Candidates, Credibility, and Re-election Incentives

RICHARD VAN WEELDEN

Department of Economics, University of Chicago

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I study elections between citizen-candidates who cannot make binding policy commitments before taking office, but who are accountable to voters due to the possibility of re-election. In each period a representative voter chooses among heterogeneous candidates with known policy preferences. The elected candidate chooses the policy to implement, and how much rent-seeking to engage in, when in office. As the voter decides both which candidate to elect and, subsequently, whether the candidate is retained, this framework integrates elements of electoral competition and electoral accountability. I show that, in the best stationary equilibrium, when utility functions are concave over policy, non-median candidates are elected over candidates with policy preferences more closely aligned with the voter. In this equilibrium, there are two candidates who are elected at some history, and the policies these candidates implement in office do not converge. This divergence incentivizes candidates to engage in less rent-seeking, increasing voter welfare.

Keywords: political competition; repeated elections; endogenous candidates; divergent platforms; rent-seeking.

1. INTRODUCTION

In elections, voters choose between competing candidates for office. What attributes should voters look for in order to select the candidates with the greatest incentive to act in the voters’ interest once elected? Are centrist candidates always preferable or might there be advantages to electing candidates who have non-median policy preferences? Elections do not, in general, produce centrist outcomes. In American elections there are usually two candidates, one Republican and one Democrat, with a realistic chance of winning election. These candidates not only run on different platforms, but are individuals with policy preferences that appear to differ both from each other as well as from the median voter. I show that, in a dynamic setting, these features of American elections can be explained as a consequence of two standard assumptions about individual preferences and the electoral process: (1) utility functions are strictly concave with respect to the implemented policy, and (2) there is an opportunity for shirking by the elected candidate.

The authority the electorate delegates to elected representatives creates a very important principal-agent relationship with several interesting features. First, in addition to the concerns, inherent in all agency relationships, of rent-seeking or lack of effort by the agent, both the voters and the candidates care about the policy implemented in ideological space. Moreover, voters interact with their representatives through elections. This means that voters cannot design a compensation scheme for elected officials, but are able to both select the candidate to represent them, and, subsequently, decide whether that candidate is to be retained. The presence of re-election possibilities creates a dynamic agency problem in which voters choose both which candidate to elect, and the re-election standard, with the goal of incentivizing candidates to implement ideologically desirable policies with minimal rent-seeking. In the absence of re-election incentives, it would, of course, be optimal to delegate to an agent whose preferences are as closely aligned with the electorate as possible. However, in a dynamic setting, I show that biased agents –
that is, candidates who hold non-median policy preferences – are selected over unbiased ones.

I consider a model in which a (representative) voter chooses from a continuum of potential candidates with known ideal points in every period. Further, I assume there is some opportunity for shirking, such as corruption or lack of effort, on the part of the elected candidate. I model the rents that the elected candidate could secure at the voter’s expense as a transfer from the electorate to the candidate. The game is infinitely repeated and I focus on pure strategy subgame perfect equilibria in which the candidates’ platforms are history independent. Since the elected candidate will come up for re-election, it is possible for the voter to induce the candidate to engage in less than full rent-seeking by making his re-election conditional on his behavior in office. Of course, if the voter always elects a candidate with median policy preferences, all that a deviating candidate can be punished with is the lack of future rents from holding office. So the amount of rent-seeking in equilibrium must be strictly positive. If utility functions are concave, however, equilibria exist in which non-median candidates are elected that result in higher voter welfare. In such an equilibrium, the elected candidate implements a policy slightly in the direction of his ideal point while engaging in significantly less rent-seeking than a median candidate would. If he deviates, the voter elects a candidate on the other side of the median and allows that candidate to implement a policy slightly in the direction of his ideal point. Since utility functions are concave over policy, the difference between the two policies, even if they are very close to the voter’s ideal point, has a significant disciplining effect on candidates with preferences different than the voter. Consequently, the disutility to the voter from a non-median policy is more than off-set by the decrease in rent-seeking. Further, I show that the best stationary equilibrium involves exactly two candidates, symmetric about the median, who are elected at some continuation history, and that these candidates implement different platforms if elected. This provides, to my knowledge, the first result in the literature with endogenous candidates, diverging platforms, and candidates who implement a platform other than their ideal point if elected. \(^1\)

This framework provides many interesting insights. First, it provides a theory of the platforms that candidates could credibly promise to implement when in office, and how this varies with the candidates’ policy preferences. Second, the voter elects non-median candidates, and the platforms different candidates implement, if elected, do not converge. Moreover, while the platforms do not converge, the desire to secure re-election causes candidates to pursue more moderate policies than they would in the absence of electoral concerns. Finally, even if the voter has complete control over the type of candidate to elect, in the best stationary equilibrium, there are only two types of candidates that are ever elected at any history. All other candidate types, including more centrist candidates, would implement strictly less desirable platforms in office, and so are never elected.

While allowing the voter to select any candidate type is useful for determining the optimal candidates in the absence of external constraints on the available candidates, in real-world elections voters must choose between a small number of candidates for office. That the voter has access to a large number of candidates is not necessary for the core.

\(^1\)Feddersen et al. (1990) allows for endogenous candidates, motivated only by holding office, who can make binding commitments, and shows that the candidates locate at the median in the pure strategy equilibrium. Roemer (2001) specifies the objective of the party to be that of the average (or median) member, assuming that all citizens belong to one of the two parties. In addition, Palfrey (1984) shows that the possibility of third party entry can induce divergent platforms between the two incumbent parties.
insights of this paper – only one candidate from each side of the political spectrum is required. This means that the underlying mechanism can be applied in a more standard model of two-party elections. I show this by considering a repeated two-party election in which each party is made up of moderate and polarized types, and the challenger is randomly drawn from the out-party in each period. I show that a centrist candidate is always defeated by a polarized candidate, since the polarized candidate will engage in less rent-seeking if elected. Since centrist candidates are never elected, the voter’s welfare is higher when the fraction of polarized candidates in each party is larger. As the fraction of polarized candidates in each party converges to 1, this equilibrium converges to the best stationary equilibrium with two candidates – which is the best stationary equilibrium even if all candidate types are available in every period.

This paper advances the literature by integrating elements of electoral competition with insights from political agency models. Since the seminal results of Downs (1957),2 a large literature has emerged to explain the lack of convergence in candidate platforms.3 These models typically make two key assumptions: the parties are exogenously specified and the parties can make binding commitments to implement any platform. As such, the question of why parties represent the interests they do – as opposed, for example, to having a median voter party – is not addressed. On the other end of the spectrum, citizen-candidate models (Osborne and Slivinski 1996, Besley and Coate 1997), which endogenize the candidates’ decision of whether or not to run for office, assume that candidates cannot make binding commitments, and so will always implement their ideal policy if elected. There is then a large gap, in the existing theories of political competition, between the polar opposite assumptions of perfect commitment with exogenous parties and endogenous parties with no commitment.

Neither the assumption that candidates can make binding commitments to any platform or that candidates will always implement their most-preferred platform if elected is particularly natural: elected officials have considerable discretion over which policies to pursue when in office, but their behavior is surely influenced by the desire to be re-elected. To capture this requires a model in which the incumbent comes up for re-election. The previous literature on repeated elections, however, has assumed either that all candidates are identical (e.g., Ferejohn 1986) or that candidates’ types are private information (e.g., Duggan 2000, Bernhardt et al. 2009). As such, while this literature provides interesting insights about how re-election pressures affect the policy choices of incumbent candidates and which candidates are likely to secure re-election, it cannot address the question of which candidates will be elected in the first place or who will replace a defeated incumbent. In this paper, I study the impact of re-election incentives on political competition between candidates of known type, and use this to determine which candidates will be elected. This allows me to integrate the study of two central elements of elections – that voters can choose which candidates have the opportunity to hold office, as well as incentivizing the incumbent candidate to work for re-election – into a single framework. Moreover, combining these elements generates a model in which both the candidates and the platforms are endogenously determined.

That re-election concerns can influence the choices of incumbent politicians is well known. It has been shown that re-election incentives can discipline homogeneous candidates in a model of moral hazard (Barro 1973, Ferejohn 1986), moderate the policies that exogenously specified and ideologically motivated candidates implement in office

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2See also Hotelling (1929) and Black (1958).
(Alesina 1988), and allow candidates to credibly promise to implement policies that differ from their ideal point (Aragones et al. 2007). In this paper, I build on the framework of repeated elections introduced in Duggan (2000). Duggan (2000) considers an infinitely repeated elections model in which the candidates’ preferences are private information. He shows that re-election pressures induce partial, but not complete, policy compromise, with compromising incumbents implementing a policy between their own and the median voter’s ideal point. Since the degree of policy compromise is determined, in part, by the expected policy if the incumbent is defeated, Bernhardt et al. (2009) show that the incumbent will compromise more if his potential replacement is drawn from the other side of the political spectrum. As such, Bernhardt et al. (2009) establish that a two-party system, in which the challenger is drawn from the party representing the opposite side of the ideological spectrum, rather than from the entire population of candidates, is welfare enhancing. Bernhardt and Camara (2012) further explore how voter welfare relates to the distribution of candidates in each party.

My model differs from the above papers in that I allow voters to choose among candidates with known, and heterogeneous, policy preferences rather than assuming that candidate types are private information. I integrate the rent-seeking motivation (Ferejohn 1986) with ideological heterogeneity (Duggan 2000) and show that non-median candidates are elected in the best stationary equilibrium. By electing a candidate with non-median preferences, it is possible to punish a shirking candidate with a policy further from his ideal point, and, as in Bernhardt et al. (2009), this decreases shirking. I show that, when utility functions are strictly concave and players are reasonably patient, not only is rent-seeking reduced, but the effect of this improved disciplining overwhelms the increased scope for shirking from electing a non-congruent candidate. Consequently, polarized candidates are elected over median ones. This is in contrast to the above papers on repeated elections in which, all else equal, more moderate candidates are electorally advantaged.

For the above theoretical mechanism to make sense, of course, elected candidates must be concerned about policy outcomes after leaving office. As candidates are citizens themselves it is certainly natural that they should care about the policy in future periods. Moreover, they have a reason to care even taking a more cynical view of candidates’ motivations. If candidates care about policy only to the extent that it creates a “legacy” for themselves, this legacy is surely eroded if the successor immediately reverses the implemented policy. Some evidence that concerns about future policies do impact the decisions of policymakers comes from the timing of judicial retirements. If, as argued in the literature, judges often time their retirement so that the party that appointed them can appoint a successor likely to share their judicial philosophy, it stands to reason that similar considerations would impact the choices of elected politicians.

In addition, the prediction that voters elect non-median candidates because they engage in less rent-seeking is consistent with voters having a tendency not to “trust” candidates who propose moderate policies – something that has received much attention in the recent literature. It has been argued that non-median policies can signal that the politician is honest or of high character (Kartik and McAfee 2007, Callander and Wilke 2007), motivated by policy concerns (Callander 2008), or more likely to be competent (Carrillo and Castanheira 2008). There is no signaling in my model, however, and the

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4 See also Testa (2012), who shows that polarization between long-lived parties can provide incentives to short-lived candidates, who would otherwise be unaccountable.
5 See Hagle (1993), Spriggs and Walbeck (1995), and Hansford et al. (2010), though there are dissenting findings such as Yoon (2006).
effect identified here is quite different. The previous literature has argued that voters may be suspicious of candidates who claim to be moderate, as announcing a moderate policy could be a sign of opportunism or dishonesty. I show instead that a centrist voter may rationally prefer a non-centrist candidate to one she knows to be moderate, even if non-centrist candidates are neither more competent nor more honest than moderate ones on average. Simply because of the incentives provided by re-election pressures, candidates with preferences less similar to the median voter implement platforms more to the median voter’s liking.

Moreover, there is some evidence that moderate candidates may shirk more in office, engaging in more corruption or exerting less effort. For example, prior to the mani pulite investigations in Italy, the alternative to the centrist, but corrupt, coalition led by the Christian Democrats and the PSI was the Communist party, that was mired in fewer corruption scandals but ideologically extreme. Similarly, the centrist Liberal Party in Canada was defeated due to corruption scandals in the early twenty-first century. More generally, there is a small empirical literature on the relationship between polarization and corruption, with some studies finding that polarization decreases corruption (Testa 2010, Brown et al. 2011), and others that polarization increases it (Eggers 2011). Furthermore, interpreting rent-seeking as a lack of effort by the elected candidate, Halberstam and Montagnes (2012) present evidence that less centrist legislators work harder on behalf of their constituents. I discuss the empirical implications of the model further in the Conclusions.

Finally, as this paper considers the agency relationship between the electorate and policymakers, it is related to a large literature on agency problems. Banks and Sundaram (1993, 1998), Martinez (2009), and Schwabe (2011), who all consider the problem of motivating heterogeneous agents to exert costly effort in a dynamic setting, are especially related. My model differs in that heterogeneity is over ideology rather than competence and considers a setting in which agents’ types are public information. My results are also related to the literature on the advantages of biased agents (e.g., Rogoff 1985, Dewatripont and Tirole 1999). I identify another potential benefit of biased agents: since biased agents can be more effectively disciplined, they can be incentivized to engage in less shirking. This mechanism could, potentially, be applied in other environments. For example, in a committee setting, the principal may optimally delegate to members with non-congruent policy preferences in order to incentivize the members to exert more effort.

The paper proceeds as follows: In Section 2, I describe the model; Section 3 describes stationary subgame perfect equilibria with and without divergent platforms; Section 4 presents the results for the baseline model; and Section 5 presents the results for two-party elections with a randomly drawn challenger. Section 6 concludes. I extend the model to allow for endogenous candidacy and imperfect monitoring in the Appendix, which also contains the proofs of the results.

2. MODEL

I assume there is a single “representative” voter who chooses among heterogeneous candidates in every period. These candidates differ in their type, reflecting their most preferred policy in a one dimensional policy space. I normalize the type of the voter to be 0 and assume there are a continuum of candidates of known types, $\kappa \in [-1, 1]$. To highlight that the results depend on the heterogeneity of the candidates’ preferences as opposed to the increased competition from additional candidates, I further assume that there are an infinite number of candidates of each type. In particular, this guarantees
that the voter always has the option of selecting a different candidate with the same policy preferences as herself in every period. Formally, each candidate can be indexed as

\[ k = (\kappa, i) \in [-1, 1] \times \mathbb{N}, \]

where \( \kappa \) represents the type and \( i \) indexes which type-\( \kappa \) candidate \( k \) is.

In every period the voter chooses one candidate to elect.\(^6\) The elected candidate then chooses the policy to implement from a one-dimensional policy space, \( x \in [-1, 1] \), as well as how much rent-seeking, \( m \in [0, M] \), to engage in. These rents could reflect outright corruption, such as taking bribes, or simply a lack of effort – either taking time off work or not addressing difficult but important non-ideological issues – by the candidate when in office. To simplify the analysis I assume that utilities are linear in the amount of rents secured, but I show that the main results go through when utility functions are concave over rents. I refer to a policy-rents pair, \( p = (x, m) \), as a platform.

The utility to the voter when platform \( p = (x, m) \) is implemented in the stage game is

\[ u(p) = -|x|^\lambda - m, \]

where \( \lambda > 1 \) reflects the degree of concavity. That utility functions are strictly concave with respect to policy will be the key assumption driving the results. Candidates also have policy preferences, and the elected candidate benefits from securing rents for himself. I assume that candidates, like the voter, have utility functions that are concave over policy.\(^7\) So the utility to a candidate of type \( \kappa \) who implements platform \( p = (x, m) \) in office is

\[ \bar{g}_\kappa(p) = -|\kappa - x|^\lambda + \gamma m, \]

where \( \gamma > 0 \). If the candidate is not in office his utility is identical to the voter’s utility, except he has ideal point \( \kappa \). The candidate’s utility is then

\[ g_\kappa(p) = -|\kappa - x|^\lambda - m. \]

In particular, \( g_0(p) = u(p) \). In specifying the preferences of the candidates out of office to be the same as the voter, I am following in the citizen-candidate tradition (Osborne and Slivinski 1996, Besley and Coate 1997) in which candidates are viewed as citizens with policy preferences of their own. As candidates only take action if they are elected by the voter, what is important for the results is that an elected candidate has preferences over the policy implemented both when in, and after leaving, office.

Notice that, as the elected candidate benefits from engaging in rent-seeking, there is some misalignment of interests between the voter and the elected candidate, even if the candidate shares the voter’s policy preferences. The parameter \( \gamma \) reflects the relative trade off, for the elected candidate and the voter, between rents and policy. As the benefits of rent-seeking accrue to the candidate, while the costs are disbursed across the electorate, it is likely that \( \gamma > 1 \), although it is not necessary to assume this. Moreover, since rents may be secured in a manner that generates a large deadweight loss (e.g., the creation of

\(^6\)I do not allow candidates to communicate with the voter before the election. Since candidates’ have no private information, adding (non-binding) communication would add additional complexity to the model without generating additional insights.

\(^7\)While I assume that the concavity of utility functions with respect to policy, \( \lambda \), is the same for the candidates and for the voter, this is not necessary. For appropriate parameters, divergent platforms are welfare enhancing if either the voter’s utility function is strictly concave or if candidates’ utility functions are strictly concave. Moreover, the more concave the utility function of the voter/candidates, the less patience is necessary for divergence to be welfare enhancing. The details are available from the author upon request.
monopolies, the introduction of tariffs, inefficient tax loopholes or subsidies), rent-seeking can produce a substantial disutility for voters. And, if rent-seeking is interpreted as a lack of effort or an unwillingness to confront unrepresentative groups – that is, candidates receive benefit $\gamma M$ from holding office, but this benefit is eroded if the candidate works excessively long hours, denies favors to family and friends, or takes actions that antagonize powerful lobby groups or vocal minorities – the costs to voters are no doubt very high.

Note also that, in this model, candidates do not value being in office for its own sake but as a means to an end: being elected allows a candidate to implement a policy more to his liking and to secure rents for himself. I could allow candidates to derive some intrinsic benefit, $\phi \geq 0$, from holding office as long as this benefit is not so large that candidates would be willing to forgo all rent-seeking in order to secure re-election. Provided that $\phi < \frac{1-\delta}{\delta} \gamma M$, the problem of disciplining elected candidates is nontrivial, and the results go through unchanged.

The timing of the game is as follows:
1. In each period the voter selects a candidate, $k_t$.
2. The elected candidate implements platform $p_t = (x_t, m_t)$ for that period.
3. The voter observes the $p_t$ implemented.
4. The game is repeated with the voter deciding whether to re-elect the incumbent. If not, the voter chooses which candidate to elect instead.

I assume that the game is infinitely repeated with discount factor $\delta < 1$. Since each period in this model corresponds to an election cycle, $\delta$ is expected to be significantly less than 1, and the analysis will be concerned with more than just the limiting properties as $\delta \to 1$.

In each period the voter chooses a candidate, $k_t$, and the elected candidate chooses platform $p_t = (x_t, m_t)$. A history then consists of the previously elected candidates and the platforms they implemented when in office. That is, a history is $h^t = (k_0, p_0), \ldots, (k_{t-1}, p_{t-1})$.

Define $h^t_k = (h^t, k)$ to be the history in which candidate $k$ is elected after history $h^t$, $H^t$ and $H^t_k$ to be the set of all $t$ period histories $h^t$ and $h^t_k$ respectively, and $H = \cup_{t \geq 0} H^t$ and $H_k = \cup_{t \geq 0} H^t_k$. Restricting attention to pure strategies, a strategy for the voter is to elect one candidate at every history,

$s_v : H \rightarrow \{-1, 1\} \times N,$

$s_v(h^t) = k_t$.

Similarly, a strategy for each candidate consists of the platform he would implement, if elected by the voter, at every history,

$s_k : H_k \rightarrow \{-1, 1\} \times [0, M],$

$s_k(h^t_k) = p_t$.

For any profile of strategies, $s$, each player’s utility is just the discounted average utility in the repeated game. The payoff for the voter is then

$U(s) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u(p_t),$

and for each candidate $k$ the payoff is

$U_k(s) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_k(k_t, p_t),$. 
where
\[ u_k(k_t, p_t) = \begin{cases} \bar{g}_\kappa(p_t) & \text{if } k = k_t, \\ g_\kappa(p_t) & \text{if } k \neq k_t. \end{cases} \]

I restrict attention to subgame perfect equilibria in pure strategies. In addition, since the objective of this paper is to determine not only the equilibrium platforms, but also the candidates elected, I introduce the notion of credible candidates. A type of candidate is credible at a given history, \( h^t \), in a subgame perfect equilibrium, \( s^* \), if that type is elected at some history, \( h^t h^\tau \), where \( h^t h^\tau \) is the concatenation of history \( h^t \) followed by \( h^\tau \). That is, credible types are those who are elected at some history, either on or off the equilibrium path, in equilibrium \( s^*|_{h^t} \).

**Definition 1. (Credible Candidates)** A candidate type, \( \kappa \), is credible at history \( h^t \), in subgame perfect equilibrium \( s^* \), if there exists a history, \( h^t h^\tau \), at which a candidate of type \( \kappa \) is elected in equilibrium \( s^* \).

Before proceeding with the analysis I pause to discuss some of the main elements of the model. The most striking feature is that I have specified a single representative voter who can select from a continuum of candidates. The assumption of a single voter not only simplifies the analysis – in particular, it abstracts away from the issue of how voters coordinate on specific candidates – but also shows that policy divergence can be welfare enhancing even with a homogeneous electorate.

It is possible to extend the model to allow for a large number of voters with heterogeneous preferences. Because there is only one dimension of policy disagreement among voters, and preferences are single-peaked in the policy dimension, the existence of a representative voter is equivalent to assuming that the median voter is decisive. In this way, the representative voter is to be thought of as representing the median voter in a large, and possibly diverse, electorate. Van Weelden (2012) considers two-candidate elections with a heterogeneous electorate, and shows that divergent platforms are welfare enhancing for any symmetric distribution of voter preferences with sufficient patience.

Similarly, the assumption of a continuum of candidates, while highly stylized, reflects the idea that there are many potential candidates who desire to hold office, so, if voters wanted to elect a candidate with a certain set of preferences, such a candidate could be found. I make this explicit in Appendix A.2, by assuming there are a continuum of potential candidates who make the decision of whether or not to run for office. Moreover, while allowing the voter to choose any candidate type determines the optimal candidates in the absence of external constraints, non-median candidates are electorally advantaged in two-party elections as well. I consider a repeated two-party election with a randomly drawn challenger in Section 5.

Finally, as the game is infinitely repeated, candidates are not term-limited and so can be re-elected forever. While many offices, such as the Presidency, are term-limited, there are many others that are not. There is no limit on the number of terms that can be served in the U.S. House or Senate, or, in many states, as Governor. In fact, the results of this paper go through with short-lived, or alternatively term-limited, candidates provided the candidates care about the policy implemented after they leave

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8It is often argued that parties represent relatively extreme policies while the voters themselves are more moderate. See Fiorina et al. (2006): “[P]olarized political ideologues are a distinct minority of the American population. For the most part Americans continue to be ambivalent, moderate, and pragmatic, in contrast to the cocksure extremists and ideologues who dominate our political life.” (Preface to 2nd Edition)
office. In this case, although the voter cannot reward the incumbent with re-election, she can elect a candidate of the same type as the incumbent in the next period if he implements a desirable platform, and elect a candidate from the opposite side of the median otherwise. With reasonable patience, non-median candidates are elected in the best stationary equilibrium (Van Weelden 2010).

3. STATIONARITY AND THE DIVERGENT PLATFORMS EQUILIBRIUM

I now turn to analyzing the equilibria of this game. If the game were not repeated, it would have a unique equilibrium: all candidates implement their ideal policy with maximal rent-seeking, $M$, if elected, so the voter elects a candidate of type 0. This would, of course, form a subgame perfect equilibrium if played at every history. Moreover, as there is complete information, and past play is not relevant for the future payoffs for any player, this is the unique Markov Perfect Equilibrium of the repeated game. As the purpose of studying the repeated game is to consider the impact of re-election pressures on platform choices, I consider equilibria in which the voter conditions her re-election decision on the candidate’s behavior in office.

Without any restrictions on the set of equilibria, however, any feasible voter payoff can be supported in a subgame perfect equilibrium if players are sufficiently patient. This includes equilibria in which the voter’s most preferred platform is implemented in every period along the equilibrium path. Even here, however, there is an advantage to delegating to non-median candidates: the minimal level of patience for which it is possible to support the voter’s most preferred platform in all periods with non-median candidates elected on the equilibrium path is lower than with median candidates elected on path. An analysis of non-stationary equilibria is included in Appendix A.1.

Rather than consider all equilibria, I take the standard approach in the literature on repeated elections (Duggan 2000, Banks and Duggan 2008, Bernhardt et al. 2009, 2011) and focus attention on equilibria in which the candidates’ strategies are stationary. However, in order to let the candidate’s re-election be contingent on his behavior in office, I allow the voter’s strategy to depend on the utility she received in the last period, as well as the candidate’s identity. The voter can then reward the incumbent for good behavior with re-election and choose the appropriate candidate to elect as punishment otherwise. This still incorporates elements of stationarity, however, as she gives each candidate a clean slate and evaluates him based only on her utility in the last period.

**Definition 2. (Stationarity)** A subgame perfect equilibrium, $s^*$, is stationary if, for all histories $h$,

1. the candidate the voter elects depends only on the candidate elected in the last period, $k_{t-1}$, and the utility the voter received from the candidate’s platform, $u(p_{t-1})$. That is, there exists a function $\bar{s}_v : ([-1,1] \times \mathbb{N}) \times \mathbb{R} \rightarrow [-1,1] \times \mathbb{N}$ such that $s^*_v(h^*) = \bar{s}_v(k_{t-1}, u(p_{t-1}))$.

2. for all candidates $k$, the platform they implement if elected does not depend on the history at which they are elected. That is, $s^*_k(h^*_k)$ is a constant function.

*In particular, it is always possible for the voter to punish a deviating candidate by keeping him out of office forever. For example, the voter’s strategy could involve always electing a candidate whose index is one higher than the incumbent whenever the incumbent is not re-elected.*
Because candidates’ types are public information, the voter does not learn about the elected candidate during his tenure in office. As such, this is a stationary environment, and it is natural to focus on equilibria in which the candidate’s platform, and the voter’s response to his platform, are independent of history. Moreover, stationary equilibria have many attractive features. First, they reflect an environment in which the candidates (i.e. parties) have platforms that are stable over time – something observed in advanced democracies. Stationarity also rules out equilibria in which the voter induces good behavior by the candidate with the threat of extremely bad outcomes that harm both the voter and candidate in future periods. As such, stationary equilibria are renegotiation proof. Finally, because candidates’ strategies do not vary with history, stationary equilibria are comparable to the equilibria analyzed in previous work on repeated elections, and provide a natural benchmark for comparison with static models of political competition.

I begin by considering equilibria in which the voter always elects a candidate who shares her policy preferences, $\kappa = 0$, and such candidates always implement the median policy, $x = 0$. For the voter to be willing to condition her re-election decision on the candidate’s behavior in office she must be indifferent between re-electing the incumbent and kicking him out of office. Hence, the level of rent-seeking will not be affected by whether or not the incumbent is re-elected. The minimum amount of rent-seeking that can be supported with median candidates is then determined from the equation

$$\bar{g}_0(0, m_0) = (1 - \delta) \gamma M + \delta g_0(0, m_0).$$

That is, $m_0$ is the level of rent-seeking that makes a type 0 candidate indifferent between being elected and securing rents $m_0$ in every period (which gives the candidate utility $\bar{g}_0(0, m_0)$ in every period) and securing maximal rents today with some other candidate engaging in that level of rent-seeking in all future periods (which gives the candidate utility $\gamma M$ in the first period, and $g_0(0, m_0)$ in all future periods). It is straightforward to show that the minimal level of rent-seeking is then $m_0 = (1 - \delta)\gamma M/(\gamma + \delta)$, which is strictly positive. I refer to the best stationary equilibrium in which the voter always delegates to type 0 candidates as the Median Candidates Equilibrium.

**Median Candidates Equilibrium**

The following strategies constitute a subgame perfect equilibrium: The voter elects a candidate of type 0 at every history. If the incumbent is of type 0, the voter re-elects him if and only if the platform he implemented gave her utility at least $u(0, (1 - \delta)\gamma M/\gamma + \delta)$, and elects a different type 0 candidate otherwise. Candidates of type $\kappa = 0$ implement their ideal policy while securing rents $m_0 = (1 - \delta)\gamma M/(\gamma + \delta)$, and all other candidates implement their ideal policy with full rent-seeking.

While this equilibrium takes advantage of the competition between candidates, it does not exploit the advantages of heterogeneity in candidate preferences. Consider a candidate of type $\kappa > 0$, where $\kappa$ is small, and consider the most desirable platform such a candidate would be willing to implement to secure re-election. Note that the most desirable platform, from the voter’s perspective, would trade off policy and rents in the

\[10\] If the candidates’ types were not observed by the voter, she would update about the candidate based on his behavior in office. See Martinez (2009) and Schwabe (2011) for models in which voters update about the incumbent over his entire tenure, and so must condition on all periods he was in office when deciding whether to re-elect him.
CANDIDATES, CREDIBILITY, AND RE-ELECTION INCENTIVES

same ratio for the candidate and the voter. As utilities are linear with respect to rents, the ratio of marginal utilities over rent-seeking is a constant, $\gamma$. So the ratio of the marginal utilities for the candidate and the voter with respect to policy must also be equal to $\gamma$. This implies that the candidate implements a platform $(x, m)$ that satisfies,

$$\frac{-\frac{\partial g_\kappa(x, m)}{\partial x}}{\partial u(x, m)} = (\frac{\kappa - x}{x})^{\lambda - 1} = \gamma.$$

Consequently, candidate $\kappa$ will implement policy

$$x_\kappa = \frac{\kappa}{1 + \gamma^{\frac{1}{1-\lambda}}} = \frac{\gamma^{\frac{1}{1-\lambda}} \kappa}{1 + \gamma^{\frac{1}{1-\lambda}}}.$$

As the candidate cares about policy, the punishment for not securing re-election is harshest if the candidate who would replace him comes from the opposite side of the political spectrum. I then consider the equilibrium in which, if a candidate of type $\kappa$ does not secure re-election, a candidate of type $-\kappa$ is elected and implements policy $-\kappa/(1 + \gamma^{\frac{1}{1-\lambda}})$ in all future periods. The lowest level of rent-seeking that can be supported can then be calculated from the equation

$$g_\kappa(x_\kappa, m_\kappa) = (1 - \delta)\gamma M + \delta g_\kappa(-x_\kappa, m_\kappa).$$

Solving for the level of rent-seeking, $m_\kappa$, the voter’s utility from candidates of type $\kappa$ is

$$w(\kappa) = -\frac{(1 - \delta)\gamma M}{\gamma + \delta} + \frac{1}{\gamma + \delta} \left(1 + \gamma^{\frac{1}{1-\lambda}}\right)^{-\lambda}\left[\delta((1 + 2\gamma^{\frac{1}{1-\lambda}})^{\lambda} - \gamma^{\frac{1}{1-\lambda}}) - (1 + \gamma^{\frac{1}{1-\lambda}})\right]^{\kappa \lambda - \lambda}.$$

As such, voter utility is increasing in $\kappa$ when $\delta > \frac{(1 + \gamma^{\frac{1}{1-\lambda}})}{(1 + 2\gamma^{\frac{1}{1-\lambda}})^{\lambda} - \gamma^{\frac{1}{1-\lambda}}}$. I assume a stronger condition, $\delta \geq \delta \equiv (1 + 2\gamma^{\frac{1}{1-\lambda}})^{1-\lambda}$. When $\delta \geq \delta$, not only is welfare increasing in $\kappa$, but no candidate of type in $[0, \kappa]$ would have an incentive to implement an equally desirable platform if a deviation is punished with a type $-\kappa$ candidate elected. Consequently, the threat of electing a polarized candidate cannot be used to induce a moderate candidate to implement a platform as desirable as the polarized candidate’s platform. This will guarantee that the best stationary equilibrium involves exactly two candidates. Also, because $\gamma > 0$ and $\lambda > 1$, the assumed level of patience, $\delta$, is strictly less than 1. I show in Appendix A.5 that it is significantly less than 1 for reasonable parameters.

Delegating to candidates of type $\kappa > 0$ has both advantages and disadvantages to the voter. First, the candidate implements policy $x_\kappa > 0$ in every period, resulting in a utility loss to the voter. Additionally, the elected candidate has greater potential gains from deviating along the policy dimension as the equilibrium policy will not correspond to the candidate’s ideal policy, increasing the amount of rents required to prevent the candidate from deviating. However, the candidate also has more to lose by deviating, since if he fails to secure re-election the policy in the future moves from $x_\kappa$ to $-x_\kappa$. This increased disciplining effect makes the incumbent willing to decrease rent-seeking to secure re-election. Because the implemented policy, $x_\kappa$, is proportional to $\kappa$, the advantages and disadvantages are both proportional to $\kappa^{\lambda}$. So, when $\delta \geq \delta$, the voter’s welfare is increasing in $\kappa$, and will continue to increase in $\kappa$, until either the rents secured
by the elected candidate are driven to 0 or the $\kappa \leq 1$ constraint binds.\footnote{That it will be optimal to drive rents to 0 is due to the quasilinear functional form assumption, which I relax in section 4.} I then define
\[
\bar{\kappa} = \min\{\kappa : \bar{g}_\kappa(x_\kappa, 0) \geq (1 - \delta)\gamma M + \delta g_\kappa(-x_\kappa, 0)\} > 0,
\]
to be the minimum divergence in candidate preferences for which rents are driven to 0. When $\bar{\kappa} < 1$, the level of rent-seeking is 0 for all $\kappa \in [\bar{\kappa}, 1]$, so voter welfare is maximized with the most moderate policy that can be supported. For each $\kappa \in [\bar{\kappa}, 1]$, define $x(\kappa)$ to be the policy that makes a type $\kappa$ candidate indifferent between implementing $x(\kappa)$ in every period with no rent-seeking, and implementing his ideal point with full rent-seeking today with $-x(\kappa)$ implemented in all future periods,
\[
\bar{g}_\kappa(x(\kappa), 0) = (1 - \delta)\gamma M + \delta g_\kappa(-x(\kappa), 0).
\]
I show in Appendix A.3 that $x(\kappa)$ has a unique minimizer in $[\bar{\kappa}, 1]$. This corresponds to the most moderate policy that can be supported in a stationary equilibrium with no rent-seeking. The optimal degree of divergence in candidate preferences is then
\[
k^* = \begin{cases} 
\arg \min_{\kappa \in [\bar{\kappa}, 1]} x(\kappa) & \text{if } \bar{\kappa} < 1, \\
1 & \text{otherwise.}
\end{cases}
\]
Similarly, the amount of policy divergence and rent-seeking are defined as
\[
x^* = \begin{cases} 
x(\kappa^*) & \text{if } \bar{\kappa} < 1, \\
\frac{1}{1+\gamma\bar{\kappa}} & \text{otherwise,}
\end{cases}
\]
\[
m^* = \begin{cases} 
0 & \text{if } \bar{\kappa} < 1, \\
\frac{1}{\gamma+\bar{\kappa}} \left( 1 - \delta \right) \gamma M - \frac{2(1+2\gamma\bar{\kappa})^{\bar{\kappa}} - 1}{(1+\gamma\bar{\kappa})^{\bar{\kappa}}} & \text{otherwise.}
\end{cases}
\]
Finally,
\[
u^* = u(x^*, m^*) = -(x^*)^{\bar{\kappa}} - m^*,
\]
is the utility to the voter when the policy implemented is $x^*$ and the elected candidate secures rents $m^*$. This determines the optimal divergence in platforms and candidate preferences. I refer to this equilibrium as the Divergent Platforms Equilibrium.

**Divergent Platforms Equilibrium**

Suppose $\delta \geq \bar{\delta}$. The following strategies constitute a subgame perfect equilibrium in which the voter receives utility $u^* > u(0, m_0)$.

(a) The voter elects a candidate of type $\kappa \in \{-\kappa^*, \kappa^*\}$ at every history.

(b) For all histories $h^t$, if $k_t$ is type $\kappa^*$ or $-\kappa^*$ they are re-elected ($s_{\kappa^*}(h^t, (k_t, p_t)) = k_t$) if $u(p_t) \geq u^*$; if $u(p_t) < u^*$ and the incumbent, $k_t$, is type $\kappa^*$ ($-\kappa^*$) the voter elects a candidate of type $-\kappa^*$ (respectively $\kappa^*$) at history $(h^t, (k_t, p_t))$.

(c) All candidates of type $\kappa^*$ implement platform $(x^*, m^*)$, and all candidates of type $-\kappa^*$ implement platform $(-x^*, m^*)$, at any history in which they are elected.

(d) Candidates of type $\kappa \notin \{-\kappa^*, \kappa^*\}$ implement platform $(\kappa, M)$ if elected.

The Divergent Platforms Equilibrium highlights the advantage of having candidates with heterogeneous preferences. When $\delta \geq \bar{\delta}$ the voter receives a strictly higher payoff in this equilibrium than in the Median Candidates Equilibrium.
4. RESULTS

4.1. Efficiency of Divergent Platforms

I now show that the Divergent Platforms Equilibrium is the best stationary subgame perfect equilibrium. Recalling that the voter’s payoff in this equilibrium is $u^*$, this is shown in the following result.

**Theorem 1.** *(Efficiency of Divergent Platforms Equilibrium)* If $\delta \geq \bar{\delta}$,

1. there does not exist a stationary, pure strategy, subgame perfect equilibrium that gives the voter a payoff higher than $u^*$.
2. in any stationary, pure strategy, subgame perfect equilibrium, $s^*$, in which the voter’s payoff is $u^*$, if candidate $k$ is elected at any history, $k \in \{s^*_k(h^t) : h^t \in H\}$, then either $s^*_k(h^t_k) = (x^*, m^*)$ and $k$ is type $\kappa^*$ or $s^*_k(h^t_k) = (-x^*, m^*)$ and $k$ is type $-\kappa^*$.

The first part of Theorem 1 shows that there is no stationary equilibrium that generates a higher payoff to the voter than the Divergent Platforms Equilibrium. The second part establishes that the equilibrium is unique up to whether a type $\kappa^*$ or $-\kappa^*$ candidate is elected in the initial period, and the behavior at certain off-path histories. Of course, for a candidate of type $\kappa^* (-\kappa^*)$ to have an incentive to implement platform $(x^*, m^*)$ (respectively $(-x^*, m^*)$), if he deviates to his ideal point, the voter must elect a candidate of type $-\kappa^* (\kappa^*)$, as in the Divergent Platforms Equilibrium.

Theorem 1 shows that only candidates of type $\kappa^*$ and $-\kappa^*$ can be elected at any history of the best stationary equilibrium. In any stationary equilibrium any candidate of type $\kappa \notin \{-\kappa^*, \kappa^*\}$ would strictly prefer to implement his ideal point while securing full rents in the current period, followed by any continuation equilibrium that gives the voter utility $u^*$, to implementing any platform that gives the voter payoff $u^*$ and securing re-election for himself. Consequently, if elected, such a candidate must implement a platform that gives the voter utility strictly less than $u^*$. Recognizing this, at all histories, the voter has a strict incentive not to elect any candidate of type $\kappa \notin \{-\kappa^*, \kappa^*\}$. Recalling the definition of credible types leads to the following immediate corollary.

**Corollary 1.** At all histories of any stationary, pure strategy, subgame perfect equilibrium in which the voter’s utility is $u^*$, there are exactly two credible candidate types, $\kappa^*$ and $-\kappa^*$.

The above results show that the best stationary subgame perfect equilibrium, the Divergent Platforms Equilibrium, incorporates some of the central features of American elections. There are exactly two types of candidates who are ever elected, and these candidates implement non-converging platforms. Moreover, while the platforms do not converge, the candidates implement more moderate platforms than they would in the absence of re-election incentives. While the incumbent candidate is re-elected forever along the equilibrium path, I show in Appendix A.2 that there is positive turnover on path when the incumbent’s platform is imperfectly observed.

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12For example, whether the incumbent is re-elected if they implemented a platform that gives utility greater than $u^*$, and the platform that would be implemented by candidates not elected at any continuation history.
In the *Divergent Platforms Equilibrium*, the voter follows a very simple cutoff strategy: she re-elects the incumbent if and only if the utility she received is at least $u^*$. That is, the voter’s re-election decision is consistent with “retrospective voting”, which is often used by political scientists to describe voter decisions (e.g., Fiorina 1981, Duggan 2000). In this equilibrium, the voter sets the standard for re-election that results in the highest payoff from any pure strategy, stationary equilibrium. This is then similar to the equilibrium considered by Ferejohn (1986), who looks for the optimal standard for re-election when candidates are homogeneous. In my paper, the voter’s decision is more complicated, as she not only needs to determine the standard for re-election, but also which candidates are willing to meet it. In Ferejohn’s model, the threatened punishment of not re-electing the incumbent is always credible because the candidates are identical, so the voter is indifferent between them. Here, there is heterogeneity in the candidates, but, in equilibrium, the voter must still be indifferent between the candidates who are elected at different histories. If the voter strictly preferred to re-elect the incumbent, the threat of switching candidates would not be credible and the incumbent could engage in full rent-seeking with impunity.

Notice one more attractive feature of the *Divergent Platforms Equilibrium*. Because candidate strategies are stationary, the voter chooses a static best response at every history. As such, the equilibrium does not require the voter to consider the effect of her vote on future candidate decisions, and the discount factor of the voter is irrelevant for the analysis. Even if the voter were completely myopic, the results would go through unchanged. What matters for the result is that the voter recognizes that relatively non-centrist candidates will engage in less rent-seeking when in office.

Finally, while I have assumed that there is only one dimension of policy disagreement, this can be extended to two or more dimensions. If utility functions are additively separable over policy dimensions, and concavity is the same in each dimension, then the voter’s utility in an equilibrium with candidates of type $(\kappa_1, \kappa_2)$ and $(-\kappa_1, -\kappa_2)$ is the same as in an equilibrium with one dimension of policy disagreement and candidates of type $\kappa = (\kappa_1^\lambda + \kappa_2^\lambda)^\frac{1}{\lambda}$ and $-\kappa$. Hence the best stationary equilibria are those that involve two candidates, $(\kappa_1, \kappa_2)$ and $(-\kappa_1, -\kappa_2)$, for any $(\kappa_1, \kappa_2)$ that satisfies $\kappa_1^\lambda + \kappa_2^\lambda = (\kappa^*)^\lambda$. These equilibria have two-candidates in the sense that, after a candidate of type $(\kappa_1, \kappa_2)$ deviates to $(\kappa_1, \kappa_2, M)$, a candidate of type $(-\kappa_1, -\kappa_2)$ must be elected in the next period. This makes the punishment for deviating as harsh as possible. As different equilibria that maximize the voter’s utility involve different elected candidates, however, other candidates may be elected at different (off-path) histories of such an equilibrium. Finally, when voter preferences are heterogeneous, the optimal dimension of polarization in a two-candidate equilibrium varies with the polarization in the electorate. See Van Weelden (2012) for details.

### 4.2. Comparative Statics

I now turn to analyzing the properties of the *Divergent Platforms Equilibrium*. The first result of this section considers the role of the concavity of utility functions in the divergent platforms result.

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13In an equilibrium with two candidates who aren’t symmetric around the voter, for the threat of switching candidates to be credible, the candidate who implements the more moderate policy must also secure more rents in equilibrium.

14For the purposes of this discussion, assume that $\kappa^* < 1$. Multiple dimensions of policy disagreement can also increase the feasible levels of divergence in candidate ideal points.
Theorem 2. (Concavity and Divergent Platforms)

(1) For any level of patience, $\delta \in (0, 1)$, provided utility functions are sufficiently concave, the best stationary equilibrium involves non-median candidates and divergent platforms:

$$\lim_{\lambda \to \infty} \bar{\delta} = 0.$$ 

(2) For any level of patience, $\delta \in (0, 1)$, the Median Candidates Equilibrium is the best stationary equilibrium if $\lambda = 1$ and $\gamma \geq 1$.

The first part of Theorem 2 shows that, regardless of the level of discounting, divergence is beneficial provided utilities are sufficiently concave. The second, that unless candidates place a low weight on rents, the assumption that utility functions are strictly concave ($\lambda > 1$) is necessary for the best stationary equilibrium to involve divergence.15

Interestingly, in static models, such as in the literature on probabilistic voting (e.g., Ledyard 1984, Kamada and Kojima 2010), concavity often creates additional pressures for convergence. Here, in a dynamic environment, concavity is the key assumption that generates divergent platforms. I now consider how the candidates elected and the platforms implemented vary with parameters in the Divergent Platforms Equilibrium.

Theorem 3. (Properties of Divergent Platforms Equilibrium) If $\delta > \bar{\delta}$ and $\kappa^* < 1$, the optimal amount of divergence is increasing in the opportunity for rent-seeking ($\frac{\partial \kappa^*}{\partial M} > 0$ and $\frac{\partial x^*}{\partial M} > 0$), and candidates’ taste for rents ($\frac{\partial \kappa^*}{\partial \gamma} > 0$ and $\frac{\partial x^*}{\partial \gamma} > 0$), but decreasing in patience ($\frac{\partial \kappa^*}{\partial \delta} < 0$ and $\frac{\partial x^*}{\partial \delta} < 0$).

When there is more opportunity for the elected candidate to secure rents for himself, there is greater benefit to deviating, and hence greater polarization of candidates and policies in the best equilibrium. Similarly, if candidates have greater taste for rents, relative to policy, greater divergence is necessary to prevent shirking. Finally, greater patience moderates the policies implemented, as well as the preferences of the candidates who implement them, in the best equilibrium.

While $\kappa^*$ is decreasing in $\delta$, it is bounded away from 0.16 As $\delta \to 1$ the payoff to the voter converges to the payoff from her bliss point, but the preferences of the elected candidates do not converge to the preferences of the voter. That the implemented platform converges to the voter’s ideal point as $\delta$ goes to 1 is not surprising, as this is true even without heterogeneity in candidate preferences. With divergent platforms, however, the utility of the voter approaches the utility from her ideal point at a much faster rate. Non-median candidates can be punished more harshly than median ones, and future periods become relatively more important when players become more patient. Consequently, the ratio of the utility loss to the voter in the Divergent Platform Equilibrium to the utility loss in the Median Candidates Equilibrium goes to 0 as players become very patient.

15When $\gamma < 1$, divergent platforms can be welfare enhancing even if $\lambda = 1$. For such parameters, rents have little value to politicians, but, in the Median Candidates Equilibrium, candidates receive positive rents anyway. It is then “cheaper” to compensate non-median candidates by allowing them to move policy even when utility functions are not concave because policy is more important, relative to rents, for politicians than voters.

16Here it is important that candidates not receive an intrinsic benefit, $\phi > 0$, from holding office. If they did incentivizing candidates not to shirk would be trivial with high levels of patience.
Theorem 4. (Patient Players)

(1) Platforms converge to the voter’s ideal point as candidates become patient:
$$\lim_{\delta \to 1} (x^*, m^*) = (0, 0).$$

(2) The candidates elected in equilibrium do not converge:
$$\lim_{\delta \to 1} \kappa_* = \min\{((\lambda - 1)\gamma M)^{\frac{1}{\gamma}}, 1\} > 0.$$

(3) There is faster convergence to the voter’s ideal point with divergent platforms:
$$\lim_{\delta \to 1} u(0, 0) - u^* - u(0, m_0) = 0.$$

Before concluding this section, I discuss the robustness of the quasilinear functional form assumption. When utilities are concave over rents there is still a benefit to policy divergence, but, unlike with quasilinear utility functions, equilibrium rent-seeking is always strictly positive. Consider the utility functions $\bar{g}_k(p) = -|\kappa - x^\lambda + \gamma m^{\nu_1}|$, $g_k(p) = -|\kappa - x^\lambda - m^{\nu_2}|$, and $u(p) = g_0(p) = -|x^\lambda - m^{\nu_2}|$. When $\nu_1 < 1 < \nu_2$ utility functions are concave over rents. Now, define $m_0$ to be the minimum level of rents that can be supported without heterogeneity, $\bar{g}_0(0, m_0) = (1 - \delta)\gamma M + g_0(0, m_0)$, and $\gamma_0$ to be the associated ratio of marginal utilities in rent-seeking between candidate and voter, $\gamma_0 = \frac{\nu_1}{\nu_2} m_0^{\nu_1 - \nu_2}$. By the previous argument, the utility of the voter is increasing in $\kappa$ (at 0) when
$$\delta \geq (1 + 2\gamma_0 \frac{\nu_1}{\nu_2})^{1-\lambda}.$$ 
So divergence is always beneficial when $\lambda$ is sufficiently large or when $M$ is sufficiently high. However, as the voter’s strategy depends only on the utility she received, the candidate’s platform trades off policy and rents in the same ratio as the voter,
$$\left(\frac{\kappa - x}{\kappa}\right)^{\lambda - 1} = \frac{\nu_1}{\nu_2} m^{\nu_1 - \nu_2}.$$ 
So, when $\nu_1 < \nu_2$, the amount of rent-seeking in the best equilibrium, $m_\kappa$, is strictly positive.

5. TWO-PARTY ELECTIONS WITH A RANDOM CHALLENGER

So far I have assumed that the voter can elect any candidate type she wants, which is useful for determining the optimal candidates in the absence of external constraints on the available candidates. However, in real-world elections, many elements outside the model – most obviously parties and primaries – influence the candidates who will stand for office, reducing the amount of choice voters have. In this section I show that the underlying theoretical mechanism can be applied to two-party elections in which the representative voter chooses between the incumbent and a randomly drawn challenger from the out-party. Since the voter may not find the challenger acceptable, the incumbent will recognize that the threat of being replaced after a deviation is not always credible. Since the incentive to elect non-median candidates is due to the improved ability to discipline such candidates, this decreases the incentive to elect candidates with non-median preferences. Still, I show that with sufficient patience, or sufficient concavity, the voter receives higher utility in equilibria in which only non-median candidates are
CANDIDATES, CREDIBILITY, AND RE-ELECTION INCENTIVES

This shows that the results about the efficiency of divergent platforms do not depend on the voter having the ability to elect any candidate type, but also hold in a more standard two-party election. Moreover, in considering the case in which the types of the candidates are publicly known at the time of election, but the incumbent is unsure who the challenger will be when deciding which platform to implement, this provides an intermediate case between the assumption in the baseline model that candidate types are publicly known, and the assumption in the previous literature (Duggan 2000, Bernhardt et al. 2009) that candidate types are private information.

There are two “parties”, L and R. I assume that party R consists of candidates of type $\kappa \geq 0$ and candidates of type 0, and party L consists of candidates of type $-\kappa$ and 0. At time 0, the voter can elect a candidate from either party and choose whether to elect a candidate of type $-\kappa$, 0, or $\kappa$. The elected candidate then chooses the platform to implement. In all future periods the voter can elect either the incumbent or the challenger, with the challenger drawn at random from the party not in office. That is, the challenger is $-\kappa$ ($\kappa$) with probability $\alpha \in (0, 1)$ if the incumbent is party R (L) and type 0 with probability $1 - \alpha$. The voter observes the platform the incumbent implemented, and the type of the challenger, before deciding whether or not to re-elect the incumbent. I assume a defeated incumbent will never be a candidate again. The timing is:

1. In each period, the voter elects a candidate, $k_t$.
2. The candidate implements platform, $p_t$, which is publicly observed.
3. The type of the challenger, $\kappa_{t+1}$, is revealed.
4. The game is repeated with the voter deciding whether to elect the incumbent or the challenger.

I focus on symmetric, stationary, pure strategy equilibria and consider when voter welfare is higher with non-converging platforms and non-median candidates than with converging platforms.

**Definition 3. (Stationary, Symmetric Public Perfect Equilibrium)** A public perfect equilibrium, $s^*$, is symmetric and stationary if there exist $(x_\kappa, m_\kappa)$ and $m_0$ such that, for all histories $h^*$,

1. The candidate the voter elects depends only on the candidate elected in the last period, $k_{t-1}$, the utility received from the candidate’s platform, $u(p_{t-1})$, and the type of the challenger, $\kappa_t$. That is, there exists a function $s_v$ such that $s_v(h^t) = s_v(k_{t-1}, u(p_{t-1}), \kappa_t)$.

2. All candidates of type $\kappa$ implement platform $(x_\kappa, m_\kappa)$, all candidates of type 0 implement platform $(0, m_0)$, and all candidates of type $-\kappa$ implement $(-x_\kappa, m_\kappa)$, at all histories in which they are elected.

First consider the case in which $\kappa = 0$, so that both parties are made up of median candidates. It is then possible to support the Median Candidates Equilibrium from Section 3: all candidates implement platform $(0, \frac{(1-\delta)\gamma M}{\gamma + \delta})$, and are re-elected if and only if the utility they provide to the voter is at least $-\frac{(1-\delta)\gamma M}{\gamma + \delta}$. As there is no uncertainty as to whether the desired challenger type will be available, the voter can always punish a deviating candidate. If $\kappa > 0$ this is no longer true if the voter is unwilling to elect non-median candidates. However, since the voter is indifferent between platform $(0, \frac{(1-\delta)\gamma M}{\gamma + \delta})$
and platform \((x_{\kappa}, \frac{(1-\delta)\gamma M}{\gamma + \delta} - x_{\kappa})\), if
\[
\bar{g}_{\kappa}(x_{\kappa}, \frac{(1-\delta)\gamma M}{\gamma + \delta} - x_{\kappa}) \geq (1-\delta)\gamma M + \delta \left[ \alpha g_{\kappa}(-x_{\kappa}, \frac{(1-\delta)\gamma M}{\gamma + \delta} - x_{\kappa}) - (1-\alpha)g_{\kappa}(0, \frac{(1-\delta)\gamma M}{\gamma + \delta}) \right],
\]
there exists an equilibrium in which the voter holds candidates of type \(\kappa\) and \(-\kappa\) to the same standard as median candidates, and elects any challenger, regardless of his type, if the incumbent deviated. Straightforward algebraic calculations show that this holds if and only if \(\delta \geq \delta_0\), for some \(\delta_0 \in (0, 1)\). For these parameters, the voter is indifferent between median and non-median candidates, and the equilibrium when \(\kappa > 0\) provides the same utility to the voter as the Median Candidates Equilibrium. Moreover, since a median candidate shares the same preferences as the voter if not in office, no symmetric, stationary equilibrium in which median candidates are ever elected can give the voter a payoff higher than in the Median Candidates Equilibrium.

However, for appropriate parameters, there exist equilibria that generate higher utility in which only type \(\kappa > 0\) and \(-\kappa\) candidates are ever elected. To see this, note that, if the voter does not replace a candidate of type \(\kappa\) who deviated unless a type \(-\kappa\) is available, the incumbent will be re-elected with probability \(1 - \alpha\) if he deviates, and probability 1 if he doesn’t. As such, to maintain the candidates’ incentives, it must be that
\[
\bar{g}_{\kappa}(x_{\kappa}, m_k) \geq (1-\delta)\gamma M + \delta [\alpha g_{\kappa}(-x_{\kappa}, m_k) + (1-\alpha)\bar{g}_{\kappa}(x_{\kappa}, m_k)].
\]
I refer to the best equilibrium with only candidates of type \(\kappa\) and \(-\kappa\) ever elected, and fraction \(\alpha\) polarized candidates in each party, as the \((\alpha, \kappa)\)-Divergence Equilibrium for the model with a random challenger. As \(\alpha\) decreases, the elected candidate has less to fear from deviating, as he knows he is less likely to be replaced, and so voter welfare in the \((\alpha, \kappa)\)-Divergence Equilibrium is lower. However, I have previously established (Theorems 2 and 4) that the welfare advantage from divergent platforms is very large if \(\delta\) or \(\lambda\) is high. As such, for any \(\alpha\), voter welfare is higher in the \((\alpha, \kappa)\)-Divergence Equilibrium than the Median Candidates Equilibrium if players are sufficiently patient or utilities are sufficiently concave.

**Theorem 5.** (Divergent Platforms with Random Challenger) For any \(\alpha \in (0, 1)\) and \(\kappa \in (0, 1]\), there exists \(\delta(\alpha, \kappa) \in (0, 1]\) such that, for all \(\delta > \delta(\alpha, \kappa)\), the voter’s welfare is strictly higher in the \((\alpha, \kappa)\)-Divergence Equilibrium than the Median Candidates Equilibrium. Moreover, for any \(\alpha \in (0, 1)\), there exists a \(\kappa_\alpha \in (0, 1]\) such that
\[
\lim_{\lambda \to \infty} \delta(\alpha, \kappa_\alpha) = 0.
\]

This means that, no matter how difficult it is to find an appropriate candidate to elect as punishment, as long as such a candidate can be found with strictly positive probability, divergent platforms are welfare enhancing if candidates are sufficiently patient. Furthermore, extreme patience is not necessary for divergent platforms to be beneficial. For any level of patience, if utility functions are sufficiently concave, the voter’s welfare is higher in the \((\alpha, \kappa)\)-Divergence Equilibrium with the optimal degree of polarization in candidate preferences than in the Median Candidates Equilibrium. So, even if the challenger is randomly generated, and the voter cannot guarantee that it will
be sequentially rational to remove a deviating candidate, non-median candidates and non-converging platforms can be welfare enhancing.

Finally, as the welfare of the voter is increasing in $\alpha$, she benefits when the fraction of candidates in the two parties with polarized preferences increases. In fact, as $\alpha \to 1$, the $(\alpha, \kappa^*)$-Divergence Equilibrium converges to the Divergent Platforms Equilibrium from Section 4. Moreover, Theorem 1 has established that this is the best stationary equilibrium no matter how many different candidate types are available. So, not only does the mechanism underpinning the divergent platforms result apply in two-party elections, but, if the two parties are made up entirely of polarized candidates with the appropriate level of polarization, even if other parties or candidates were available, in the best stationary equilibrium, the voter would only ever elect candidates from those two parties.

6. CONCLUSION

I have presented a model of repeated elections in which a representative voter chooses among heterogeneous candidates of known type in every period. I have shown that, when there is an opportunity for shirking by elected candidates, and utility functions are concave over policies, non-median candidates are elected over median ones in the best stationary equilibrium. Furthermore, in the best stationary equilibrium, there are two candidates, symmetric about the median, who implement non-converging platforms and have non-converging preferences that are ever elected at any history. These predictions are broadly consistent with what is observed in US elections: there are two credible candidates with different ideal points and these candidates implement non-converging platforms that are more moderate than their own ideal point if elected. It is also, to my knowledge, the first model to derive all of these implications.

Models of political competition generally make very stark assumptions: candidates can make binding commitments to any platform or cannot credibly commit to implement any policy other than their ideal point; candidates are exogenously specified and compete by choosing platforms or are endogenous but always implement their ideal policy if elected. Moreover, models that study how elected candidates respond to re-election incentives generally assume that candidates’ types are private information before they take office, and so abstract from the role of elections in determining which candidates will have the opportunity to hold office. This paper bridges the gap between these literatures by studying a model of repeated elections with candidates of known, and heterogeneous, preferences. As such, I incorporate two central elements of elections, the voters’ selection of candidates, and the incumbent’s response to re-election pressures, into the same framework. I consider which platforms candidates would be willing to implement in order to secure re-election, and how this varies with the policy preferences of the candidates. This, in turn, determines which candidates are elected in the best equilibrium. Therefore, by studying the impact of re-election pressures on the behavior of candidates of known type, both the candidates and the platforms are endogenously determined.

Of course, in my analysis, I have made another very stark assumption. While the previous literature on repeated elections has assumed that candidates’ policy preferences are private information, I have assumed instead that voters know the candidates’ preferences with certainty before electing them. The truth is somewhere between these two extremes. While the analysis would be more complicated, it is likely that rich insights could be derived from a setting in which voters observe an informative, but imperfect,
signal about the types of different candidates. First, since candidates of different types would be elected on the equilibrium path, this would likely generate interesting comparative statics about how rent-seeking varies with the ideological positions of the candidates. Moreover, if voters are unsure of the candidates’ policy preferences, candidates would compete to win office, and secure re-election, not through binding policy commitments, but rather by trying to convince voters that they are a desirable type – which, if centrist candidates engage in more rent-seeking, may not mean a type whose preferences are closely aligned with the median voter. Studying what voters learn about candidates from their earlier careers and the electoral process, as well as how re-election incentives influence the incumbent’s choices when voters have partial information about the candidates, is an important avenue for future research.

Another important avenue for future research is to study the predictions of the model empirically. The model predicts that, all else equal, non-median candidates are elected over median ones because they engage in less rent-seeking. This negative relationship between rent-seeking and polarization is apparent across, and in asymmetric, equilibria. However, testing this prediction is complicated because the optimal level of polarization is endogenous, and changes in parameters could affect both the amount of polarization and rent-seeking. So testing the underlying mechanism involves studying whether, for appropriate polarization levels and holding parameters constant, candidates who are ideologically closer to the median voter in their constituency do engage in more rent-seeking, through corruption or by exerting less effort.

A natural place to study this question is in a legislative body such as the House or Senate. Because many legislators are elected simultaneously, it should be possible to control for changes in parameters that would affect both polarization and rent-seeking. Moreover, much work has been done on measuring the ideology of legislators in the United States (e.g., McCarty, Poole and Rosenthal 2006), and there is an empirical literature that measures legislative effort by the amount of discretionary federal funds legislators secure for their constituency (e.g., Snyder and Stromberg 2010). Halberstam and Montagnes (2012) offer some preliminary evidence that moderate politicians exert less effort in office. They argue that more ideologically extreme candidates are elected in Congressional races concurrent with the Presidential election, and that these candidates subsequently secure more discretionary federal funds for their district. This is consistent with the prediction that less centrist candidates work harder on behalf of their constituents. It would be interesting to see the empirical relationship between ideological polarization and electoral accountability explored further, and this paper develops a promising framework to interpret the findings.

APPENDIX A.

Appendix A.1. Non-Stationary Subgame Perfect Equilibria

In the main text I showed that the Divergent Platforms Equilibrium is the best stationary equilibrium. In this section I consider non-stationary subgame perfect equilibria. As expected, with sufficient patience, any feasible voter payoff can be supported in a subgame perfect equilibrium. However, the benefits of delegating to non-median candidates are still apparent in non-stationary equilibria: for intermediate levels of patience, the only equilibria that result in the voter’s ideal platform being implemented in every period along the equilibrium path involve non-median candidates being elected along the equilibrium path.

As is standard in the literature on repeated games (e.g., Abreu 1988) the first step in determining which outcomes can be supported is to determine the harshest possible equilibrium punishment. In order to reduce the number of cases to consider, I assume that $\gamma M > 1$ – that is, the available rents are large
enough that a median candidate would prefer to implement any platform while securing full rents to the median policy with no rents – though the case in which $\gamma M < 1$ is similar.

**Example: The Worst Equilibrium**

Suppose $\delta \geq \frac{\gamma M}{1 + (\gamma + 1)M}$. Then the following strategies constitute a SPE:

At time-0 the voter elects a median candidate, $k_0$, in the first period, and re-elects him as long as he implements a policy $p \in \{-1, 1\}$, and a different candidate of type 0 is elected otherwise. All candidates of type $\kappa \leq 0$ implement platform $(-1, M)$ at all histories in which they are elected, and all candidates of type $\kappa > 0$ implement platform $(1, M)$ at all histories.

Notice that the lowest possible payoff to the voter in any stage game comes from policy $x \in \{-1, 1\}$ and full rent-seeking $m = M$. As this platform is implemented in every period in equilibrium, this is the worst possible equilibrium for the voter. It is also the worst possible equilibrium for any candidate of type $\kappa \geq 0$. I now show that every possible voter payoff is supportable in equilibrium with sufficient patience.

**Example: All Voter Payoffs Possible in Equilibrium with Sufficient Patience**

Suppose $\delta \geq \frac{\gamma M}{1 + (\gamma + 1)M}$. Then for all $u \in \{ u(p) : p \in [-1, 1] \times [0, M] \}$ the following strategies constitute a SPE:

The voter elects a type 0 candidate, $k_0$, in the first period, and re-elects him as long as he implements a policy with $u(p) = u$. Candidate $k_0$ implements $p' = \arg \max \{ g_0(p) : u(p) = u \}$ if he has always been elected and implemented platform $p'$ in the past, and $(-1, M)$ at any other history in which he is elected. Any deviation is punished with reversion to the *Worst Equilibrium*, with a candidate other than $k_0$ elected in the first period. All other candidates implement their strategy from the *Worst Equilibrium*.

Notice that the bound $\delta \geq \frac{\gamma M}{1 + (\gamma + 1)M}$ comes from the minimum patience necessary for type 0 candidates to prefer to implement platform $(0, 0)$ in every period rather than engaging in full rent-seeking in this period with the *Worst Equilibrium* in all future periods. When $\gamma M > 1$, the candidate would have a strict incentive to implement a platform that gives the voter any other payoff. So the voter’s most preferred platform can be supported with median candidates elected along the equilibrium path if and only if $\delta \geq \frac{\gamma M}{1 + (\gamma + 1)M}$. I now show that less patience is required to induce a non-median candidate to implement the voter’s most preferred platform along the equilibrium path. The reason for this is quite simple. If the elected candidate is type $\kappa > 0$, there are greater gains from deviating in terms of policy (from 0 to $\kappa$), but the punishment is harsher if defeated (moving from $x = 0$ to $x = -1$ has a larger impact on a candidate of type $\kappa$ than one of type 0). When $\kappa$ is small, the disciplining effect is first order while the increased benefits from deviating are second order, so less patience is required.

**Example: Advantage of Non-Median Candidates**

Let $\kappa \geq 0$ and define $\delta^*(\kappa)$ to be the minimum $\delta$ such that

$$g_\kappa(0, 0) \geq (1 - \delta)\gamma M + \delta g_\kappa(-1, M).$$

That is, $\delta^*(\kappa)$ is the minimum discounting necessary to support the voter’s most preferred platform every period in an equilibrium with the *Worst Equilibrium* used as punishment. As this is the harshest possible punishment, this determines whether the voter’s most preferred platform can be supported in every period along the equilibrium path with a candidate of type $\kappa$ elected. Note that

$$\delta^*(\kappa) = \frac{\gamma M - g_\kappa(0, 0)}{\gamma M - g_\kappa(-1, M)} = \frac{\gamma M + \kappa}{(\gamma + 1)M + (1 + \kappa)^\gamma}.$$

Moreover, $\delta^*(0) = \frac{\gamma M}{1 + (\gamma + 1)M}$ and

$$\frac{\partial \delta^* (0)}{\partial \kappa} = \frac{-\lambda \gamma M}{(1 + (\gamma + 1)M)^2} < 0.$$

So there exist $\delta < \frac{\gamma M}{1 + (\gamma + 1)M} = \delta^*(0)$ and $\kappa' > 0$ such that the following strategies result in the voter’s most preferred platform implemented in every period:

The voter elects type $\kappa'$ candidate $k_0$ in the first period, and re-elects him as long as he implements $p_t = (0, 0)$. Candidate $k_0$ implements $(0, 0)$ if he has always been elected and implemented platform $(0, 0)$ in the past, and $(1, M)$ at any other history in which he is elected. Any deviation is punished
with reversion to the Worst Equilibrium. All other candidates implement their strategy from the Worst Equilibrium.

Therefore, with sufficient patience, it is possible to support any feasible payoff to the voter. However, the highest feasible voter payoff can be supported for a greater range of $\delta$ when non-median candidates are elected on the equilibrium path.

Appendix A.2. Extension: Imperfect Monitoring and Citizen Candidates

In the baseline model, I studied whom the representative voter would elect if she had all candidates available to choose from, and, in Section 5, I applied the underlying mechanism to two-party elections. In this section I consider a repeated citizen-candidate framework (Besley and Coate 1997, Osborne and Slivinski 1996) in which individuals must pay a cost to run for office. As such, while the voter must choose from a subset of the potential candidates, this subset is determined by the candidates’ decisions of whether to run for office. Of course, with pure strategies and complete and perfect information, as is assumed in the baseline model, there is no source of randomness in outcomes, and candidates who are certain to lose would have no reason to pay the cost of running for office. In this section I allow the voter’s utility to be non-deterministic, so, while more moderate policies and less rent-seeking lead to higher utility, on average, there are still shocks (such as shocks to the economy) that affect the voter’s utility. I assume the voter does not observe the platform the candidate implemented but only the utility she derived from it. That is, the voter observes

$$u_t = u(p_t) + \epsilon_t,$$

where $\epsilon_t$ is an i.i.d. draw from some distribution. I show that, when the voter’s utility is non-deterministic, and the costs of running for office are sufficiently small, there will be two candidates who run for office and are elected with positive probability in every period. So, unlike in the baseline model, the incumbent is defeated with positive probability along the equilibrium path.

In period-0, I assume that the voter can elect any candidate in $[-1,1] \times N$, and in all subsequent periods she must choose among the candidates who decided to run for office. The voter can only elect candidates who have run for office, and, to guarantee that at least one candidate runs at all histories, I assume that if no candidate runs for office in the current period all players receive payoff of $-\infty$. I denote by $K_t$ the set of candidates who run for office in period $t$.

In each period that a candidate runs for office he must pay a cost $c > 0$. I assume that candidates who have run for office, and, to guarantee that at least one candidate runs at all histories, I assume that if no candidate runs for office in the current period all players receive payoff of $-\infty$. I denote by $K_t$ the set of candidates who run for office in period $t$.

In order to maintain the following order of play in each stage game:

1. The voter selects candidate $k_t \in K_t$, where $K_0 \equiv [-1,1] \times N$.
2. The elected candidate, $k_t$, implements platform $p_t \in [-1,1] \times [0,M]$. All candidates simultaneously decide whether or not to run for office in the next period, $r^*_k \in \{R,N\}$, with $K_{t+1}$ the candidates who decide to run.
3. The noise $\epsilon_t$ is realized and $u_t = u(p_t) + \epsilon_t$ is publicly observed.
4. This game is repeated with the voter deciding which candidate in $K_{t+1}$ to elect.

Histories and strategies are defined as before, with a few exceptions. First, the voter only observes utility levels but not platforms, $h^t = ((k_0,u_0,K_1),\ldots,(k_{t-1},u_{t-1},K_t))$, and can only elect a candidate who has run for office, $s_t(h^t) \in K_t$. The candidates observe the public history as well as the previous platforms implemented, $p^t = (p_0,\ldots,p_{t-1})$, and candidates’ strategies, $s_t = (s^*_1,\ldots,s^*_t)$, consist of the decision of whether or not to run at each history, $s^*_k(h^t,p^t,k) \in \{R,N\}$, and which platform to implement if elected, $s^*_k(h^t,p^t,k) \in [-1,1] \times [0,M]$. Candidate payoffs are the same as before except that they must pay the cost of running for office, $c$.

As in the baseline model I restrict attention to equilibria in which the candidates’ strategies, and the voter’s response to the utility she receives from the implemented platform, are independent of history. Note that, although the voter imperfectly observes the candidate’s action, as candidates’ types are public information, the voter does not update her beliefs about the candidate’s type over his tenure in office.
As such, this is a stationary environment and it makes sense to focus on stationary equilibria. Unlike the baseline model, I restrict attention to equilibria in which the same two candidates run for office at every history. By assuming that the same candidates run for office in every period the incumbent knows that, even if he is defeated, he will still pay the cost of running for office in future periods. So, provided the costs are small enough, they do not affect the equilibrium platforms. It also means that, in an equilibrium with positive turnover, a defeated candidate will be elected again with positive probability. I look for two-candidate stationary pure strategy public perfect equilibria in which these candidates’ types are symmetric about the median.

**Definition A4. (Stationary, Symmetric, Two-Candidate Public Perfect Equilibrium)** A public perfect equilibrium, \( \pi^* \), is a stationary, symmetric, two-candidate equilibrium if, for all histories \( h^t \) and \( p^t \),

1. the candidate the voter elects depends only on the candidate elected in the last period, \( k_{t-1} \), the utility received, \( u_{t-1} \), and the candidates available, \( K_t \). That is, there exists \( s_t \), such that \( s_t^* (h^t) = s_t (k_{t-1}, u_{t-1}, K_t) \).
2. there are two candidates, \( k_1 \) and \( k_2 \), of types \( \kappa \geq 0 \) and \( -\kappa \), who run for office at every history, and no other candidate runs for office at any history. Moreover, for candidates \( k \in \{k_1, k_2\} \), \( s_t^* (h^t, p^t, k) \) is a constant function.

If the candidates are to have an incentive to implement a platform other than their ideal point, the voter must still condition her decision on the outcome in the previous period. Consequently, the incumbent is rewarded for luck (Achen and Bartels 2004, Wolters 2007), and, depending on the realization of the noise, may be defeated. In order to make the analysis tractable, and to parameterize the noise for comparative statics, I assume \( \varepsilon_t \) has support \([-\beta, \beta] \) with density function

\[
f(y) = F^t(y) = \begin{cases} \frac{\beta - |y|}{\beta^2}, & \text{if } y \in [-\beta, \beta], \\ 0, & \text{otherwise.} \end{cases}
\]

Notice that \( \varepsilon_t \) is a continuous, symmetric, mean-0 random variable, and that the probability of observing utility level \( u_t \), given implemented platform \( p_t \), is decreasing in \( |u_t - p_t| \). The parameter \( \beta \) determines the amount of noise in policy-making, with higher \( \beta \) reflecting greater randomness. That \( \varepsilon_t \) does not have full support not only simplifies the analysis but also biases against generating turnover in equilibrium. If the noise had full support then, regardless of the platform implemented and the standard for re-election, the incumbent would be defeated with positive probability.

I now turn to describing the best stationary, symmetric two-candidate equilibria with positive noise. Since the voter observes only the utility she receives, in equilibrium, the elected candidate’s platform will trade off rents and policy in the same ratio for both the candidate and the voter. So, when \( \kappa \) and \( \beta \) are not too large, because rents enter utilities linearly, the implemented policies will be the same as in the perfect monitoring case, \( x_{e} \) and \( -x_{e} \). The lowest level of rent-seeking that can be supported with candidates \( \kappa \) and \( -\kappa \) can then be determined as in the baseline model. When \( \delta \geq \delta \) the voter’s welfare is increasing in \( \kappa \) provided the noise is not too large. As in the perfect monitoring case, the welfare of the voter increases until the rents are driven to 0 or until the constraint \( \kappa \leq 1 \) binds. I then define \( \kappa^\ast \) to be the unique degree of polarization that maximizes the voter’s utility, with \( (x_{e}^\ast, m_{e}^\ast) \) the associated platform and \( u_{e}^\ast = u(x_{e}^\ast, m_{e}^\ast) \) the associated utility. I define the **Divergent Platforms Equilibrium** with imperfect monitoring to be the equilibrium with the optimal amount of divergence and the voter re-electing the incumbent if and only if her utility is above some threshold level. For the voter to have this choice, of course, it is necessary that the costs of running are low enough that both candidates are incentivized to run for office rather than concede the election to the other candidate. I now show that the **Divergent Platforms Equilibrium** is the best stationary, symmetric, two-candidate public perfect equilibrium.
Theorem A6. (Divergent Platforms with Imperfect Monitoring) For all \( \delta \geq \bar{\delta} \) there exists \( \bar{\beta}(\delta) > 0 \) such that, for all \( \beta \in (0, \bar{\beta}(\delta)) \), there exists \( \bar{c}_\beta > 0 \), and a unique \( \kappa^*_\beta > 0 \), such that, for all \( c \in (0, \bar{c}_\beta) \), the best stationary, symmetric, two-candidate equilibrium consists of candidates of type \( \kappa^*_\beta \) and \( -\kappa^*_\beta \), gives the voter utility \( u^*_\beta = u(x^*_\beta, m^*_\beta) \), and involves re-election probability \( q^*_\beta \in (1/2, 1) \). This can be achieved in an equilibrium in which the voter re-elects the incumbent if and only if \( u \geq \bar{u}_\beta \), where \( q^*_\beta = F(u^*_\beta - \bar{u}_\beta) \). Moreover,

1. greater noise results in lower welfare and more turnover,
\[
\frac{\partial u^*_\beta}{\partial \beta} < 0; \quad \frac{\partial q^*_\beta}{\partial \beta} < 0.
\]
2. if \( c = 0 \), the equilibrium converges to the perfect monitoring case as the noise disappears,
\[
\lim_{\beta \to 0} (x^*_\beta, m^*_\beta) = (x^*, m^*); \quad \lim_{\beta \to 0} \kappa^*_\beta = \kappa^*; \quad \lim_{\beta \to 0} q^*_\beta = 1.
\]

The proof is included in the Supplemental Appendix. This Theorem shows that there is a unique degree of polarization, \( \kappa^*_\beta > 0 \), in candidate preferences that maximizes voter welfare.\(^{17}\) Moreover, these candidates are incentivized to implement the most desirable platforms if the voter uses a “retrospective voting” strategy of re-electing the candidate if and only if she receives utility at least \( \bar{u}_\beta \) from the implemented platform. In addition, there is an incumbency advantage, but one that is less extreme than with perfect monitoring. Since the density of the noise goes to 0 continuously, whatever the standard for re-election, the elected candidate will implement a platform that results in a positive probability of being defeated. As re-election is uncertain, provided the costs of running are not too high (\( c \in [0, \bar{c}_\beta] \)), both candidates have an incentive to run for office in each period.

In addition, part (1) shows that higher levels of noise lead to worse platforms being implemented and a higher probability of the incumbent being defeated. As the probability that the incumbent is defeated falls when there is less noise, the costs of running must also be lower for the challenger to run. In the limit, running for office must be costless.\(^{18}\) Part (2) shows that, as the noise and cost of running disappear, this equilibrium converges to the equilibrium with perfect monitoring. As such, although this section only considered equilibria with two symmetric candidates, as the noise disappears, the Divergent Platforms Equilibrium converges to the best stationary equilibrium of the perfect monitoring game.

Appendix A.3. \( \kappa^* \) and \( x^* \)

I now define the degree of divergence in candidate preferences and policies, \( \kappa^* \) and \( x^* \), used in the construction of the Divergent Platforms Equilibrium. For any \( \kappa \geq \bar{\kappa} \), define \( x(\kappa) \) to be the unique solution to
\[
(1 - \delta)M - \delta(\kappa + x(\kappa))^\lambda = -(\kappa - x(\kappa))^\lambda.
\]
Differentiating with respect to \( \kappa \) and re-arranging implies that
\[
\delta(\kappa + x(\kappa))^\lambda - 1 + (\kappa - x(\kappa))^\lambda - 1|x'(\kappa) = (\kappa - x(\kappa))^\lambda - 1 - \delta(\kappa + x(\kappa))^\lambda - 1.
\]
Therefore, \( x'(\kappa) = 0 \) if and only if
\[
\delta(\kappa + x(\kappa))^\lambda - 1 = (\kappa - x(\kappa))^\lambda - 1,
\]
or equivalently
\[
x(\kappa) = \frac{1 - \delta}{1 + \delta} \bar{\kappa}.
\]

\(^{17}\)While the degree of polarization is unique, there is non-uniqueness in whether a type \( \kappa^*_\beta \) or \( -\kappa^*_\beta \) candidate is elected in the initial period and, in each subsequent period, after a set of utility levels that occur with probability 0 along the equilibrium path are realized.

\(^{18}\)If there is a positive cost to running, there must be positive turnover in equilibrium in order for the challenger to be incentivized to run for office. The best stationary two-candidate equilibrium, when \( c > \bar{c}_\beta \), involves the minimum turnover necessary to induce both candidates to run.
Plugging this into the original expression for \( x(\kappa) \) implies that
\[
(1 - \delta)\gamma M = \frac{2\delta(1 - \delta^{1/\gamma})}{(1 - \delta^{1/\gamma})^\lambda} \kappa^\lambda,
\]
which has a unique solution,
\[
\kappa^{**} = \frac{1 + \delta^{1/\gamma}}{2} \left( \frac{(1 - \delta)\gamma M}{\delta(1 - \delta^{1/\gamma})} \right)^{1/\lambda}.
\]
Further, differentiating with respect to \( \kappa \) again,
\[
[\delta(\kappa + x(\kappa))^{\lambda - 1} + (\kappa - x(\kappa))^{\lambda - 1}]x''(\kappa) = (\lambda - 1)[(\kappa - x(\kappa))^{\lambda - 2} - \delta(\kappa + x(\kappa))^{\lambda - 2}] > 0,
\]
when \( \kappa = \kappa^{**} \). So \( x''(\kappa^{**}) > 0 \) and \( \kappa^{**} \) is a local minimum. Combining these two observations, it follows that \( x(\kappa) \) is decreasing for \( \kappa < \kappa^{**} \) and increasing for \( \kappa > \kappa^{**} \). Hence there exists a unique minimizer,
\[
\kappa^* = \arg \min_{\kappa \in [\tilde{\kappa}, 1]} x(\kappa).
\]

Appendix A.4. Proofs

Before proceeding to the main results I show a simple lemma: any candidate not of type \( \kappa^* \) would strictly prefer to implement his ideal policy while securing full rents in the current period with \((-x^*, m^*)\) being implemented in every future period to implementing any platform that gives the voter utility \( u^* \) in every period.

**Lemma A1.** If \( \delta \geq \delta \) then for all \( \kappa \neq \kappa^* \) and any \((x, m)\) such that \( u(x, m) \geq u^* \),
\[
\tilde{g}_e(x, m) < (1 - \delta)\gamma M + \delta g_e(-x^*, m^*).
\]

**Proof.** There are three cases to consider: candidates of type \( \kappa < 0 \), \( \kappa \in [0, \min\{\tilde{\kappa}, 1\}] \), and \( \kappa \in [\tilde{\kappa}, 1]\backslash\{\kappa^*\} \).

First consider \( \kappa \in [\tilde{\kappa}, 1]\backslash\{\kappa^*\} \). If this set is non-empty, this implies that \( \kappa^* \geq \tilde{\kappa} \) and so \( m^* = 0 \).

Further, candidates of type \( \kappa \) are indifferent between implementing platform \((x, 0)\) in every period and implementing their ideal point with full rent-seeking today and having \((-x(\kappa), 0)\) implemented in all future periods. In addition, as \( x(\kappa) \) has a unique minimizer,
\[
x^* < x(\kappa) \leq \frac{\kappa}{1 + \gamma^{1/\gamma}},
\]
from the definition of \( \tilde{\kappa} \). So for candidates of type \( \kappa \), for any platform \((x, m)\) with \( u(x, m) \geq u(x(\kappa), 0) \) it follows that
\[
\tilde{g}_e(x, m) \leq \tilde{g}_e(x(k), 0) = (1 - \delta)\gamma M + \delta g_e(-x(\kappa), 0) < (1 - \delta)\gamma M + \delta g_e(-x^*, m^*).
\]

As \( u(x(\kappa), 0) < u^* \), this completes the proof for all \( \kappa \in [\tilde{\kappa}, 1]\backslash\{\kappa^*\} \).

Now consider \( \kappa \in [0, \min\{\tilde{\kappa}, 1\}] \). I now consider the most desirable platform, from the voter’s perspective, that candidate \( \kappa \) would be willing to implement. Note that the payoff from the most desirable platform can be no higher than the payoff from policy \( x_\kappa = \kappa/(1 + \gamma^{1/\gamma}) \) and the associated level of rents given by
\[
\tilde{g}_e(x_\kappa, m_\kappa) = \gamma m_\kappa - (\kappa - x_\kappa)^\lambda = (1 - \delta)\gamma M - \delta (m^* + (\kappa + x^*)^\lambda) = (1 - \delta)\gamma M - \delta g_e(-x^*, m^*).
\]

Defining \( w(\kappa) = -m_\kappa - x_\kappa^\lambda \) to be the welfare to the voter from a type-\( \kappa \) candidate, I must show that
\[
w(\kappa) = -(1 - \delta)M + \frac{1}{\gamma}[\delta((\kappa + x^*)^\lambda + m^*) - (1 + \gamma^{1/\gamma})^{1-\lambda}\kappa^\lambda] \]
is strictly less than \( u^* \). First consider the case where \( \tilde{\kappa} \geq 1 \). Then \( \kappa^* = 1 \) and \( x^* = 1/(1 + \gamma^{1/\gamma}) \).

Therefore
\[
w'(\kappa) = \frac{\lambda}{\gamma}[\delta((\kappa + x^*)^{\lambda - 1} - (1 + \gamma^{1/\gamma})^{1-\lambda}\kappa^{\lambda - 1}].
\]
and as 
\[ \delta \geq \delta = (1 + 2\gamma \frac{1}{1 + \lambda})^{1 - \lambda} = (1 + \gamma \frac{1}{1 + \lambda})^{1 - \lambda}(\frac{\kappa}{\kappa + \frac{1}{1 + \lambda}})^{\lambda - 1} > (1 + \gamma \frac{1}{1 + \lambda})^{1 - \lambda}(\frac{\kappa}{\kappa + x^*})^{\lambda - 1}, \]
this implies that \( w(\kappa) \) is increasing on the interval \([0, 1)\). Thus, as \( w(1) = u^* \) when \( \kappa \geq 1 \), it follows that \( w(\kappa) < u^* \) for the interval \( \kappa \in [0, \min\{\bar{\kappa}, 1\}) \).

Now consider the case where \( \bar{\kappa} < 1 \). Then, from the definition of \( x^* \), it is immediate that 
\[ x^* \leq x(\kappa) = \frac{\bar{\kappa}}{1 + \gamma \lambda}, \]
and \( m^* = 0 \). Then
\[ w(\kappa) - u^* = -(1 - \delta)\gamma^{-1} + \frac{1}{1 + \lambda} - (1 + \gamma \frac{1}{1 + \lambda})^{1 - \lambda} \kappa \lambda. \]

As the previous analysis established that \( w(\kappa) - u^* < 0 \) when \( x^* = \frac{\kappa}{1 + \gamma \lambda} \), and \( w(\kappa) - u^* \) is clearly increasing in \( x^* \), it follows that \( w(\kappa) - u^* < 0 \) for all \( \kappa \in [0, \bar{\kappa}) \) when \( \bar{\kappa} < 1 \). Hence, for all \( \kappa \in [0, \min\{\bar{\kappa}, 1\}) \), \( w(\kappa) < u^* \) and so
\[ g_\kappa(x, m) < (1 - \delta)\gamma M + g_\kappa(-x^*, m^*). \]

Finally, note that for all \( \kappa < 0 \) the payoff from platform \((-x^*, m^*) \) in future periods is strictly higher than for candidate \(-\kappa > 0 \). Hence, as candidate \(-\kappa \) does not strictly prefer any platform which gives the voter \( u^* \) to his ideal point today followed by \((-x^*, m^*) \) in future periods, candidates of type \( \kappa \) must strictly prefer to implement their ideal point.

Clearly, by symmetry, Lemma 1 also implies that no type other than \(-\kappa^* \) is willing to implement a platform that gives the voter utility \( u^* \) if the punishment is that \((x^*, m^*) \) would be implemented in all subsequent periods. I now prove Theorem 1.

**Proof of Theorem 1.** Part (1). I proceed by contradiction. Suppose there exists a stationary SPE that gives the voter utility \( u' > u^* \). Let \( s^* \) be a SPE such that \( U(s^*) = u' \). By stationarity, the voter has the same options available to her at all histories, and so must receive the same payoff, \( u' \) in every period regardless of the history. Without loss of generality, assume the candidate elected in the first period, \( k_0 \), is of type \( \kappa_0 \geq 0 \). Now consider the continuation equilibrium after that candidate deviated to his ideal policy with maximal rent-seeking, \( s^*[k_0, \kappa_0, M] \). Note that in this continuation equilibrium the voter receives utility \( u \) in all periods, and it is without loss of generality to restrict attention to equilibria in which, if \( u(p_0) = u' \), candidate \( k_0 \) is re-elected. Recall, by Lemma 1, that no candidate would be willing to implement a platform that gives the voter utility \( u' \) if the outcome path from the continuation equilibrium if not re-elected has \((-x^*, m^*) \) implemented in all future periods. Thus, the continuation equilibrium must give candidate \( k_0 \) a payoff strictly lower than \((-x^*, m^*) \). Hence, there exists a platform \((x^1, m^1) \) implemented along the equilibrium path of \( s^*[k_0, \kappa_0, M] \) that gives
\[ g_{x_0}(x^1, m^1) < g_{x_0}(-x^*, m^*). \]

Combined with the fact that
\[ u(x^1, m^1) = u' > u^* = u(x^*, m^*), \]
it follows that \( x^1 < -x^* \) and \( m^1 < m^* \). This clearly implies that \( m^* > 0 \) and so \( \kappa^* = 1 \) and \( x^* = \frac{\lambda}{1 + \gamma \lambda} \). Now let \( k_1 \) be the candidate who implements platform \((x^1, m^1) \) and let \( \kappa_1 \) be his type. Since candidate \( k_1 \) is evaluated strictly on the utility he provides to the voter, he must select a platform which trades off policy and rent-seeking in the same ratio as the voter when \( m^1 < M \). However, because \( \kappa_1 \in [-1, 1] \),
\[ |x^1| \leq \frac{|\kappa_1|}{1 + \gamma \lambda} \leq \frac{1}{1 + \gamma \lambda}, \]
Hence, it is not possible to have \( x^1 < -x^* \), which generates a contradiction. Therefore, there cannot exist a stationary SPE which gives the voter utility greater than \( u^* \).

**Part (2).** Now consider the SPEs that give the voter utility \( u^* \). Modifying the above argument to reflect that now \( g_{x_0}(x^1, m^1) \leq g_{x_0}(-x^*, m^*) \) and \( u(x^1, m^1) = u(-x^*, m^*) \), implies that \( x^1 \leq -x^* \) and \( m^1 \leq m^* \). Similarly, as above, \( x^1 < -x^* \) can be ruled out, and so \((x^1, m^1) = (-x^*, m^*) \). Hence, by
Lemma 1, it follows that \( \kappa_0 = \kappa^* \) is the only candidate with \( \kappa_0 \geq 0 \) who could be elected, and, by symmetry, \(-\kappa^*\) is the only candidate with \( \kappa < 0 \) who could be elected. Therefore, no type \( \kappa \notin \{-\kappa^*, \kappa^*\} \) is elected at any history of the equilibrium. Finally, as all players’ utility functions are strictly concave with respect to policy, if a candidate of type \( \kappa^* \) implements a platform other than \((x^*, m^*)\), or a type \(-\kappa^*\) candidate implements a platform other than \((-x^*, m^*)\), this platform must give the voter utility less than \( u^* \) and that candidate will never be elected. ■

Proof of Theorem 2. I first show that \( \bar{\delta} \) goes to 0 as \( \lambda \to \infty \). Note first that \( \log \bar{\delta} = (1 - \lambda) \log(1 + 2\gamma \sigma \theta) \), and

\[
\frac{\partial \log \bar{\delta}}{\partial \gamma} = \frac{2\gamma \sigma \theta}{1 + 2\gamma \theta} > 0,
\]

so \( \bar{\delta} \) is increasing in \( \gamma \), and it is sufficient to show that \( \bar{\delta} \) goes to 0 when \( \gamma \geq 1 \). Also, when \( \lambda \geq 2 \), \( \gamma \sigma \theta \geq \gamma^{-1} \) and so

\[
0 < \bar{\delta} < (1 + 2\gamma^{-1})^{1-\lambda}.
\]

As \( (1 + 2\gamma^{-1})^{1-\lambda} \) clearly goes to 0 as \( \lambda \to \infty \), it follows that, for all \( \gamma > 0 \),

\[
\lim_{\lambda \to \infty} \bar{\delta} = 0.
\]

Now consider the case in which \( \lambda = 1 \) and \( \gamma \geq 1 \). Then, since the candidate is evaluated by the utility the voter receives, in any equilibrium in which the voter’s welfare is higher than \( u' > -M \), \((x, m) = (0, u')\) is a best response for any candidate elected at any history in equilibrium. Further, for any platform \((x, m)\),

\[
u(x, m) = -|x| - m = u(0, |x| + m).
\]

and for all \( \kappa \),

\[
g_\kappa(x, m) \geq g_\kappa(0, |x| + m).
\]

It is therefore without loss of generality to restrict attention to equilibria in which the policy is \( x = 0 \) in every period. Then, for any \( \delta \in (0, 1) \), the lowest level of rent-seeking that can be supported in a equilibrium with a candidate of type \( \kappa \) elected is determined from,

\[
\bar{\delta} = \gamma m - |\kappa| = (1 - \delta)\gamma M - \delta (m + |\kappa|).
\]

Thus,

\[
m^*_\kappa = \frac{(1 - \delta)\gamma M + |\kappa|}{\gamma + \delta},
\]

which is increasing in \( |\kappa| \) and equal to \( m_0 \) when \( \kappa = 0 \). Hence, when \( \lambda = 1 \) and \( \gamma \geq 1 \), for any \( \delta \in (0, 1) \), the best stationary equilibrium is the Median Candidates Equilibrium. ■

Proof of Theorem 3. I begin by calculating some comparative statics on \( \kappa^{**} \) and \( x^{**} \). The equations

\[
x^{**} = \frac{1 - \delta}{1 + \delta} \frac{\kappa^{**}}{1 + \delta} x^{**},
\]

\[
\kappa^{**} = \frac{1 + \delta^{1/\gamma} ((1 - \delta)\gamma M)^{1/\gamma}}{2 (1 - \delta)^{1/\gamma}},
\]

immediately imply that

\[
\frac{\partial \kappa^{**}}{\partial M} > 0, \quad \frac{\partial x^{**}}{\partial M} > 0,
\]

and

\[
\frac{\partial \kappa^{**}}{\partial \gamma} > 0, \quad \frac{\partial x^{**}}{\partial \gamma} > 0.
\]
I now show that $\kappa^{**}$ is strictly decreasing in $\delta$, which will immediately imply that $x^{**}$ is also strictly decreasing. Since

$$(\kappa^{**})^\lambda = \frac{(1 + \delta \frac{1}{\lambda})^\lambda}{2\lambda} \frac{(1 - \delta)\gamma M}{\delta(1 - \delta \frac{1}{\lambda})},$$

it is sufficient to show that

$$\frac{(1 - \delta)(1 + \delta \frac{1}{\lambda})^\lambda}{\delta(1 - \delta \frac{1}{\lambda})}$$

is decreasing in $\delta$. Define $\rho = \delta \frac{1}{\lambda}$. Taking the log and differentiating with respect to $\rho$ gives

$$\frac{\rho(1 - \delta - \lambda)}{1 - \rho} + \frac{1}{1 - \rho} = \frac{(1 - \delta - \lambda)\rho}{\rho(1 - \rho^2)}.$$ 

This means that $\kappa^{**}$ is decreasing if

$$f(\rho) \equiv (\rho - 1)(1 - \rho^2) - \rho(\lambda + 1)(1 - \rho^2) > 0.$$ 

Note that $f(1) = 0$, and for all $\rho \in (0, 1)$,

$$f'(\rho) = (\rho + 1)[(\lambda - 1)\rho^2 - 1 + \lambda \rho^2] < 0,$$

as $-(\lambda - 1)\rho^2 - 1 + \lambda \rho^2$ increases in $\rho$ and equal to 0 when $\rho = 1$. So $f(\rho) > 0$ for all $\rho < 1$ and it follows that $\kappa^{**}$ is decreasing in $\delta$. Therefore

$$\frac{\partial \kappa^{**}}{\partial \delta} < 0, \quad \frac{\partial x^{**}}{\partial \delta} < 0.$$

Thus, $k^{**}$ and $x^{**}$ are increasing in $M$ and $\gamma$ and decreasing in $\delta$. Finally, since $\kappa^* < 1$ it follows that $\kappa^* = \kappa^{**}$ and $x^* = x^{**}$, which completes the proof.

**Proof of Theorem 4.** I now consider the limit as $\delta \to 1$. Since $\lim_{\delta \to 1} \bar{\kappa} = 0$, it follows when $\delta$ is close to 1 that $\kappa^* = \min\{\kappa^{**}, 1\}$, $m^* = 0$, and the policy is $x^* = x^{**}$ if $\kappa^{**} < 1$ and $x^* = x(1)$ otherwise. Recall that

$$(\kappa^{**})^\lambda = \frac{\gamma M[(1 - \delta)(1 + \delta \frac{1}{\lambda})^\lambda]}{2\delta \lambda(1 - \delta \frac{1}{\lambda})},$$

and since the top and bottom of the fraction both converge to 0 as $\delta$ goes to 1, applying L’Hopital’s rule implies that

$$\lim_{\delta \to 1} (\kappa^{**})^\lambda = \lim_{\delta \to 1} \frac{\gamma M[(1 + \delta \frac{1}{\lambda})^\lambda + (1 - \delta)(1 + \delta \frac{1}{\lambda})^\lambda - (1 + \delta \frac{1}{\lambda})^\lambda]}{\lambda - 1} = \gamma M(\lambda - 1).$$

Consequently,

$$\lim_{\delta \to 1} \kappa^* = \lim_{\delta \to 1} \min\{\kappa^{**}, 1\} = \min\{\gamma M(\lambda - 1)\frac{1}{\lambda} - 1\}.$$ 

Next, consider the limiting policy, $x^*$. First note that $x^* \leq x(1)$ and $x(1)$ solves

$$-(1 - x(1))^\lambda = (1 - \delta)\gamma M - \delta(1 + x(1))^\lambda,$$

so $x^*$ obviously goes to 0 as $\delta$ goes to 1. More interestingly, dividing by $m_0 = (1 - \delta)\gamma M/(\gamma + \delta)$ implies that

$$-(m_0^{-1/\lambda} - x(1)m_0^{-1/\lambda})^\lambda = \gamma + \delta - \delta(m_0^{-1/\lambda} + x(1)m_0^{-1/\lambda})^\lambda.$$

Re-arranging this equation, it follows that

$$\delta[(m_0^{-1/\lambda} + x(1)m_0^{-1/\lambda})^\lambda - (m_0^{-1/\lambda} - x(1)m_0^{-1/\lambda})^\lambda] \leq \frac{(\gamma + \delta)(\gamma M + 1)}{\gamma M},$$

and so $\lim_{\delta \to 1}[(m_0^{-1/\lambda} + x(1)m_0^{-1/\lambda})^\lambda - (m_0^{-1/\lambda} - x(1)m_0^{-1/\lambda})^\lambda]$ is finite. Given that $m_0^{-1/\lambda}$ goes to $\infty$ as $\delta$ goes to 1, this implies that $x(1)m_0^{-1/\lambda}$ goes to 0. Therefore,

$$\lim_{\delta \to 1} \frac{u(0, x) - u^*}{u(0, m_0) - u(0, m_0)} = \lim_{\delta \to 1} \frac{(x^*)^\lambda}{m_0} = 0.$$
\[
\delta \text{ which is finite. Moreover, since } \delta > 0. \text{ It is sufficient to show that } \lim_{x \to \infty} \delta \frac{1}{x} = 0. \text{ As this is clearly less than 1, I restrict attention to } \delta \text{ for which the best equilibrium involves } m_* = 0 \text{ and solve for the minimum divergence necessary to maintain incentives with no rent-seeking, } x(\kappa). \text{ Note that } x(\kappa) \text{ solves }
\]

\[
-\{1 - (1 - \alpha)\delta\}(\kappa - x(\kappa)) = (1 - \delta)\gamma M - \delta\alpha(\kappa + x(\kappa))^\gamma,
\]

and so

\[
\delta\alpha[\gamma M - \delta\alpha(\kappa + x(\kappa))^\gamma] = (1 - \delta)\gamma M + (1 - \delta)(\kappa - x(\kappa))^\gamma < (1 - \delta)(\gamma M + \kappa^\gamma).
\]

Now note that, dividing by \(m_0 = (1 - \delta)\gamma M/\gamma\) this implies that

\[
\delta\alpha[(\kappa m_0)^\frac{1}{\gamma} + m_0^{-\frac{1}{\gamma}}x(\kappa)]^\gamma - (\kappa m_0)^\frac{1}{\gamma}m_0^{-\frac{1}{\gamma}}x(\kappa))^\gamma < \frac{(\gamma M + \kappa^\gamma)(\gamma + \delta)}{\gamma M}.
\]

Taking limits, this means that

\[
\lim_{\delta \to 0} \left[\left(\frac{(\kappa m_0)^\frac{1}{\gamma} + m_0^{-\frac{1}{\gamma}}x(\kappa)]^\gamma - (\kappa m_0)^\frac{1}{\gamma}m_0^{-\frac{1}{\gamma}}x(\kappa))^\gamma}{\gamma M}\right]\right] = \frac{(\gamma M + \kappa^\gamma)(\gamma + 1)}{\gamma M} ,
\]

which is finite. Moreover, since \(\kappa m_0^{-\frac{1}{\gamma}}\) goes to \(\infty\), \(x(\kappa)m_0^{-\frac{1}{\gamma}}\) goes to 0, so it follows that

\[
\lim_{\delta \to 0} \frac{x(\kappa)^\gamma}{m_0} = 0.
\]

As the welfare in the \((\alpha, \kappa)\)-Divergence Equilibrium is \(-x(\kappa)^\gamma\) when \(\delta \geq \delta(\alpha, \kappa)\), I conclude that there exists \(\delta(\alpha, \kappa) \in (0, 1)\) such that, for all \(\delta > \delta(\alpha, \kappa)\), the welfare in the \((\alpha, \kappa)\)-Divergence Equilibrium, is higher than in the Median Candidates Equilibrium.

I conclude by showing that, for all \(\alpha \in (0, 1)\), there exists a \(\kappa_\alpha \in (0, 1)\) such that \(\lim_{\lambda \to \infty} \delta(\alpha, \kappa_\alpha) = 0\). It is sufficient to show that \(\lim_{\lambda \to \infty} \delta(\alpha, 1) = 0\) for all \(\alpha \in (0, 1)\). Note that, for all strictly positive \(\alpha, \delta, \) and \(x\),

\[
\lim_{\lambda \to \infty} \delta\alpha(1 + x)^\gamma = \infty.
\]

It then follows that

\[
\lim_{\lambda \to \infty} x(1) = 0.
\]
As the voter’s utility from the Median Candidates Equilibrium is $u(0, m_0) < 0$ which is constant in $\lambda$, but the utility from $\kappa = 1$ candidates goes to 0 as $\lambda$ gets large, it follows that, for all $\alpha \in (0, 1)$,

$$\lim_{\lambda \to \infty} \bar{\delta}(\alpha, 1) = 0.$$ 

Appendix A.5. Examples of $\bar{\delta}$

The following table shows $\bar{\delta}$ for different values of $\gamma$ and $\lambda$. Provided that $\lambda$ is reasonably large, the lower bound on $\delta$ is not very restrictive.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\lambda$</th>
</tr>
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<tbody>
<tr>
<td>0.2</td>
<td>2, 2.5, 3, 3.5, 4, 5</td>
</tr>
<tr>
<td>0.5</td>
<td>0.20, 0.12, 0.07, 0.04, 0.02, 0.01</td>
</tr>
<tr>
<td>1</td>
<td>0.33, 0.19, 0.11, 0.06, 0.04, 0.01</td>
</tr>
<tr>
<td>5</td>
<td>0.71, 0.46, 0.28, 0.17, 0.10, 0.03</td>
</tr>
<tr>
<td>10</td>
<td>0.83, 0.58, 0.38, 0.23, 0.14, 0.05</td>
</tr>
<tr>
<td>50</td>
<td>0.96, 0.81, 0.61, 0.42, 0.27, 0.11</td>
</tr>
<tr>
<td>100</td>
<td>0.98, 0.88, 0.69, 0.50, 0.34, 0.14</td>
</tr>
<tr>
<td>500</td>
<td>0.99+, 0.95, 0.84, 0.68, 0.51, 0.24</td>
</tr>
</tbody>
</table>

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