Optimal Sales Contracts with Withdrawal Rights

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Abstract

We introduce ex post participation constraints in the standard sequential screening model. This captures the presence of consumer withdrawal rights as, for instance, mandated by EU regulation of “distance sales contracts”. With such additional constraints, the optimal contract is static and, unlike with only ex ante participation constraints, does not elicit the agent’s information sequentially. This holds whenever differences in ex ante and ex post outside options are below a positive upper bound. Welfare effects of mandatory withdrawal rights are ambiguous. Since it is insufficient in our setting to consider only local incentive constraints, we develop a novel technique to identify the relevant global constraints.

Keywords: Sequential screening, dynamic mechanism design, participation constraints, Mirrlees approach
JEL codes: D82, H57

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1 Introduction

On the 12th of December 2011 the European directive 2011/83/EU was adopted, harmonizing earlier legislation on "distance sales contracts". These contracts govern internet and mail order sales to consumers in the EU, a market which in 2014 is expected to surpass 200 billion dollars in sales and to represent an average market share exceeding 7%. As the share of internet sales is expected to rise steadily in the coming years, the economic impact of the legislation increases further.¹

Governing distance sales contracts, the directive mandates a withdrawal right for consumers of two weeks. When the consumer exercises his right of withdrawal, all contractual obligations are terminated and the seller reimburses all payments received from the consumer. In sections 37 and 47 the directive indicates that the goal of the withdrawal right is to ensure that consumers in internet shops can condition their purchase on the same information as in traditional bricks-and-mortar stores: “Since in the case of distance sales, the consumer is not able to see the goods before concluding the contract, he should have a right of withdrawal” and “In order to establish the nature, characteristics and functioning of the goods, the consumer should only handle and inspect them in the same manner as he would be allowed to do in a shop.”

Hence, regulators view internet consumers at the following informational disadvantage. While a consumer who buys on the internet, signs the sales contract before being able to ascertain the nature of a good, a similar consumer who buys at a traditional store, signs his or her sales contract after obtaining this information.

Motivated by this observation, we investigate the economic effects of withdrawal rights on optimal sales contracts. To compare the optimal selling contracts under the two different selling modes, we model the selling problem of a traditional store as a static screening problem in the tradition of Baron and Myerson (1982), where the buyer, before signing the contract, has received all relevant private information. It is well known that in the static screening problem, a posted price contract is the optimal selling contract. To capture the view of the EU regulation, we model internet sales as a sequential screening

¹Figures taken from http://www.retailresearch.org/onlineretailing.php (last retrieved 22.10.2014). According to The Economist (edition of July 13th 2013) 90% of growth in retail sales expected until 2016 in Britain, Germany and France will be online.
problem in the sense of Courty and Li (2000), where the consumer learns additional private information about his valuation after signing the contract.\textsuperscript{2}

For the standard sequential screening model, the optimal selling contract is dynamic in that it screens the buyer over time.\textsuperscript{3} In this model the buyer has only an ex ante outside option and is bound by the contract even if ex post, after new information has arrived, this imposes losses on him. We argue that the inclusion of withdrawal rights, as mandated by the EU regulation, is equivalent to introducing ex post participation constraints in the sequential screening model, implying that the buyer can sustain no (or only limited) losses ex post. The main result of our paper is that, even though sequential screening is still feasible with ex post participation constraints, the seller no longer benefits from it. Instead, the optimal selling contract is static and coincides with the optimal posted price contract in the static screening model.

Exploring the limits of our result, we derive an explicit upper bound on the difference in the ex ante and ex post outside option so that the static contract remains optimal, and a further lower bound on this difference, above which the ex post outside option is irrelevant. Therefore, we can view the sequential screening models with either only an ex ante or only an ex post outside option as two extremes of a unified framework. In addition, this upper bound implies that our result holds also when the agent’s ability to post a non–refundable bond ex ante is limited. Hence, we consider our result relevant for economic environments where due to explicit or implicit regulatory restrictions on contracting, outside options or bonding opportunities are restricted.

One such possible environment is the employment relation. In most countries, employees have the legal right to resign from the contract at any time, and labor protection laws

\textsuperscript{2}Our motivation for studying a monopoly setup is to capture the consumer’s weak bargaining position which the European Court of Justice views as inseparably linked with directive: “the system of protection introduced by the Directive is based on the idea that the consumer is in a weak position vis-à-vis the seller or supplier, as regards both the bargaining power and his level of knowledge” (Joined Cases C-240/98 to C-244/98 Ocano Grupo Editorial and Salvat Editores [2000] ECR I-4941, §25).

\textsuperscript{3}The (strict) optimality of sequential screening in the absence of ex post participation constraints figures most prominently in Courty and Li (2000), but also features in Baron and Besanko (1984), Battaglini (2005), Boleslavsky and Said (2013), Dai et al. (2006), Esö and Szentes (2007a, b), Inderst and Peitz (2012), Hoffmann and Inderst (2011), Krähmer and Strausz (2011a), Nocke et al. (2011) and Pavan et al. (2014).
prohibit the employer from demanding, either ex- or implicitly, a non-refundable signing bond by the employee that restrains the worker’s withdrawal decision.\footnote{E.g., the California Labor Code Section 402 explicitly states “No employer shall demand, exact, or accept any cash bond from any employee or applicant”. Likewise, employment bonds are prohibited under German law, including the retainerment of unpaid wages after a worker’s resignation (see ruling BAG 06.09.1989 - 5 AZR 586/88). Some important but context specific exceptions exist such as “training bonds”, where the employer makes a costly, non-specific human capital investment in the employee, or deposits when the employee is entrusted with the employer’s property.} It seems also natural that, similar to the internet consumer, employees learn important private information about the value of the job shortly after signing the contract and joining the firm such as the general working atmosphere between colleagues. Hence, when outside options in the form of alternative job opportunities before and after learning such information remain comparable—which seems plausible when resignations are not observable or signal little information to alternative employers—then an employer offering an employment contract to a potential employee faces a sequential screening problem with ex post participation constraints. In employment relationships withdrawal options also arise when employees obtain a personal leave of absence from their current employer before accepting a job offer from a new employer. Such leaves of absence are for example prevalent in civil services (including academia). They enable employees to return after learning unfavorable information about the new job. Clearly, the withdrawal rights we have in mind are not relevant in all labor settings. For example, the employee’s outside options after resigning may differ greatly from his outside options before starting the job. This will especially be the case when quitting is observable and is interpreted as a negative signal about the employee such as in the market of young professionals. If this difference in outside options exceeds the aforementioned lower bound, it remains optimal to sequentially screen the agent’s private information.

A further application is the procurement relationship where the ability to withdraw arises implicitly. Unfavorable private ex post information, such as cost overruns, may force the supplier to file for bankruptcy before completing the contract. Limited liability on the supplier’s side and bankruptcy law restrict the procurer’s ability to extract payments or seize assets from an insolvent supplier. From the procurer’s perspective, bankruptcy thus constitutes an ex post outside option of the contractor, which limits the losses a contractor can bear. The procurement industry is well aware of this problem, and it is
common to require “performance bonds” which are paid up-front and returned to the supplier only upon contract completion. Cash-constraints and imperfect capital markets, however, place limits on such bonds.\(^5\)

To shed light on our result that an ex post participation constraint eliminates the benefits of sequential screening, it is easiest to consider the case that the seller’s costs are zero, so that trade is always efficient, and the seller offers the buyer a menu of option contracts. An option contract consists of an *up-front payment by* the buyer, and gives the buyer the option to purchase the good at a pre-specified *exercise price* after having observed his true valuation. Our result that with ex post participation constraints the seller does not benefit from screening the buyer sequentially means that offering a menu containing different option contracts is not optimal.

To gain intuition for this, assume to the contrary that, at the optimum, different ex ante buyer types select different option contracts. Observe first that when the buyer’s true valuation happens to equal the exercise price, the buyer obtains a net payoff of zero from consumption. Therefore, with ex post participation constraints, the seller cannot demand a positive up-front fee, because this would cause an ex post loss if the buyer’s true valuation equals the exercise price. This then means that any option contract from the menu is individually rational for any ex ante type.

Now consider the contract in the menu with the highest exercise price. This contract generates less surplus than any other contract and, by incentive compatibility, yields any type who picks it, a weakly higher rent than any of the more efficient contracts. But this implies that the seller is better off excluding this contract from the menu so that the buyer must pick one that generates more surplus, while paying him lower rents (but, as argued, is also individually rational). By this argument it is optimal to delete any but the most efficient contract from the menu. Therefore, with ex post participation constraints, it is optimal not to screen ex ante types.\(^6\)

\(^5\) In addition, for the case that a supplier breaches the contract and quits, the law often explicitly allows courts to reduce penalties that are considered as out of proportion. See, e.g., the US Uniform Civil Code §2-718: “A term fixing unreasonably large liquidated damages is void as a penalty”; or the German Civil Code §343: “If a payable penalty is disproportionately high, it may on the application of the obligor be reduced to a reasonable amount by judicial decision.”

\(^6\) This argument fails if there are only ex ante participation constraints. In this case, the seller charges
The above reasoning only applies to option contracts. Our main conceptual contribution is to derive sufficient conditions under which option contracts are indeed optimal. As we will argue, this is equivalent to showing that the optimal contract is deterministic. In the absence of ex post participation constraints, the optimality of deterministic contracts can be established by considering a relaxed problem in the spirit of Mirrlees, which only considers the “local” ex ante incentive constraints. Under appropriate regularity conditions, the solution to the relaxed problem is automatically deterministic and globally incentive compatible.\(^7\) We show that in our case, such a local approach does not work, because one cannot find a regularity condition so that the solution to the corresponding relaxed problem is automatically deterministic.\(^8\) Instead, we develop a novel technique to identify a different relaxed problem, which involves global constraints, whose solution solves the original problem under an appropriate regularity condition.\(^9\) In addition to the familiar monotone hazard rate which requires the ratio of an ex ante type’s cumulative distribution and the same ex ante type’s density to be monotone, this condition requires that also the “cross-hazard rate”, i.e., the ratio of an ex ante type’s cumulative distribution and any other ex ante type’s density is monotone.

Although the main objective of our analysis is to investigate the implications of withdrawal rights for optimal dynamic contracting, we complement our positive analysis with discussing normative aspects of the EU withdrawal rights regulation. We first discuss the origins of the withdrawal regulation and its classification by legal scholars as a right of withdrawal due to absence of the good. This indicates that, in the eyes of the legislators, a major goal behind the EU regulation was to safeguard the consumer’s right of choice by ensuring that distance sales contracts do not erode the consumer’s informational pot-the high (strictly positive) up-front fee for the contract with the lowest exercise price so that it is acceptable only for the buyer who is most optimistic about his future valuation. The more pessimistic buyers would make an expected loss from this contract. Hence, only offering the most efficient contract in the menu would violate ex ante participation constraints of all but the most optimistic type.

\(^7\) Courty and Li (2000) were the first to identify an appropriate regularity condition for sequential screening models, which requires a modified hazard rate to be monotone. Most of the literature mentioned in footnote 3 adopts these or closely related conditions.

\(^8\) Battaglini and Lamba (2014) argue that the failure of the local approach is typical for dynamic mechanism design problems.

\(^9\) Our technique requires to consider sequential screening with finite ex ante types. However, as our result holds for any number of ex ante types, we can regard the continuous types case as a limiting case.
position relative to traditional shopping contexts. Since our result implies that withdrawal rights level the playing field between internet and traditional sales, we confirm that they achieve this goal. We also show however that the welfare effects of a mandatory withdrawal right are ambiguous. In addition to hurting firms, they may also hurt consumers. Hence, despite the fact that the regulation limits the seller’s ability to price discriminate and, therefore, her ability to extract rents from the consumer, it may lead to lower consumer rents. This is so because withdrawal rights may induce the seller to opt for larger economic distortions, resulting in a smaller overall surplus. While our model stresses that “fit uncertainty” is a key issue in online retail, in practice also moral hazard and adverse selection on the seller’s side are likely to affect the welfare effects of mandatory withdrawal rights. While incorporating moral hazard and adverse selection in our model raises conceptual issues whose full treatment is beyond the scope of our paper, we discuss informally whether mandatory withdrawal rights are an efficient way to mitigate agency problems on the seller’s side.

The rest of this paper is organized as follows. The next section introduces the setup and derives the principal’s problem. In Section 3, we solve the principal’s problem for the case that she offers a menu of option contracts. Moreover, we extend our result to settings with less stringent ex post participation constraints and costly returns. In Section 4, we discuss the normative effects of withdrawal rights and provide an informal discussion of how agency problems on the seller side and competition affect these welfare conclusions. In Section 5, we allow for general, including stochastic, contracts. Section 6 concludes. All proofs that do not appear in the main text are relegated to the appendix.

2 The setup

Consider a potential buyer (he) and a seller (she), who has a single unit of a good for sale. The buyer’s valuation of the good is \( \theta \in [0, 1] \) and the seller’s opportunity costs are commonly known to be \( c \in [0, 1) \). Trade is therefore efficient for at least the valuation \( \theta = 1 \). The terms of trade specify whether the good is exchanged and payments from the buyer to the seller. The parties are risk neutral and have quasi-linear utility functions. That is, the seller’s profit is payments minus her opportunity costs, and the buyer’s utility is valuation minus payments.
At the time of contracting about the terms of trade, no party knows the buyer’s true valuation, \( \theta \), but the buyer has private information about its distribution. After the seller offers the contract, the buyer privately learns his true valuation \( \theta \). Formally, there are two periods. In period 1, the buyer knows his valuation is distributed according to distribution function \( G_i \) with non–shifting support \([0,1]\), where \( i \) is drawn from the set \( I \equiv \{1, \ldots, n\} \) with probability \( p_i > 0 \). We refer to \( i \) as the buyer’s ex ante type. In period 2, the buyer observes his ex post type \( \theta \) which is drawn according to \( G_i \). While the buyer’s ex ante and ex post types are his private information, the distributions of ex ante and ex post types are common knowledge.\(^{11}\)

The seller’s problem is to design a contract that maximizes her expected profits. The main novelty of our analysis is to consider the case in which the buyer has a withdrawal right after having observed his ex post type. It is well–known that in the absence of such a withdrawal right, the optimal contract is dynamic in the sense that it conditions non-trivially both on the buyer’s ex ante and ex post private information. The main result of this paper is that this is no longer true in the presence of a withdrawal right. In the next section, we first show this result for the case that the seller offers the buyer a menu of option contracts. In section 5, we show that the result extends to the class of all contracts.

### 3 Option contracts

In this section, we consider the case that the seller offers the buyer a menu of option contracts. Under an option contract \((F, R)\), the buyer pays the seller the up–front fee \( F \in \mathbb{R} \) in period 1 and receives the option to buy the good at the exercise price \( R \in [0,1] \) in period 2 after having learned \( \theta \). We say that a menu of \( n \) option contracts,

\[
(F, R) = ((F_1, R_1), \ldots, (F_i, R_i), \ldots, (F_n, R_n)),
\]

is incentive compatible if choosing option contract \((F_i, R_i)\) from the menu is optimal for buyer type \( i \). As is well known, such option contracts are, under certain regularity

\(^{10}\)In the context of distance sales, the good is shipped to the buyer who learns \( \theta \) upon inspecting and trying out the good.

\(^{11}\)Our assumption that ex post types are continuous and ex ante types are discrete is for technical convenience only. Note that we allow for an arbitrary number of ex ante types.
conditions, optimal without withdrawal rights and equivalent to deterministic contracts.\textsuperscript{12}

When buyer type $i$ has chosen contract $(F_j, R_j)$ and observed his valuation $\theta$, he exercises the option only if $\theta$ exceeds the exercise price. Hence, the contract yields him the ex post utility

\[
V_j(\theta) = \begin{cases} 
-F_j + (\theta - R_j) & \text{if } \theta \geq R_j \\
-F_j & \text{otherwise.}
\end{cases}
\] (1)

Buyer type $i$’s ex ante utility from the contract $(F_j, R_j)$ is

\[
U_{ji} = -F_j + \int_{R_j}^{\theta} \theta - R_j \, dG_i(\theta).
\]

Thus, the menu is incentive compatible if for all $i, j \in I$:

\[
U_{ii} \geq U_{ji}. \tag{IC_{ij}}
\]

Our main objective is to analyze the case in which the buyer has a withdrawal right. This means that, after having observed his valuation $\theta$, the buyer has the choice between exercising his option as specified in the contract, or withdrawing from it and obtaining his outside option of 0. The withdrawal right effectively guarantees the buyer a utility of 0 for any realization of his ex post valuation. Accordingly, with withdrawal rights, the contract needs to satisfy the ex post individual rationality constraints:

\[
F_i \leq 0. \tag{IR_{ex}}
\]

Effectively, the presence of a withdrawal right prevents the seller from using option contracts with a positive up-front fee. Put differently, withdrawal allows the buyer to reclaim any payment he might have made ex ante. Our notion of withdrawal rights therefore captures Article 13 of the EU directive which states “The trader shall reimburse all payments received from the consumer” and Article 25 which states that “Any contractual terms which directly or indirectly waive or restrict the rights resulting from this Directive shall not be binding on the consumer”\textsuperscript{13}.

\textsuperscript{12}In Section 5, we explicitly show the equivalence between incentive compatible direct mechanisms that are deterministic and an incentive compatible menu of option contracts.

\textsuperscript{13}In Section 3.3, we consider the case in which the seller can retain some ex ante payments and show that our result that sequential screening is not beneficial is still true if the amount the seller can retain is not too large, e.g. due to cash constraints on the buyer’s side.
3.1 Optimal option contracts with withdrawal rights

The seller’s payoff from an option contract is the expected payment minus the cost of the sale, or, equivalently, the difference between the option contract’s aggregate surplus and the buyer’s utility. Hence, if the buyer’s ex ante type is \( i \), the seller’s conditional expected payoff is

\[
W_i = F_i + (1 - G_i(R_i))(R_i - c) = \int_{R_i}^{1} \theta - c \ dG_i(\theta) - U_{ii}.
\]

Thus, the optimal menu of option contracts \((F^{xp}, R^{xp})\) solves the problem

\[
P^{\alpha} : \max_{(F,R)} \sum_i p_i W_i \quad s.t. \quad (IC_{ij}), \ (IR^{xp}_i) \text{ for all } i, j \in I.
\]

We now show that with withdrawal rights, the seller optimally offers a degenerate menu of option contracts, consisting of a single option contract only. That is, \( P^{\alpha} \) exhibits a solution with \( F_i = F \) and \( R_i = R \) for all \( i \in I \). We refer to such a menu as static. A static menu of option contracts yields the seller a payoff

\[
\bar{W} = \sum_i p_i (F + (1 - G_i(R))(R - c)) = F + (1 - \bar{G}(R))(R - c),
\]

where \( \bar{G}(\theta) \equiv \sum_i p_i G_i(\theta) \) is the ex ante distribution over types.

Since a static menu of option contracts is trivially incentive compatible, the optimal static menu of option contracts, \((\bar{F}, \bar{R}) = ((\bar{F}, \bar{R}), \ldots, (\bar{F}, \bar{R}))\) maximizes \( \bar{W} \) subject to the ex post individual rationality constraints \( \bar{F} \leq 0 \). It is evident from inspection that the seller optimally sets \( \bar{F} = 0 \) and chooses a price \( \bar{R} \) that solves

\[
\max_{\bar{R}} (1 - \bar{G}(R))(R - c).
\]

We assume that an optimal static menu exists, which means a maximizer \( \bar{R} \) exists.

We are now in the position to state our main result for option contracts.

**Proposition 1** If the buyer has a withdrawal right so that the seller has to respect the ex post individual rationality constraints \((IR^{xp}_i)\) for all \( i \in I \), then an optimal menu \((F^{xp}, R^{xp})\) of option contracts consists of a single contract only: \((F^{xp}_i, R^{xp}_i) = (0, \bar{R})\) for all \( i \in I \).
To demonstrate the result, we first consider an arbitrary feasible menu \((F, R)\) and argue that the seller is at least as well off by offering to each type only the option contract in the menu with the smallest exercise price larger than costs but with an up–front fee of zero.

More specifically, let \(k = \arg \min \{ R_i \mid R_i \geq c \} \) indicate the option contract in the menu \((F, R)\) with the smallest exercise price larger than costs. Define the static menu \((\tilde{F}, \tilde{R})\) with \((\tilde{F}_i, \tilde{R}_i) = (0, R_k)\) for all \(i \in I\). Note first that the static menu \((\tilde{F}, \tilde{R})\) is evidently feasible with withdrawal rights. We now show that, conditional on any ex ante type \(i\), the seller obtains a (weakly) larger profit under the static menu \((\tilde{F}, \tilde{R})\) than under the original one \((F, R)\).

First consider buyer types \(i\) who, under the sequential menu, chooses a contract that exhibits an exercise price below costs: \(R_i < c\). By (2) the seller’s profit from such a buyer type is

\[
W_i = F_i + (1 - G_i(R_i))(R_i - c),
\]

which is negative since \(F_i\) is negative and costs exceed the exercise price. In contrast, the seller’s profit from buyer type \(i\) under the static menu is non–negative.

Next, consider the other buyer types \(i\) who, under the original menu, choose a contract that displays an exercise price above costs: \(R_i \geq c\). Since the original menu is incentive compatible by assumption, buyer type \(i\)’s ex ante utility from contract \((F_i, R_i)\) exceeds his utility from contract \((F_k, R_k)\), that is, \(U_{ii} \geq U_{ki}\). Observe further that the buyer’s utility from the contract \((\tilde{F}_i, \tilde{R}_i) = (0, R_k)\) is smaller than from the contract \((F_k, R_k)\) because they display the same exercise prices, but, since \(F_k \leq 0\) by \((IR^{xp}_k)\), the latter has a (weakly) smaller up–front fee. Hence,

\[
U_{ii} \geq U_{ki} \geq \tilde{U}_{ii},
\]

where \(\tilde{U}_{ii}\) denotes buyer type \(i\)’s ex ante utility from the contract \((\tilde{F}_i, \tilde{R}_i) = (0, R_k)\). Moreover, since \(R_k\) is the menu’s smallest exercise price exceeding costs, the contract \((F_i, R_i)\) yields a smaller surplus than the contract \((\tilde{F}_i, \tilde{R}_i) = (0, R_k)\):

\[
\int_{R_i}^{1} \theta - c \, dG_i(\theta) \leq \int_{R_k}^{1} \theta - c \, dG_i(\theta).
\]

\footnote{If such a \(k\) does not exist, then the menu \((F, R)\) yields the seller a loss, because she sells her good below cost, and a static menu with \((F, R) = (0, c)\) does better.}
The two previous inequalities imply that the seller’s profit from the option contract 
\((F_i, R_i)\) is smaller than from \((\tilde{F}_i, \tilde{R}_i)\):

\[
W_i(F_i, R_i) = \int_{R_i}^{1} \theta - c \ dG_i(\theta) - U_{ii} \leq \int_{R_i}^{1} \theta - c \ dG_i(\theta) - \tilde{U}_{ii} = W_i(\tilde{F}_i, \tilde{R}_i).
\]

Intuitively, the option contract \((\tilde{F}_i, \tilde{R}_i)\) yields the seller a larger profit, because it yields both a higher surplus and requires a smaller rent to be paid to the buyer.

We conclude that the static menu \((\tilde{F}, \tilde{R})\) yields the seller a (weakly) larger profit than any feasible menu \((F, R)\). As a result, a static menu consisting of a single option contract with a zero up–front fee must be optimal. Since the seller’s profit from such a menu is \((1 - \bar{G}(R))(R - c)\), the optimal menu exhibits \(F_i = 0\) and \(R_i = \bar{R}\) as given by (3). This establishes Proposition 1.

### 3.2 Effects of withdrawal rights

To better understand the role of withdrawal rights, we next compare our optimal menu of option contracts to the optimal menu when the buyer does not have withdrawal rights. This will also allow us to discuss the possible welfare effects of the EU withdrawal rights regulation outlined in the introduction.

#### 3.2.1 Optimal option contracts without withdrawal rights

When the buyer does not have a withdrawal right, the seller has to respect only ex ante individual rationality constraints, which in terms of option contracts becomes

\[
U_{ii} = -F_i + \int_{R_i}^{1} \theta - R_i \ dG_i(\theta) \geq 0. \quad (IR_{2a})
\]

Courty and Li (2000) study the problem without withdrawal rights for the case with a continuum of ex ante types and identify natural conditions so that the seller’s problem can be solved by the “local Mirrlees” approach. Translated into our setting with discrete ex ante types, this means that the optimal menu of option contracts obtains from solving a relaxed problem where only the (ex ante) individual rationality constraint for the type \(i = n\), and the local incentive constraints \(IC_{i,i+1}\) are considered. One of the identified conditions is that the distributions \(G_i\) are ordered in the sense of first order stochastic dominance. In this case, the solution to the relaxed problem represents also a solution
to the original problem if the obtained exercise prices are monotonically increasing in the buyer’s ex ante type $i$.

Applying the local Mirrlees approach to our setup yields exercise prices $R_{i}^{xa}$ for buyer types $i$ that are implicitly given by the equations

$$R_{i}^{xa} - c \equiv 0, \quad \text{and} \quad R_{i}^{xa} - c \equiv h_{i}^{xa}(R_{i}^{xa}) \quad \forall i = 2, \ldots, n,$$  

(4)

where

$$h_{i}^{xa}(\theta) \equiv \frac{p_{1} + \ldots + p_{i-1}}{p_{i}} \cdot \frac{G_{i}(\theta) - G_{i-1}(\theta)}{g_{i}(\theta)}$$

(5)

is a modified hazard rate that measures the degree of the price distortion due to asymmetric information.\footnote{Courty and Li (2000) present a continuous version of this modified hazard rate, while Dai et al. (2006) present it for the case with two ex ante types. Baron and Besanko (1984) were the first to interpret the second factor as an informativeness measure of the ex ante information. Pavan et al. (2014) refer to this measure as an impulse response function and show that it plays a crucial role for dynamic settings in general.}

A sufficient condition that ensures the existence and uniqueness of a solution to (4) is that $h_{i}^{xa}(\theta)$ is non-negative (which obtains when $G_{i-1}$ dominates $G_{i}$ in the sense of first order stochastic dominance) and concave in $\theta$. Hence, the remaining question is under which conditions the exercise prices $R_{i}^{xa}$ are increasing in $i$. A sufficient condition to obtain this ordering is that $h_{i}^{xa}(\theta)$ is increasing in $i$.

Given the exercise prices, the optimal up-front fees are then pinned down by the binding individual rationality constraints for type $n$,

$$F_{n}^{xa} \equiv \int_{R_{n}^{xa}}^{1} \theta - R_{n}^{xa} dG_{n}(\theta),$$

(6)

and by the binding incentive constraints $IC_{i,i+1}$ for the other types $i < n$:

$$F_{i}^{xa} \equiv F_{i+1}^{xa} + \int_{R_{i}^{xa}}^{1} \theta - R_{i}^{xa} dG_{i}(\theta) - \int_{R_{i+1}^{xa}}^{1} \theta - R_{i+1}^{xa} dG_{i}(\theta).$$

(7)

We summarize these considerations in the next lemma which is a restatement of the result of Courty and Li (2000) with discrete ex ante types.
Lemma 1 (Courty and Li (2000)) Suppose $G_{i-1}$ dominates $G_i$ in the sense of first order stochastic dominance for all $i = 2, \ldots, n$, that $h_i^{xa}(\theta)$ is concave in $\theta$ and increasing in $i$. Then, if the seller has to respect only the ex ante individual rationality constraints ($IR_{i}^{xa}$), the optimal menu of option contracts is given by $(F_i^{xa}, R_i^{xa}) \equiv ((F_1^{xa}, R_1^{xa}), \ldots, (F_n^{xa}, R_n^{xa})).$

Hence, in contrast to the case with withdrawal rights, the optimal menu without withdrawal rights screens sequentially in that it offers different option contracts to different ex ante types. Moreover, it violates all ex post individual rationality constraints ($IR_{i}^{xp}$) because the fact that $R_i^{xa} < R_{i+1}^{xa}$ implies that:

$$0 < F_n^{xa} \leq \ldots \leq F_1^{xa}. \quad (8)$$

This ordering also reveals the intuition why, in the absence of withdrawal rights, offering the optimal static menu from Proposition 1 is not optimal. Observe that a reduction of the exercise price increases the buyer’s “ex post information rent” which amounts to his total utility net of the up-front payment. The reduction raises, moreover, the surplus as long as the price still remains above costs. Therefore, if all buyer types were offered the option contract $(0, \bar{R})$ from the optimal static menu, the seller could reduce the exercise price for type 1, thereby increasing his ex post information rent, and at the same time impose an appropriate up-front fee that exactly extracts type 1’s gain in ex post information rent. Under first order stochastic dominance, such a modification is incentive compatible because any other type is less optimistic about his valuation than type 1 so that such a type’s gain in ex post information rent in response to a price decrease is smaller than type 1’s.

Conversely, one may ask why, with withdrawal rights, it is not optimal to screen sequentially. First note that the seller can, in principle, induce the same buying behavior as under the sequential menu $(F_i^{xa}, R_i^{xa})$, but to satisfy ex post individual rationality, this requires her to decrease all up-front fees $F_i$ by the fixed amount $F_1^{xa}$. Therefore, it is feasible to sequentially screen the buyer also in the presence of withdrawal rights, but as we have shown, it is not optimal to do so. In this sense, Proposition 1 is an optimality result rather than an implementation result. The reason why sequential screening is not optimal is implicit in the previous paragraph. Withdrawal rights prevent the seller from using the up-front fee to extract the additional surplus created by sequential screening.
3.3 Bonds and differences in outside options

In the analysis so far, we assumed that, by withdrawing from the contract, the buyer can obtain his outside option of zero and therefore avoid any losses ex post. In other words, the seller cannot require the buyer to post a non–refundable bond when the contract is signed. As we have argued above, this captures consumer withdrawal rights that fall under the EU regulation of internet sales. In other economic applications, withdrawal rights may be less stringent however. In this subsection, we therefore examine the limits of our results by studying environments in which the agent has an ex post withdrawal right but can post positive bonds ex ante. We first show that such environments are equivalent to a setting in which the agent’s ex ante outside option exceeds his ex post outside option. Subsequently, we show that our main result is restrictive to the extent that it requires the maximum bond the agent can post to be below a certain, strictly positive bound. This clearly limits the universality of our results, but as argued in the introduction, in some important economic applications such bounds exist for legal reasons or because the agent is cash–constrained.

We start by assuming that the buyer has a (normalized) ex ante outside option of zero and an ex post outside option equal to $-B < 0$. Thus, a menu of option contracts is ex post individually rational if and only if $V_i(\theta) \geq -B$ for all $i, \theta$, which by (1) is equivalent to

$$F_i \leq B \quad \text{for all } i \in I.$$  (9)

In contrast, the ex ante individual rationality constraint ($IR_{xa}^i$) remains unaffected.

Alternatively, we can interpret the constraint (9) as representing a situation in which the buyer does have an ex post outside option of zero, but in period 1 the seller can demand an up–front payment up to the amount $B$, which she retains when the buyer withdraws in period 2. Effectively, it is as if the buyer pays a non–refundable bond $F_i$ in period 1 and decides in period 2 whether to consume at the exercise price or not, knowing that the payment $F_i$ is sunk.

We now argue that our result that the static contract is optimal still holds when $B$ is strictly positive but not too large:
Proposition 2 Let

\[ B^{xp} \equiv \min_i \int_{\bar{R}}^{1} 1 - G_i(\theta) \, d\theta. \]

If the maximal bond \( B \) is smaller than \( B^{xp} \), or, equivalently, if the buyer’s ex post outside option is larger than \(-B^{xp}\), then the static menu \((F^{xp}, R^{xp})\) with \((F_i^{xp}, R_i^{xp}) = (B, \bar{R})\) for all \( i \in I \) is optimal.

Because \( B^{xp} > 0 \), our result that sequential screening is not beneficial with ex post individual rationality constraints is robust.\(^{16}\) It extends to cases in which posting a limited bond is possible, or in which the seller’s ex post outside option is not too small.

To see Proposition 2, note first that if we continue to disregard the ex ante individual rationality constraint \((IR_{xa}^{i})\) and solve problem \(P^o\) but with the adapted ex post individual rationality constraint \((9)\) instead of \((IR_{xp}^{i})\), then the arguments leading to Proposition 1 imply that the solution corresponds again to a static menu with the single price \( \bar{R} \), but now with the up–front fee \( F_i = B \). For this solution, it follows that the ex ante utility of type \( i \) is

\[ U_{ii} = -B + \int_{\bar{R}}^{1} 1 - G_i(\theta) \, d\theta. \]

Hence, for \( B \leq B^{xp} \), the solution satisfies automatically the ex ante individual rationality constraint \((IR_{xa}^{i})\) for any \( i \), implying Proposition 2.

Taking the opposite approach and solving the model with the ex ante individual rationality constraint \((IR_{xa}^{i})\) while disregarding the ex post individual rationality constraint \((9)\) yields the solution of Lemma 1 (under the appropriate distributional assumptions of the lemma). Recall from (8) that the ex ante type 1 pays the largest up–front fee, and with (6) and (7), we obtain

\[ F_{1}^{xa} = \sum_{i=1}^{n} \int_{\theta_{i}^{xa}}^{\theta_{i+1}^{xa}} 1 - G_i(\theta) \, d\theta \equiv B^{xa}, \]

where \( \theta_{n+1}^{xa} \equiv 1 \). Since \( F_{1}^{xa} \geq F_{i}^{xa} \) for all \( i \in I \), the solution satisfies the neglected ex post individual rationality constraint \((9)\) whenever \( B \geq B^{xa} \).

It follows that as we vary the maximal bond \( B \), we obtain the sequential screening models with ex ante and ex post individual rationality constraints as two extremes: the model with ex ante constraints for \( B \geq B^{xa} \) and the model with ex post constraints for \( B \leq B^{xp} \).

\(^{16}\)Note that the bound \( B^{xp} \) does not converge to zero as the number of ex ante types \( n \) increases.
3.4 Costly withdrawal

Until now we abstracted from any costs of withdrawal. In practice, however, withdrawal from a contract may involve some costs. For instance, returning retail goods involves transportation costs, and the EU directive allows these costs to be borne by the buyer.\(^\text{17}\) In this subsection, we show that introducing withdrawal costs has a similar effect as introducing differences between the buyer’s ex ante and ex post outside option as discussed in the previous subsection. In particular, the optimal menu remains static if the costs of withdrawal are small relative to the expected surplus generated under the optimal static menu.

More specifically, suppose that the buyer incurs some cost \(k \geq 0\) when he returns the good to the seller. (In general, \(k\) may be a fixed cost of quitting the relation.) With return costs, there are now three options concerning the good’s allocation, each leading to a different aggregate surplus:

1. The good is sent to the buyer with some ex post valuation \(\theta\), who keeps it and thereby generates the aggregate surplus \(\theta - c\).
2. The good is sent to the buyer, but he returns it and thereby generates the surplus \(-k\).
3. The good is not sent to the buyer at all, which generates a surplus of 0.

We start by deriving the optimal option menu under the assumption that the seller always sends the good to the buyer for inspection. Let

\[
\bar{R}^k = \arg \max_R (1 - \bar{G}(R - k))(R - c),
\]

and

\[
K = \min_{i \in I} \int_{R^k - k}^{1} 1 - G_i(\theta) \, d\theta.
\]

The next lemma states that if return costs are smaller than \(K\), then it is optimal for the seller to simply offer the good at the price \(\bar{R}^k\):

**Lemma 2** Suppose return costs \(k\) are smaller than \(K\), and that it is optimal for the seller to send the good to all ex ante buyer types. Then an optimal menu of option contracts consists of a single contract only: \((F_i, R_i) = (0, \bar{R}^k)\) for all \(i \in I\).

\(^{17}\)Article 6.1(i) of the directive states "the consumer will have to bear the cost of returning the goods in case of withdrawal".

17
To see the result, consider a buyer, who after learning his ex post type $\theta$ contemplates exercising his option to buy the good. If he decides not to exercise his option, he now has to incur the return cost $k$. Hence, under an option contract $(F_j, R_j)$, buyer type $i$ keeps the good if $\theta \geq R_j - k$, implying the ex post utility

$$V^k_i(\theta) = \begin{cases} -F_j + (\theta - R_j) & \text{if } \theta \geq R_j - k \\ -F_j - k & \text{otherwise,} \end{cases}$$

and the ex ante utility

$$U^k_{ji} = -F_j + \int_{R_j - k}^{1} \theta - R_j \ dG_i(\theta) - G_i(R_j - k)k.$$ 

Because the buyer incurs the return cost $k$ when returning the good, the ex post individual rationality constraints now only guarantee that the buyer’s ex post utility does not fall below $-k$: $V^k_i(\theta) \geq -k$ for all $i, \theta$. As before, this is equivalent to $F_i \leq 0$ for all $i$. Hence, the constraint $(IR^{xp}_i)$ remains unchanged. Moreover, the definitions of incentive compatibility $(IC_{ij})$ and ex ante individual rationality $(IR^{xa}_i)$ also remain the same. However, in contrast to the model without return costs, ex post individual rationality does no longer imply ex ante individual rationality because the buyer may end up with the negative utility $-k$ associated with returning the good ex post. Hence, as in the previous subsection, return costs create a wedge between the ex post and ex ante individual rationality constraints. For this reason, also with costly returns, we have to consider explicitly the ex ante individual rationality constraint $(IR^{xa}_i)$.

Given a buyer type $i$, an incentive compatible menu $(F, R)$ generates the surplus

$$S^k_i = \int_{R_i - k}^{1} \theta - c \ dG_i(\theta) - G_i(R_i - k)k$$

so that the seller’s conditional expected payoff from a buyer type $i$ is $W^k_i = S^k_i - U^k_{ii}$.

Consequently, for the case that the good is always sent to the buyer, we obtain the optimal menu of option contracts with return costs as a solution to problem $P^o$ but with the adjusted payoff functions $V^k_i$, $U^k_{ji}$, $W^k_i$ and the explicit inclusion of the additional constraint $(IR^{xa}_i)$. Yet, if we ignore $(IR^{xa}_i)$, then Proposition 1 directly implies that the solution is given by the static menu $(F_i, R_i) = (0, \bar{R}^k)$. We now show that if $k \leq K$, then this solution automatically satisfies $(IR^{xa}_i)$ so that the static menu is indeed also optimal with return costs.
To see this observe that the option contract \((F_i, R_i) = (0, \bar{R}^k)\) yields buyer type \(i\) a utility of

\[
U_{ii} = \int_{\bar{R}^k - k}^{1} \theta - \bar{R}^k dG_i(\theta) - G_i(\bar{R}^k - k)k = \int_{\bar{R}^k - k}^{1} 1 - G_i(\theta) d\theta - k,
\]

where the second equality follows from integration by parts. Hence, if \(k \leq K\), then \(U_{ii} \geq 0\) so that the static menu is ex ante individually rational. This establishes Lemma 2.

Lemma 2 derives the optimal contract under the assumption that the seller sends the good to the buyer for each ex ante type. Because returning the good is costly, the seller may, however, find it suboptimal to send the good to ex ante types who are likely to return the good. Instead, she may prefer to “screen ex ante types by participation” and not send the good to all ex ante types.

To study this possibility, we introduce the following notation. Given a subset \(J \subseteq I\) of ex ante buyer types, denote by \(\bar{G}_J \equiv \sum_{j \in J} p_j G_j\) the average distribution over types in \(J\). Moreover, let

\[
\bar{R}_J = \arg \max_R (1 - \bar{G}_J(R - k))(R - c).
\]

We now state the problem of the seller who wants to send the good only to ex ante types \(j\) in some set \(J \subseteq I\). In this case, the seller needs to induce the types \(i \in I \setminus J\) not to participate. Hence, the seller must ensure that these types do not obtain a positive utility from choosing the contract \((F_j, R_j)\) of some ex ante type \(j \in J\) who does receive the good. This yields the additional “screening by participation” constraint

\[
U_{ji}^k \leq 0 \quad \text{for all } i \in I \setminus J, j \in J.
\]  

An optimal menu of option contracts under which the good is sent only to the ex ante types in \(J\) is a solution to the following program:

\[
P^J: \max_{(F, R)} \sum_{j \in J} p_j W_{i_j}^k \quad \text{s.t.} \quad (IC_{ij}), (IR_{it}^{zp}), (IR_{it}^{ra}), (IC_J).
\]

We finally show that when costs satisfy an analogous condition as the one in Lemma 2, then an optimal menu of option contracts that screens by participation is still static in the sense that it does not screen between the ex ante types who do receive the good.
Proposition 3 Let

\[ K^J \equiv \min_{j \in J^*} \int_{R_j, -k}^1 1 - G_j(\theta) \, d\theta. \]

If return costs \( k \) are smaller than \( K^J \), and it is optimal for the seller to send the good to ex ante types in \( J^* \), then the static menu \((F^k, R^k)\) with \((F^k_j, R^k_j) = (0, \bar{R}_{J^*})\) for all \( j \in J^* \) is optimal.

4 Normative perspectives

Our analysis so far has focused on the implications of withdrawal rights for optimal dynamic contracting. Because such withdrawal rights are mandatory for distance sales contracts in the EU, this section complements our positive analysis and examines the normative aspects of withdrawal rights in the context of retailing. We first ask whether withdrawal rights serve the maintained goals of EU legislators and, second, whether, in our model, their mandatory character is justifiable from a welfare economic perspective. We conclude with a largely informal discussion on how the presence of agency problems on the seller side and competition would affect the welfare effects of withdrawal rights.

4.1 Legislator’s goals of withdrawal rights

From its conception in the early 1990s, the regulation of distance sales contracts was guided by the principle to “safeguard the consumer’s right of choice” (Commission of the European Communities, 1992, p.308). At a time where internet sales were still at their infancy, regulators were concerned that distance sales contracts weaken the consumer’s informational position. We cite further from this initial proposal: “The basic principle is that the use of new technologies must not lead to a reduction in the information provided”.

In their taxonomy of withdrawal rights, Kalls and Lurger (p.156ff, 1998) deduce from this basic principle a legal “right of withdrawal due to absence” (“Abwesenheitsrücktrittsrecht”). Because the good is physically absent when the consumer signs a distance sales contract, he has less information than in a traditional shop. Kalls and Lurger argue that

\[ \text{The European Commission released a first proposal for a definition of “distance sales” and the requirement of a withdrawal right in 1992. An amended proposal entered into EU law through the directive 97/7/EC in 1997. The directive 2011/83/EU updates and replaces this earlier directive.} \]
a withdrawal right rectifies this lack of information. In line with this idea, Section 47 of
the EU directive says: “In order to establish the nature, characteristics and functioning
of the goods, the consumer should only handle and inspect them in the same manner as
he would be allowed to do in a shop. For example, the consumer should only try on a
garment and should not be allowed to wear it.”

In the spirit of the maxim that “the consumer should only try on a garment”, our
model captures the consumer’s lack of information as uncertainty about the subjective
product fit. Our result that a withdrawal right leads to an outcome as if the good were
not absent therefore suggests that the regulation achieves its underlying goal to close the
buyer’s informational gap in distance sales.

Another, often articulated rationale for mandatory withdrawal rights is that in pro-
viding a cooling–off period, they protect consumers from the influence of aggressive sales
tactics or buyer remorse. This rationale seems however not to have played a major role
with respect to the regulation of distance sales. Even though the EU regulation does
refer to “surprise element and/or psychological pressure”, it only does so in the explicit
context of off-premises (doorstep) sales and not for distance sales contracts. The minutes
of a hearing concerned with harmonizing the regulation of doorstep selling and distance
sales contracts confirm this view: “situations in distant and direct selling are said to be
completely different: In the Doorstep Selling Directive, the aspect of surprise plays an
important role as it is usually the direct seller who initiates the business contact. How-
ever, in the Distance Selling Directive, the lack of information which is caused by the
distance between customer and seller is addressed” (EC2000, p.3). Along the same lines,
Loos (2009) argues that psychological considerations do not justify withdrawal rights for

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19 This is consistent with findings from case studies that 70% of returns are due to a poor fit to the
consumer’s needs rather than defects (see Posselt et al., 2008). Similarly, Heiman et al. (2002) argue that
the main purpose of withdrawal rights, in contrast to performance warranties, is to protect the consumer
when the product does not fit her personal needs.

20 We stress that the view that withdrawal rights should be granted to improve consumer information
is not universally accepted among legal scholars, because withdrawal rights violate the principle “pacta

21 For a Law reference, see Kalls and Lurger (1998). In Economics, projection bias (Loewenstein et al.,
2003) or strategic naivety (Inderst and Ottaviani, 2013) have been shown to provide a justification for
mandatory minimum withdrawal rights.
distance sales, because consumers conduct such transactions at their own initiative and without time pressure.

4.2 Welfare effects of withdrawal rights

The previous considerations suggest that the desire to safeguard the consumer’s informational position as compared to non-distance sales was indeed a main driver behind the EU regulation. While a withdrawal right improves the consumer’s information, the main question from a welfare economic point of view is how it affects welfare and consumer rents. In this subsection, we study the welfare implications of introducing withdrawal rights in our model. We will show that the welfare effects are, in general, ambiguous, and that, in fact, the withdrawal rights regulation may reduce welfare and hurt consumers.

We compare the parties’ utilities and aggregate surplus under the optimal contracts with and without withdrawal rights. We begin with the straightforward observation that the seller is (weakly) worse off when withdrawal rights are introduced. This follows simply from the fact that with withdrawal rights she faces more constraints. Even though straightforward, this observation clarifies that, in a sequential screening setup, the seller has no incentive to offer a withdrawal right voluntarily.

In contrast, the effect on the aggregate surplus and on the buyer’s expected utility is ambiguous. Both with and without withdrawal rights, exercise prices are inefficiently distorted away from marginal costs and the overall welfare effect depends on the magnitude of these distortions. To see this more formally, the difference in aggregate surplus conditional on an ex ante type $i$ is

$$\Delta_i = \int_{\hat{R}}^{1} \theta - c d G_i(\theta) - \int_{R_{xa}^i}^{1} \theta - c d G_i(\theta) = \int_{\hat{R}}^{R_{xa}^i} \theta - c d G_i(\theta)$$

so that the regulation changes the aggregate surplus by $\Delta = \sum_i p_i \Delta_i$. The sign of $\Delta_i$ depends on the ordering of $R_{xa}^i$ and $\hat{R}$. Only for type 1 this ordering is unambiguous, since $R_{xa}^1 = c < \hat{R}$. But for $i > 1$, it depends on the details of the model whether $R_{xa}^i$ is smaller or larger than $\hat{R}$.  

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22A further motivation for the new directive was harmonizing national regulation so as to reduce legal uncertainty in cross-border transactions. On a related note, Loos (2009) views the directive as a (questionable) attempt to foster cross-border trade. Moreover, Ben–Shahar and Posner (2011) speculate that the regulation may have been influenced by lobbying, as it raises entry costs.
Figure 1: Dead weight loss and welfare effects of withdrawal rights with two types.

The left panel in Figure 1 illustrates the welfare effects with two ex ante types. We may interpret the downward sloping curve \( R_i(q) = G_i^{-1}(1 - q) \) as a usual (inverse) demand function where \( q \) denotes the ex ante probability of trade under \( G_i \). When \( G_1 \) first order stochastically dominates \( G_2 \), the curve \( R_1(q) \) lies above the curve \( R_2(q) \). Conditional on type 1, withdrawal rights cause the deadweight loss given by area \( |\Delta_1| \) due to the price increase from \( R_1 = c \) to \( \bar{R} > c \). The graph depicts the case in which \( R_2 > \bar{R} \) so that, conditional on ex ante type 2, withdrawal rights induce a welfare gain of \( \Delta_2 \). The regulation is welfare enhancing whenever \( p_2 \Delta_2 \geq p_1 |\Delta_1| \).

The welfare comparison is clear-cut for the extreme case that there is no private ex ante information. Absent ex ante private information, it is well-known that without withdrawal rights, the seller can extract all gains of trade, despite the buyer’s ex post private information (see Harris and Raviv, 1978). Consequently, allocations are efficient. In contrast, Sappington (1983) shows that if the seller has to respect the ex post individual rationality constraints implied by withdrawal rights, then full rent extraction is not possible. As a result, allocations are distorted. Hence, when there is little ex ante private information, withdrawal rights are welfare reducing.

To shed light on less extreme cases, the right panel in Figure 1 illustrates the change in welfare (\( \Delta \)), profits (\( \Delta W \)), and buyer rents (\( \Delta U \)) for the specification

\[
p_1 = p_2 = 1/2, \quad c = 1/4, \quad G_2(\theta) = \theta, \quad G_1(\theta) = \theta^{1+\alpha}, \quad \alpha \geq 0.
\]
For $\alpha = 0$, we have $G_1 = G_2$ so that there is no relevant ex ante private information. Hence, the previous paragraph explains why the graph starts with $\Delta$ and $\Delta W$ negative and $\Delta U$ positive. For $\alpha \approx \infty$, ex ante type 1 is virtually ensured to have valuation $\theta = 1$ so that for any price $\bar{R} < 1$ he always buys. As a result, withdrawal rights do not affect the surplus from type 1 too negatively ($\Delta_1 \approx 0$). In this case, the buyer’s gain from the regulation outweighs the seller’s loss, and it is therefore socially beneficial ($\Delta > 0$).

In conclusion, our analysis demonstrates that already with two ex ante types the welfare effects of a mandatory withdrawal right are ambiguous. In addition to hurting firms, they may also hurt consumers. The underlying reason is that, even though withdrawal rights limit the seller’s ability to price discriminate, they may induce the seller to opt for larger economic distortions, resulting in a smaller overall surplus. Hence, while our analysis confirms that withdrawal rights accomplish the regulatory goal to close the buyer’s informational gap in distance sales, this goal may be questionable from a welfare point of view.

4.3 Alternative agency problems and the role of market power

Our model identifies the buyer’s private information about product fit as the key source of inefficiency. While this is in line with the aforementioned empirical observations of Posselt et al. (2008) and Heiman et al. (2002) (see footnote 19), another potential source of inefficiency in online retail is that the seller may have private information about her quality, giving rise to moral hazard or adverse selection problems. For example, a seller may exaggerate product descriptions, delay delivery, or try to sell refurbished goods as brand new ones. We now explore to what extent concerns about agency problems on the seller side may provide additional justifications for mandating withdrawal rights.

To incorporate elements of moral hazard, suppose that the buyer’s valuation for the good is $q\theta$, where $q \geq 0$ is the quality level chosen by the seller at some cost $k(q)$. Consider first the case that $q$ is observable ex ante before the buyer accepts the contract, for example, due to the availability of online consumer reviews about past purchases.\footnote{We focus on the case that $q$ is not contractible because otherwise no moral hazard problem arises since $q$ can be made part of the contract. In a related model with observable quality and a similar payoff structure, Inderst and Tirosh (2012) consider competition between a high and low quality seller and show that the high (low) quality seller offers an inefficiently generous (restrictive) returns policy, and argue...}
a given quality level $q$, the seller’s maximum (gross) profits with and without mandatory withdrawal rights, $W_w(q)$ and $W_n(q)$, can be computed as in our former analysis where quality was exogenously normalized to $q = 1$. Whether the seller chooses a higher quality level with or without mandatory withdrawal rights is determined by how marginal profits, $W_w'(q)$ and $W_n'(q)$, compare to one another. It is tedious but conceptually straightforward to demonstrate that the comparison depends on the model details and is, in general, not clear-cut. In particular, the comparison of welfare and consumer rents remains ambiguous for the same reasons as in the previous section: the trade-off between the pricing distortions imposed by the seller with and without mandatory withdrawal rights is non-trivial and can go either way. Therefore, when quality is observable, it is in general not clear whether mandatory withdrawal rights lead to a more or less efficient quality choice by the seller.

Consider next the case that the seller’s quality is *ex ante unobservable* but becomes known to the buyer only *ex post* after delivery. Intuitively, this could severely restrict trade because a rational buyer anticipates that, due to his inability to verify quality up front, the seller may be tempted to deliver low quality. A mandatory withdrawal right will mitigate this problem because the threat of the buyer returning a low quality product makes it credible that the seller chooses appropriate quality. However, when the seller is free to offer any contract, she may have sufficient private incentives to offer return policies similar to the mandatory one in order to commit herself credibly to high quality.\footnote{For a formalization of this argument in a setting without ex ante private information, see Bester and Krähmer (2012).} Therefore, it is a priori again not clear whether moral hazard concerns that arise from unobservable quality strengthen or weaken the case for mandatory withdrawal rights.

Next to moral hazard, the buyer’s inability to examine quality at the time of sale may also give rise to adverse selection, as it may attract low quality sellers to online selling platforms. Because mandatory withdrawal rights are likely to deter low quality sellers to enter, they may mitigate adverse selection problems by improving the average quality in the market.\footnote{A similar argument has been made to advocate the imposition of minimum quality standards in adverse selection markets (see Leland, 1979). In this sense, withdrawal rights could be broadly interpreted as part of a minimum quality.} On the other hand, the related literature on performance warranties has ambiguous effects.\footnote{On the other hand, the related literature on performance warranties has ambiguous effects.}
suggests that a privately informed seller has incentives to offer return rights voluntarily to signal high quality.\textsuperscript{26} In particular, Grossman (1981) shows that these private incentives may be sufficient to even overcome the adverse selection problem.\textsuperscript{27} Hence, assessing the welfare implications of mandatory withdrawal rights requires to understand how they interact with the seller’s private signaling incentives. In so far as signaling efforts are often wasteful and justify the regulation of contracting practices (Aghion and Hermalin, 1990), regulating withdrawal rights could prevent excessive signaling. As Ben-Shahar and Posner (2011) suggest, this may then call for maximum rather than minimum withdrawal rights or even their ban.\textsuperscript{28} Properly assessing how this multitude of effects interact with one another ultimately requires a mechanism design approach with an informed principal.

Finally, we point out that withdrawal rights may not only mitigate agency problems, but can also create them. Griffis et al. (2012) for example document that nearly 20\% of all consumers may be buying products with the specific intention of returning them after satisfactory use. This problem of “retail borrowing” or “de–shopping” is an expression of moral hazard on the buyer’s side which becomes clearly more prevalent the more generous return rights are.

To conclude our discussion on the normative aspects of the regulation, we discuss the role of our assumption that the seller is a monopolist. Although we motivated this assumption in footnote 2 with the intention to capture market power, it of course does so in an extreme form. At the other extreme of perfect competition, the regulation would indeed be superfluous. Under perfect competition, sellers would compete to an efficient contract that leaves all rents to the consumer. In terms of option contracts, this implies that they

\textsuperscript{26}This may be one reason why many online retailers offer return rights that go beyond the legally mandated minimum.

\textsuperscript{27}A vast literature studies the role of performance warranties as quality signals. (Next to Grossman, 1981, see Lutz, 1989, Mann and Wissink, 1990, Shieh, 1996. For a survey see Emons, 1989.) To our knowledge, only Moorthy and Srinivisaan (1995) analyze the role of an \textit{unconditional} return right as a quality signal. While the signaling logic is similar, the economic difference between return rights and warranties is that a warranty becomes effective only when product failure can be verified while return rights can be exercised at will, even if nothing is wrong with the good.

\textsuperscript{28}Another reason for constraining the generosity of refunds is indirectly suggested by Matthews and Persico (2007) who argue that sellers may offer withdrawal rights voluntarily to dissuade buyers from acquiring information ex ante, and this may lead to excessive refunds that generate an inefficiently large number of returns.
would offer a single option contract with an up-front fee of zero and an exercise price equal to marginal cost $c$. Because this option contract is equivalent to a sales contract with a price $p = c$ and a full withdrawal right for the consumer, perfect competition effectively leads sellers to offer withdrawal rights voluntarily, rendering regulation superfluous.

In contrast, our analysis showed that a monopolist would not offer a withdrawal right voluntarily. We strongly conjecture that this will also be the case for imperfect competition when sellers still have some market power to engage in intertemporal price discrimination. Moreover, based on our analysis the welfare effects of withdrawal rights seem similar for less extreme forms of market power. Market power, by definition, means that prices are inefficiently distorted away from marginal costs, and our analysis shows that the overall welfare effect of mandating withdrawal rights depends on how they impact the relative rather than the absolute magnitude of these distortions. Hence, we expect similar trade-offs as in the monopoly case to drive the welfare effects under weaker forms of imperfect competition. Indeed, since we see no straightforward argument for why market power systematically affects the relative size of the distortions with, relative to without, the regulation, we do not expect a monotone relationship between the welfare effect of the regulation and the degree of market power.

5 General contracts

In Section 3, we showed that it is suboptimal for the seller to elicit the buyer’s ex ante information when she offers a menu of option contracts. In this section, we show that under appropriate regularity conditions this result remains true when we allow the seller to choose an arbitrary contract. This section will also show that the setting with menus of option contracts considered in the previous section restricts generality only to the extent that it rules out stochastic trading rules.

To find an optimal contract in the class of all contracts, we apply the revelation principle for sequential games (e.g., Myerson 1986), which states that the optimal contract

\footnote{A full formal analysis of this point is beyond the scope of the current paper. We are unaware of a fully-fledged analysis of sequential screening in an oligopolistic framework even without ex post participation constraints. Such an analysis is non-trivial, because competition transforms the contracting problem into a model of common agency where different sellers compete in contracts for the consumer.}
can be found in the class of direct and incentive compatible contracts which, on the equilibrium path, induce the buyer to report his type truthfully. Formally, a direct contract
\[(x, t) = (x_j(\theta'), t_j(\theta'))_{j \in I, \theta' \in [0, 1]}\]
requires the buyer to report an ex ante type \(j\) in period 1, and an ex post type \(\theta'\) in period 2. A contract commits the seller to a selling schedule \(x_j(\theta')\) and a transfer schedule \(t_j(\theta')\).

If the buyer’s true ex post type is \(\theta\) and his period 1 report was \(j\), then his utility from reporting \(\theta'\) in period 2 is
\[v_j(\theta'; \theta) \equiv \theta x_j(\theta') - t_j(\theta').\]
With slight abuse of notation, we denote the buyer’s period 2 utility from truth-telling by
\[v_j(\theta) \equiv v_j(\theta; \theta).\] (10)
The contract is incentive compatible in period 2 if it gives the buyer an incentive to announce his ex post type truthfully:
\[v_j(\theta) \geq v_j(\theta'; \theta) \quad \text{for all } j \in I, \theta, \theta' \in [0, 1].\] (11)
If the contract is incentive compatible in period 2, the buyer announces his ex post type truthfully no matter what his report in the first period.\(^{30}\) Hence, the contract induces the buyer to announce his ex ante type truthfully, and is thus incentive compatible in period 1 if
\[\int_0^1 v_i(\theta) \, dG_i(\theta) \geq \int_0^1 v_j(\theta) \, dG_j(\theta) \quad \text{for all } i, j \in I.\] (12)
When the buyer has a withdrawal right, then, after having observed his valuation \(\theta\), the buyer has the choice between continuing with the trade as specified in the contract, or withdrawing from it and obtaining his outside option of 0. Accordingly, with withdrawal rights, the contract needs to satisfy the ex post individual rationality constraints:\(^{31,32}\)
\[v_i(\theta) \geq 0 \quad \text{for all } i \in I, \theta \in [0, 1].\] (13)
\(^{30}\)Because the buyer’s period 2 utility is independent of his ex ante type, a contract which is incentive compatible in period 2 automatically induces truth-telling in period 2 also off the equilibrium path, that is, if the buyer has misreported his ex ante type in period 1.
\(^{31}\)Put differently, if the seller offered a contract for which the buyer would make an ex post loss for some \(\theta\), then the buyer would withdraw from the contract for such a \(\theta\), and effectively enforce the terms of trade \(x_i(\theta) = t_i(\theta) = 0\). For a formal derivation of this point we refer to the online appendix.
\(^{32}\)For simplicity, we consider in this section only the case that the ex post and ex ante outside options are the same. Clearly, ex post individual rationality then implies ex ante individual rationality. We can use the same arguments as in Section 3.3 to show that the result in this section is robust if the difference between ex ante and ex post outside option is not too large.
If the buyer’s ex ante type is $i$, the seller’s conditional expected payoff is the difference between aggregate surplus and the buyer’s utility:

$$w_i = \int_0^1 [\theta - c]x_i(\theta) - v_i(\theta) \, dG_i(\theta).$$  \hfill (14)

To state the seller’s problem, we proceed in a standard fashion and first eliminate transfers from the problem. As usual, incentive compatibility in the second period is equivalent to (i) monotonicity of the selling schedule,

$$x_i(\theta) \text{ is increasing in } \theta \text{ for all } i \in I; \quad (MONT_i)$$

and (ii) “revenue equivalence”, which means that the buyer’s utility is determined by the selling schedule up to his utility at the lowest valuation, $v_i(0)$. We can use “revenue equivalence” to eliminate transfers and obtain the seller’s problem as a choice problem over the selling schedule $x$ and the vector $v = (v_i(0))_{i \in I}$. Formally, the first period incentive constraints (12) and the ex post individual rationality constraints (13) can respectively be re-written as

$$\int_0^1 [x_i(\theta) - x_j(\theta)] [1 - G_i(\theta)] \, d\theta + v_i(0) - v_j(0) \geq 0, \quad (IC_{ij}^x)$$

$$v_i(0) \geq 0, \quad (IR_i)$$

and the seller’s objective becomes

$$w(x, v) = \sum_{i \in I} p_i \int_0^1 [\theta - c - h_i(\theta)] x_i(\theta) \, dG_i(\theta) - p_i v_i(0).$$

The following lemma summarizes.

**Lemma 3** The seller’s problem can be written as follows:

$$\mathcal{P} : \max_{x, v} w(x, v) \quad \text{s.t.} \quad (MONT_i), \ (IC_{ij}^x), \ (IR_i) \quad \text{for all } i, j \in I.$$

As our main result, we will establish conditions so that $\mathcal{P}$ exhibits a static solution that does not condition on the buyer’s ex ante type. More precisely, we will show that in this case a solution to $\mathcal{P}$ is given by the optimal static contract $(\bar{x}, \bar{v})$ which has $\bar{v}_i(0) = 0$ for all $i \in I$ and displays the selling schedule

$$\bar{x} \equiv (x_1(\theta), \ldots, x_n(\theta)), \quad x_i(\theta) \equiv \bar{x}(\theta) = 1_{[\bar{R}, 1]}(\theta) \quad \text{for all } i \in I,$$

where $1$ denotes the indicator function and $\bar{R}$ is given by (3).\(^{33}\)

\(^{33}\)Because a static contract is trivially incentive compatible in period 1, the optimal static contract
5.1 Deterministic contracts

We begin by deriving the solution to problem $\mathcal{P}$ for the case that the seller is restricted to choose a deterministic contract which exhibits selling schedules

$$x_i(\theta) \in \{0, 1\} \text{ for all } i \in I, \theta \in \Theta.$$ 

It turns out that the case with deterministic contracts is essentially equivalent to the case with option contracts considered in the previous section. To see this, we can use an insight of the sequential screening literature that an incentive compatible, deterministic contract can be indirectly implemented by a menu of option contract. The next lemma establishes this fact.

**Lemma 4** For any direct, incentive compatible, deterministic contract $(x, t)$, there is an equivalent incentive compatible menu $(F, R)$ which implements the same outcome as the direct contract and vice versa.

The equivalence is a direct consequence of the fact that incentive compatibility in period 2 entails the monotonicity constraint $(MON_i)$. For deterministic selling schedules, this implies that there exists a cutoff $R_i$ in the unit interval where the schedule $x_i$ jumps from 0 to 1. This cutoff corresponds to the exercise price of the option contract, and the utility of the lowest valuation type, taken negatively, $-v_i(0)$, corresponds to the up-front fee $F_i$.

Lemma 4 makes clear that the restriction to menus of option contracts in the previous section amounts precisely to ruling out non-deterministic trading probabilities. Moreover, in light of Lemma 4, Proposition 1 directly implies that in the class of direct, incentive compatible, deterministic contracts, the optimal static contracts is a solution to the principal’s problem.

**Proposition 4** If the seller can offer only deterministic contracts, then the optimal static contract $(\bar{x}, \bar{v})$ is a solution to $\mathcal{P}$.

$(\bar{x}, \bar{v})$ maximizes $w(x, v)$ subject to the $(MON_i)$, $(IC^*_i)$, $(IR_i)$ and the additional constraints $x_i = \bar{x}$ and $v_i(0) = \bar{v}(0)$ for all $i \in I$. This is a standard unit good screening problem, and it is well known from, for example, Riley and Zeckhauser (1983), that the seller’s optimal selling policy is to offer the good at a take-it-or-leave-it price. It is straightforward to see that the optimal take-it-or-leave-it price in our setting is given by $\bar{R}$ as defined in (3).
5.2 Stochastic contracts

We now turn to the case that the seller can also choose stochastic schedules

\[ x_i(\theta) \in [0,1]. \]

Our main question in this subsection is when the optimal contract, within the set of all contracts, is deterministic. In standard screening problems, the optimality of deterministic contracts is typically ensured by regularity conditions. It should therefore not be too surprising that also in our setting we need to impose additional distributional assumptions. In what follows, we first of all impose the usual smoothness assumptions that the probability density \( g_i(\theta) = G'_i(\theta) \) exists, is differentiable and strictly positive for all \( \theta \in [0,1] \). Our key regularity condition is:

**Condition R** The cross hazard rate between the types \( i \) and \( j \) and the hazard rate of type \( i \), defined as

\[ h_{i,j}(\theta) \equiv \frac{1 - G_i(\theta)}{g_j(\theta)}, \quad \text{and} \quad h_i(\theta) \equiv h_{i,i}(\theta), \]

are decreasing in \( \theta \) for all \( i, j \).

We can now state the main result of this section.

**Theorem 1** If condition R holds, the seller’s problem \( \mathcal{P} \) has a deterministic solution. Moreover, the optimal selling schedule is given by \( \bar{x} \).

The second part of the theorem simply re-states Propositions 1 and 4 that the optimal deterministic contract corresponds to the optimal static contract and does not depend on the ex ante type. Therefore, the interesting question is why the optimal deterministic contract is indeed a solution to \( \mathcal{P} \).

\(^{34}\) As can be seen from (5), cross-hazard rates are an essential part of the modified hazard rate, informativeness measure, or impulse response function and, hence, play a prominent role in the literature on dynamic mechanism design. We are however not aware that their role has been noted before.

\(^{35}\) Because \( h_{ij}(\theta) > 0 \) for all \( \theta < 1 \) and \( h_{ij}(1) = 0 \), a cross hazard rate is always decreasing close to \( \theta = 1 \). Condition R, therefore, requires it to be decreasing on the entire interval \( \theta \in [0,1] \). A sufficient condition to obtain Condition R is that densities \( g_i \) are increasing, or, equivalently, that the cumulative distributions \( G_i \) are convex. Condition R is therefore satisfied for large families of distributions. A concrete example is \( G_i(\theta) = \theta^{a_i} \) with \( 1 \geq a_1 > \ldots > a_n \).
In standard screening problems, the optimality of deterministic contracts obtains when the so-called “local Mirlees” or “first order” approach is valid (see Strausz 2006). This approach considers a relaxed problem where the monotonicity constraints are neglected and only the local incentive compatibility constraints are imposed while all global incentive constraints are ignored. Regularity conditions then guarantee that the solution to the relaxed problem is a solution to the original problem and, moreover, that the solution is deterministic. Even though our regularity condition R displays some similarity to those of standard screening problems, the first order approach turns out to fail in our setting.\footnote{Battaglini and Lamba (2014) argue that the first order approach “often” fails in dynamic screening problems.} The reason is that the solution to the relaxed problem associated with the first order approach cannot be shown to be monotone in our setting.

We now sketch our alternative approach which identifies a set of (global) incentive constraints, different from the local ones, which does allow us to verify that the optimal static contract solves the principal’s problem.\footnote{In our working paper versions (Krähmer and Strausz 2011b, 2014) we present a constructive but lengthy procedure by which to identify the exact relevant constraints. Here we provide instead a much shorter albeit indirect proof.} To define the set of relevant incentive constraints, for each $i$, let $\theta_i$ be implicitly given by\footnote{Because, by condition R, the hazard rate is decreasing, $h_i(1) = 0$, and $c \in [0, 1)$, $\theta_i$ is unique, exists, and lies in between $c$ and 1.}

$$\theta_i = c + h_i(\theta_i).$$

Observe that $\theta_i$ corresponds to the optimal monopoly price the seller would charge if he knew the buyer’s ex ante type is $i$. We now label the ex ante types according to the order of monopoly prices:

$$c < \theta_n \leq \ldots \leq \theta_i \leq \ldots \leq \theta_1 < 1.$$

Define by $C^*$ the set of incentive constraints so that no type $\theta_i$ above $\bar{R}$ has an incentive to mimic a type $\theta_j$ below $\bar{R}$:

$$C^* \equiv \{i \in I \mid \theta_i \geq \bar{R}\} \times \{j \in I \mid \theta_j < \bar{R}\}.$$
constraints that correspond to $C^*$, and moreover, we ignore the monotonicity constraints:

$$\mathcal{R} : \max_{x,v} w(x,v) \text{ s.t. } (IC_{ij}^v), (IR_k) \text{ for all } (i, j) \in C^*, k \in I.$$ 

We have:

**Proposition 5** Let condition $R$ hold. Then the optimal deterministic contract $(\bar{x}, \bar{v})$ is a solution to problem $\mathcal{R}$.

Because the optimal deterministic contract satisfies all neglected constraints, it is also a solution to the original problem $\mathcal{P}$, and this establishes Theorem 1.

To shed light on Proposition 5, consider the case with three types and the ordering $\theta_3 < \bar{R} < \theta_2 < \theta_1$. If ex ante types were publicly known, the seller would offer each type the good at price $\theta_i$ in period 2. With ex ante types being private information, there are two natural ways in which the seller could approximate the public information outcome, rather than offering the optimal static contract. First, the seller could pay type $i = 2$ and $i = 3$ appropriate up-front fees and allow them to buy the good at the exercise price $\theta_i$ corresponding to the public information price. While up-front fees can be constructed to make this incentive compatible, this would induce deterministic selling schedules, which, as we have seen in the previous subsection, is suboptimal.

More subtly, the seller could deviate from the optimal static contract by offering type $i = 2$ a stochastic option contract that allows him to choose in period 2 whether to get the good for sure at the public information price $\theta_2$, or to get it with some positive probability $\hat{x}$ smaller than 1 for a smaller price $\hat{\theta}$, where $\hat{x}$ and $\hat{\theta}$ are chosen to maintain the incentive constraint $IC_{21}$. Without assumptions on the distributions, it is well-known that in optimization problems in which the allocation has to satisfy certain integral constraints such as our incentive constraints, non-deterministic deviations of this sort can, in general, be profitable (see Samuelson (1984) or Manelli and Vincent (2007)). However, as we show in our proof, in the presence of our regularity condition $R$, such stochastic deviations are not profitable for the seller.

An example in which condition $R$ and our Theorem 1 indeed fails is presented in Heumann (2013) who considers a setup where the seller controls both the design of the contract and the sequential revelation of the buyer’s private information, and has to respect ex post participation constraints. This yields an optimal, multi-period information
structure in which the analog to our regularity condition \( R \) is violated and for which stochastic non-static contracts are optimal.\(^{39}\) Finally, we note that Theorem 1 may also fail when the seller, instead of a single unit, may sell an arbitrary quantity of the good and costs or benefits are non-linear in quantity.

We conclude this section by pointing out that our techniques and results extend readily to settings with multiple buyers. For the unit good auction model in which the buyers’ private information about their valuation arrives sequentially, Esö and Szentes (2007b) show that, when there are only ex ante individual rationality constraints, the optimal mechanism is a sequential auction where the winner’s price depends not only on the final bid but also on information provided by bidders in an initial round. In contrast, it follows from our result that the optimal mechanism with ex post individual rationality constraints is equivalent to the static Myerson (1981) auction that is optimal for the seller when he faces the buyers after they received all their private information. Hence, with ex post individual rationality constraints, the optimal mechanism is simpler, and the seller does not benefit from a sequential mechanism.

6 Conclusion

This paper shows that, in environments where an agent obtains private information dynamically, stringent ex post participation constraints eliminate the value of sequentially eliciting the agent’s information. Instead, a simple contract that conditions only on the agent’s final information is optimal.\(^{40}\)

Such stringent participation constraints arise when, due to limited resources or regulatory constraints, the principal’s ability to contractually demand up-front payments or impose penalties for quitting the relationship are limited. We focused on one such environment, the online retail market in Europe, where mandatory withdrawal rights lead to

\(^{39}\)Bergemann and Wambach (2013) also construct a sequential disclosure policy and a mechanism which does sequentially screen the buyer, and which does respect stronger than ex ante participation constraints. Compared with us, these authors use a weaker concept of ex post individual rationality which only requires ex post individual rationality \textit{conditional} on the information disclosed.

\(^{40}\)In a similar vein, Kovac and Krähmer (2013) show that a static mechanism can be optimal in a sequential optimal delegation environment in which, unlike in the current work, monetary transfers between the principal and the agent are not feasible.
stringent ex post participation constraints. As discussed in the introduction such participation constraints may also arise in employment and procurement relationships. Yet also in financial markets, similar limitations exist. For instance, the Dodd-Frank Act in the US bans excessive pre-payment penalties for mortgage contracts. Since a pre-payment of a mortgage effectively represents a withdrawal of the consumer from a mortgage contract, these regulatory measures limit penalties for quitting the relationship.

Focusing on the withdrawal rights as mandated by the EU regulation, we argue that they are in line with the the regulator's original intention to safeguard the informational position of the consumer as compared to more traditional sales. We however also show that the welfare effects are ambiguous and both firms and consumers may be hurt by the regulation.

An additional implication of our result is that mandatory withdrawal rights achieve a level playing field between internet shops and traditional stores. Although not an explicit goal of the EU directive, the growing success of online markets may make traditional stores and regulators wary of any "unfair" advantage of online retailers that is not directly related to efficiency. Hence, similar to the current call for the Marketplace Fairness Act in the US, which is to limit "unfair" tax advantages for internet stores over traditional stores, withdrawal rights may limit "unfair" informational advantages vis-à-vis the consumer of internet stores over traditional ones.

A Appendix

Proof of Lemma 1: Follows directly from Courty and Li (2000).

Proof of Proposition 1: Follows from the discussion in the main text.

Proof of Proposition 2: Follows from the discussion in the main text.

Proof of Lemma 2: Follows from the discussion in the main text.

Proof of Proposition 3: Lemma 2 implies that the static menu with \((F_i, R_i) = (0, \bar{R}_J)\) for all \(i \in I\) solves the relaxed version of problem \(P^{J^*}\), where we ignore the constraint \((IC_J)\). Let \(W^{J^*}\) represent the objective of \(P^{J^*}\) evaluated at the static menu with \((F_i, R_i) = (0, \bar{R}_J)\) for all \(i \in I\). Now suppose, in contradiction to our claim, that the static menu is not a solution to the original problem \(P^{J^*}\). Then there is a non–empty set \(\bar{I} \subset I \setminus J^*\) of ex
ante types for which the static menu violates constraint \((IC_J)\). Also the value of program \(P^{J^*}\) must be less than \(W^{J^*}\), because \(W^{J^*}\) is the value of the relaxed program. But the static menu with \((F_i, R_i) = (0, \bar{R}_J)\) for all \(i \in I\) satisfies all constraints of program \(P^{J^*} \cup \bar{I}\). It yields the seller strictly more than \(W^{J^*}\), because she now also receives a positive payoff from ex ante types \(i \in \bar{I}\). Hence, it is not optimal for the seller to send the good only to ex ante types in \(J^*\), a contradiction. Q.E.D.

**Proof of Lemma 3:** We use the following lemma which characterizes incentive compatibility in period 2. (The proof is standard and therefore omitted.)

**Lemma A.1** A direct contract \((x, t)\) is incentive compatible in period 2, i.e. satisfies \((11)\), if and only if for all \(i \in I\), the functions \(v_i\) as given by \((10)\) are absolutely continuous and

\[
\begin{align*}
x_i(\theta) & \text{ is increasing in } \theta, & (MON_i) \\
v_i(\theta) & = \int_0^\theta x_i(z) \, dz + v_i(0). & (RE)
\end{align*}
\]

Now, recall that problem \(\mathcal{P}\) is given as

\[
\mathcal{P} : \quad \max_{(x, t)} \sum_{i \in I} p_i w_i \quad s.t. \quad (11), (12), (13).
\]

We first show that the constraints \((11), (12), (13)\) are equivalent to the constraints \((MON_i), (IC_i^\nu), (IR_i)\) as stated in Lemma 3 and \((RE)\) as stated in Lemma A.1. Indeed, by Lemma A.1, \((11)\) is equivalent to \((MON_i)\) and \((RE)\). By \((RE)\), we obtain

\[
\int_0^1 v_j(\theta) \, dG_i(\theta) = \int_0^1 \int_0^\theta x_j(z) \, dz \, g_i(\theta) \, d\theta + v_j(0)
\]

\[
= - \left[ \int_0^\theta x_j(z) \, dz \cdot [1 - G_i(\theta)] \right]_0^1 + \int_0^1 x_j(\theta)[1 - G_i(\theta)] \, d\theta + v_j(0)
\]

\[
= \int_0^1 x_j(\theta)[1 - G_i(\theta)] \, d\theta + v_j(0),
\]

where we have used integration by parts in the second line. Thus, since \(u_{ji} = \int_0^1 v_j(\theta) \, dG_i(\theta)\), \((19)\) implies that \((12)\) is equivalent to \((IC_i^\nu)\). Moreover, because \(x_i\) is non–negative, \((RE)\) implies that \(v_i(\theta)\) is increasing in \(\theta\), and hence \((13)\) is equivalent to \((IR_i)\). In sum, this shows that \((11), (12), (13)\) are equivalent to \((MON_i), (IC_i^\nu), (IR_i)\), and \((RE)\).
Finally, we can eliminate constraint (RE) by inserting it in the objective: (19) for \( j = i \) yields

\[
\int_0^1 v_i(\theta) \, dG_i(\theta) = \int_0^1 x_i(\theta) h_i(\theta) \, dG_i(\theta) + v_i(0). 
\]

(20)

Plugging this in (14) yields

\[
w_i = \int_0^1 [\theta - c - h_i(\theta)] x_i(\theta) \, dG_i(\theta) - v_i(0),
\]

(21)

and hence, we obtain the objective as stated in Lemma 3. Q.E.D.

**Proof of Lemma 4** Since the “vice versa” statement follows directly from the revelation principle, we prove the lemma only in one direction. In light of Lemma A.1, consider an incentive compatible, deterministic contract \((x, t)\). Because the contract is deterministic, condition \((MON_i)\) implies the existence of a cutoff \( R_i \in [0, 1] \) so that \( x_i(\theta) = 1_{[R_i, 1]}(\theta) \) a.e., where \( 1 \) denotes the indicator function. Moreover, let \( F_i = -v_i(0) \), and define \((F, R) = ((F_1, R_1), \ldots, (F_i, R_i), \ldots, (F_n, R_n))\). By \((RE)\) and (1), the buyer’s utility from submitting report \( j \) in period 1 under the direct contract is the same as choosing \((F_j, R_j)\) from the menu of option contracts. Therefore, because \((x, t)\) is incentive compatible in period 1, the menu \((F, R)\) is incentive compatible and implements the same outcome as the direct contract. Q.E.D.

**Proof of Proposition 4** Follows directly from Proposition 1 and Lemma 4. Q.E.D.

**Proof of Proposition 5** We break up the proof in three steps.

**Step 1:** We begin by showing that \( C^* \) is non-empty. To see this, we show that

\[
\bar{R} \in [\theta_n, \theta_1].
\]

(22)

Since densities \( g_i(\theta) = G_i'(\theta) \) exist, \( \bar{R} \) as a solution to (3) satisfies the first order condition

\[
1 - \sum_{i \in I} p_i G_i(\bar{R}) - (\bar{R} - c) \sum_{i \in I} p_i g_i(\bar{R}) = 0.
\]

(23)

Now suppose that, contrary to the claim, \( \bar{R} < \theta_n \). (Similar arguments apply to the claim \( \bar{R} > \theta_1 \).) Then, because the hazard rate is decreasing and since \( \bar{R} < \theta_n \leq \theta_i \) for all \( i \in I \), (15) implies that \( \bar{R} < \theta_i = c + h_i(\theta_i) < c + h_i(\bar{R}) \) so that \((\bar{R} - c) g_i(\bar{R}) < 1 - G_i(\bar{R}) \). Multiply this inequality with \( p_i \) and sum over \( i \in I \) to get \((\bar{R} - c) \sum_{i \in I} p_i g_i(\bar{R}) < 1 - \sum_{i \in I} p_i G_i(\bar{R}) \), a contradiction to (23). This establishes (22) and directly implies that \( C^* \) is non-empty.
Step 2: Next, we consider the auxiliary problem $\mathcal{R}^0$ which differs from $\mathcal{R}$ in that we set $v_i(0)$ exogenously equal to 0:

$$\mathcal{R}^0 : \max_x w(x, 0) \quad \text{s.t.} \quad (IC^0_{ij}) \quad \text{for all } (i, j) \in C^*.$$ 

We show:

$$\bar{x} \text{ is a solution to } \mathcal{R}^0.$$  \hspace{1cm} (24)

Indeed, by the Kuhn–Tucker theorem for function spaces (see Luenberger, 1969, p.220), a selling schedule $x$ solves $\mathcal{R}^0$ if and only if

(i) there are multipliers $\lambda_{ij} \leq 0$ associated to constraint $IC^0_{ij}$, and

(ii) $x$ maximizes the Lagrangian

$$\mathcal{L}^0 = \sum_{k \in I} \int_0^1 p_k[\theta - c - h_k(\theta)]x_k(\theta)g_k(\theta)d\theta - \sum_{(i, j) \in C} \lambda_{ij} \int_0^1 [x_i(\theta) - x_j(\theta)][1 - G_i(\theta)]d\theta$$

(iii) and, moreover, $\lambda_{ij} = 0$ only if the inequality in $IC^0_{ij}$ is strict.

Note that by definition of $C^*$, we can write $\mathcal{L}^0$ as

$$\mathcal{L}^0 = \sum_{k \in I} \int_0^1 \Psi_k(\theta, \lambda)x_k(\theta)g_k(\theta) d\theta,$$  \hspace{1cm} (25)

where\footnote{The argument $\lambda$ in $\Psi_k$ represents the vector $\{\lambda_{ij}\}_{(i, j) \in C^*}$.}

$$\Psi_k(\theta, \lambda) = \begin{cases} p_k[\theta - c - h_k(\theta)] - \sum_{j: \theta_j < \bar{R}} \lambda_{kj} h_k(\theta) & \text{if } \theta_k \geq \bar{R} \\ p_k[\theta - c - h_k(\theta)] + \sum_{i: \theta_i \geq \bar{R}} \lambda_{ik} h_k(\theta) & \text{if } \theta_k < \bar{R}. \end{cases}$$  \hspace{1cm} (26)

We now show (i) to (iii) for $x = \bar{x}$. Since $\bar{x}$ trivially satisfies $IC^0_{ij}$ with equality, condition (iii) is redundant. Moreover, by point–wise maximization, a selling schedule maximizes $\mathcal{L}^0$ if $x_k(\theta)$ is set to 1 whenever $\Psi_k(\theta, \lambda)$ is positive, and $x_k(\theta)$ is set to 0 otherwise. Therefore, a sufficient condition for $\bar{x}$ to maximize the Lagrangian is that for all $(i, j) \in C^*$ there is a $\lambda_{ij}$ so that

$$\lambda_{ij} \leq 0 \quad \text{and} \quad \Psi_k(\bar{R}, \lambda) = 0 \quad \forall k \in I; \quad \text{and} \quad \Psi_k(\theta, \lambda) \text{ is increasing in } \theta \quad \forall k \in I.$$  \hspace{1cm} (27, 28)}

To see (27), we write the system of equations $\Psi_k(\bar{R}, \lambda) = 0$, $k \in I$, in the $L$ unknowns $\lambda_{ij}$, $(i, j) \in C^*$, in matrix notation. Let $\lambda = (\lambda_1, \ldots, \lambda_L) \in \mathbb{R}^L$ be the (column) vector
consisting of the multipliers \( \lambda_{ij}, (i, j) \in C^* \). Moreover, define the (column) vector \( b = (b_1, \ldots, b_n) \) by

\[
b_k = p_k g_k(\bar{R})(\bar{R} - c) - p_k[1 - G_k(\bar{R})].
\]  

(29)

To simplify notation, we omit the argument \( \bar{R} \) in what follows. Therefore, by (26), after multiplying \( \Psi_k(\lambda) = 0 \) by \( g_k \), we obtain that

\[
\theta_k \geq \bar{R} : \Psi_k(\lambda) = 0 \iff \sum_{j : \theta_j < \bar{R}} [1 - G_k] \lambda_{kj} = b_k,
\]  

(30)

\[
\theta_k < \bar{R} : \Psi_k(\lambda) = 0 \iff \sum_{i : \theta_i \geq \bar{R}} -[1 - G_i] \lambda_{ik} = b_k.
\]  

(31)

In matrix notation, (30) and (31) write

\[
A\lambda = b,
\]  

(32)

for the following \( n \times L \) matrix \( A \): Let \( a_\ell \) be the \( \ell^{th} \) column vector of \( A \in \mathbb{R}^{n \times L} \). Consider an index \( \ell \) with \( \lambda_\ell = \lambda_{ij} \). Then, by inspection of (30) and (31), the \( i^{th} \) row of \( a_\ell \) is equal to \( 1 - G_i \) and the \( j^{th} \) row of \( a_\ell \) is equal to \( -(1 - G_i) \) and all other rows of \( a_\ell \) are equal to 0:

\[
a_\ell = \begin{pmatrix}
0 \\
\vdots \\
1 - G_i \\
\vdots \\
-(1 - G_i) \\
\vdots \\
0
\end{pmatrix} \quad \left\{ \begin{array}{c}
\leftarrow i \\
\leftarrow j
\end{array} \right.
\]  

(33)

Therefore, (27) is equivalent to the existence of a \( \lambda \leq 0 \) (componentwise) so that \( A\lambda = b \). By Farkas’ lemma this is equivalent to:

for all \( y \in \mathbb{R}^n \) there is an \( \ell \in \{1, \ldots, L\} \) so that \( a_\ell \cdot y > 0 \) or \( b \cdot y \geq 0 \),

(34)

where “\( \cdot \)” indicates the scalar product. To prove (34), it is sufficient to show that \( a_\ell \cdot y \leq 0 \) for all \( \ell \in \{1, \ldots, L\} \) implies

\[
b \cdot y \geq 0.
\]  

(35)
To this aim, suppose \( a_\ell \cdot y \leq 0 \) for all \( \ell \in \{1, \ldots, L\} \). Because each \((i, j) \in C^* \) is associated with some \( \ell \), it follows that for each \((i, j) \in C^* \) there exists an \( \ell \) such that \( a_\ell \cdot y = G_i(y_i - y_j) \leq 0 \). Consequently, \( y_i \leq y_j \) for all \((i, j) \in C^* \). Hence, by definition of, \( C^* \):

\[
\max_{i : \theta_i \geq R} y_i \leq \min_{j : \theta_j < R} y_j. \tag{36}
\]

Now observe that \( b_k \geq 0 \) if and only if \( \tilde{R} \geq \theta_k \). Hence,

\[
b \cdot y = \sum_{i : \theta_i \geq R} b_i y_i + \sum_{j : \theta_j < R} b_j y_j \geq \max_{i : \theta_i \geq R} y_i \cdot \sum_{i : \theta_i \geq R} b_i + \min_{j : \theta_j < R} y_j \cdot \sum_{j : \theta_j < R} b_j \geq \max_{i : \theta_i \geq R} y_i \cdot \sum_{k \in I} b_k, \tag{37}
\]

where the last inequality follows by (36). Finally observe that the final term is zero, because \( \sum_{k \in I} b_k = 0 \) by (23). This establishes (35) and completes the proof of (27).

It remains to show (28). Let \( \lambda_{ij} \leq 0, (i, j) \in C^* \) be the multipliers from the proof of (27) that solve \( \Psi_k(\tilde{R}, \lambda) = 0 \) for all \( k \in I \). Recall the definition of \( \Psi_k \) in (26). Observe first that the hazard rate \( h_k(\theta) \) is decreasing and \( p_k[\theta - c] \) is strictly increasing, hence it follows that \( p_k[\theta - c - h_k(\theta)] \) is strictly increasing. Now consider \( k \) with \( \theta_k < \tilde{R} \). The fact that \( \lambda_{ik} \leq 0 \) and decreasing cross hazard rates \( h_{ik}(\theta) \) imply that \( \sum_{i : \theta_i \geq R} \lambda_{ik} h_{ik}(\theta) \) is increasing in \( \theta \). Hence, \( \Psi_k(\theta, \lambda) \) is strictly increasing in \( \theta \) for \( k \) with \( \theta_k < \tilde{R} \). Next, consider \( k \) with \( \theta_k \geq \tilde{R} \), and re–write \( \Psi_k(\theta, \lambda) \) as

\[
\Psi_k(\theta, \lambda) = p_k[\theta - c] - \left( p_k + \sum_{j : \theta_j < R} \lambda_{kj} \right) h_k(\theta). \tag{38}
\]

By (27), \( \Psi_k(\tilde{R}, \lambda) = 0 \). Since \( \tilde{R} \geq c \), this implies that

\[
p_k + \sum_{j : \theta_j < R} \lambda_{kj} = \frac{p_k[\tilde{R} - c]}{h_k(\tilde{R})} \geq 0. \tag{39}
\]

The decreasing hazard rate \( h_k(\cdot) \) therefore implies that \( (p_k + \sum_{j : \theta_j < R} \lambda_{kj}) h_k(\theta) \) is decreasing. Due to the term \( p_k[\theta - c] \), it then follows that (38) is strictly increasing in \( \theta \). This establishes (28). Hence, we have shown (24), and this completes the proof of Step 2.

Step 3: Finally, we prove the actual claim of Proposition 5. By the Kuhn–Tucker theorem, we have to show that there are multipliers \( \lambda_{ij} \leq 0, (i, j) \in C^* \), and \( \mu_k \leq 0, k \in I \), so that

\[42\text{To see this, recall that } \theta_k \text{ is given as the root of the function } p_k[\theta - c - h_k(\theta)]. \] Because of the monotone hazard rate, this function is increasing, and so we have that \( \tilde{R} \leq \theta_k \) if and only if \( p_k[\tilde{R} - c - h_k(\tilde{R})] \leq 0 \Leftrightarrow b_k \leq 0 \).
\((\bar{x}, \bar{v})\) maximizes the Lagrangian

\[
\mathcal{L} = \sum_{k \in I} \left[ \int_0^1 p_k[\theta - c - h_k(\theta)]x_k(\theta)g_k(\theta) \, d\theta - p_kv_k(0) \right] - \sum_{(i,j) \in C^*} \lambda_{ij} \left[ \int_0^1 [x_i(\theta) - x_j(\theta)][1 - G_i(\theta)] \, d\theta + v_i(0) - v_j(0) \right] - \sum_{k \in I} \mu_kv_k(0)
\]

\[
= \sum_{k \in I} \int_0^1 \left[ p_k[\theta - c - h_k(\theta)] - \sum_{j : \theta_j < \bar{R}} \lambda_{kj}h_k(\theta) + \sum_{i : \theta_i \geq \bar{R}} \lambda_{ik}h_{i,k}(\theta) \right] x_k(\theta)g_k(\theta) \, d\theta
\]

\[
- \sum_{k \in I} \left\{ p_k + \sum_{j : \theta_j < \bar{R}} \lambda_{kj} - \sum_{i : \theta_i \geq \bar{R}} \lambda_{ik} + \mu_k \right\} v_k(0),
\]

where \(\lambda_{ij} = 0\) or \(\mu_k = 0\) only if the respective constraints are not binding. Now, let \(\lambda_{ij} \leq 0\), \((i, j) \in C^*\) be the multipliers from the proof of (27) in Step 2 that solve \(\Psi_k(\bar{R}, \lambda) = 0\) for all \(k \in I\) and define

\[
\mu_k = \begin{cases} 
-p_k - \sum_{j : \theta_j < \bar{R}} \lambda_{kj} & \text{if } \theta_k \geq \bar{R} \\
-p_k + \sum_{j : \theta_j \geq \bar{R}} \lambda_{ik} & \text{if } \theta_k < \bar{R} 
\end{cases}
\]

Then the curly brackets in (41) are zero, and the Lagrangian \(\mathcal{L}\) is identical to the Lagrangian \(\mathcal{L}^0\) in (25), which, by (24) in Step 2, is maximized by \(\bar{x}\). Therefore, \((\bar{x}, \bar{v})\) maximizes \(\mathcal{L}\). It remains to be shown that \(\mu_k \leq 0\). Since \(\lambda_{ik} \leq 0\), the claim is trivial for \(k\) with \(\theta_k < \bar{R}\). For \(k\) with \(\theta_k \geq \bar{R}\), recall from (39) that \(-p_k - \sum_{j : \theta_j < \bar{R}} \lambda_{kj} \leq 0\). This establishes Step 3 and completes the proof. Q.E.D.

**References**


