Should day care be subsidized?

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In an economy with distortionary taxes on labour, can subsidies on day care, financed by increased taxes, raise welfare by encouraging women with small children to work? We show, within a stylized life-cycle framework, that the Ramsey optimal policy consists in equalizing consumption/leisure wedges over the life cycle. A simple way to implement this is to make day care expenses tax deductible. Modifying and calibrating our model to fit some key facts about labour supply in Germany, we find that the reform that maximizes a distribution-neutral social welfare function involves subsidizing day care at a rate of 50 percent and leads to a near doubling of labour supply for mothers with small children. The overall welfare gain from this reform corresponds to a 0.4 percent increase in consumption.

Keywords: Female labour force participation, Germany, day care, fiscal policy

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1. INTRODUCTION

In both Europe and North America, government support for preschool day care is an issue high on the political agenda. At its Barcelona meeting in 2002, the European Union called on member countries to “remove disincentives to female labour force participation and strive . . . to provide child care by 2010 to at least 90% of children between 3 years old and the mandatory school age and at least 33% of children under 3 years of age.”¹ In the United States from 2002 to 2007, the fraction of four-year-olds attending state-funded preschool education rose from 14 to 22 percent. At the same time, U.S. state funding for preschool education rose from 2.4 to 3.7 billion dollars.² In Canada, partly inspired by the $5-a-day day care policy introduced in Quèbec in 1997, all major parties in the 2006 federal election campaign put forward proposals for a national day care programme.

Is public funding for preschool day care a good idea? We approach this question from an optimal taxation perspective, essentially in the spirit of Ramsey (1927). This means that we ask whether day care subsidies can contribute to a more efficient allocation of resources.

There are of course several alternative justifications for government funding of day care whose validity may be worth exploring. One is that subsidized day care might raise fertility and thereby contribute to solving the problems generated by an aging population.

¹ Source: European Council (2002)
Another is that it might promote early learning, especially among disadvantaged children. Meanwhile, day care subsidies tend to encourage female labour force participation, which might be desirable either because it promotes equality within the household or because it leads to a more efficient allocation of resources. Here we focus entirely on the consequences of day care subsidies for the allocation of resources.

We make the case for subsidizing day care in two ways. The first is to develop a stylized life-cycle framework where the demand for day care arises from the assumption that every hour of work makes it necessary for a parent to purchase an hour of day care per small child. A non-trivial optimal taxation problem arises from the assumption that the government has an exogenous spending requirement and that lump-sum taxes are ruled out but linear taxes and subsidies are allowed. Thus the consumption/leisure choice is distorted and the problem is to minimize the total welfare cost of this distortion.

Our main finding in that context is that the optimal policy consists in equalizing the consumption/leisure wedge over the life cycle, and that the most straightforward way to accomplish this is to make day care expenses tax deductible. The intuitive reason for deductibility being optimal is that if everyone is taxed as a percentage of market labour income, then they will potentially face very different tax rates on effective labour income, i.e. labour income net of day care expenses, and that is not optimal. Another way to think about it is the following. If day care is not subsidized or tax deductible and marginal taxes rates are not conditional on the number of small children, then the presence of small children reduces the ratio of marginal private to marginal social returns to working net of day care expenses. This means that labour supply is more distorted the more small children are present, and this is not consistent with minimizing the total distortion. What is optimal is to even out the distortion. This is essentially a tax smoothing argument.

Having established this theoretical result, we then construct an argument for subsidized day care in a different way, one more empirically and quantitatively oriented. What we do is to build and calibrate a model designed to capture the key features of labour supply in Germany. There are several strong reasons for studying Germany in the context of day care policy. One, as emphasized in a recent OECD survey of Germany (OECD, 2008), is that “fewer mothers with small children are employed in Germany than in other countries and those who work do so for fewer hours”. In fact, this feature of the data is particularly pronounced in western Germany. As we show in Figure 1, West German women with small children supply on average 60 percent fewer hours than other West German women of the same age. There is no corresponding difference between fathers with small children and other men.

A second reason to consider (western) Germany is that, until recently, availability of subsidized day care is very limited there. On average, across the OECD countries for

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3 Knowles (2006) shows that rising women’s wages relative to men’s wages has strengthened the relative bargaining position of women within U.S. households. Since the introduction of day care subsidies raises the effective wage for women, it is likely to have the same effect.

4 Interestingly, Israel made day care expenses tax deductible starting in the fiscal year 2010, following a decision by their Supreme Court. See Segal (2009). A child care tax credit system is also incorporated into, for instance, the French tax code and the federal tax code of the United States.

5 Merz (2004) studies female labour supply trends in western Germany and finds that although participation has risen somewhat, hours per female employee actually fell between 1980 and 2000, the net result being almost no change in weekly market hours worked per woman. It is worth noting that, in Germany as a whole, including the new Bundesländer, female weekly or annual market hours worked per person did increase markedly after 1990, but that is largely because of the addition of new women from the east who were already working more hours.
which data are available, 23 percent of 0-3 year-olds are in formal day care. In Germany as a whole, that number is just 9 percent, and in western Germany it is less than 3 percent. Meanwhile, for 4-6-year-olds, Germany does not stand out in terms of participation rates in some form of subsidized preschool organized activity. However, in contrast to France or the Scandinavian countries, western German Kindergartens typically offer only a half-day service that does not include lunch. The number of full-time subsidized day care spots for children 3 years or older corresponds to just 16 percent of these children.\(^6\)

A third reason to study Germany is that its day care policy is currently in transition. In 2004, the SPD-Green federal government enacted the *Tagesbetreuungsausbaugesetz*; this legislation requires that, each year, 1.5 billion euros have to be invested in day care, and local authorities are responsible for supplying sufficient day care for children under the age of three. The CDU/CSU-SPD government that took office in 2005 continued in the same spirit. In 2007 the German government announced that it plans by 2013 to triple the number of subsidized day care spots for small children to 750,000.\(^7\) In 2008, the *Kinderförderungsgesetz* was passed in the *Bundestag* (federal parliament), establishing a right to a day care spot for every child from the age of one.\(^8\) Recent reforms in Germany have also included moves towards making day care expenses tax deductible.\(^9\) Our work contributes to an evaluation of this and other day care finance reforms in Germany.

When calibrating our model to (western) Germany, it is important to capture the way that labour supply varies with the presence (or not) of small children in the household. In the data, the extent of this variation is strongly associated with gender and marital status.\(^10\) Consequently, our quantitative model, unlike our stylized model, features heterogeneity with respect to these features.

Meanwhile, the current German tax system is very far from linear and there is a generous system of means-tested social assistance payments. Hence we modify the model environment to be broadly consistent with the existing framework of German fiscal policy. To focus sharply on the effects of day care finance reform, the policy changes that we consider keep other aspects of the tax and transfer system unchanged, subject only to government solvency. In this environment, our deductibility theorem no longer applies. We therefore consider other policies as well, namely proportional day care subsidies at rates varying from 0 to 100 percent. As it turns out, there are subsidy rates that lead to larger welfare gains than deductibility. In the range of subsidy rates we consider, a 50 percent subsidy delivers the highest welfare gains whether we use a social welfare function that is designed not to attach any value to redistribution relative to the status quo\(^11\) or equal weights.

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\(^7\) Source: Deutsche Welle (2007).

\(^8\) For more details on these reforms and also appropriate references to German legislation, see the website of the German federal ministry of families, seniors, women, and youth, http://www.bmfsfj.de/.


\(^10\) When talking about marital status we are not interested in whether anyone is legally married, only whether they are living in a relationship that is economically equivalent to marriage. Therefore we will use the terms “married” and “cohabiting” interchangeably throughout the paper.

\(^11\) This is achieved by choosing Pareto weights that are inversely proportional to the marginal utility of consumption in the initial (pre-reform) situation.
Our main quantitative findings can be summarized as follows. A 50 percent subsidy leads to substantial increases in the labour supply of mothers with small children.\textsuperscript{12} The reform leads to a 71 percent increase in labour supply (hours worked) for married mothers and a 125 percent increase for single mothers with small children. Meanwhile, though the reform involves spending 0.4 percent of GDP on day care subsidies, it involves a shift up of the labour income tax schedule of just 0.1 percentage points. The reform also has significant welfare effects. The overall welfare gains correspond to a 0.4 percent increase in consumption. Among couples, the gains correspond to a 0.4 percent increase; for single women this number is 0.3 percent, and for single men it is $-0.1$ percent (a welfare loss).

The reason why single men lose from the reform is that, in the model, they have no children,\textsuperscript{13} so the only way in which they are affected is through the tax increase. This tax increase is not large. One reason (though not the only one) why it is so small is that the reform encourages single mothers to start working instead of living on social assistance. Single women gain a bit less than couples precisely because they are the ones forgoing social assistance in response to the reform. Meanwhile, most of the increased public expenditure implied by the reform is financed neither by people choosing to rely less on social assistance, nor by tax rate increases, but by the fact that labour supply goes up, generating more tax revenue.

Our work is related to several strands of the literature. First, there is an empirical literature documenting that subsidized day care may encourage labour supply of mothers. The seminal work of Heckman (1974) presents strong evidence from the United States that day care subsidies do indeed increase female labour supply; more recently, in a case more directly relevant to this paper, Wrohlich (2006) finds that expanding availability of subsidized day care increases maternal employment in Germany. Like Heckman (1974), Wrohlich (2006) uses an econometric approach; Bick (2010) draws the same qualitative conclusion as she does in the context of a calibrated life-cycle model, though he finds a smaller effect than Wrohlich.\textsuperscript{14}

Another strand considers the role of public policy in determining the choice between home and market production. Lindbeck (1982) argues that subsidies to day care in many countries has contributed to the increased labour supply of women, to some extent counteracting the rise in tax rates that have been required to finance these subsidies.\textsuperscript{15} The reason for Lindbeck’s conclusion is that he thinks, as we do, of market provided day care as a close substitute for day care at home. Rosen (1997), on the other hand, considers, in the context of a model conceptually similar to Lindbeck’s, whether high day care subsidies can be justified and concludes that they cannot. This conclusion hinges on imposing a rather low upper limit on the degree of substitutability between market and home produced day care. Another important reason why our conclusions differ from

\textsuperscript{12} The effects of reform are compared to an initial situation where day care is neither subsidized nor tax deductible.

\textsuperscript{13} In the data, only 1.2 percent of single men have a small child at any given point in time; in the model, we set this number to zero.

\textsuperscript{14} The contrasting conclusions drawn by Wrohlich and Bick is that the latter assumes that day care is not necessary even when both parents are working. Instead he assumes that the mother suffers a utility cost from being away from her children and that this utility cost can be avoided by purchasing day care. Our quantitative results lie between those of Wrohlich and those of Bick.

\textsuperscript{15} Bergstrom and Blomquist (1996) argue that the effect on labour supply may be so large that subsidizing day care is self-financing; a similar result is found in Lundholm and Ohlsson (1998).
Rosen’s is that the tax smoothing considerations central to our analysis are absent from his static model.

A third strand of the literature is concerned, as we are, with understanding female labour supply. Much of that literature has focused on the remarkable rise of female labour force participation since the 1950s in the United States. Prominent contributions include Jones et al. (2003) who discuss the importance of the declining gender wage gap, Greenwood et al. (2005) who talk about the impact of adopting household appliances, and Olivetti (2006) who focuses on the increasing returns to experience. More recently, Attanasio et al. (2008) have emphasized the importance of reduced day care costs in explaining observed changes in U.S. female participation rates.

Other related papers consider the role of fiscal policy in explaining differences in labour supply across time and across countries, starting with Prescott (2004). Of particular relevance to the present paper is the idea that differences in labour-supply-promoting public spending may be an important factor accounting for differences in labour supply across countries, especially the difference between Scandinavia and (the rest of) continental Europe; this idea is explored in Olovsson (2009), Ragan (2005), and Rogerson (2006).

Our work is also closely related to the growing literature on quantitative evaluation of fiscal policy reforms in dynamic models with heterogeneity. A particularly relevant part of the literature are those papers that deal with multi-member households, such as Chade and Ventura (2002) and Güner et al. (2010) who look at the effects of income tax reforms on labour supply and Erosa et al. (2008) who look at the effects of parental leave policies.

The paper is organized as follows. Section 2 presents the environment, proves the optimality of equalizing consumption/leisure wedges and discusses how that can be implemented by making day care expenses tax deductible. Section 3 provides a quantitative assessment of optimal day care reform in Germany. Section 4 concludes.

2. THEORY

Consider a $T$-period life-cycle model where at age $s$ the representative agent has $b_s$ small children. The demand for day care arises from the following assumption: for every unit of time that the agent works, it needs to purchase day care for each small child. Beyond that, agents in the model are indifferent to the presence of children; time spent with children is like leisure spent without children, no better and no worse. The only significance of children in the model is that they require constant supervision and so day care is required when the parent is working.

The resource cost of day care is $d$ per unit of time and child. The government levies age-dependent linear taxes on labour income (at rates $\tau_s$) to finance exogenous government purchases $G$ and age-dependent linear subsidies on day care (at rates $\theta_s$). There is a world asset market that enables agents to transform one unit of the age $s$ good into $1 + r$ units of the age $s + 1$ good and vice versa without any non-negativity restrictions on the amount saved. Each agent’s initial endowment of assets is zero. Factor prices are exogenous.

Formally, an agent solves

$$\max \sum_{s=1}^{T} \beta^s [u_s(c_s) + v_s(h_s)],$$

where $u_s: \mathbb{R}_+ \rightarrow \mathbb{R}$ is an increasing, differentiable and concave function and $v_s: \mathbb{R}_+ \rightarrow \mathbb{R}$ is a decreasing, differentiable and concave function for each $s$, subject to the life-time
budget constraint (associated with the Lagrange multiplier $\lambda$)

$$\sum_{s=1}^{T} \prod_{t=1}^{s-1} (R_{t+1})^{-1} [c_s + (1 - \theta_s)db_s h_s] = \sum_{s=1}^{T} \prod_{t=1}^{s-1} (R_{t+1})^{-1} [1 - \tau_s]w_s h_s,$$

where $c_s$ denotes consumption, $h_s$ denotes hours worked, and $w_s$ denotes age-specific productivity. $R_{s+1}$ is the after-tax gross rate of return between $s$ and $s+1$. Denoting the age-specific capital income tax rate by $\xi_s$, we have

$$R_s = 1 + (1 - \xi_s)r.$$

Notice that we allow the utility of consumption $u_s(c)$ and the disutility of labour, $v_s(h)$, to depend on age; this enables us to establish a slightly more general result than otherwise.

Assuming an interior solution, labour supply and consumption are characterized by the agent’s first order conditions which are

$$\beta_s u_{c_s} - \lambda \prod_{t=1}^{s-1} (R_{t+1})^{-1} = 0$$

and

$$\beta_s v_{h_s} + \lambda \prod_{t=1}^{s-1} (R_{t+1})^{-1} [(1 - \tau_s)w_s - (1 - \theta_s)db_s] = 0.$$

where we define

$$u_{c_s} = \frac{\partial u_s(c_s)}{\partial c_s}$$

and

$$v_{h_s} = \frac{\partial v_s(h_s)}{\partial h_s},$$

thus suppressing the $s$ argument of $u$ and $v$.

The economy as a whole faces the following constraint:

$$\sum_{s=1}^{T} (1 + r)^{-s} [c_s + db_s h_s + G] = \sum_{s=1}^{T} (1 + r)^{-s} w_s h_s,$$

where $G$ denotes government consumption.

2.1. Ramsey government

The Ramsey government maximizes

$$\sum_{s=1}^{T} \beta^s [u_s(c_s) + v_s(h_s)]$$

subject to (2.5), associated with the Lagrange multiplier $\mu$, and the implementability constraint

$$\sum_{s=1}^{T} \beta^s [u_s(c_s) + v_s(h_s)] = 0$$

associated with Lagrange multiplier $\varphi$. This constraint, together with Equation (2.5) fully characterizes the set of competitive equilibria, as in Lucas and Stokey (1983).

The first order conditions with respect to consumption $c_s$ and labour supply $h_s$, respectively, are

$$\beta^s u_{c_s} [1 + \varphi (1 - \sigma_s(c_s))] - \mu (1 + r)^{-s} = 0,$$

and

$$\beta^s v_{h_s} \left[ 1 + \varphi \left( 1 + \frac{1}{\varepsilon_s(h_s)} \right) \right] + \mu (1 + r)^{-s} [w_s - db_s] = 0,$$
for $s = 1, 2, \ldots, T$, where $\sigma_s$ is defined via
\begin{equation}
\sigma_s(c) = \frac{\partial^2 u_s(c)}{\partial c \partial c} \cdot c \tag{2.10}
\end{equation}
and $\varepsilon_s$ is the Frisch (1959) elasticity of labour supply, defined via
\begin{equation}
\varepsilon_s(h) = \frac{\partial v_s(h)}{\partial h} \cdot \frac{1}{h} \cdot \frac{\partial^2 v_s(h)}{\partial h^2} \cdot 1 \cdot h. \tag{2.11}
\end{equation}

2.2. Ramsey optimum

We now establish that, under certain conditions, the Ramsey optimal allocation equalizes the consumption/leisure wedge over the life cycle. The precise definition of this wedge, which is equal to one in a Pareto optimal allocation, is as follows:\footnote{Notice that this wedge is only well-defined for those $s$ such that $(w_s - db_s) > 0$, i.e. such that the wage exceeds the cost of day care. This is the rationale for the production of day care outside the family; for those $s$ where it does not hold, the Ramsey optimum dictates that no day care should be consumed.}
\begin{equation}
W_s = -\frac{v_s(h_s)}{w_s - db_s}. \tag{2.12}
\end{equation}
To see under what conditions it is optimal to equalize $W_s$ over the life cycle, combine the Ramsey government’s first order conditions with respect to consumption (Equation 2.8) and labour supply (Equation 2.9) for an agent at ages $s$ and $t$ to obtain
\begin{equation}
W_t = W_s \cdot \frac{1 + \varphi (1 + 1/\varepsilon_s(h_s))}{1 + \varphi (1 + 1/\varepsilon_t(h_t))} \cdot \frac{1 + \varphi (1 - \sigma_t(c_t))}{1 + \varphi (1 - \sigma_s(c_s))}. \tag{2.13}
\end{equation}
This establishes the following Proposition.

**Proposition 1.** If (1) $\varepsilon_s(h) = \varepsilon$ for all $s$ and $h$, (2) $\sigma_s(c) = \sigma$ for all $s$ and $c$, then the Ramsey allocation satisfies
\begin{equation}
W_t = W_s \tag{2.14}
\end{equation}
for all $s$ and $t$.

As is always the case, the Ramsey optimal allocation can be implemented in many different ways. Generically, in order to implement the Ramsey optimal policy, we need one policy instrument per decision. If more instruments are available, the optimal policy is not unique. In the present setting, we have two decisions ($c_s$ and $h_s$) and three policy instruments ($\tau_s$, $\theta_s$ and $\xi_s$) per agent and age. Since $\theta_s$ and $\tau_s$ both affect the same wedge $W_s$, they are not uniquely determined. Nevertheless, we can characterize the set of optimal policies in the following way. The government should either subsidize day care or give agents more favorable tax treatment in periods when more small children are present. To give a more precise description of the set of optimal policies, it is useful to let $\theta_s = \theta$ for all $s$. Because of the indeterminacy, this involves no loss of generality. Then, as we will demonstrate below, there are, under the assumptions of Proposition 1, only three possibilities that are consistent with Ramsey optimality. One is for the subsidy rate to be low, $\theta < \tau_s$ for all $s$. In this case, the tax rate $\tau_s$ should be lower in those periods where a larger number of small children are present. Another possibility is the converse,
\( \theta > \tau_s \) for all \( s \), and a higher tax rate in those periods where a larger number of small children are present.

A final possibility is to set \( \tau_s = \theta \) for all \( s \). This means that the tax rate is constant over the life cycle—surely an advantage from the point of view of simplicity. One way to describe this policy is that it involves making day care expenses tax deductible and equalizing tax rates over the life cycle. If taxes cannot vary over the life cycle, then this is the uniquely optimal policy. This is what we mean when we say that it is optimal to make day care expenses tax deductible.

To see that these are the only three possibilities consistent with Ramsey optimality, combine the agent’s and the Ramsey government’s first order conditions with respect to labour supply \( h_s \), (2.4) and (2.9), to obtain

\[ \tau_s = \chi_s - (\chi_s - \theta) \cdot \frac{db_s}{w_s} \]  

(2.15)

where

\[ \chi_s = \frac{\varphi(1/\varepsilon_s(h_s) + \sigma_s(c_s)(1 - x_s/c_s))}{\pi + \varphi(1 + 1/\varepsilon_s(h_s))}. \]  

(2.16)

Notice that under the assumptions of Proposition 1 we have \( \chi_s = \chi > 0 \) for all \( s \). Given our assumption that \( w_t > db_t \) for all \( s \), this means that Equation (2.15) only allows three possibilities: either \( \theta < \tau_s < \chi \) for all \( s \), \( \theta > \tau_s > \chi \) for all \( s \) or \( \theta = \tau_s = \chi \) for all \( s \). Meanwhile, Equation (2.15) implies that

\[ \tau_t - \tau_s = - (\chi - \theta) \left( \frac{db_t}{w_t} - \frac{db_s}{w_s} \right). \]  

(2.17)

It follows that, as described above, there are only three possibilities that are consistent with a Ramsey optimum. They are

1. \( \tau_v > \theta \) for all \( v \). Then \( \tau_t < \tau_s \) whenever \( b_t/w_t > b_s/w_s \).
2. \( \tau_v < \theta \) for all \( v \). Then \( \tau_t > \tau_s \) whenever \( b_t/w_t > b_s/w_s \).
3. \( \tau_v = \theta \) for all \( v \).

We will now discuss the significance of the assumptions of Proposition 1. Suppose Assumption (1) does not hold and that the Frisch elasticity of labour supply varies over the life cycle. Then it follows from Equation (2.15) that for any fixed \( \theta_s = \theta \), taxes should be set so that when the Frisch elasticity is lower (all else equal), the agent should face a higher tax rate, i.e. that \( \tau_s \gtrless \tau_t \) if \( \varepsilon_t \gtrless \varepsilon_s \), a natural feature of Ramsey optimal taxation.\(^{17}\) Suppose on the other hand that Assumption (2) does not hold and that \( \sigma_s \) varies over the life cycle; then Equation (2.15) implies that \( \tau_s \gtrless \tau_t \) if \( \sigma_s \gtrless \sigma_t \), all else equal.

So far, we have only discussed labour taxes and day care subsidies. It turns out that, under the assumptions of Proposition 1, optimal capital income taxes are zero. To see this, combine the agent’s and the Ramsey government’s first order conditions with respect to consumption \( c_s \) (Equations 2.3 and 2.8) for \( s - 1 \) and \( s \) to obtain

\[ \xi_s = \left( \frac{1 + \varphi \sigma_s(c_s)}{1 + \varphi \sigma_{s-1}(c_{s-1})} - 1 \right) \cdot \frac{1 - r}{r}. \]

\(^{17}\) This has previously been discussed in a life-cycle setting by Erosa and Gervais (2002).
Hence if we have \( \sigma_s(c_s) = \sigma \) for all \( s \) then \( \xi_s = 0 \) for all \( s \) so that capital income taxes are zero in the Ramsey optimum.

We end this section with a discussion of the factors that determine the magnitude of the effects on allocations and welfare from optimal reform. What that magnitude depends on is the degree to which the consumption/leisure wedge \( W_s \) is not equal across different values of \( s \) in the initial situation. When this wedge is nearly equal over the life cycle, there is not much for day care subsidies to accomplish; when it varies a lot over the life cycle, optimal day care subsidies have large effects on labour supply and on welfare. We now consider what conditions might lead the wedge to vary over the life cycle. Note that, by the agent’s first order condition, we can express it as the ratio of the marginal private to the marginal social return to working, i.e.

\[
W_s = \frac{(1 - \tau_s)w_s - (1 - \theta_s)db_s}{w_s - db_s}.
\]

Important factors that lead to \( W_s \) being different for different values of \( s \) include the unit day care cost, the spacing of children and the initial level of taxation. To see this in more detail, consider an initial situation where \( \theta_s = 0 \) and \( \tau_s = \tau \). Then the wedge is given by

\[
W_s = \frac{(1 - \tau)w_s - db_s}{w_s - db_s}.
\]

Evidently the wedge is constant as a function of \( s \) if \( \tau = 0 \) so there is no rationale for day care finance reform in an economy with zero labour taxes. If \( \tau > 0 \) then the wedge is a decreasing function of \( db_s \) which means that the higher the hourly day care cost per child and the greater the number of children, the more severe is the consumption/leisure distortion. This in turn means that the effects of optimal reform will be greater the greater the extent to which having children is confined to a small part of the life cycle.

2.3. Discussion

The main optimal taxation result that we have established in this paper says that consumption/leisure wedges should be equalized across ages.\(^{18}\) This means that our work is related to the uniform commodity taxation result of Sandmo (1974). What he shows is that if utility is separable in labour and commodities and homogeneous (of arbitrary degree) in the commodities, then commodities should be taxed uniformly. We prove a somewhat more general result in an environment where labour is supplied in several periods. Under assumptions that imply Sandmo’s and also assuming that the Frisch elasticity of labour supply is constant and age-independent, then labour supply at different ages should be taxed uniformly. Perhaps the most important novel feature of our work, however, is not the small generalization of Sandmo’s theoretical result, but that our approach allows us to interpret uniform consumption/leisure wedges as giving favorable tax treatment to households with small children.

The connection to the efficiency-in-production result of Diamond and Mirrlees (1971) is looser. The result of that paper is that technical rates of substitution should be equalized across sectors. That result does not apply to our environment, since labour

\(^{18}\) In Domeij and Klein (2010), an earlier version of this paper, we prove a further result. Allowing for heterogeneous agents, we establish conditions, similar to those of Proposition 1, under which all agents should face the same tax rates.
inputs from different individuals are perfect substitutes. Technical rates of substitution are exogenously fixed and thus either happen to be equal or cannot be equalized.

Our work is also related to that of Erosa and Gervais (2002) who characterize Ramsey optimal taxes in a lifecycle environment. In that context it is worth noting that the optimality of age-independent tax rates and subsidies hinges on the Frisch elasticity being constant. If it is not, taxes and subsidies should typically be age-dependent, a feature of Ramsey optimal policy that is emphasized in their work.

3. A QUANTITATIVE ASSESSMENT OF OPTIMAL DAY CARE REFORM

In this section we provide a quantitative assessment of optimal day care reform. For this assessment to be credible, we need to extend the model laid out in Section 2. The extensions go in two directions; the first introducing heterogeneity among agents and the other introducing empirically relevant features of the German tax and transfer system, namely social assistance payments and progressivity of the income tax code.

Heterogeneity is important because, as we saw in Section 2.2, it is pre-reform variation in labour wedges that makes it socially beneficial to subsidize child care. This variation can be across different ages, as we have seen already, but as we pointed out in Footnote 18, it can be shown that, under conditions similar to those of Proposition 1, it is socially beneficial to smooth labour wedges across distinct agents as well. Also, we want to capture the way that labour supply varies with the presence (or not) of small children in the household. In the data, the extent of this variation is strongly associated with gender and marital status. For these reasons, we introduce households containing either (1) a single male, (2) a single female or (3) a cohabiting couple. Meanwhile, in order to generate heterogeneity with respect to both wage levels and the steepness of the age-wage profile, we let individuals belong to either of two education groups.

Progressive taxes and social assistance are important because the purpose of this paper is to provide an assessment of day care finance reform, keeping other features of the tax and transfer system in place, and the effects of reform are likely to depend on what those other features are. The relevance of social assistance is particularly clear; if day care subsidies can persuade people who otherwise would have lived on benefits to go to work, this affects the degree to which day care subsidies are self-financing. Meanwhile, a realistic specification of the marginal income tax schedule helps ensure that our predictions about post-reform behaviour are based on actual incentives.

How robust is the conclusion of Proposition 1 (making day care expenses tax deductible is optimal) to the extensions we have introduced here? As mentioned above, heterogeneity across agents is not much of a problem. For the result to go through, however, we need to assume that Pareto weights are inversely proportional to the marginal utility of consumption; the interpretation of this assumption is that it removes any redistributational motives on the part of the government. Extending the proof to allow for households with two adult members (married couples) is also possible.\footnote{The calibrated model features overlapping generations as opposed to just a single life cycle; the theoretical results survive this extension as well, provided that the Pareto weights of the generations decline geometrically at a rate given by the households’ subjective discount factor $\beta$.}

However, social assistance benefits and progressive taxes put us in a new situation as far as the optimal policy is concerned. In this context, there are many policy changes one might want to consider, for instance a flat tax reform. However, this paper is (only) about day care finance reform and so the policy changes that we consider in this section all keep...
the rest of the tax and transfer system unchanged, subject only to government solvency. With this restriction on the set of reforms, tax deductibility of day care expenses is not necessarily optimal anymore. We still consider such a policy, where what we mean by deductibility is that both taxes and transfers are computed on the basis of earnings less day care expenses. In addition, we also consider subsidizing day care at various rates.

3.1. The model

The economy is populated by $I$ households, some of which are lifelong couples and some of which are lifelong singles. The number $i$ identifies a particular household and conveys enough information to describe it completely; in particular it specifies whether the household contains a single male, a single female or a cohabiting couple. It also identifies the level of education of each adult member and the birthdays of their children (if any).

To simplify the notation, we suppress the birth cohort. Singles solve

$$
\max \sum_{s=1}^{T} \beta^s \left[ u^s_i(c^s_i) + v^s_i(h^s_i) \right],
$$

subject to the life-time budget constraint

$$
\sum_{s=1}^{T} (1 + r)^{-s+1} \left[ c^s_i + (1 - \theta)db^s_i h^s_i \right] = \sum_{s=1}^{T} (1 + r)^{-s+1} y^s_i(h^s_i),
$$

where $y^s_i$ is disposable labour income (after-tax earnings plus social assistance payments).

A couple consists of a man and a woman and it solves

$$
\max \sum_{s=1}^{T} \beta^s \left[ u^s_i(c^s_i) + v^s_i(h^{i,m}_s, h^{i,f}_s) \right],
$$

subject to the life-time budget constraint

$$
\sum_{s=1}^{T} (1 + r)^{-s+1} \left[ c^s_i + (1 - \theta)db^s_i \min\{h^{i,m}_s, h^{i,f}_s\} \right] = \sum_{s=1}^{T} (1 + r)^{-s+1} y^s_i(h^{i,m}_s, h^{i,f}_s),
$$

where the superscript $m$ represents the man and $f$ represents the woman. (Again, we suppress dependence on birth cohort.) The idea behind the $\min\{h^{i,m}_s, h^{i,f}_s\}$ expression in the budget constraint is that day care has to be purchased for every hour that both parents work in the market and that shift work is not allowed.

3.2. Calibration

We now describe how we calibrate the demographic features of the model (including the arrival of children), wages, the tax and transfer system, preferences and the per-unit day care cost.

20 By government solvency we mean that the (single) intertemporal government budget constraint is satisfied. Thus the government’s budget is not necessarily balanced in each period or for any particular cohort.

21 When adopting a policy of day care subsidies in practice, a question that arises is whether to make the subsidy contingent on working, so that the subsidy cannot be used to enjoy child-free leisure. In the context of our model, this is not a relevant consideration, since we assume that leisure without children is the same as leisure with children. But in real life the question of course does arise, and the logic of our model strongly suggests that it would make sense for work to be a prerequisite for making use of subsidized child care.
3.2.1. Demographics. The model population is assumed to be of constant size over time, and is designed to reflect the current German population in terms of the number of singles and married couples and the distribution of children. The data that we use for this purpose are from the database (G)SOEP, see http://www.diw.de.22 A household is defined in this context as a set of people living at the same address. Two adults living in the same household are counted as a couple if (1) they are the two eldest in the household, (2) they are of the opposite sex, and (3) the age difference is strictly less than 20 years. (For details, see Appendix A.1.) In SOEP data from 1984 to 2004, about 81 percent of people between the ages of 23 and 65 are cohabitors in this sense; this is what we assume for the model population as well. Half of the single people in the model are assumed to be male, and the other half female.

We now describe how we calibrate the number and spacing of births and replicate the observed heterogeneity in this respect. (For more details than we provide in this section, see Appendix A.2.) We assume that the length of a time period is six years. Adults are assumed to live for \( T = 10 \) periods, the last three periods being spent in retirement. We think of the first period of adult life as 23-28 years of age. Children live with their parents for three periods, and we assume that they require constant supervision only during their first period of life. Newborn children may arrive when the parents are aged 1-4 (corresponding to 23-46 years). At age 1, the maximum number of newborns is 2, reflecting the fact that young parents very rarely have more than two children. For ages 2-4, the maximum number of newborns is 3. In addition, we set an upper limit on completed fertility at 3 children. This gives rise to 34 different possible profiles and hence our model population has 34 different child profile types for single women and couples respectively. (For single men, we ignore the small fraction (1.2 percent) who have small children at a given point in time and assume that they are all permanently childless.) To determine population shares of these types, we proceed as follows. First we use SOEP data to compute the probabilities of children arriving as a function of family characteristics such as age, number of existing children, and marital status. Then we simulate a large population on the basis of these probabilities. Based on these populations, we can determine the population shares of each of the 34 child profiles for single women and couples, respectively.

By construction, these population shares ensure that we match the following features of our SOEP sample: 3.5 percent of single women and 13.5 percent of couples have small (0-5 years old) children at any given point in time. These population shares also imply that 17.4 percent of the model population is between 0 and 17 years old. Reassuringly, according to Eurostat, the corresponding number for the entire population of Germany was 18.2 percent in 2004.

3.2.2. Wages, taxes and transfers. Disposable labour income in the model, \( y_i \), equals household labour earnings minus taxes plus social assistance payments. We now describe how we calibrate each of these terms. To generate a wage distribution that reflects some key features of its empirical counterpart, we let wages depend on age, marital status, gender and education. Each individual is classified as highly educated if he or she has strictly more than 12 years of schooling. About 34 percent of single males and 32 percent of single females are highly educated in this sense. In 70 percent of couples, both spouses have 12 years of education or less. In 12 percent of couples, the

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22 For a more detailed description of this dataset, see Appendix A.
man is highly educated and the woman is not; in six percent of couples, this situation is reversed. Among the remaining 12 percent of couples, both spouses are highly educated.

Wage profiles are calibrated on the basis of a regression, specific to each gender-education combination, of the log wage and dummies for age and marital status. Among the results from these regressions, a few are worth mentioning here. For young workers, the education and male gender premia are small, but the age-wage profile is much steeper for educated workers and somewhat steeper for men than for women. For all but low educated women, there is a marriage premium of 5-7 percent. For more details, see Appendix A.4.

We model the German labour income tax schedule following the description in OECD (2005). For example, the average and marginal tax rates for a single individual with one child are displayed in Figure 2. Consumption is taxed separately from labour income at the rate 15.8 percent, taken from Carey and Tchilinguirian (2000). We model transfers following the description of German social assistance policies in Adema and Kahl (2003). For example, a single woman with one small child receives €1133 per month and a married couple with one child 5 years or younger and one child older than 5 receives €1360 per month. Transfers are generally clawed back at a rate of 85 percent as household income rises; the exception is single mothers, who face a clawback rate of 75 percent. See Appendix B for details.

3.2.3. Preferences and the cost of day care. We assume that the utility of consumption takes the form

$$u^i_s(c) = n^i_s \left( \frac{c}{\eta^i_s} \right)^{1-\sigma}.$$ 

The number $n^i_s$ represents the number of household members. The consumption equivalents, $\eta^i_s$, are calculated using the OECD consumption equivalence scale. According to this scale, the first adult counts as one unit, the second adult as 0.7 and each child as 0.5.

Meanwhile, the disutility of labour for singles is given by

$$v^i_s(h) = \psi^i_s \frac{h^{1+1/\varepsilon}}{1 + 1/\varepsilon}$$

and that for couples is given by

$$v^i_s(h^m, h^f) = \psi^i_{s,m} \frac{(h^m)^{1+1/\varepsilon}}{1 + 1/\varepsilon} + \psi^i_{s,f} \frac{(h^f)^{1+1/\varepsilon}}{1 + 1/\varepsilon} + \alpha^i \cdot I_{h^f > 0 \& h^m > 0}$$

The reciprocal of the intertemporal elasticity of substitution for consumption, $\sigma$, is set to 2. The parameter $\varepsilon$, representing the Frisch elasticity of labour supply, is set to 0.5; see Domeij and Flodén (2006) and Pistaferri (2003). The parameter $\beta$ is set to 0.974 $$\approx 0.83$$ and the interest rate $r$ is set so that the subjective and market discount rates are equal; $r = 1/\beta - 1.$

23 In the data, the wage is defined as the ratio of detrended labour earnings to hours worked.

24 The value added tax in Germany is 19 percent on most goods and services and 7 percent for food. Day care is not subject to value added tax.

25 See Section 3.4 for a discussion of the implied uncompensated labour supply elasticities for different groups.
Table 1: Disutility of labour parameter values

<table>
<thead>
<tr>
<th></th>
<th>Cohabiting</th>
<th>Single</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Females</td>
<td>Males</td>
</tr>
<tr>
<td>Age 23-46</td>
<td>290</td>
<td>182</td>
</tr>
<tr>
<td>Age 47-64</td>
<td>480</td>
<td>230</td>
</tr>
</tbody>
</table>

Table 2: Hours per year

<table>
<thead>
<tr>
<th>Ages</th>
<th>Data 23-46</th>
<th>Data 47-64</th>
<th>Model 23-46</th>
<th>Model 47-64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohabiting women</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without children</td>
<td>1437</td>
<td>1185</td>
<td>1437</td>
<td>1188</td>
</tr>
<tr>
<td>Cohabiting women</td>
<td>494</td>
<td>–</td>
<td>523</td>
<td>–</td>
</tr>
<tr>
<td>with children</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohabiting men</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without children</td>
<td>1978</td>
<td>1908</td>
<td>1979</td>
<td>1911</td>
</tr>
<tr>
<td>Cohabiting men</td>
<td>2015</td>
<td>–</td>
<td>2079</td>
<td>–</td>
</tr>
<tr>
<td>with children*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single women</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without children</td>
<td>1653</td>
<td>1759</td>
<td>1655</td>
<td>1759</td>
</tr>
<tr>
<td>Single women</td>
<td>619</td>
<td>636</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>with children</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single men (without</td>
<td>1666</td>
<td>1803</td>
<td>1667</td>
<td>1804</td>
</tr>
<tr>
<td>children)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Not a calibration target.

Note: A child in this context is one below the age of 6.
Data source: SOEP

The disutility of labour parameters $\psi^i$, $\psi^{i,m}$ and $\psi^{i,f}$ are set so as to match average labour supply by marital status (cohabiting or single) and age group (23-46 and 47-64) for those without small children. Parameter values are given in Table 1. The labour market participation cost $\alpha^i$ and the day care cost parameter $d$ are set so as to match the labour supply of cohabiting and single mothers with small children and is discussed further below. The results of the calibration are shown in Table 2.

As can be seen in Table 2, we match our calibration targets reasonably well. We also match the labour supply of married men with small children, even though this is not an explicit calibration target. In particular, we replicate the fact that married men with small children work more than married men without small children. Meanwhile, Figures 3 and 4 show that the model also does a pretty good job of capturing the complete life-cycle profile of hours worked, though the lifecycle hours profile of married women with small children is a partial exception. Given that wages increase with age, it is very difficult to replicate the fact that hours for this group decline with age.\footnote{To a large extent, this feature of the data, the sources of which are discussed in detail in Appendix A.3, relies on information from older cohorts (those born before the 1960s) who behaved rather differently.}
The parameter \( d \) determining the cost of day care corresponds to €2.77 per hour (at 2004 prices); recall that this number is chosen so that single mothers with small children work as much on average in the model as in the data. This cost corresponds to 23% of the wage of an average young (age 23-28) woman with 12 years of education or less.

How does this number compare with the available direct evidence on day care costs in Germany? Documenting these costs is somewhat difficult because of the small size of the German day care sector. Wrohlich (2006) says that private day care costs are about €800 per month. Meanwhile, the monthly fee for full-time subsidized day care is, again according to Wrohlich (2006), about €161 per month for young children and €110 per month for children 3 years or above. Using these numbers, as well as estimated probabilities of getting access to subsidized care and informal care, Wrohlich (2006) reports an expected monthly cost of full-time day care of €664 per month for children less than 3 years old and €354 per month for children at least 3 years old. If we take a weighted average for the 0-5-year-old group as a whole, and assume that full-time care corresponds to 168 hours per month, we get an expected hourly price of €3.03. This number is remarkably close to the €2.77 that we assume in order to replicate empirical labour supply profiles.

It is also worth noting that in the pre-reform steady-state equilibrium, about 0.45 percent of GNP is spent on day care. There is no data on the amount of private spending on day care services in Germany. However, according to OECD (2006), public expenditure on day care and pre-primary education amounted to 0.45 percent of GDP in 2004.

The parameter \( \alpha \) defining the utility cost of both spouses working is chosen so that married mothers with small children work as much on average in the model as in the data. It is set to zero for couples without small children, but is positive for couples with small children. The reason why this parameter cannot be set to zero for all couples is that the means-tested transfer system significantly weakens the incentives for single mothers to work; yet many of them do work. In order to match their labour supply, the hourly cost

from the younger cohorts whose behaviour we are interested in capturing. Since our data set ends in 2004, we have very few observations on 43-year olds born in the 1960s. It is not clear whether the downward sloping age-hours profile would survive exposure to newer data.

27 Her source for the cost of private day care is www.tagesmutter.de, a childminder advertising website. The source for the price of subsidized day care is SOEP, wave 2002.

28 To get some perspective on these numbers, it is useful to consider evidence on hourly costs of day care from the United States and Sweden. According to the U.S. Census Bureau (2005), the average ratio of day care costs to earnings for full-time working mothers is about 20 percent in the United States. In Sweden, the national average cost of day care corresponds to about 33 percent of the wage (inclusive of social insurance contributions) of a young woman (age 25-34) who works in the day care sector. (Source: SCB, 2010.)
of day care cannot exceed €2.77 per hour. Meanwhile, for a given hourly cost of day care, married women with small children, very few of whom enjoy any transfer payments on account of their husbands’ earnings, face stronger incentives to work than single women do. To counteract this effect, which we must do to match the facts, we need a mechanism that reduces the labour supply of married mothers with small children. The mechanism that we have adopted follows Güner et al. (2010), and with its help we are able to match our calibration targets very closely.

Our calibration is further validated by the following observation. Before any reform, 31 percent of single mothers in the model receive social assistance. The corresponding number in the data, according to Adema and Kahl (2003), was 26 percent in the year 2000. The case of couples is perhaps a little bit more problematic. According to the aforementioned source, 4.7 percent of unmarried couples with children and 1.9 percent of married couples with children received social assistance in 2000; in the model, no couples receive any social assistance. The reason why couples receive no social assistance in the model is that there is not enough wage heterogeneity; the very bottom of the distribution is not represented there. However, the majority of social assistance recipients are single mothers, and the model clearly captures that fact.

3.3. Computational considerations

Since no household faces any uncertainty, solving for the optimal labour supply profile is conceptually straightforward. Essentially it boils down to solving a set of first-order conditions. However, because of the progressive labour income tax system and the means-tested system of social assistance, the marginal benefit of working is a non-monotonic function of labour supply. This means that the marginal benefit and marginal cost curves will intersect at more than one point, and all the points of intersection must be considered as candidate solutions. In addition, there is always the corner solution (zero hours) to consider. These considerations apply at any particular age. But we are interested in solving for an entire age-hours profile consisting of seven periods. Suppose there are three candidate subintervals at each age (not points, since the precise point of intersection at a given age depends on labour supply at the other ages); then there are in principle $3^7 = 2187$ combinations to consider. Some of these can be ruled out a priori, but enough of them need to be taken seriously that it takes about 45 minutes or more to solve the model for a given set of parameter values.
3.4. Elasticities

As described above, we have assumed that the Frisch elasticity preference parameter is constant and common to all agents in the model. Meanwhile, there is plenty of empirical evidence that different groups respond rather differently to variations in wages. Is our model consistent with that evidence? At a qualitative level, the answer is clearly “yes.” The labour supply response to a wage change, or to a change in the cost of day care, depends not only on the Frisch elasticity preference parameter but on other factors as well. In particular, if two spouses have the same Frisch elasticity preference parameter, then the low-wage spouse will respond more to a given percentage increase in her own wage than the high-wage spouse will respond to the same percentage change in his own wage. The reason is that a given percentage change in the wage of the low-wage spouse has a lower percentage impact on the total human wealth of the household than does the same percentage change in the wage of the high-wage spouse. With a smaller wealth effect, the labour supply effect is larger.\footnote{We owe this point to Gustavo Ventura.}

This difference in response is even more pronounced in the presence of small children. The choice now has to be made whether to let both spouses work or let one of them stay at home at least part of the time. It is clear that if anyone should reduce labour supply in this case, it should be the one with the lower wage, usually the woman. Meanwhile, suppose her wage is \( w \), the unit day care cost is \( d \) and the number of small children is \( b \). The marginal pecuniary benefit of the woman’s work is now \( w - db \). If this is a small positive number, or even zero or negative then a given percentage increase in \( w \) leads to a much larger (potentially infinite) percentage increase in \( w - db \) and consequently a large response in hours worked. Notice that this argument works for single women versus single men as well. If single women have lower wages than single men, then their value of \( w - db \) is also smaller and hence a given percentage increase in \( w \) or percentage decrease in \( d \) leads to a larger percentage increase in \( w - db \).

In order to find out what the labour supply elasticities are in the model, we carry out the following computational experiments in the context of the pre-reform model economy. For wage elasticities, we perturb the wage in each six-year period separately; specifically, we increase it by 10 percent.\footnote{We have also computed results from smaller perturbations with similar though not identical results.} The (arc) elasticity is then computed as the percentage change in hours worked in that period divided by 10. This number is computed for each time period and each agent. The numbers reported in Table 3 are averages over ages and...
types within a given group. As can be seen there, wage elasticities are much higher for married women than for married men, and this is especially true for married women with small children. The smallest labour supply elasticity is found for single men; the highest labour supply elasticity is found for single women with small children.

The numbers reported in Tables 3 are broadly in line with estimates from the empirical literature. Although Blundell and MaCurdy (1999) reported elasticities for men barely above zero, more recent studies such as Pistaferri (2003) and Domeij and Flodén (2006) report numbers in the range 0.2-0.7 for men. Meanwhile, Blundell and MaCurdy (1999) reported a median elasticity of 0.78 for married women. Note that these studies do not distinguish between those who have small children and those who do not.

We also compute elasticities with respect to day care costs. For this purpose, we reduce costs by 10, 50, and 90 percent and compute labour supply responses in those periods during which agents have small children. Again, the arc elasticities are averaged across ages and agents within a given group, and the results are presented in Table 4. Here too we find larger responses for women than for men. In fact for married men we get negative labour supply responses to reductions in day care costs and hence positive elasticities. This makes perfect sense: not only does cheaper day care make male leisure cheaper relative to female leisure, but, probably more importantly, there is a wealth effect associated with the income generated by the wife working more. Notice also that small changes have disproportionately smaller effects than large changes do. This is related to various aspects of the social assistance system that pushes agents to corner solutions, and it takes a big change in conditions to induce them to move away from a corner. It is also related to the fact that a decrease in \( d \) may be small enough in magnitude that \( w - db \) remains negative for some agents. For these agents, there is of course no labour supply response to such a small change in \( d \) at all. Meanwhile, they may well respond to a large change in \( d \).

The numbers reported in Table 4 are broadly consistent with the available evidence. Powell (1997), using data from Canada, reports an elasticity of female labour supply with respect to day care costs of –0.32. Averett et al. (1997), using NLSY data from the United States, find an elasticity of –0.78. Ribar (1992), using data from the Survey of Income Program Participation in the United States, finds an elasticity of –0.74. Attanasio et al. (2008), using a structural approach, find that a 15 percent decrease in child care costs should increase female labour force participation by six percentage points.
Table 3: Labour supply wage elasticities

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>W/o small children</th>
<th>With small children</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohabiting women</td>
<td>0.51</td>
<td>0.43</td>
<td>1.37</td>
</tr>
<tr>
<td>Cohabiting men</td>
<td>0.37</td>
<td>0.36</td>
<td>0.40</td>
</tr>
<tr>
<td>Single women</td>
<td>0.45</td>
<td>0.26</td>
<td>10.31</td>
</tr>
<tr>
<td>Single men</td>
<td>0.24</td>
<td>0.24</td>
<td>–</td>
</tr>
</tbody>
</table>

Notes: A child in this context is one below the age of 6. Elasticities are computed from responses to a one-period 10 percent wage increase.

Table 4: Labour supply day care cost elasticities for parents with small children

<table>
<thead>
<tr>
<th>Day care cost change</th>
<th>Cohabiting women</th>
<th>Cohabiting men</th>
<th>Single women</th>
</tr>
</thead>
<tbody>
<tr>
<td>−10%</td>
<td>−0.02</td>
<td>0.00</td>
<td>−0.02</td>
</tr>
<tr>
<td>−50%</td>
<td>−0.63</td>
<td>0.03</td>
<td>−0.57</td>
</tr>
<tr>
<td>−90%</td>
<td>−1.47</td>
<td>0.06</td>
<td>−1.41</td>
</tr>
</tbody>
</table>

Note: The elasticity is the percentage change in labour supply divided by the percentage change in unit cost. For example, if the day care cost is reduced by 50 percent, labour supply increases for cohabiting women with small children by 31.5%, but falls for cohabiting men with small children by 1.5%.

3.5. Effects of day care reform

We now consider the effects of day care finance reform. We design the reforms in such a way that those already born at the moment of reform are not affected; they pay their taxes according to the old system. We adopt this approach in order to avoid any issues of intergenerational redistribution. In particular, we want to avoid the result that the initial old and middle-aged lose from the reform simply because they pay for it but get nothing in return. Such a result would not be particularly interesting. Because of the grandfather clause of our reform, no agent experiences any transition. Nevertheless, we solve for the transition at the aggregate level; this is necessary in order to compute the intertemporal government budget balance. The reforms are financed by a vertical shift of the marginal tax schedule for those income levels where taxes are paid. Because of the grandfather clause, this vertical shift only applies to those born after the reform; cohorts born prior to the reform pay taxes according to the old schedule.
When evaluating welfare gains for a group of households, we use two distinct sets of weights. In the first, weights are inversely proportional to the marginal utility of consumption; this removes any redistributional motives on the part of the government, as noted above. In the second, we use equal weights. This gives rise to two distinct social welfare functions; we will call them the weighted and unweighted SWF, respectively.

Since we don't know a priori which reform is optimal in this context, we consider a range of linear subsidies as well as policy of letting child care expenses be tax deductible. The welfare effects of these reforms are depicted in Figure 5 which shows the equivalent consumption increase for each of the major groups (all, married couples, single women and single men) as a function of the subsidy rate from 0 to 100 percent. The figure shows that a 50 percent subsidy maximizes both the weighted and the unweighted SWF. (Deductibility is not the best policy from the point of view of either welfare criterion.) We now examine the effects of this reform in more detail.

The overall welfare gain associated with increasing the subsidy from zero to 50 percent corresponds to a 0.4 percent increase in consumption when the weighted SWF is used; 0.5 if the unweighted SWF is used. For couples, single women and single men, the reform leads to a gain, computed using the weighted SWF, corresponding to an increase in consumption of 0.4 percent, 0.3 percent and −0.1 percent, respectively. With an unweighted SWF these numbers are 0.5, 0.5 and 0.1.

Underlying the aggregated welfare gains, there is considerable heterogeneity, the details of which are depicted in Figure 6. Only 44 percent support 50 percent subsidies over zero subsidies. Half the opposition consists of those 28 percent of the population who never have children. Among those who do have children at some point, all single women support the reform and so do 60 percent of married couples. The opposition among married couples comes from those who do not use day care, neither before nor after the reform. Who are they? One group consists of those who have two or three small children in a single six-year period but no newborns in any other periods. 18 percent of couples have children according to this pattern, and for these couples even subsidized child care is so expensive that they choose not to make use of it. Another group that is mostly opposed to reform is the set of couples where the husband is well educated and the wife is not; this group accounts for 12 percent of couples and 89 percent of this group is opposed to reform.

Increasing the subsidy rate beyond 50 percent would be even less popular than moving from zero to 50 percent because of the required tax increase. Only 37 percent would support an increase from 50 percent to 90 percent; 42 of married and 28 percent
Table 5: Hours per year

<table>
<thead>
<tr>
<th>Ages</th>
<th>Pre-reform</th>
<th>50% subsidy</th>
<th>Deductibility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>23-46</td>
<td>47-64</td>
<td>23-46</td>
</tr>
<tr>
<td>Cohabiting women without children</td>
<td>1437</td>
<td>1188</td>
<td>1422</td>
</tr>
<tr>
<td>Cohabiting men without children</td>
<td>1979</td>
<td>1911</td>
<td>1959</td>
</tr>
<tr>
<td>Cohabiting men with children</td>
<td>2079</td>
<td>–</td>
<td>2029</td>
</tr>
<tr>
<td>Single women without children</td>
<td>1655</td>
<td>1759</td>
<td>1646</td>
</tr>
<tr>
<td>Single men (without children)</td>
<td>1667</td>
<td>1804</td>
<td>1667</td>
</tr>
</tbody>
</table>

Note: A child in this context is one below the age of 6.

of single women support such a move. (Obviously no single men support it.) In addition to those single women who have no children, highly educated single women who have exactly one child at age 23-28 disapprove of moving from 50 to 90 percent; this group does however support a move from a zero to a 50 percent subsidy. Though the opposition to a 90 percent subsidy rate is strong, such a policy would, according to our model, increase labour supply dramatically. When day care is not subsidized, the labour supply of married women with small children is just 25 percent of that of married men with small children. After the introduction of a 90 percent subsidy, that number is 68 percent, almost a tripling. For single women with small children, a 90 percent subsidy would increase their labour supply by about 160 percent, implying that single women would work slightly more when they have small children than when they do not. On the other hand, the increased demand for highly subsidized day care gives rise to a tax increase by 1.3 percentage points. This is what makes a 90 percent subsidy less popular than the 50 percent subsidy.

The effects of a 50 percent subsidy on labour supply are summarized in Table 5. Hours worked by mothers with small children almost double. Aggregate labour supply increases by 1.0 percent. Meanwhile, GNP increases by 0.7 percent and resources allocated to day care go up from 0.45 percent of GNP to 0.81 percent. Recall that, pre-reform, 31 percent of single mothers receive social assistance payments in the model (26 percent in the data). Post-reform, this number is just 12 percent.
3.5.0.1. Effects of deductibility. What are the effects of making day care expenses tax deductible in this environment? The gains are considerably smaller than those associated with a 50 percent subsidy. 34 percent would support this policy over the status quo of no subsidy and no deductibility. The overall welfare gains of deductibility correspond to a consumption increase of 0.2 percent (weighted and unweighted). The numbers for married couples and single men are 0.2 and -0.1, respectively. For single women, the gains are 0.4 percent (weighted) and 0.6 percent (unweighted). The tax schedule is shifted up by a tiny bit more than as a result of a 50 percent subsidy, contributing to the unpopularity of deductibility. Meanwhile, the effects on labour supply are still fairly significant. Among single mothers with small children, labour supply increases by 73 percent; on the other hand, among married mothers with small children, hours only go up by 11 percent.

Though neither tax deductibility nor any subsidy rate different from zero commands majority support, it seems reasonable to suppose that day care subsidies might still emerge in a democratic system as a result of some horse trading. The reason for saying that is that the losers lose very little compared to what the gainers gain. Among the losers, the maximum loss associated with a 50 percent day care subsidy corresponds to a 0.09 percent reduction in consumption. Among the winners, the median gain corresponds to an increase in consumption of 0.93 percent.

4. CONCLUDING REMARKS
In this paper we have shown, within a heterogeneous-agent life-cycle framework, that the Ramsey optimal policy is to make day care expenses tax deductible. Calibrating our model to Germany, we found that subsidizing day care would make German mothers work significantly more, leading to a noticeable welfare gain by moving the economy toward a more efficient trade-off between consumption and leisure.

What other policy reforms might enhance efficiency and encourage mothers to work? Güner et al. (2010) consider the hypothetical replacement in the United States of the progressive income tax system there by a proportional one. What they find is that such a reform would increase married female labour supply by 9 percent and raise aggregate welfare by an amount corresponding to a 1.3 percent increase in consumption. Our framework has similar implications. We find that making labour income taxes proportional in Germany would raise married female labour supply by 7 percent and the aggregate (weighted) welfare gain corresponds to a 1.4 percent rise in consumption. However, this reform has a much more modest effect on the labour supply of mothers.
with small children than day care finance reform does. For married mothers, hours are predicted to go up by 17 percent when taxes are made proportional; for single mothers, hours go down by 7 percent.

In our analysis we have not considered the possible effects of day care on child welfare and development. OECD (2006) summarizes the literature, reporting mostly positive effects, though Baker et al. (2008) reports some evidence of negative effects from the province of Québec. More recent studies focus on Scandinavia, which has seen the largest expansion of subsidized day care in the world. Havnes and Mogstad (2011) find evidence of positive effects on children’s long-run outcomes, whereas no statistically significant effects are found in Denmark and Sweden (Gupta and Simonsen, 2010 and Gruber et al., 2010). Thus there appears to be no strong counter-argument based on child development to the efficiency case for day care subsidies that we make in this paper.

A. DATA

Our main source of data is the German Socio-Economic Panel (SOEP) for the years 1984-2004. More information about SOEP can be found at http://www.diw.de/english/soep/29012.html. In this dataset, we look only at data from western Germany. In SOEP, all data are annual. In the model, a period is six years. This means that we have to perform some transformations into six-yearly data; see below.

A.1. Measurement of marital status

Conceptually, we are not interested in whether anyone is legally married, only whether they are living with one other person in a relationship that is in relevant ways similar to marriage. The approach we adopt is an imperfect attempt to capture that notion. What we do is to group people who belong to the same household (live at the same address) and try to pick out among the adult members a pair of individuals who appear to be in a marriage-like relationship. If the household has just one adult member, the situation is clear: we then consider the sole household member to be single. On the other hand, a cohabiting couple is defined as a man and a woman living at the same address who are 17 years or less apart in age and are either

1 in a two-adult household
2 in a three-adult household and the third adult is more than 17 years older or younger than any member of the couple and the oldest member of the couple is at least 30 years old or
3 in a four-adult household and all other adults are more than 17 years older or younger than any member of the couple and the oldest person in the couple is at least 30 years old

A single person living in a multi-adult household is defined as a man or a woman who is either

1 in a same-sex cohabitation relationship defined as above except for the “man and a woman” clause
2 in a three-adult household where the other two constitute either a cohabiting couple or are in a same-sex cohabitation relationship

Individuals in multi-adult households who fall into neither of these categories are not classified according to marital status and are thrown out of the sample.

The exclusion of same-sex couples is there because we are interested in using marital status information to draw inferences about the probability of the arrival of children, and the exclusion of couples 18 years apart or more in age is there to exclude single parents living with their children from being assigned as married.

Evidently our approach excludes some couples that, conceptually speaking, are married in the economic sense of that term. Nevertheless, we take the view that our approach yields an acceptable approximation.

A.2. Probabilities of having children

As described in the main text, the distribution of children among households in the model is based on simulating large populations of single women and couples on the basis of probabilities of newborn children arrivals that are taken from the data. Based on these populations, we can then determine the population weights of each of the 34 child profiles.

The probabilities of having (acquiring\(^{31}\)) 0, 1, 2 and 3 children (more than that is not allowed in any given six-year period) for single women and couples are set so as to match the number of young (less than six years old) children that these categories of household have (possess) as a function of the age of the parents and the number of children already present in the household, i.e. those children that are between the ages of

\(^{31}\) It is of course not idiomatic in English to speak of a household “acquiring” a child, but in this context it is crucial to distinguish between possession and acquisition, and here we are talking about acquisition and not possession.
six and seventeen. Since the fraction of young parents who have more than two children is tiny, we force this fraction to be zero.

In the SOEP files Xkind (where X is a letter representing the year), there is an entry corresponding to each child in the sample with information on birthyear and a number identifying the household. We then merge this data with the information on marital status, age, and, if unmarried, the gender of each apparent parent (any adult in the same household as the child), and remove those adult-children pairs that are such that the child is less than 18 years younger than the parent. We then consider those children that are between 0 and 5 years old; these are considered “newborn” for the purpose of the calibration. The probabilities of having 0, 1, 2 and 3 new children for potential parents categorized by marital status, age, gender and number of children aged 6-17 are then simply given by the corresponding fractions in the data, e.g. the fraction of 23-28-year-old single women with two children aged 6-17 who have exactly one child between 0 and 5.

A.3. Life-cycle hours profiles

The data on hours are based on the SOEP variable “average hours worked per week”, called, for example, BP41 in 1985 and NP47 in 1997. What we would like to do is to match the life-cycle profile of hours worked for the first cohort to be affected in a major way by day care reform, i.e. those who are young today. For obvious reasons, there is no data on the entire life-cycle profile of hours for this cohort. Therefore, we use the entire SOEP panel from 1984 to 2004 and regress hours on age and cohort dummies representing the decade of birth. The data presented in Figure 1 are the predicted values for the cohort born in the 1960s.

A.4. Wages

We begin by using annual data to compute a wage trend which we then remove before constructing six-yearly data. For this purpose, we compute log annual wages by dividing individual annual labour earnings by annual hours worked and then take logs. (The names of these SOEP variables are e11l01XX and i111l0XX, where the XX stands for the year.) We then remove observations in the bottom 10 percentiles and the top 1 percentile on a yearly basis. This involves keeping wages that were between €4.44 and €23.19 in 1984 and between €7.23 and €57.72 in 2004. We then regress these remaining observations on year dummies. We then detrend annual log earnings by subtracting the year-specific
### Table A1: Wage profiles

<table>
<thead>
<tr>
<th>Age Group</th>
<th>≤ 12 years of schooling</th>
<th>&gt; 12 years of schooling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Females</td>
<td>Males</td>
</tr>
<tr>
<td>23-28</td>
<td>0.000</td>
<td>0.051</td>
</tr>
<tr>
<td>29-34</td>
<td>0.139</td>
<td>0.134</td>
</tr>
<tr>
<td>35-40</td>
<td>0.179</td>
<td>0.227</td>
</tr>
<tr>
<td>41-46</td>
<td>0.195</td>
<td>0.277</td>
</tr>
<tr>
<td>47-52</td>
<td>0.203</td>
<td>0.307</td>
</tr>
<tr>
<td>53-58</td>
<td>0.223</td>
<td>0.295</td>
</tr>
<tr>
<td>59-64</td>
<td>0.222</td>
<td>0.260</td>
</tr>
<tr>
<td>Cohab. premium</td>
<td>-0.015</td>
<td>0.066</td>
</tr>
</tbody>
</table>

*Note: A single highly educated woman in the age group 47-52 makes 51.8 percent more per hour than a 23-28-year-old single woman with less than 13 years of education. If she is married, she makes another 4.9 percent more. Other numbers in the table are interpreted analogously.*

The trend component for each observation is estimated, and then the exponential function is applied to obtain yearly detrended earnings.

The next step is to add subsequent observations to obtain total earnings over a six-year period, and similarly with hours worked. Thus we obtain six-yearly wages. We then remove observations involving less than 6000 hours worked over a six-year period and individuals born before 1940. Then we only retain wages in the 10th-99th percentile range. After taking logs, we then regress the remaining observations on dummies for age and marital status separately for each gender/education group (highly educated men etc.). The regression coefficients are presented in Table A1.

Finally, we use wage data to construct a nominal anchor that enables us to translate figures in euros (e.g. in the tax and transfer system) into model units. The anchor is the wage of a young single woman with 12 years of education or less. To compute it, we use annual data on wages for those working at least 1000 hours, remove observations below the 10th percentile and above the 99th by year. Then we regress log wages for women with 12 years or less of education on dummies for age, calendar year and marital status. The anchor is then constructed as the predicted wage from this regression in 2004 for young single women. The number we get is €11.74.
B. TAX AND TRANSFER SYSTEM

B.1. Tax system

The German tax system is modeled following the description in OECD (2005). Spouses are assessed jointly using the income splitting method. We define taxable income, \( x \), as earnings less a basic allowance which consists of three parts. First, there is an allowance of €1308 for single parents. Second, there is a work-related allowance of €920 per employed person. Third, there is lump-sum allowance of €36 for singles and €72 for couples. The tax liability, \( T \), is then calculated as follows. Let \( y = \frac{(x - 7664)}{1000} \) and \( z = \frac{(x - 12739)}{10000} \).

\[
T = \begin{cases} 
0 & \text{if } x \leq 7664, \\
(793.10y + 1600)y & \text{if } 7664 < x \leq 12739, \\
(265.78z + 2045)z + 1016 & \text{if } 12739 < x \leq 52152, \\
(0.48x - 10410) & \text{if } 52152 < x.
\end{cases}
\]

These formulae are used directly to calculate the tax liability for a single individual. For couples, we apply these formulae on half the taxable income and then double the resulting amount to arrive at the tax liability. A “solidarity surcharge” (Solidaritätszuschlag) is then levied at 5.5 percent of the tax liability subject to an exemption limit of €972 for singles and €1944 for couples. Total tax payments are equal to the tax liability plus the solidarity surcharge. Note finally that we do not model social security contributions or benefits. The reason is that a large part of benefits are tied to contributions in a more or less actuarially fair way which means that the system of social insurance contributions and benefits is not distortive in the way that income taxes and public purchases are.

B.2. Social assistance

German social assistance policies are modeled following the description in Adema and Kahl (2003).

The first component is a universal child benefit (Kindergeld) of €1848 per child and year.\(^{32}\) The second component is a means-tested child rearing benefit of €3684 per year for children below 2 years of age. The third and final component consists of additional means-tested benefits, including a standard payment and allowances for children, housing, heating and special needs. We can summarize the second and third components of social

\(^{32}\) Legally, the child benefit is treated as a tax credit, but in cases where the tax liability is less than the tax credit, the difference is paid out as a cash transfer. The child benefit is thus equivalent to a direct cash transfer, and we have therefore chosen to model it as such.
benefits (as we model them) in the following equations. A single parent without children receives a benefit equal to €7044 per year. A couple without children receives €10920 per year. A single parent receives (on average per year in a six-year period)

\[
€10356 + €1396 \cdot # \text{ of small children} + €936 \cdot # \text{ of other children} \tag{B.23}
\]

and a couple with children receives

\[
€12312 + €1228 \cdot # \text{ of small children} + €936 \cdot # \text{ of other children} \tag{B.24}
\]

The sum of these means-tested social assistance payments are reduced by 85% of earnings net of taxes except when it comes to single mothers; their clawback rate is just 75 percent. These clawbacks are incorporated into the model; others are not. For instance, there are earnings limits above which the clawback rate is 100 percent; incorporating this feature only affects a very narrow range of earnings and would complicate the analysis considerably without affecting the results much. In addition, there are asset limits, but the value of the home or car may be disregarded; for that reason, we have chosen not to incorporate asset limits into the model.

REFERENCES


Figure 1

Average hours worked over the life cycle in Germany. Solid lines refer to people without small children, dashed lines to people with small children. For details, see Appendix A.3.
Figure 2

Average and marginal tax rates for a single individual with one child.
Figure 3
Average hours worked over the life cycle in the data and in the model for cohabiting men and women. Solid lines refer to the data, dashed lines to model predictions.
Figure 4
Average hours worked over the life cycle in the data and in the model for single men and women. Solid lines refer to the data, dashed lines to model predictions.
Figure 5

Welfare gains associated with day care subsidies between 0 and 100 percent. Solid lines represent gains computed using a social welfare function (SWF) with weights inversely proportional to the marginal utility of consumption; dashed lines refer to gains computed using a SWF with equal weights. The diamond (computed using equal weights) and the solid dot (computed using unequal weights) represent the welfare gains associated with tax deductibility.
The distribution of welfare gains associated with a 50 percent subsidy. In the lower right-hand panel, the solid line represents married couples and the dashed line represents single women. All single men lose as a result of day care subsidies.