rho \left( x^* - \frac{x}{\alpha} \right) < \frac{x^*}{(1 - \rho)u(x^*)},$$

(49)

and

$$\frac{\frac{x}{\alpha}}{(1 - \rho)u \left( \frac{x}{\alpha} \right)} < \frac{x^*}{(1 - \rho)u(x^*)},$$

(50)

for $0 < \alpha < 1$. Therefore, from (17), (19), (40) and (47), if $\alpha$ falls, so that there are fewer good buyers in the population, this acts to relax incentive constraints, expand the region of the parameter space in which incentive constraints do not bind, and shrink the region where incentive constraints bind. For this result, it is important that $-\frac{x'}{u(x)} < 1$, which implies that $\frac{x}{\alpha}$ is increasing in $\alpha$.

\footnote{The result goes the other way if $-\frac{x''}{u(x)} > 1$.}
seems counterintuitive, as we might expect that the presence of more bad buyers would make good buyers worse off, and therefore give them a greater incentive to default, thus tightening incentive constraints. But this is not the case, as the indirect effect dominates. Thus, if

$$\beta \geq \frac{x^*}{(1 - \rho)u(x^*)},$$

then from (17), (40), and (49), incentive constraints do not bind in a symmetric equilibrium or in any asymmetric equilibrium with $0 < \alpha < 1$. But, since $\tilde{x} < x^*$, consumption is smaller in limited information meetings in the DM in asymmetric equilibria than in the symmetric equilibrium, and consumption is the same in full information meetings. Therefore, welfare is higher in the symmetric equilibrium in this case.

If

$$\beta < \frac{\tilde{x}}{(1 - \rho)u(\tilde{x})},$$

then given (50), the incentive constraint binds in the symmetric equilibrium, and both incentive constraints bind in an asymmetric equilibrium. But from (46) and (18), consumption is the same in full information meetings in the symmetric and asymmetric equilibria, but is smaller in limited information meetings in the asymmetric equilibrium. Therefore, welfare is higher in the symmetric equilibrium.

In the case where

$$\beta \in \left[ \frac{\tilde{x}}{(1 - \rho)u(\tilde{x})}, \frac{x^*}{(1 - \rho)u(x^*)} \right],$$

it is difficult to make comparisons between symmetric and asymmetric equilibria.

Thus, we can at least make clear statements about symmetric vs. asymmetric equilibria if the discount factor is high, or if it is low. For a middle range of discount factors, it certainly seems intuitive that symmetric equilibria should dominate in welfare terms, but we cannot show it.

**Multiplicity of Asymmetric Equilibria** One feature of asymmetric equilibria that we did not see with symmetric equilibria is that, for given $\alpha$, there can be multiple equilibria. From (40), (44), (45), and (47), if

$$0 < \rho < \frac{u(\hat{x}) - u(x^*)}{\hat{x} - x^*},$$

then for high $\beta$, neither incentive constraint binds, for middle levels of $\beta$ the limited-information incentive constraint does not bind while the other incentive
constraint does, and for low levels of $\beta$ both incentive constraints bind. However, if
\[
\frac{\frac{1}{\alpha} u(x) - u(x^*)}{x^*} < \rho \leq \frac{u(x^*) - x^*}{u(x^*) - x^* + \frac{x}{\alpha}},
\] (51)
then for middle levels of $\beta$ two equilibria exist – the equilibrium where neither incentive constraint binds and the one where both incentive constraints bind, as the following proposition summarizes.

**Proposition 5** Suppose (51) holds and
\[
\beta \in \left[ \frac{x^*}{(1 - \rho)u(x^*) + \rho(x^* - \frac{x}{\alpha})}, \frac{\frac{x}{\alpha}}{(1 - \rho)u(x^*)} \right].
\] (52)
Then, for given $\alpha$, there exist two equilibria, one where neither incentive constraint binds, and one where both incentive constraints bind.

**Proof.** Consider the necessary and sufficient conditions for existence of equilibria with neither incentive constraint binding, (40), and both incentive constraints binding, (47). The set defined by (51) and (52) is the set satisfying (40) and (47). Then, any $(\rho, \beta)$ satisfying (51) and (52) is such that both (40) and (47) are satisfied, so both equilibria exist.

For asymmetric equilibria, there is generally a multiplicity of equilibria corresponding to different levels of $\alpha$. The above proposition states that there can be another dimension to multiplicity in that, for given $\alpha$, multiple equilibria can exist, due to the fact that multiplicities can work through incentive constraints.

**5.1.5 Discussion: Private Credit and No Government Debt**

In asymmetric equilibria, limited commitment of course plays a key role, but limited recordkeeping introduces an additional friction that gives novelty to the results. In these equilibria, asymmetric information in limited information meetings in the $DM$ is important, but not because would-be borrowers are intrinsically different, or because of hidden actions. Buyers who wish to borrow from sellers behave differently because they receive different treatment in the future, which affects their future payoffs and willingness to default. This is why there is nothing that good borrowers can do to distinguish themselves from bad ones. There is no action that a good borrower can take that a bad borrower would for some reason choose not to take, and which would thus reveal the good borrower’s type.

The result is an equilibrium in which bad borrowers sometimes get loans, and then proceed to default on those loans. Good borrowers do not default, because they have too much to lose from being treated like bad borrowers in the future. Bad borrowers default because they have nothing to lose from doing so.
5.2 Asymmetric Equilibria with Government Debt

Our goals in this section are to show that there can exist asymmetric equilibria with government debt, in which some buyers always default on their tax liabilities while others do not, and to establish under what conditions such equilibria exist. We can then make comparisons to what occurs in the absence of government debt, as analyzed in the previous subsection. Does government debt affect the equilibrium allocation in particular types of asymmetric equilibria? Does it rule out the existence of asymmetric equilibria?

In this section, we take the same approach as for symmetric equilibria, in constructing equilibria in which the quantity of bonds issued by the government is just sufficient to drive out private credit. In this case however, we are constructing equilibria for given $\alpha$, and then characterizing such equilibria and determining under what conditions they exist. In these equilibria, there is a mass $\alpha$ of good buyers who trade government bonds for goods in limited-information and full-information meetings in the $DM$, and who always pay their taxes in the $CM$. There is also a mass of $1 - \alpha$ of bad buyers, who are able to trade in limited-information meetings in the $DM$ if they mimic the behavior of good buyers, but cannot trade in full-information $DM$ meetings. These bad buyers always default on their tax liabilities.

As with symmetric equilibria, we assume that the government is able to confiscate government debt at the beginning of the $CM$ from sellers or buyers who have traded in the previous $DM$. Again, the rationale is that a technology exists which allows private agents to post government debt as collateral in a private credit contract, and this collateral can be seized in the event of default. If we permit the government to have the same powers as private sector agents, this means that we allow the government the latitude to write laws specifying contingencies under which government debt can be seized, and allow it to follow through when those contingencies arise.

As in the symmetric case, we construct an equilibrium with government debt in which consumption in the $DM$ by good buyers is $x = \beta B$, where $B$ now denotes the quantity of debt the government issues per good buyer. Then, the continuation utility $\hat{v}$ for the bad buyer is given by (22). The incentive constraint for the good buyer depends on whether $\hat{v} = 0$ (the bad buyer chooses autarky) or $\hat{v} > 0$. In the former case, the behavior of bad buyers is irrelevant for good buyers, while in the latter case the bad buyers default on their tax liabilities, so the good buyers will bear the tax burden associated with servicing government debt held by bad buyers. If $\hat{v} = 0$, then the incentive constraint for good buyers is given by (21), and $v$ is determined by (23). However, if $\hat{v} > 0$ then a good buyer’s incentive constraint is

$$\frac{B(1-q)}{\alpha} \leq v - \hat{v},$$

and the continuation value for a good buyer is given by

$$v = -qB + u(\beta B) - \frac{\beta B(1-q)}{\alpha} + \beta v.$$
As well, whether or not bad buyers hold government debt in equilibrium, optimization by good buyers implies that the bond price \( q \) is determined by (24).

The equilibria we construct have a similar structure of off-equilibrium punishments to the symmetric case. Assume that, in full information meetings in the DM, a seller understands that, if he or she were to engage in exchange with a bad buyer – either in the form of an outright exchange of government debt for goods, or a lending arrangement with government debt as collateral – then the government would either seize the government debt from the seller, or seize the buyer’s collateral, at the beginning of the next CM. Then, the seller’s off-equilibrium expected utility from accepting a contractual offer from a buyer who the seller knows is bad, is strictly dominated by refusal of the offer.\(^5\)

We assume that the government seizes no assets in connection with limited information exchange, as the seller is uninformed about the buyer’s type in those meetings, so there is no direct gain from dissuading the seller from trading in limited information meetings in the DM. To keep things simple, we do not search for optimal government policies in this context. We only want to show that there is potential for the introduction of government debt to improve the equilibrium allocation.

5.2.1 Incentive Constraint Does Not Bind

If the good buyer’s incentive constraint does not bind, then we construct an equilibrium where solving (25) gives \( \beta B = x^* \), and from (24), \( q = \beta \). Then, if a bad buyer mimics the behavior of a good buyer, by purchasing \( B \) government bonds in the CM, exchanging all of these bonds for consumption in limited information meetings in the DM, and holding the bonds until the next CM in the event that there is a full information meeting in the DM, the continuation value for the bad buyer is

\[
\hat{v} = \frac{\rho [u(x^*) - x^*]}{1 - \beta} > 0.
\] (55)

Therefore, bad buyers will mimic the behavior of good buyers in limited information meetings in the DM, as this is preferable to autarky. Then, from (54) and (55),

\[
v - \hat{v} = \frac{(1 - \rho)u(x^*) - \frac{x^*(1 - \beta)}{\alpha} - (1 - \rho)x^*}{1 - \beta}.
\] (56)

\(^5\)It is important to notice that in order to support these equilibria, it is sufficient that collateral – government debt – be seized only when a bad buyer actually trades in full information meetings and defaults on those loans. If the bad buyer does not get to trade there is no need to seize his or her holdings of government debt to support equilibria with endogenous default. The bad buyer can redeem those bonds in the CM to consume CM goods. This is not an irrelevant detail because in order to support default in equilibrium, it is necessary that the punishment for default is not too severe.

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To determine existence of this equilibrium, it is sufficient to check the incentive constraint (53), so from (53) and (56) we obtain

$$ \beta \geq \frac{x^*}{\alpha(1 - \rho)u(x^*) + [1 - \alpha(1 - \rho)]x^*}, $$

(57)

Inequality (57) is then a sufficient condition for an equilibrium of this type to exist for given $\alpha$.

### 5.2.2 Incentive Constraint Binds and $\hat{\nu} > 0$

Next, consider the case in which the asymmetric equilibrium with government debt involves a binding incentive constraint, and $\hat{\nu} > 0$, so that bad buyers strictly prefer to mimic good buyers in limited information meetings in the DM. If (53) binds, then the quantity of consumption in DM meetings is $x = \beta B$, and the price of government bonds in the CM is $q = \beta u'(x)$. Then, from (53), and (54), $x$ solves

$$ x[1 - \beta u'(x)] = \alpha \beta (1 - \rho)[u(x) - x]. $$

(58)

A unique solution to (58) exists, and we require that the incentive constraint bind, or $x < x^*$. Thus, from (58), a necessary condition for an equilibrium of this type to exist is

$$ \beta < \frac{x^*}{\alpha(1 - \rho)u(x^*) + [1 - \alpha(1 - \rho)]x^*}. $$

(59)

Further, we require $\hat{\nu} > 0$. As in our analysis in the symmetric equilibrium case, suppose that $-\frac{xu''(x)}{w(x)}$ is constant. Then, as we showed above, if $-\frac{xu''(x)}{w(x)} \geq 1 - \rho$, then $\hat{\nu} > 0$, and if $-\frac{xu''(x)}{w(x)} < 1 - \rho$ then $\hat{\nu} > 0$ if and only if $x > \tilde{x}$ where $\tilde{x}$ solves

$$ -\tilde{x}u'(\tilde{x}) + \rho u(\tilde{x}) + (1 - \rho)\tilde{x} = 0. $$

(60)

Then, since the left-hand side of (58) is increasing in $x$ and the right-hand side is decreasing in $x$, (58) and (60) imply $x > \tilde{x}$ if and only if

$$ \beta > \frac{\tilde{x}}{\alpha(1 - \rho)[u(\tilde{x}) - \tilde{x}] + \tilde{x}u'(\tilde{x})}. $$

(61)

### 5.2.3 Incentive Constraint Binds and $\hat{\nu} = 0$

In this case, $x = \beta \frac{B}{\bar{\alpha}}$, since bad buyers do not hold bonds, and $q = \beta u'(x)$. Then, from (53), and (54), $x$ solves (6). A necessary condition for this equilibrium to exist is that $x < x^*$, which from (6) gives (7). As well, bad buyers must not prefer to mimic the behavior of good buyers in limited information meetings in the DM, i.e.

$$ -xu'(x) + \rho u(x) + (1 - \rho)x \leq 0. $$

(62)
But (62) holds if and only if \(-\frac{\nu''(x)}{u'(x)} < 1 - \rho\) and \(x \leq \tilde{x}\), where \(\tilde{x}\) is defined by (60). But from (6) and (60), \(x \leq \tilde{x}\) is equivalent to

\[
\beta \leq \frac{\tilde{x}}{u(\tilde{x})}
\]

Therefore (7), \(-\frac{\nu''(x)}{u'(x)} < 1 - \rho\), and (63) are necessary and sufficient for this equilibrium to exist.

### 5.3 Effects of Government Debt in Asymmetric Equilibria

We have two goals in this subsection. The first is to determine how the trading of government debt affects exchange and welfare for given \(\alpha\). In other words, given an economy with a particular fraction of defaulting buyers in the population, how will the existence of government debt affect what is consumed in limited-information and full-information DM meetings? Our second goal is to determine the effects of trading in government debt on the existence of particular asymmetric equilibria. Does government debt encourage or discourage default? For our welfare analysis we use the welfare measure in (48).

First, for given \(\alpha\), compare the properties of the equilibrium (equilibria) that exists (exist) without government debt, and with government debt, in alternative regions of the parameter space. Let \(x_{LN}\) and \(x_{FN}\) denote consumption in limited-information and full-information meetings, respectively, in an asymmetric equilibrium without government debt. Similarly, \(x_{LD}\) and \(x_{FD}\) are consumption quantities in the DM when government debt is traded.

**Proposition 6** If \(\beta \geq \frac{x^*}{\alpha(1-\rho)u(x^*) + [1-\alpha(1-\rho)]x^*}\), then \(x_{LN} < x_{LD} = x^*\) and \(x_{FN} \leq x_{FD}\). Welfare is higher with government debt than without it.

**Proof.** From (57) an asymmetric equilibrium with government debt exists in which the incentive constraint does not bind, so \(x_{LD} = x_{FD} = x^*\). If (40) holds, then an asymmetric equilibrium without government debt exists in which \(x_{LN} = \tilde{x}\) and \(x_{FN} = x^*\), so \(x_{LN} < x_{LD}\) and \(x_{FN} = x_{FD}\), and welfare is higher in the equilibrium with government debt. If (44) and (45) hold, then an asymmetric equilibrium without government debt exists in which \(x_{LN} = \tilde{x}\) and \(x_{FN} < x^*\). Therefore, \(x_{LN} < x_{LD}\) and \(x_{FN} < x_{FD}\) in this case, and welfare is higher in the equilibrium with government debt. Finally, if (47) holds, then an equilibrium without government debt exists in which \(x_{LN} < \tilde{x}\) and \(x_{FN} < x^*\). Therefore \(x_{LN} < x_{LD}\) and \(x_{FN} < x_{FD}\) in this case, and welfare is higher in the equilibrium with government debt. \(\blacksquare\)

**Proposition 7** If \(\beta \leq \frac{\tilde{x}}{u(\tilde{x})}\) and \(\beta < \frac{\tilde{x}}{(1-\rho)u(\tilde{x})}\), then \(x_{LN} < x_{LD}\), \(x_{FN} = x_{FD}\), and welfare is higher with government debt than without it.

**Proof.** From (63), an equilibrium exists with government debt in which there is a binding incentive constraint, where \(x = x_{LD} = x_{FD}\) solves (6). From
(47), an equilibrium also exists without government debt where \( x = x_{FN} \) also solves (47), so \( x_{FD} = x_{FN} \). But in the equilibrium without government debt, \( x_{LN} = \alpha x_{FN} = \alpha x_{FD} = \alpha x_{LD} < x_{LD} \), as both incentive constraints bind in the equilibrium without government debt. ■

Figure 3 shows the subdivision of the parameter space, according to which asymmetric equilibria exist, with and without government debt. First, Proposition 5 deals with regions 1, 2, and 3 in Figure 3, where an asymmetric equilibrium with government debt exists in which the incentive constraint does not bind. In region 1, there exists an equilibrium without government debt in which neither incentive constraint binds. In region 2, there exist two equilibria without government debt; in one of these equilibria neither incentive constraint binds, and in the other both incentive constraints are binding. In region 3, there exists an equilibrium with no government debt where both incentive constraints bind. In regions 1, 2, and 3, the equilibrium with government debt dominates any equilibrium without government debt that exists.

[Figure 3 here.]

Second, Proposition 6 deals with region 12 in Figure 3. In that subset of the parameter space, an equilibrium without government debt exists in which the incentive constraint binds, and bad buyers choose autarky rather than defaulting. This equilibrium dominates the equilibrium without government debt in which both incentive constraints bind, which also exists in region 12.

In Propositions 5 and 6, we have shown that there exist subsets of the parameter space for which issuing government debt will improve matters in asymmetric equilibria, for given \( \alpha \). First (Proposition 5), for any \( \alpha \) with \( 0 < \alpha < 1 \), there is a subset of the parameter space in which there exists an equilibrium with government bonds for which the incentive constraint does not bind, and in which there is more exchange in the DM, and higher welfare, than in any asymmetric equilibrium without government bonds that exists in that subset. Second (Proposition 6), for any \( \alpha \) with \( 0 < \alpha < 1 \), there is a subset of the parameter space in which there exists an equilibrium with government debt for which the incentive constraint binds, in which there exists an equilibrium without government debt in which both incentive constraints bind, and in which there is less exchange in the DM and therefore lower welfare in the equilibrium without government debt.

In regions 4 through 11 in Figure 3, it is more difficult to draw conclusions, though we know that, by continuity, the introduction of government debt must be welfare improving for sufficiently large \( \alpha \). This follows from the fact that government debt always improves matters in symmetric equilibria – the special case where \( \alpha = 1 \).

In the equilibrium with government bonds in which the incentive constraint does not bind (regions 1, 2, and 3 of Figure 3), the welfare improvement from trade in government bonds does not result from effects on default behavior. Indeed, if such an equilibrium exists for given \( \alpha \), then there exists an equilibrium without government bonds for the same \( \alpha \). The key welfare-improving effect
is that introducing government bonds solves the adverse selection problem in limited-information meetings in the $DM$. With government bonds, good buyers are no longer charged a default premium in limited-information exchange, and more goods are traded. However, in the equilibrium with government bonds, good buyers suffer because bad buyers always default on their taxes. Thus, good buyers have to bear the entire burden of paying the net interest on the government debt.

In an asymmetric equilibrium with government debt in which the incentive constraint binds and bad buyers choose autarky ($\hat{\theta} = 0$), in region 12 of Figure 3, government debt improves the allocation for two reasons. First, as we outlined for the case where the incentive constraint does not bind, government debt solves the adverse selection problem in limited information meetings in the $DM$. Second, in region 12 of Figure 3, bad buyers do not default in equilibrium, but instead choose autarky. Therefore, the behavior of bad buyers has no effect on good buyers.

Next, government debt can act to mitigate default in that it can kill some undesirable asymmetric equilibria. Note in particular that, without government bonds, an asymmetric equilibrium exists for any $\alpha \in (0, 1)$ and $(\beta, \rho) \in (0, 1) \times (0, 1)$. However, from (57), (59), (61), and (63), with government bonds, there is a subset of the parameter space where an asymmetric equilibrium does not exist, for given $\alpha$, where parameters satisfy

$$\rho < 1 + \frac{xu''(x)}{u'(x)}$$

and

$$\frac{\hat{x}}{u(\hat{x})} < \beta \leq \frac{\hat{x}}{\alpha(1 - \rho)[u(\hat{x}) - \hat{x}] + \hat{x}u'(\hat{x})}$$

This subset comprises regions 8, 9, and 10 in Figure 3. Note further that this subset expands as $\alpha$ decreases, so government debt more successfully does away with default the more severe the default problem is.

We are not saying that government debt is useful because it leads to nonexistence of equilibrium. Indeed, we showed that, with government debt, a symmetric equilibrium always exists (i.e. $\alpha = 1$). Another way to state the idea in the previous paragraph is that there are some regions of the parameter space for which the introduction of government debt implies that an asymmetric equilibrium exists only if $\alpha$ is sufficiently large. Thus, government debt eliminates some equilibria with large numbers of defaulters.

In asymmetric equilibria government debt can improve welfare for three reasons. First, just as in symmetric equilibria, government debt acts to alter the payoff to defaulting for buyers, and therefore relaxes incentive constraints. Second, government debt can solve an adverse selection problem in private credit market arrangements subject to limited information. Effectively government debt permits all credit to be secured, which can support equilibria with more exchange and higher welfare. Third, government debt can eliminate equilibria with high default rates. Thus, government debt can mitigate default, and can
enhance the ability of private sector agents to defend themselves against default, thus increasing the volume of credit.

The interpretation of the private decentralized exchange in the model as collateralized credit, with government debt serving as collateral, is consistent with how markets in repurchase agreements – basically collateralized short-term credit arrangements – work in practice, and with some features of the recent financial crisis. From Martin et al. (2012), for example, we know that in times of credit market dysfunction, such as the recent financial crisis, government-related securities are relatively more valuable as collateral. In our model, credit market dysfunction is an endogenous phenomenon, and this dysfunction creates a role for government debt as collateral.

6 Conclusion

In this model, government debt can act to discourage default, and to reduce the quantity of equilibrium default. Effectively, government debt can be interpreted as playing a role as collateral in private credit contracts, and the fact that – like good collateral – government debt can be easily seized, helps to discipline agents who potentially or actually default on private or public debts. The model uses ideas from the monetary theory literature, in which memory and recordkeeping play a critical role in giving government liquidity a role in exchange. Government debt matters in this model because of limited memory, in conjunction with weak punishments for default. This creates the possibility that agents can borrow even if they have defaulted in the past, and in that context the economy is non-Ricardian – government debt matters.

A novelty in this limited commitment model is that default can occur in equilibrium, which opens up interesting possibilities for future work. For example, this model could be extended to consider the coexistence of secured and unsecured private credit arrangements. As well, it is possible to use this model environment to analyze endogenous fluctuations in which the aggregate default rate varies over time. Such a model would be useful for studying financial panics.

7 References


