Holdouts in Sovereign Debt Restructuring: A Theory of Negotiation in a Weak Contractual Environment

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Abstract

Why is it difficult to restructure sovereign debt in a timely manner? In this paper we present a theory of the sovereign debt restructuring process in which delay arises as individual creditors hold-up a settlement in order to extract greater payments from the sovereign. We then use the theory to analyze recent policy proposals aimed at ensuring equal repayment of creditor claims. Strikingly, we show that such collective action policies may increase delay by encouraging free-riding on negotiation costs, even while preventing hold-up and reducing total negotiation costs. A calibrated version of the model can account for observed delays, and finds that free riding is quantitatively relevant: whereas in simple low-cost debt restructuring operations collective mechanisms will reduce delay by more than 60%, in high-cost complicated restructurings the adoption of such mechanisms results in a doubling of delay.

JEL CODES: D23, D78, F34, K12, K33

1 Introduction

Negotiations to restructure sovereign debt are time consuming, on average taking more than six years to complete. Such delays are puzzling because they are costly to all parties: Sovereign debtors in default face disruption in their access to world capital markets, while

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creditors suffer large losses in the value of their investments. Why are delays so long? In this paper, we develop a theory of sovereign debt restructuring negotiations based on the observation that it takes place in a weak contractual environment, where the sovereign cannot commit to making identical settlement offers to all creditors. Delay then arises endogenously due to a strategic holdout effect whereby creditors delay entering into a settlement in the expectation of better terms at a later date.

Motivated by a number of recent cases and the ensuing policy debate,\(^1\) we use the theory to examine whether strategic holdout is overcome by collective action mechanisms, i.e. policies which bind holdout creditors to agreements negotiated by a group of earlier settling creditors. Our most striking result is that collective action mechanisms can actually increase delay due to an offsetting free-rider effect in which creditors free-ride on the negotiation efforts of other creditors. In a calibrated version of the model, we show that collective action mechanisms can more than double delay for an empirically significant subset of restructurings.

To understand the mechanism underlying our theory, note that the legal environment governing sovereign debt restructuring gives each individual creditor the power to disrupt a sovereign’s access to world capital markets; thus, individual creditors can veto a sovereign’s attempts to regain normal credit market access. As the sovereign is also unable to commit not to increase settlement payments, this leads to the strategic holdup effect: Individual creditors have an incentive to delay entering into negotiations in the hope of exploiting their power to ‘veto’ market access later in the restructuring process and extract a higher settlement. In addition to establishing that there can be delay in equilibrium, we derive a number of comparative dynamics results. Consistent with the concerns of policy-makers, we show that if markets become more fragmented by a rise in the number of creditors, competition to be the last to settle intensifies, and delay is increased.

Recent policy proposals have tried to eliminate strategic holdout by encouraging collective restructuring negotiations that ensure equal repayment of creditor claims. Such ‘collective action mechanisms’ include both the proposed re-introduction of bondholder councils (which enforced equal repayment during the earlier era of bond lending), and the introduction of collective action clauses into bond contracts, which have recently become the norm under New York law. We demonstrate that the incentive to engage in strategic holdout is eliminated by these mechanisms. Interestingly, the introduction of collective action mechanisms may increase delay because the imposition of common settlement terms intensifies the incentive for creditors to free ride on the costs of negotiation borne by earlier settling creditors.

We then calibrate the model in order to assess the magnitude of the strategic holdout and free rider effects. For plausible estimates of creditor bargaining power, the strategic holdout incentive can produce delays of six or more years, in line with the data. We document a wide range of bargaining costs across different debt restructuring episodes and show that, when the model is calibrated to the range of negotiation costs observed in practice, the free riding incentive is quantitatively relevant. For complicated restructuring operations, where the costs of collective negotiation are high, we find the introduction of a collective action

\(^1\)For example, compare the position of the US Treasury (Taylor 2002, 2007) and the November 2010 announcement by European Finance Ministers (Eurogroup 2011), with that of the International Monetary Fund (Krueger 2001, 2002a,b).
mechanism can result in a doubling of delay. For simpler restructuring operations, where costs are low, we find that collective mechanisms will reduce delay by more than 60%.

Our main contribution to the literature on sovereign debt restructuring is the introduction of a fully specified extensive-form dynamic model of entry and settlement, in which delays arise due to collective action problems among creditors. Our calibration of the model is novel both in terms of the data used as well as the underlying game. Our theory contrasts with models of delay in bargaining between a sovereign and a single creditor (Yue 2006, Bi 2007, and Benjamin and Wright 2008), and models of collective action problems among creditors that do not produce delay (Kletzer 2002, Haldane et al. 2003, Weinschelbaum and Wynne 2005, and Wright 2001 and 2005). Our study of the ease of restructuring complements Bolton and Jeanne’s (2007) analysis of the decision of a sovereign to issue debt that is exogenously ‘easy’ or ‘hard’ to restructure. Although many policy makers advocate a reduction in delay on normative grounds, we caution that we cannot draw normative conclusions without also studying its effect on ex-ante incentives to borrow appropriately and avoid default (see instead Benjamin and Wright 2008, Pitchford and Wright 2008 and, in corporate finance, Bolton and Scharfstein 1996).

Delay in our model stems from a lack of commitment, unlike much of the theoretical literature on delay in bargaining which stresses private information (See Grossman and Perry 1986, Fudenburg, Levine and Tirole 1985, and Bai and Zhang 2008 among others). Our theory is closer in spirit to the literature on timing games, such as the war of attrition, in which delay occurs due to a sequence of payoffs which exogenously increases as the number of players who remain in the game falls (e.g. Hendricks, Weiss and Wilson 1988, Haig and Cannings 1989, Bulow and Klemperer 1999 and Kapur 1995). By contrast, our theory of bargaining in a weak contractual environment where past bargains are sunk generates a rising sequence of payoffs endogenously, as in the simple discrete time two-player model in Menezes and Pitchford (2004). Finally, our theory departs from the literature on multi-plaintiff settlement (e.g. Spier 1992, 2003A,B and 2007, and Daughety and Reinganum 2003 and 2005) by assuming a weak contractual environment, which also rules out the ingenious “divide and conquer” solution to the holdout problem devised by Che and Spier (2008).

The rest of this paper is organized as follows. After some background on the institutions governing sovereign debt restructuring in Section 2, Section 3 presents our theory and establishes our main qualitative results about delay. Section 4 then presents quantitative results for a calibrated version of the model, while Section 5 establishes the robustness of our results to a number of alternative assumptions on the way bargaining is carried out, the ability of sovereigns to influence the restructuring process, and to various forms of asymmetry amongst creditors. Section 6 concludes while an appendix collects proofs of main results, and an ancillary appendix provides further details on the calibration, and on the extensions and robustness exercises.


2 Background

Sovereign debt negotiations take place in a “weak contractual environment” which we characterize by five key features. The first is fundamental to the problem of sovereign default:

(I) Sovereigns lack the ability to commit to contracts.

The absence of an international bankruptcy court, combined with immunity from legal action in their own (and other countries’) jurisdictions—due to the Doctrine of Sovereign Immunity—meant that sovereigns could not be bound by contracts they signed. The passage of legislation like the Foreign Sovereign Immunity Act of 1976—which allowed lawsuits against sovereigns in the United States—has weakened this doctrine, but it remains very difficult for creditors to collect on favorable judgments even when assets are outside a nation’s borders. A spectacular example is of the Swiss trading firm Noga’s many failed attempts to seize Russian assets as various as sailing ships, jet fighter planes, uranium shipments, embassy bank accounts and art exhibits.2

The sovereign’s inability to commit means that it cannot bind itself to settle on the same or inferior terms with holdout creditors. In a prominent example of the inability to commit, Argentina filed documents with the U.S. Securities and Exchange Commission in 2004 stating that it would not pay holdout creditors, and passed domestic legislation prohibiting the re-opening of the exchange offer (the so-called “Padlock Law”) only to make a new exchange offer in 2010.3 Of the many examples in which holdout creditors have secured better settlement terms, the most well known are Elliott Associates, L.P. v. Banco de la Nacion and The Republic of Peru,4 where the holdout creditor Elliott Associates received a 58m settlement on bonds with a face value of 21m that it had purchased for 11m; Elliott v. Republic of Panama (1997),5 where they received roughly twice the settlement; and CIBC Bank (Kenneth Dart) v. Brazil (1995).6

These successes are a primary motivation for creditor holdout. The reason holdouts are able to secure larger settlements is due to a second feature of this environment:

(II) All creditors must settle before the sovereign can regain normal credit market access.

Historically, the London Stock Exchange refused to list a sovereign’s new money bonds until it had settled with all creditors. This is reflected in an absence of borrowing by defaulting countries in the historical record (Tomz 2007). Since the passage of the Foreign


Sovereign Immunity Act, a variety of newer legal tactics have been used to disrupt credit market access. The most famous example was in the case of Elliott Associates and Peru, discussed above, where Elliott Associates obtained an injunction to stop Peru’s bankers releasing funds to pay interest on Brady bonds issued as part of its restructuring. This brought Peru to the brink of default on these new bonds and forced a settlement. More importantly, this tactic makes it impossible for a sovereign to make payments on any new bonds issued while holdout creditors remain, and thus impossible to issue new bonds. Such lawsuits have become increasingly common: The World Bank and International Monetary Fund (2007), report forty-seven court cases against a total of eleven highly indebted poor countries, while Argentina was faced with over one hundred lawsuits following its 2001 default (Gelpern 2005).7

A sovereign’s inability to commit to discriminate against holdout creditors, combined with its need to settle with every creditor, rules out the “divide and conquer” strategies studied by Che and Spier (2008). However, the sovereign is able to discriminate in favor of holdout creditors because:

(III) A settlement exchanges defaulted debt for an immediate cash payment (or its equivalent) and expunges any future legal rights on the defaulted debt.

Sovereign debt is typically restructured through an ‘exchange offer’ in which the old debt is exchanged for some combination of cash and some new securities, as in Peru’s 1993 restructuring which included about 4bn in cash payment and new debt with a face value of 4bn (and market value of about 2bn as its cash equivalent). It might, at first, seem contradictory that a sovereign in default has access to cash. Whereas bankrupt domestic corporations have limited liquid assets, sovereigns typically have sufficient wealth to repay, but choose not to. In some cases, countries have used their resources to secretly buy back their defaulted debt on secondary markets. For example, in the 1980s Mexico re-purchased $8bn (Cohen and Verdier 1995) and Peru repurchased $1.7bn (Alfaro 2006), while Ecuador is alleged to have recently engaged in secret buybacks (Miller 2009, Porzecanski 2010).

The fact that creditors who settle early with the sovereign give up their old securities means they do not have access to legal mechanisms to prevent the sovereign paying higher settlements to holdout creditors later in the process.

In addition to avoiding creditor holdout, another motivation for improving creditor coordination is to share the costs of arranging settlement:

(IV) Creditors incur substantial transactions costs, some of which are difficult to verify.

The evidence on negotiation costs shows that they vary substantially from one restructuring to the next, are often very large, and are hard to verify and share amongst creditors. In the case of collective negotiations, the task of reconciling creditor claims is especially costly and appears to fall mostly on a small number of lead creditors. We defer a detailed discussion of the evidence to the calibration Section 4 below (with more details in the ancillary appendix) and simply note that some banks have declined to participate in negotiations

7See Gelos, Sahay and Sandleris (2004) and Richmond and Dias (2007) for a debate on the ease of capital market reaccess in the late 1990s.
due to their costs, while holdout creditors like Elliott Associated routinely complain about other holdout creditors who free-ride on the substantial cost of litigation.

The above features of the environment indicate that creditors may find it desirable, but very difficult, to coordinate. This is borne out by the evidence:

(V) Creditor efforts to coordinate have been frequent, but often ineffective.

Following the defaults of the early 19th Century, bondholders in London organized themselves into competing *ad hoc* bondholder committees (see Esteves 2007 for a review). That is, debt renegotiation involved competing bondholder committees—if not competing individual creditors. We refer to restructuring in such an environment as an Individual Settlement Process. In response to this competition, the Council (later Corporation) of Foreign Bondholders was established in 1868 to represent all British bondholders. The Corporation’s power derived largely from the fact that the London Stock Exchange deferred to it when deciding whether to list new bond issues. We refer to restructuring under such a regime as a Collective Settlement Process. This process is similar to that undertaken by Bank Advisory Committees which organize collective negotiations to restructure commercial bank loans.

The resurgence in sovereign bond issues has led to a return to an individual settlement process with *ad hoc* bondholder committees such as the Global Committee of Argentina Bondholders (GCAB). This has prompted a policy debate advocating different forms of collective settlement processes for bondholders, ranging from an international bankruptcy court (Krueger 2002a,b) to more modest changes in bond contracts (Taylor 2002, 2007). Sovereign bond issues under New York law now include engagement and collective action clauses specifying the procedures by which creditors organize and negotiate with the sovereign, and impose common settlement terms on other creditors. Euro-area government bonds will include these clauses starting in 2013 (Eurogroup 2011).

3 Model

In this section, we present our basic model, emphasizing how it captures the features described in section 2. For simplicity we assume that all creditors are symmetric, and adopt a very simple bargaining protocol. Both of these assumptions are relaxed in Section 5.

3.1 Environment

There are $N$ creditors and a sovereign. All players have complete information, are risk neutral, and have a common discount rate $r$. The game begins at time $t = t_N = 0$, after the sovereign defaults on its debt, and does not end until the sovereign has settled with all creditors. Then, the sovereign is able to re-access world capital markets and gain $V$, the surplus gross of all settlement payments (feature II).

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8The Bank of New York was unwilling to act as agent in Argentina’s 2004 restructuring due to the “size and complexity” of the deal (“Pre-match betting against Argentina” Financial Times, January 10, 2005). Devlin (1989, p.200) reports that some small banks wrote off their claims “in face of unwanted costly and protracted negotiations.”

9“Elliott’s activist chief has no time for cheats”, Financial Times, 10th April 2006, p.4.
The structure of the game is presented in Figure 1. At the beginning, the sovereign is in default with each of its $N$ creditors. The creditors can avoid engagement, so the sovereign must wait for one of them to enter the settlement process. In this initial timing stage, which we denote $T_N$, each of the $N$ creditors chooses some $t \in [0, \infty)$ at which to enter the settlement stage, which we denote by $S_N$ (and which we describe in detail below). We assume that only one creditor can enter a settlement stage at a time, and that ties are resolved by a random allocation with equal probabilities. Following receipt of a settlement, the creditor exits the game at some $t = t_{N-1}$, forfeiting any future claims (feature III). The remaining $N - 1$ creditors decide when to enter the settlement process in timing stage $T_{N-1}$ on $[t_{N-1}, \infty)$ which ends when a creditor enters settlement stage $S_{N-1}$. The creditor exits the game following receipt of its settlement at some $t_{N-2}$. Timing and settlement stages are repeated until the last creditor exits.

Creditors who exit the game have no further claims and hence no further influence on outcomes. Thus, we adopt notation to keep track of the number of active players in the game at any particular point. Subgame $i$ starts with a timing stage $T_i$ where $i$ creditors remain, followed by a settlement stage $S_i$ in which one of them has entered.\footnote{As time is continuous, there are a continuum of such subgames. We let subgame $i$ refer to the first such subgame.} Subgame $i - 1$ begins once that creditor exits and there are $i - 1$ creditors remaining, etc. We let $U_i$ denote the payoff to the creditor, and $V_i$ denote the payoff to the sovereign, at the start of subgame $i$. Lowercase variables $u_i$ and $v_i$ denote the payoffs as at the end of settlement stage $S_i$, as illustrated in Figure 2.

A settlement process is a description of how $u_i$ and $v_i$ are determined in the sequence of stages $S_i$. We consider two types of process, \textit{individual} and \textit{collective}. The individual settlement process models the uncoordinated bargaining that has been typically observed in modern times. To capture this, we assume that for each and every $S_i$, $i = N, \ldots, 1$, the creditor who enters immediately engages the sovereign in a bilateral bargain that continues until an
agreement is reached. Bargaining entails that the creditor individually bears a proportional transactions cost, so that the individual creditor’s payoff is some fraction \( \theta_i \in (0, 1) \) of the bargained amount (feature IV).

The collective process has to capture the idea that a critical number \( M < N \) of creditors must agree to a settlement payment before the sovereign can borrow again. We model it as follows: The first creditor enters into stage \( S_N \), followed by the second in \( S_{N-1} \), and so on until the \((M-1)^{th}\) enters settlement stage \( S_{N-(M-2)} \). In each of these stages, no bargain takes place. Instead, we interpret entry to such \( S_i \) as an agreement to be bound by the settlement determined in the bargain between the lead creditor and the sovereign, (less transaction costs captured by \( \theta_i \)). We identify the lead creditor with the first player to enter.\(^{11}\) This creditor then bargains with the sovereign according to our protocol in settlement stage \( S_{N-(M-1)} \), and the outcome establishes the terms of the collective settlement. Following practice, note that the settlement is binding on all of the remaining \( N-M \) creditors (see feature V).

These remaining creditors have the advantage of not having to join the collective, therefore avoiding an obligation to share in the costs of negotiating the settlement.

For both settlement processes, we allow transaction costs to vary with the order of settlement \( i \). We use this feature when calibrating the model in Section 4 below. Unless otherwise stated, we maintain the assumption that bargaining costs are weakly larger for earlier creditors, which implies \( \theta_i \leq \theta_{i-1} \) for all \( i \). This captures the fact that some costs of bargaining, such as the cost of reconciling competing creditor claims, fall primarily on the lead creditor (see feature IV and the discussion in Section 4).

Regardless of whether the settlement process is individual or collective, we assume that all negotiation follows a random-offers variant of the Rubinstein (1981) bargaining game. As depicted in Figure 3, at the start of each round, Nature randomly selects whether the creditor or the sovereign makes the first offer. We let \( \alpha \) denote the probability the creditor is selected, so that \( 1 - \alpha \) is the probability that the sovereign is chosen to make an offer in any given round. The other party then accepts or rejects the offer. Acceptance ends the bargain with the creditor exiting the game. Rejection leads to a delay of \( \Delta \) (small) units of

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\(^{11}\) Although not essential for the model, it is natural to think of the first, or lead, creditor as a party who also conducts negotiations with the sovereign.
time. This is followed by another round of bargaining where the proposer is again selected randomly. Bargaining continues until an offer is accepted.

In the game with individual settlement, a creditor and the sovereign bargain according to the random-offers protocol every time a creditor enters a settlement stage. In the game with the collective process, bargaining happens only once, via the lead creditor, in stage $S_{N-(M-1)}$, when the critical number $M$ of creditors is reached. Before this number is attained, the payoffs received in $S_i$ are those which the entering creditor correctly anticipates will be obtained via the lead creditor’s bargain with the sovereign, less transactions costs.

It is convenient to define $\delta_i$ as the *expected discount factor*—equivalently the expected cost of delay—for subgame $i$: That is, $\delta_i$ values one dollar received at the end of the game, in expected dollars received at time $t_i$. We let $\beta_i$ denote the expected discount factor for the duration of $T_i$ and $S_i$, that is, the time between the start of subgame $i$ and when the creditor who enters in $T_i$, exits settlement $S_i$. Clearly, $\delta_i = \Pi_{j=1}^i \beta_j$.

### 3.2 Solution

We solve the game using backwards induction. In any arbitrary stage $i$ we first solve settlement stage $S_i$ which begins after one creditor has entered. In the case of individual settlement, payments are determined by bargaining between the sovereign and the entering creditor in every stage $i$. With collective settlement, payments are determined by the bargain between the lead creditor and the sovereign which occurs once in $S_{N-(M-1)}$, with the settlement for all other $i$ taken as given. Moving back in the tree, we then solve timing stage $T_i$ to determine which of the $i$ creditors enters the settlement process to bargain with the sovereign. Alternating back through settlement and timing stages in this way, we characterize a subgame-perfect equilibrium of the full game that is Markov in the number of active players in the game. Note that our analysis is simplified by our assumption that bargaining does not end until an agreement is reached. This means that in settlement stage
$S_i$ the creditor cannot return to timing stage $T_i$, allowing us to separate the solution of settlement stage $i$ from the solution of timing stage $i$ (we establish robustness to non-separable bargaining protocols in Section 5 below).

We characterize the solution to the full game by two lemmas which describe outcomes in the separate settlement and timing stages. Lemma 1 specifies bargaining outcomes in some settlement stage $S_j$. It takes as given the bargaining ‘pie’ which is the sovereign’s value $V_{j-1}$ measured as at the beginning of the following timing stage (see Figure 2 setting $i = j - 1$, and note that the prior stage is $S_{j-1}$). Lemma 2 details the outcome of timing stage $T_i$ taking as given the payoffs given by the bargaining Lemma for the following settlement stage $S_{i+1}$.

We first present the bargaining Lemma. Note that under individual settlement, each creditor bargains for itself. Under collective settlement, a creditor bargains on behalf of all of the $N$ creditors in the final settlement stage before the sovereign re-enters world capital markets. Otherwise, the bargaining game is identical for both individual and collective processes.

**Lemma 1 (Bargaining)** The unique subgame perfect equilibrium payoff from any bargaining stage $j$ is

1. $\alpha V_{j-1}$

for a single creditor under individual settlement or

2. $\frac{\alpha}{N} V_{j-1}$

for all $N$ creditors under collective settlement. In both cases the equilibrium payoff for the sovereign is

3. $(1 - \alpha) V_{j-1}$

and bargaining payoffs are realized without delay.

**Proof.** See the appendix section 7.1.

Note that the probability $\alpha$ with which Nature chooses a creditor to make an offer in any round determines the expected bargaining shares (1) for the creditor and (3) for the sovereign. The Lemma presents payoffs gross of transactions costs. The payoff $u_j$ to the creditor (measured as at the time of agreement and settlement) is reduced by the factor $\theta_j$ due to proportional transaction costs.

In the entry Lemma below, we study the solution of the arbitrary timing stage in which $i$ creditors make their entry decisions. The first creditor to enter obtains $u_i$ which we take as given from bargaining in the next settlement stage, as characterized in the bargaining Lemma above. All creditors who enter after the first receive the continuation value of the game, i.e. the payoff $U_{i-1}$ valued as at the beginning of timing stage $i - 1$ (see Figure 2 and note that the following subgame yields $U_{i-1}$).

In the entry Lemma, we derive the symmetric mixed strategy equilibrium of the timing game. There are also pure strategy equilibria that are necessarily asymmetric, in which players coordinate on the order of entry. We justify the focus on mixed strategies on the grounds that sovereign default is uncommon and creditors are often anonymous, so that social norms for coordinating on pure-strategy equilibria are unlikely to arise. Alternatively, our focus on mixed strategy equilibria can be justified due to (small) uncertainty creditors have regarding others’ payoffs as in Harsanyi (1973).
Lemma 2 (Entry) (i) Suppose $u_i < U_{i-1}$. The unique symmetric Markov perfect equilibrium of timing stage $i > 1$ is in mixed strategies. The expected payoff as at the beginning of timing stage $i$ is $U_i = u_i$, where all creditors randomize according to cdf

$$F_i = 1 - \exp \{-\lambda_i t\},$$

the hazard rate is

$$\lambda_i = \frac{ru_i}{(i-1)(U_{i-1} - u_i)},$$

the expected duration of the stage is

$$E[t_i - t_{i-1}] = \frac{1}{i\lambda_i} = \frac{i - 1}{i} \frac{U_{i-1} - u_i}{ru_i},$$

and the expected discount factor is

$$\beta_i = \frac{iu_i}{(i - 1) U_{i-1} + u_i}.$$  

(ii) If $u_i \geq U_{i-1}$, equilibrium has the creditors chosen with equal probability $1/i$, to enter immediately.

**Proof.** See the appendix section 7.2 ■

The Lemma treats $T_i$ as if it were a self-contained timing game with $i$ creditors in which the first to enter receives payoff $u_i$, and the remaining $i - 1$ players receive payoff $U_{i-1}$.

To understand the results, first consider part (i). Since $u_i < U_{i-1}$, each player would prefer that some other enter first, which generates a ‘war of attrition’ over entry times. Assuming symmetric mixed strategies, each player randomizes by choosing a cdf $F_i$ over the set of feasible entry times $[t_i, \infty)$, taking others’ such strategies as given. A heuristic intuition for the equilibrium cdf (4) is as follows. Consider a particular creditor, and suppose each other creditor randomizes according to $F_i$. Our creditor can enter immediately and receive $u_i$ immediately, or delay by a small interval of time $\Delta t$ and face a gamble where $u_i$ or $U_{i-1}$ could be received: Payoff $U_{i-1}$ is obtained if another creditor happens to enter within the interval. If $\lambda_i$ is the hazard rate governing one creditor’s decision to enter, the probability that one of $i - 1$ creditors will enter in $\Delta t$ is given by $(i - 1) \lambda_i \Delta t$. In equilibrium, each creditor must be indifferent between the interest foregone over the unit interval, $ru_i \Delta t$ and the gamble that some other creditor may enter first, yielding expected gain $U_{i-1} - u_i$. This indifference implies that

$$ru_i \Delta t = [U_{i-1} - u_i] (i - 1) \lambda_i \Delta t,$$

which is equivalent to (5) and consistent with the equilibrium cdf (4). The expected discount factor $\beta_i$ in (7) can easily be found by calculating $E[e^{rt}]$.

A crucial insight from this Lemma is that the expected payoff $U_i$ of the game at the start of stage $i$, is simply $u_i$, the payoff from immediate entry. This is because the creditors delay entering until any gains from delay have been eroded in expected value. When considering the solution to the individual and collective variants of the full game below, this will allow us to replace payoffs $U_j$ in future timing and settlement stages by the payoffs $u_j$. 

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Note that the creditors randomize according to the exponential distribution (4), which is memoryless. Further, the note that environment is stationary in the number of remaining creditors. These two facts ensure no creditor wishes to revise its strategy as time evolves without the entry of a creditor, and hence the strategies are subgame perfect. Following entry, and at the beginning of the next timing stage, creditors revise their strategies with both individual and collective settlement.

Now consider part (ii) with \( u_i \geq U_{i-1} \). Immediate entry yields a creditor payoff \( u_i \), whereas delay delivers \( U_{i-1} \), but discounted at instantaneous rate \( r \). All creditors therefore wish to enter immediately, and under our tie-breaking rule, they are chosen at random with equal probabilities, yielding expected payoff

\[
U_i = \frac{u_i}{i} + \frac{U_{i-1}(i-1)}{i}.
\]

In the next two subsections, we proceed by alternating between Lemma 1 to find payoffs and Lemma 2 to find delay and expected discount factors, in order to solve the full game under both individual and collective settlement processes.

### 3.3 Individual Settlement

**Proposition 3** The game with individual settlement has a unique symmetric Markov perfect equilibrium. This equilibrium is characterized by immediate entry in subgame \( i = 1 \) and positive expected delay in stages \( i > 1 \) of

\[
E[t_i - t_{i-1}] = \frac{1}{i r \theta_i} \left[ \sum_{k=0}^{i-1} (1 - \alpha)^{-k} \theta_{i-k} - i \theta_i \right].
\] (8)

Further, expected payoffs at the start of subgame \( i \) are

\[
U_i = u_i = \delta_{i-1} (1 - \alpha)^{i-1} V \theta_i,
\] (9)

for the creditor and

\[
V_i = \beta_i v_i = \delta_i (1 - \alpha)^i V,
\] (10)

for the sovereign, where \( \delta_j = \prod_{k=1}^{j} \beta_k \) and

\[
\beta_k = k (1 - \alpha)^{k-1} \theta_k \left[ \sum_{l=1}^{k} (1 - \alpha)^{k-l} \theta_{k-l+1} \right]^{-1}, \quad k \geq 1.
\] (11)

**Proof.** See the appendix section 7.3. □

Here we present the intuition behind the equilibrium payoffs (9) and (10) since direct substitution of these payoffs yields expected delay and expected discount factors. Under individual settlement, entry leads to a settlement stage in which the entering creditor and sovereign bargain. We proceed by backwards induction. Consider subgame \( i = 1 \) and, in particular, the bargain between the last creditor and the sovereign in settlement stage \( S_1 \).

Since the sovereign receives \( V \) on re-entering world capital markets, this amount constitutes the bargaining pie. From equation (1) in the bargaining Lemma, the creditor therefore receives

\[
u_1 = \alpha V \theta_1,
\] (12)
which is share $\alpha$ of the pie less transactions costs $\theta_1$. The sovereign receives

$$v_1 = (1 - \alpha) V,$$  \hspace{1cm} (13)

(see (9) and (10) respectively, for $i = 1$). Moving back to the timing stage $T_1$, with one remaining creditor, note that because there are no competing creditors, entry is immediate, so that $E[t_1 - t_0] = 0$ and $\beta_1 = 1$ (see Lemma 2 part ii).

Now consider $S_2$. In the bargain between the second-to-last creditor to enter and the sovereign, both of these parties anticipate that the total available surplus will be reduced because of the future bargain with the last creditor. In particular, the total bargaining pie is reduced to $V_1 = (1 - \alpha) V$ in anticipation of the final settlement where the sovereign pays the last creditor $\alpha V$. To solve the bargain in $S_2$, we utilize the bargaining Lemma again. The second-to-last creditor obtains a fraction $\alpha$ of the available pie $V_1$, which yields

$$u_2 = \alpha (1 - \alpha) V \theta_2$$  \hspace{1cm} (14)

after transactions costs. The sovereign receives fraction $(1 - \alpha)$ of $V_1$, or $v_2 = (1 - \alpha)^2 V$. Moving back to the timing stage $T_2$, note that each creditor would prefer the payoff $U_1 = u_1$ from being last, to the lower payoff $u_2$ from immediate entry. This leads to a timing game with positive expected delay: the corresponding expected discount factor is found by substituting $u_1$ from (12) and $u_2$ from (14) into equation (7) of Lemma 2, noting that $U_1 = u_1$, i.e.

$$\beta_2 = \frac{2 (1 - \alpha) \theta_2}{\theta_1 + (1 - \alpha) \theta_2}$$  \hspace{1cm} (15)

as in (11) of Proposition 3 for $k = 2$. Similarly, expected entry delay can be determined using Lemma 2, equation (6) as $E[t_2 - t_1] = (\theta_1 - (1 - \alpha) \theta_2) / 2r (1 - \alpha) \theta_2$. The value of the sovereign’s payoff at the beginning of subgame $i = 2$ anticipates the entry delay in the timing game between the two creditors, and is therefore

$$V_2 = \beta_2 (1 - \alpha)^2 V$$  \hspace{1cm} (16)

as in (10). In $S_3$, third-to-last creditor and sovereign bargain over $V_2$, which determines $u_3$ and hence $\beta_3$. Proceeding in this way, we can obtain the formulae for an arbitrary subgame $i$ in the Proposition.

### 3.4 Collective Settlement

As discussed in Section 3.1, the key difference between the collective and individual settlement processes is that in the collective case, bargaining only takes place between a lead creditor and the sovereign. Entry into stages $S_N$ through $S_{N-(M-2)}$ constitutes an agreement to be bound by the bargaining outcome between the lead creditor and the sovereign in $S_{N-(M-1)}$, less transactions costs. The following Proposition summarizes the outcome for the general case.

**Proposition 4** The game with collective settlement has a unique Markov perfect equilibrium. The equilibrium is characterized by immediate entry in subgames $i = N-M, \ldots, 1$ and positive expected delay in stages $i > N-M$ of
\[
E [t_i - t_{i-1}] = \frac{1}{(1 - \alpha) r \theta_i} \left[ N - M + \sum_{k=N-(M-1)}^{i-1} \theta_k - (i - 1) (1 - \alpha) \theta_i \right].
\] (17)

Further, payoffs as at the start of subgame \(i\) are

\[
U_i = u_i = \begin{cases} 
\delta_{i-1} \frac{\alpha}{N} V \theta_i & \text{if } i > N - M \\
\delta_{i-1} \frac{\alpha}{N} V & \text{if } i \leq N - M 
\end{cases}
\] (18)

for the creditors and

\[
V_i = \beta_i u_i = \delta_i (1 - \alpha) V,
\] (19)

for the sovereign, where \(\delta_i = \prod_{j=1}^{i} \beta_j\), and where the expected discount factors are

\[
\beta_i = i \theta_i \left[ N - M + \sum_{k=N-(M-1)}^{i} \theta_k \right]^{-1}, \quad i > N - M.
\] (20)

**Proof.** See the appendix section 7.4. ■

The key to understanding the collective settlement process is to understand the payoffs that emerge in the bargain between the representative and sovereign. Suppose the \(M^{th}\) creditor has entered and the lead creditor proceeds to bargain. As before, the total surplus over which the parties negotiate is \(V\), being the amount the sovereign gets from re-access to world capital markets. In contrast to the individual settlement case, the lead creditor bargains to determine joint surplus for all creditors, and this is divided \(N\) ways.\(^{12}\) The creditors as a group therefore receive \(\alpha V\) in expectation and the sovereign receives \((1 - \alpha) V\).

Thus, each *individual* creditor receives \(\alpha V/N\) gross of transactions costs. The difference between those who joined the coalition and those who did not is that only coalition members pay transactions costs. Thus, each of the \(M\) members of the coalition receives \(\alpha (V/N) \theta_j\) and the other \(N - M\) creditors (who did not join) obtain \(\alpha V/N\). Equation (18) represents the value of these payoffs measured as at the beginning of subgame \(i\), i.e. discounted with by \(\delta_{i-1}\). Equation (19) is the value of the sovereign’s payoff at the start of subgame \(i\), which is discounted by \(\delta_i\).

Expected delay and discount factors are calculated by substitution of payoffs in Lemma 2. Consider an example with \(N = 3\) creditors, where \(M = 2\) must agree on a settlement—join the coalition—before the game ends. Proceeding by backwards induction, suppose we are at the start of subgame \(i = 1\). The remaining creditor automatically receives \(\alpha (V/3)\) at this point, the settlement having been agreed by the collective action. Thus (by Lemma 2 part ii) there is no delay in this subgame and \(\beta_1 = 1\). Moving back to the settlement stage of subgame \(i = 2\), the representative’s bargain yields (current value) \(\alpha (V/3) \theta_2\) for itself, \(\alpha (V/3) \theta_3\) for the first creditor to join and \(\alpha (V/3)\) for the creditor who does not join. Moving further back to the timing stage of subgame \(i = 2\), the remaining creditors prefer to receive \(u_1 = \alpha (V/3)\) rather than \(u_2 = \alpha (V/3) \theta_2\), which generates positive delay in expectation. Substitution of

\(^{12}\)There are two ways in which bargaining by the lead creditor can be viewed. The alternative to the text is that any settlement which the lead creditor negotiates for itself is also given to \(N - 1\) others. We analyze this approach and prove equivalence of the two approaches in an ancillary appendix available online.
these values into equation (7) of Lemma 2 yields $\beta_2 = 2\theta_2 / (1 + \theta_2)$. Entry delay is found using Lemma 2, equation (8) as $E[t_2 - t_1] = (1 - \theta_2) / 2r\theta_2$. The sovereign’s value as at the beginning of subgame 2 is $V_2 = \beta_2 (1 - \alpha) V$.

At the beginning of the game there are $i = 3$ creditors. The first to enter $S_3$ is the first party to join the coalition. Its payoff is $u_3 = \beta_2 \alpha (V/3) \theta_3$ in current value terms. The remaining creditors receive the larger payoff $u_2 = \alpha (V/3) \theta_2$. Thus, there is positive expected delay as each of the three competitors compete to avoid being the first to enter. Generally, there is positive expected delay in each stage up until the representative’s bargain.

### 3.5 Strategic Holdup and Free Rider Effects

One of our key findings is that there is delay under both the individual and the collective settlement regimes. At a basic level, delay occurs with both settlement processes because payoffs looking forward are rising under both processes. The reason why this is true, however, is qualitatively different between the individual and collective regimes. The difference is seen most clearly for the case where all transactions cost terms are the same and equal to $\theta$.

In this case, all delay under individual settlement is due to what we term the *strategic holdout* effect. To understand this terminology, consider equation (10), which gives us the sovereign’s payoff at the start of subgame $i - 1$ as $V_{i-1} = \delta_{i-1} (1 - \alpha)^{i-1} V$. This quantity is the pie over which creditor and sovereign bargain in prior stage $S_i$. Note that the undiscounted surplus $(1 - \alpha)^{i-1} V$ rises as $i$ falls when we move to later settlement stages. This happens because prior settlements are sunk and so are not subtracted from the ‘final’ pie $V$. By (9) a creditor’s payoff is $u_i = \delta_{i-1} \alpha \theta (1 - \alpha)^{i-1} V$ in subgame $i$ and $u_{i-1} = \delta_{i-2} \alpha \theta (1 - \alpha)^{i-2} V$ in subgame $i - 1$. Strategic holdout occurs when a creditor delays in order to obtain the share $\alpha$ of a larger pie, $(1 - \alpha)^{i-2} V$, tomorrow, rather than a share $\alpha$ of a smaller pie, $(1 - \alpha)^{i-1} V$, today.

Under collective settlement, the strategic holdout motive is completely absent, regardless of transactions costs. This is because each creditor receives the same payoff gross of transactions costs. The motive for delay in this case is due purely to the *free rider* effect: the desire to avoid the transactions costs that are incurred in joining the collective. From (18), the payoff measured when the representative has bargained is $(\alpha/N) V \theta$ for those who join the coalition and $(\alpha/N) V$ for those who free-ride. Creditors delay entry in subgames $i > N - M$ so as to avoid the costs which diminish their gross payoff by factor $\theta$.

We summarize these results in the following Proposition:

**Proposition 5** *(Motives for Delay under Individual and Collective Settlement)* If transactions costs are uniform, delay under individual settlement is due purely to the strategic holdout motive. Regardless of transactions costs, delay under collective settlement is due to the free rider motive.

It is possible that the free-rider motive outweighs the strategic holdout motive for delay, and that the introduction of a collective action mechanism actually increases delay. Strikingly, this is possible even if total transactions costs under collective settlement are lower than under individual settlement. To see why, note that the expected discount factors from (11) under individual settlement are independent of transactions costs (and fall as $\alpha$ rises).
However, by (20), those under collective settlement decrease as $\theta$. Delay is higher under collective settlement if $\theta$ is sufficiently small relative to $\theta$. Further, if transactions costs under individual settlement are uniform, but are higher than under collective settlement, then delay is increased despite a reduction in such costs.

Of course, this begs the question of whether delay could increase in practice. This issue is addressed in Section 4, where we calibrate the model and show that collective settlement may increase delay for an empirically plausible range of parameters.

### 3.6 The Determinants of Delay: Comparative Dynamics

Since the Propositions 3 and 4 yield closed-form expressions for expected delay, it is straightforward to calculate the impact of changes in key parameters on these magnitudes. The following Proposition summarizes the results of such an exercise:

**Proposition 6 (Impact on Delay)** Consider a renegotiation game with either settlement process and with $N \geq 2$ creditors.

(a) A rise in creditor bargaining power $\alpha$ increases delay under individual settlement, and has no effect under collective settlement;

(b) A rise in current transactions costs (a fall in $\theta_N$) increases delay;

(c) A rise in future transactions costs reduces delay;

(d) A rise in the discount rate $r$ reduces delay;

(e) A rise in the sovereign’s payoff $V$ from entering world capital markets has no effect; and

(f) A rise in the number of creditors $N$ increases delay.

**Proof.** Parts (a) through (d) follow immediately from (8) for $i > 1$, and from (17) for $i \geq N - M$. Part (e) is immediate since delay expressions are independent of $V$. Part (f) is obvious. ■

Consider part (a). The strategic holdup motive for delay is increased with a rise in creditor bargaining power. It is important to stress that this is not due to the fact that creditors receive a greater share $\alpha$ of available surplus, because such a change affects all payoffs in the same proportion. The reason for increased delay, is that the undiscounted bargaining pie, $(1 - \alpha)^{i-1}V$, increases proportionately more with $\alpha$ as $i$ falls. Such a change has no impact under collective settlement, as the strategic holdup motive is absent there. Under collective settlement, all payoffs are proportional to $\alpha$: Those who join the collective receive the undiscounted payoff $(\alpha/N)V\theta_j$ and those who do not get $(\alpha/N)V$. A rise in creditor bargaining power has no effect on relative payoffs and hence no effect on delay under this regime.

A rise in the transactions cost of bargaining in $S_N$ clearly leads to a fall in $u_N$ relative to all future payoffs, and hence leads to increased delay going forward. This explains (b). For (c), note that a rise in transactions cost in some future settlement stage raises the relative payoff in $S_N$ and therefore decreases delay. In part (d), a rise in the discount rate makes
all future payoffs less valuable relative to the current payoff, and so reduces delay. It is interesting to note that by (e) the changes in the size of $V$ do not affect the incentive to delay under either process, because all payoffs are impacted proportionately the same way by such changes. Interestingly, the model predicts that country size does not affect the expected duration of settlement, regardless of the process. Part (f) follows immediately from the fact that increasing $N$ adds more stages to the beginning of the game.

Finally, note that each subgame of a renegotiation game with $N$ players is identical to a negotiation game with fewer players. Thus Proposition 6 also applied to delay in any subgame.

4 Quantitative Results

In the previous section we showed that moving from an individual to a collective process can increase the delay before agreement is reached. Here, we ask whether this occurs for reasonable values of the level of bargaining power $\alpha$, the number of creditors $N$, and the parameters governing both the total cost of bargaining, as well its distribution across creditors. The discussion of our calibration is necessarily brief. However, since all of these parameters are non-standard, we present a more detailed discussion in the ancillary appendix available online.

We calibrate bargaining power to the observed ‘holdout premium’: the return received by holdout creditors relative to that received by early-settling creditors. There is little available data on holdout premia in sovereign debt restructurings. SINGH (2003) cites claims that holdout creditors received three times the return of regular creditors in restructurings of illiquid sovereign debts that proceed to court action, but does not provide any documentation. For cases involving liquid sovereign debts, Singh reports returns in excess of 100% per year for a sample of only four defaults. Evidence from larger samples can be found in corporate debt restructurings. FRIDSON AND GAO (2002) find holdout premia of 11% in a study of 115 U.S. corporate debt restructurings from 1992 to 2000, down from the 30% estimates of ALTMAN AND EBERHART (1994) based on 202 restructurings from 1980-1992. As the incentive to holdout is determined by the expected return from doing so, and since Singh’s estimates need not be representative of those expectations, we place more weight on the returns to corporate debt restructuring and calibrate $\alpha$ to holdout premia of 10%, while also experimenting with premia of both 20% and 30%.13

The incentive to free ride depends on the size of negotiation costs, the extent to which these are compensated for by the debtor, and (in the case of a collective mechanism) their allocation across creditors. We calibrate these aspects using data from a range of sources. While some costs, like legal fees and printing expenses, are easy to verify and share between creditors, others are not. HOLLEY (1987) documents that under a collective mechanism there is typically a lead creditor who bears both the mundane costs of travel, arranging presentations, document preparation, and arranging signatures, as well as the more substantial costs of reconciling the claims of all creditors and the sovereign, and establishing criteria for the inclusion of loans within a restructuring deal. The latter can be very large in restructurings where debt monitoring has been poor (e.g. Mexico: MILOJEVIC 1985), the sovereign’s

13The resulting values for $\alpha$ are tabulated in the ancillary appendix.
debts are numerous and complicated (e.g. Mexico: Holley 1987, Kraft 1984; Argentina and Brazil: Reiffel 2003), or when the sovereign assumes responsibility for foreign debts owed by numerous private creditors within the country (e.g. Venezuela: Holley 1987). An indication of the share of costs borne by the lead creditor can be obtained from their share of syndicated bank loan fees, which McDonald 1982 shows average 75%, rising substantially for more complicated loans.

To calibrate the level of costs, we examine domestic corporate debt restructuring operations which are often cited as a model for reforms of the sovereign debt restructuring process and thus might represent a lower bound. The direct costs of corporate debt restructuring have been found to vary from as little as 0.3% of the total assets of the firm, when debts are restructured privately, to between 3 and 4.5% of assets when debts are restructured through bankruptcy, and to between 7.5 and 9.8% when firms are liquidated (Wruck 1990). As an indicator of total costs, professional fees in corporate bankruptcy proceedings typically amount to between 40% and 65% of total costs (Lopucki and Doherty 2004). In a sovereign context, professional fees at the start of the 1980s debt crisis typically ranged from 1.5% to 3.5% of the value of the restructured debt, falling to between 0.5% and 1% by the middle of the crisis, perhaps reflecting the fact that the debts had been reconciled and verified in earlier rounds (Institute of International Finance 2001). No fees were paid to creditor groups that were not recognized by the sovereign, to any creditor if the sovereign rejected the proposed restructuring (e.g. Peru between 1985 and 1996), in cases that involved litigation, or even in many successful restructuring operations (e.g. restructurings under the Brady Plan: Reiffel 2003). Gelpern and Gulati (2009) report that in several recent bond issues with collective action clauses the sovereign is not required to reimburse expenses or professional fees. Cumulating across multiple, often unsuccessful, rounds of negotiations, it appears plausible that the total costs of a sovereign debt restructuring exceed those from a corporate restructuring by a factor of two or more, of which only a modest fraction is compensated by the sovereign. To capture the wide range in uncompensated costs caused by the varying complexity of a country’s portfolio of defaulted debt, we present results for two polar cases. In a simple restructuring, we set total costs to 1% of the value of the restructured debt, of which 75% falls on the lead creditors in a collective action process, while in a complicated restructuring total costs are set to 3.5%, of which 90% falls upon the lead creditor. We view these estimates of total costs as conservative. For the individual settlement process, where the costs of reconciling and verifying multiple creditor claims do not apply, we assume that all creditors bear the same proportional cost of negotiation.

The number of creditors $N$ is a difficult parameter to calibrate, as there is considerable variance in the number and size of creditors across, as well as within, restructurings. This is further complicated since creditors frequently combine into representative groups suggesting that it is more appropriate to think of $N$ as the number of creditor groups, rather than number of creditors per se. We calibrate $N$ to capture the incentive of creditors to free ride on the efforts of the lead creditor, and of the collective more generally. An analysis of 84 bank and bondholder representative committees that operated between 1976 and 2000 yields

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14That is, beyond court awarded costs. Lopucki and Doherty (2004) note that in corporate restructuring cases, judges frequently deny some expense claims, although for typically small amounts.
a mean and median committee size of eleven. For the subset of these groups for which data are available, one member typically acted as the lead creditor, with the committee as a whole holding between one quarter and one third of the outstanding debt (Reed 1987). Under the assumption that lead creditors held larger than average shares within the committee, these numbers suggest that lead creditors held between 5 and 10 per-cent of the outstanding debt. Hence, we calibrate $N$ to 15 so that one creditor holds just less than 7% of the debt, while also experimenting with $N$ as low as 10 and as high as 20. The collective action threshold – the proportion of creditors who must join in order to effect a settlement – is set to 75% as in most recent collective action clauses (Gelpern and Gulati 2009). The number of creditors is held constant following the introduction of a collective action mechanism. This is not an unreasonable assumption for debts which are illiquid, which describes almost all debts restructured in the 1980s, and all but a modest number of emerging market debts today.

Table 1 reports delays under an individual settlement process as a function of the number of creditors, and for three values of the holdout premium. The Table shows that, for our benchmark case of a 10% holdout premium and 15 creditors, the model produces an average delay of 6.1 years which is almost exactly the median delay reported by Benjamin and Wright (2002). With 10 creditors, the average delay produced by the model ranged from 4 to 12 years, while with 20 creditors it ranged from 8 to 23 years, as holdout premia were increased from 10% to 30%. Overall, these numbers bracket the median (6) and mean (7.4) delay found in the modern data by Benjamin and Wright (2009), and lie within the range of delays reported by those authors (the maximum delay in their sample was 24 years).

Table 1 also reports the percentage increase in delay from moving to a collective action process, as a function of the number of creditors and the holdout premium for both simple and complicated restructurings. As shown in the Table, for our benchmark case and a complicated restructuring, the adoption of a collective action process results in a more than doubling of delay. As the number of creditors falls, the lead creditors’ holdings rise and they internalize more of the costs of bargaining, so that delays under a collective action process fall. However, they always remain larger than under an individual settlement process with a holdout premium of 10%. For a 20% holdout premia, the adoption of a collective action process can reduce delays by 19% or more. By contrast, in a simple debt restructuring operation, the adoption of a collective action process always reduces delays substantially: As shown in the Table, looking across all parameter values, the reduction in delay always exceeds 60%.

To summarize, we find that for a range of plausible parameter values, the model with individual settlement is able to produce delays in line with those observed with the data. For complicated restructuring operations, we find that the adoption of a collective action process more than doubles delay for our benchmark calibration, and always increases delay when lead creditor holdings are small ($N = 20$), or the expected holdout premium is 10%. For those restructuring operations which are relatively straightforward, the adoption of a collective action process always reduces delay by more than half.

Table 1: Delays in Sovereign Debt Restructuring

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15 We thank Christoph Trebesch for sharing his data on creditor committees (Trebesch 2008).
5 Robustness and Extensions

In this section we briefly sketch the results of modifying the model to include asymmetric creditors, different bargaining protocols, and initial offers by the sovereign using a series of simple examples. A more complete treatment, including proofs, is available in the ancillary appendix.

5.1 Different Bargaining Protocols

5.1.1 Endogenous Exit From Bargaining

Our bargaining game assumes that once begun, neither the creditor nor the debtor can exit bargaining prior to agreement. This considerably simplifies the analysis by making the bargaining and timing stages separable so that the full model can be solved easily by iterating between these stages. In practice, of course, both creditor and sovereign could walk away from bargaining at any point while retaining the option to resume bargaining at a later date.

We show in the ancillary appendix that adding the option to terminate bargaining without an agreement has no effect on equilibrium outcomes. The intuition for this equivalence is quite straightforward: rejection of an offer leads to a (possibly small) socially costly delay so that the parties are better off coming to an agreement without exit. As a consequence, the option to walk away has no value in equilibrium and has no effect on equilibrium outcomes (see also the discussion in Sutton 1986).

5.1.2 The Debtor’s Option to Repay in Full

In the basic model, players’ payoffs were assumed to be independent of the face value of the debt, which we denote by $b$. In practice, this may not be the case for a number of reasons.
One is that the sovereign always has the option to settle for the full outstanding debt and may sometimes wish to do so. Here, we examine this possibility.

In many contexts, the addition of an outside option has a significant effect on bargaining outcomes (e.g. Shaked 1994). However, in the ancillary appendix we prove that, in our framework, adding the debtors outside option to repay to our extensive form bargaining game serves only to cap payments to the creditor at \( b \), so that \( u_i = \delta_i \min \{ \alpha v_i, b \} \theta_i \).

If \( \alpha V < b \), it is cheaper for the sovereign to bargain with every creditor and the analysis is unaltered. If \( \alpha V > b \), it is cheaper for the sovereign to at least pay the last creditor the face value \( b \). In general we show that early creditors will bargain and receive a haircut, while later creditors will be repaid the full face value \( b \). By capping payments to later settling creditors, the presence of this outside option can decrease delays in restructuring.

5.2 Asymmetric Creditors

5.2.1 Bargaining Abilities

Creditors may differ in their ability to bargain with the sovereign in many ways: some may have greater bargaining power, while others may find bargaining less costly. For example, vulture creditors, who specialize in bringing suit against a country in default, may enjoy greater bargaining power because of their experience in litigation, but may incur greater bargaining costs because they maintain a large legal staff. In this subsection, we discuss an extension that allows for these asymmetries, and ask whether the model supports the conventional wisdom that the presence of vulture creditors increases delay.

Consider an example in which there is one *vulture* creditor, denoted by an asterisk superscript, and a *normal* creditor, with no superscript. Bargaining occurs according to our stochastic variant on Rubinstein’s game, with the vulture creditor making offers with probability \( \alpha^\ast > \alpha \) to capture the vultures presumed superior bargaining abilities. The vulture’s transactions cost parameter \( \theta^\ast \) may be either larger or smaller than that of the normal creditor, \( \theta \). As above, we justify our focus on mixed strategy equilibria as the result of either small amounts of uncertainty regarding each other’s payoffs (as in Harsanyi 1973) or the absence of social norms for coordination due to the relative rarity of sovereign default and the anonymity of many creditors.

In the ancillary appendix we show that, in the unique mixed strategy equilibrium, the normal creditor is likely to engage before the vulture, while average delay is larger than with two normal creditors independent of the vultures bargaining costs. These results are intuitive: even though high bargaining costs could leave the vulture with a smaller *absolute* payoff than the normal creditor from going last, the *relative* gain to the vulture creditor from delay is greater due to greater bargaining power and is unaffected by bargaining costs. In the ancillary appendix we show that these results generalize to a world with many vulture and many normal creditors.

5.2.2 Discount Rates

Differences in discount rates across creditors are straightforward to analyze in our framework, and the result is consistent with the 1980s empirical observation that the most impatient
creditors (that is, the least liquid banks) settled faster than other creditors. This is the result of two reinforcing effects in the model: for given bargaining payoffs a less patient creditor chooses to enter bargaining more quickly, while at the same time a less patient creditor extracts smaller amounts in bargaining which further reduces the incentive to holdout. Holding constant the payoff from bargaining, we can show that the equilibrium delay is determined by the average level of impatience across creditors. The ancillary appendix proves this result for the case with many identical patient, and many identical impatient, creditors.

5.2.3 Creditor Holdings

In contrast to our basic model, where creditors are identical in all respects, creditor holdings are, in practice, quite heterogeneous. How does this affect delay? Given that the sovereign can end negotiations by repaying in full, the natural bargaining protocol for analyzing this case is that of ‘bargaining with outside options’ discussed in Section 5.1 above.

Consider a two creditor example, first with an individual settlement process, and let creditors’ respective bondholdings be \( b \) for small, and \( B \) for large holdings, with \( b < B \). Obviously, if \( b \leq \alpha (1 - \alpha) V \), the sovereign pays-off the small creditor and the game ends without delay, while if \( \alpha V < b \), the sovereign will never settle-in-full with either creditor and results are the same as for our basic model. In the intermediate case \( \alpha (1 - \alpha) V < b < \alpha V < B \) the sovereign will want to pay the small creditor in full only if it is last to bargain, and will never wish to pay the large creditor in full. This case generates delay on average, although less than with two symmetric creditors because the cap on the small creditors payoff from holding-out reduces its incentive to do so. In addition, the large creditor bargains over a large pie if it engages first, increasing its incentive to engage quickly. To summarize, suppose we start with two creditors with symmetric holdings that are large enough that the sovereign never wants to pay in full. As heterogeneity of holdings increases, holding total debt constant, delay is initially unaffected, begins to decline as the sovereign prefers to pay off a small holdout creditor, and eventually disappears when the sovereign finds it always profitable to pay the small creditor in full.

Now consider the same example but under a collective action mechanism with constant repayment per bond (as in practice). In this case, if the sovereign repays any creditor in full, it must pay all in full. Suppose that bargaining costs are fixed as a proportion of the total repayment and are thus a greater proportion of a small creditor’s settlement. This reduces the small creditor’s incentive to engage quickly. However, as the total settlement is capped, costs are now a smaller proportion of the large creditor’s settlement, increasing its incentive to settle quickly. Both creditors adjust their strategies in response to the changing incentives of the other creditor and, strikingly, both effects exactly offset and the amount of delay observed under a collective action clause is unchanged.

This implies that, if we compare restructurings that are identical except for different degrees of asymmetry in bondholdings by creditors, the gains from moving to a collective action clause will be lowest (or most negative) for the restructuring with the most asymmetric bondholdings.
5.3 Initial Settlement Offers by the Sovereign

In the basic model, the sovereign makes offers only through bargaining, which itself is initiated by creditors. This is designed to reflect the relatively anonymous nature of many sovereign bond investments, where, in practice, the sovereign cannot easily initiate settlement negotiations. Nonetheless, it is not uncommon for a sovereign to announce an initial settlement offer. Here, we consider the effect of adding an initial stage where the sovereign makes an offer that each creditor can accept or reject.

We show that an initial payout offer will not generally eliminate delay: The only way to do so, is for the payout to exceed that which each creditor otherwise obtains. Consider individual settlement. Since the last creditor to bargain extracts $\alpha V$, to eliminate delay each creditor must be offered at least this amount, which makes for a total bill of $N\alpha V$. If $N\alpha > 1$, then this exceeds the entire proceeds from re-entering world capital markets.

Even if $N\alpha < 1$, it may not be optimal to eliminate delay: the optimal offer must trade off the decline in delay against the higher payments to early settling creditors. In the ancillary appendix we solve for the optimal offer analytically with two creditors. We find that when bargaining power $\alpha$ is below some cutoff, the sovereign finds it optimal to eliminate delay by paying out creditors, whereas if bargaining power is above another cutoff, the sovereign prefers to make no offer at all. In between, a positive initial offer serves to reduce, but not eliminate, delay.

6 Concluding Comments

In this paper we developed a model of negotiation in a weak contractual environment that reflects the key features of the sovereign debt renegotiation process. The model generates delay in equilibrium as a result of two collective action problems: a ‘strategic holdup’ effect, where each creditor delays settlement with the sovereign in order to exploit the fact that earlier settlements are sunk and the bargaining pie becomes larger; and, a ‘free rider’ effect in which creditors delay settlement to avoid sharing in negotiation costs. In our most striking result, we show that the introduction of a collective action mechanism can increase delay, as the imposition of common settlement terms across creditors intensifies the incentive for creditors to free-ride. When the model is calibrated to the range of negotiation costs observed in practice, we find that free riding is quantitatively relevant, with the introduction of a collective action mechanism more than doubling delay in complicated and costly restructuring operations. For simpler restructuring operations, where costs are low, we find that collective mechanisms reduce delay by more than 60%. This bodes well for any possible future restructuring of European debts such as in Greece where debt records are well-maintained (and hence negotiation costs should be low), foreign debts contain collective action clauses, and domestic debts can be modified ex post to allow easier restructuring (see Buchheit and Gulati 2010).

We conclude with three suggestions for future work. First, we have considered a sovereign that is already in default, abstracting from the sovereign’s decision to borrow and default in

\footnote{Indeed, in some cases, sovereigns have had to hire intermediaries to track down and contact creditors. See, for example, the discussion of Ukraine’s 2000 restructuring in IMF (2000).}
the first instance. As a result, we cannot draw normative conclusions for the introduction of collective action mechanisms, unlike Pitchford and Wright (2007) which uses numerical methods to study the welfare effect of collective action mechanisms in a related environment. Second, we considered a case in which the ability to retrade sovereign debts in secondary markets is limited, so that creditor numbers and holdings of debt cannot adjust following a change in restructuring institutions. While this is a reasonable description of almost all sovereign debt markets in the 1980s, and of many today, for some countries debt liquidity has increased substantially. For some work on the effect of secondary markets for sovereign debt see Broner, Martin and Ventura (2009). Third, in focusing on collective action problems among creditors, we have entirely abstracted from collective action problems within the debtor country. Some progress on this question has been made by Alesina and Drazen (1991).
7 Appendix

7.1 Proof of Lemma 1

**Proof.** We will prove the result for the case where one creditor bargains for all $N$, since the individual case is captured trivially by setting $N = 1$.

Let the set of subgame perfect (SGP) equilibrium payoffs for the sovereign be given by

$$X_j \equiv \{ x_j : \exists \text{ an SPE of } S_j \text{ with payoffs } (x_j, z_j) \},$$

and the set of SGP equilibrium payoffs for the creditor by

$$Z_j \equiv \{ z_j : \exists \text{ an SPE of } S_j \text{ with payoffs } (x_j, z_j) \}.$$

Note that all subgames prior to the realization of the identity of the proposer are identical, and so possess the same set of SGP equilibrium values.

Suppose that the set of SPE values is non-empty, which mean there exists a (non-trivial) sup and an inf for each set. Thus, the sovereign can do no better than $\pi_j = \sup X_j$ and no worse than $\underline{x}_j = \inf X_j$. Similarly, the creditor can do no better than $\bar{z}_j = \sup Z_j$ and no worse than $\underline{z}_j = \inf Z_j$. Consider a subgame of $S_j$ that begins after nature has determined that the creditor makes the offer. Then since the sovereign will reject all offers less than $e^{-r\Delta} \underline{x}_j$, the worst payoff it could get after rejection, the creditor’s payoff can be no greater than $\left( V_{j-1} - e^{-r\Delta} \underline{x}_j \right) / N$. Similarly, as the sovereign would accept any offer greater than $V_{j-1} - e^{-r\Delta} \bar{z}_j$, the best it could get after rejection, the creditor’s payoff can be no less than $\left( V_{j-1} - e^{-r\Delta} \bar{z}_j \right) / N$. Now consider a subgame of $S_j$ that begins after nature has determined that the sovereign makes the offer. Using the same reasoning, the creditor will reject any offer less than $e^{-r\Delta} \underline{z}_j$ which implies the sovereign’s payoff is no greater than $V_{j-1} - Ne^{-r\Delta} \underline{z}_j$. Similarly, the creditor will accept any offer greater than $e^{-r\Delta} \bar{z}_j$ which implies the sovereign’s payoff is no less than $V_{j-1} - Ne^{-r\Delta} \bar{z}_j$.

Moving back to a point in $S_j$ before nature has selected the proposer, we must have $\pi_j = (\alpha/N) \left( V_{j-1} - e^{-r\Delta} \underline{x}_j \right) + (1 - \alpha) e^{-r\Delta} \underline{x}_j$. That is, the best the creditor can do at this point in $S_j$ is the probability-weighted sum of the best it can do if it has the offer (i.e. the sovereign is pinned down to $e^{-r\Delta} \underline{x}_j$, so the creditor gets $V_{j-1} - e^{-r\Delta} \underline{x}_j$) and the best it can do if the sovereign has the offer (i.e. receive $e^{-r\Delta} \underline{x}_j$). Analogously we have $\bar{z}_j = (\alpha/N) \left( V_{j-1} - e^{-r\Delta} \bar{z}_j \right) + (1 - \alpha) e^{-r\Delta} \bar{z}_j$, as well as $\pi_j = \alpha e^{-r\Delta} \bar{z}_j + (1 - \alpha) \left( V_{j-1} - Ne^{-r\Delta} \bar{z}_j \right)$, and $\underline{x}_j = \alpha e^{-r\Delta} \underline{x}_j + (1 - \alpha) \left( V_{j-1} - Ne^{-r\Delta} \underline{x}_j \right)$. Solving these equations we find $\pi_j = \pi_j = (\alpha/N) V_{j-1}$ and $\bar{z}_j = \bar{z}_j = (1 - \alpha) V_{j-1}$. This establishes uniqueness of the SGP equilibrium values.

To complete the proof, we now exhibit SGP equilibrium strategies that attain these values. The strategies take the following form: The creditor always proposes a split in which it receives $\left( 1 - e^{-r\Delta} (1 - \alpha) \right) V_{j-1} / N$, accepts any proposal greater than or equal to $e^{-r\Delta} \alpha V_{j-1} / N$; the sovereign always proposes a split in which the creditor receives $e^{-r\Delta} \alpha V_{j-1} / N$ and accepts any proposal in which it receives greater than or equal to $e^{-r\Delta} (1 - \alpha) V_{j-1}$. Suppose nature chooses the sovereign to make the offer. The creditor will only accept an offer if it is at least as large as $e^{-r\Delta} \alpha V_{j-1} / N$, since rejection leads to $S_j$ with the payoff $e^{-r\Delta} \alpha V_{j-1} / N$. Hence, this acceptance rule constitutes a best response by the creditor. Further, the sovereign proposes the share $e^{-r\Delta} \alpha V_{j-1} / N$ for the creditor (which the
creditor immediately accepts). To see why, note that the sovereign’s payoff in this sub-game is \( (1 - e^{-r\Delta \alpha}) V_{j-1} \) which exceeds \( e^{-r\Delta} (1 - \alpha) V_{j-1} \) and which is only attainable if the sovereign proposes \( e^{-r\Delta \alpha} V_{j-1}/N \) (which we have argued the creditor always accepts). Analogous reasoning holds if nature chooses the creditor to make the offer. ■

7.2 Proof of Lemma 2

Proof. Consider part (i) with \( u_i < U_{i-1} \). First note that there does not exist a symmetric equilibrium in pure strategies. If there did, then any creditor could profitably deviate by delaying entry into the settlement process by any \( \varepsilon > 0 \).

To calculate the symmetric mixed strategy equilibrium for \( T_i, i > 1 \), note that players are indifferent between playing the mixed strategy and any pure strategy in its support. Let all other’s play according to \( F_i(t - t_i) \), and consider the pure strategy in which the creditor enters \( T \) units of time after \( t_i \)—which we normalize to 0 for convenience—provided no other player has entered by this time, and enters immediately if another has entered. The creditor receives \( e^{-rT} u_i \) with probability \( [1 - F_i(T)]^{i-1} \), which is the probability no other creditor enters the settlement process before \( T \). If one of the other \( i - 1 \) creditors does enter before \( T \), the creditor gets \( U_{i-1} \). The probability that this occurs is governed by the first order statistic for the randomized entry of the \( i - 1 \) other creditors, each of whom play according to \( F_i \), which has pdf \( (i - 1) f_i(t) [1 - F_i(t)]^{i-2} \). Thus, the expected payoff conditional on one other entering is

\[
\left[ \int_0^T e^{-rt} (i - 1) f_i(t) [1 - F_i(t)]^{i-2} dt \right] U_{i-1}.
\]

The creditor is indifferent between entering immediately and getting \( u_i \), and playing the pure strategy described above and receiving the RHS of the following equation:

\[
u_i = u_i e^{-rT} [1 - F_i(T)]^{i-1} + U_{i-1} \int_0^T e^{-rt} (i - 1) f_i(t) [1 - F_i(t)]^{i-2} dt \tag{21}\]

Since (21) is true for all \( T \), the derivative of the RHS with respect to \( T \) is zero, i.e.

\[-ru_i e^{-rT} [1 - F_i(T)]^{i-1} - [u_i - U_{i-1}] e^{-rT} f_i(T) (i - 1) [1 - F_i(T)]^{i-2} = 0. \tag{22}\]

Re-arranging and canceling terms yields the differential equation

\[-d \log (1 - F_i) /dT = ru_i / (i - 1) (U_{i-1} - u_i), \tag{23}\]

with initial condition \( F_i(0) = 0 \), which has solution

\[F^*_i(t) = 1 - \exp \{- \lambda_i t\}, \tag{24}\]

where

\[\lambda_i \equiv \frac{ru_i}{(i - 1) [U_{i-1} - u_i]} \tag{25}\]

Since the creditor is indifferent between playing \( F^*_i \) and any pure strategy, such as entering immediately in \( T_i \) and receiving \( U_i \), the expected payoff of each creditor at the start of \( T_i \) is \( U_i = u_i \). \( \beta_i \) is the expected value of \( e^{rt} \), conditional on entry by the first creditor.

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Thus, it is calculated using the density for the minimum entry time of \( i \) random variables drawn from the equilibrium density \( F_i^* \), which we denote \( g_i(t) \equiv if_i^*(t)[1 - F_i^*(t)]^{i-1} \), as 
\[
\beta_i = \int_0^\infty e^{rt} g_i(t) dt = iu_i / ((i - 1)U_{i-1} + u_i).
\]
Finally, note that by the well known properties of the exponential distribution \( E[t_i - t_{i-1}] = [i\lambda_i]^{-1} = (i - 1) [U_{i-1} - u_i] / i ru_i \).

For part (ii) with \( u_i \geq U_{i-1} \), immediate entry yields \( u_i \) and delay yields \( U_{i-1} \), discounted. Under the tie breaking rule the entering creditor is selected by randomizing over all creditors with equal probability \( 1/i \).

7.3 Proof of Proposition 3

Proof. The proof is by induction. Consider the settlement stage in subgame 1. By Lemma 1, as the sovereign receives \( V \) from concluding the bargain, the creditor expects to receive \( u_1 = \alpha V \), and the sovereign \( v_1 = (1 - \alpha) V \). Moving backwards to the timing stage, by Lemma 2, the creditor enters immediately so that delay is 0, and \( \beta_1 = 1 \), which yields \( V_1 = (1 - \alpha) V \).

Now suppose that the payoff to the sovereign from moving to the start of subgame \( i - 1 \) is given by \( V_{i-1} = \delta_{i-1} (1 - \alpha)^{i-1} V \), while the payoff to a creditor is \( U_{i-1} = \delta_{i-1}\alpha (1 - \alpha)^{i-2} V \theta_{i-1} \). Consider the settlement stage at \( i \). By Lemma 1, the creditor that has entered the settlement stage expects to receive \( u_i = \alpha V_{i-1} = \delta_{i-1}\alpha (1 - \alpha)^{i-1} V \theta_i \), while the sovereign expects to receive \( v_i = (1 - \alpha) V_{i-1} = \delta_{i-1} (1 - \alpha)^i V \). Under our assumption that \( \theta_i \leq \theta_{i-1} \), we have \( u_i < U_{i-1} \) and we can apply Lemma 2 part (i), which gives us that, in timing stage \( i \), the creditors play the symmetric mixed strategies given by \( F_i = 1 - \exp \{-\lambda_i t\} \) where, on substitution of payoffs using Lemma 2, the hazard rate is
\[
\lambda_i = \frac{r\beta_{i-1} (1 - \alpha) \theta_i}{(i - 1) [\theta_{i-1} - \beta_{i-1} (1 - \alpha) \theta_i]},
\]
implying that the expected duration of the stage is
\[
E[t_i - t_{i-1}] = \frac{1}{i\lambda_i} = \frac{i - 1}{ri} \frac{\frac{1}{\beta_{i-1}} \theta_{i-1} - (1 - \alpha) \theta_i}{(1 - \alpha) \theta_i},
\]
and the expected discount factor is \( \beta_i = i\beta_{i-1} (1 - \alpha) \theta_i [(i - 1) \theta_{i-1} + \beta_{i-1} (1 - \alpha) \theta_i]^{-1} \). Substituting the expression for \( \beta_{i-1} \) from equation (11) above yields
\[
\beta_i = i (1 - \alpha)^{i-1} \theta_i \left[ \sum_{k=1}^{i} (1 - \alpha)^{i-k} \theta_{i-k+1} \right]^{-1}.
\]
Substituting this in the expression for expected delay, we obtain
\[
E[t_i - t_{i-1}] = \frac{1}{ir\theta_i} \left[ \sum_{k=0}^{i-1} (1 - \alpha)^{-k} \theta_{i-k} - i \theta_i \right].
\]

Remark 7 Our maintained assumption that \( \theta_i \leq \theta_{i-1} \) is not necessary for delay to occur in equilibrium under an individual settlement mechanism; all that we require is that the transaction costs of later settling creditors are not too much larger than those for early settling creditors, or more specifically \( (1 - \alpha) \theta_i < \theta_{i-1} \). If this is not satisfied for some \( i \), then we will not observe delay in stage \( i \).
7.4 Proof of Proposition 4

Proof. The proof is by induction. Consider stage 1. The settlement stage $S_1$ yields payoffs $u_1 = \alpha V/N$ to the creditor. As the sovereign regains access to capital markets once it has settled with $N - M$ creditors, its payoff remains at $v_1 = V$. As there is only one creditor, there is no delay in the timing stage $T_1$, so that $\beta_1 = 1$, and $V_1 = V$.

Now suppose the above results hold for stage $i - 1$, and consider stage $i$. To begin, assume that $i < N - M$. Then settlement stage $S_i$ yields payoffs $u_i = \alpha V/N$ to the creditor, while the sovereign continues to enjoy $v_i = V$. By Lemma 2 part (ii) there is no delay, $\beta_i = 1$, $U_i = u_{i-1}$, and $V_i = V$. Instead, suppose $i = N - M > i - 1$. Then the settlement stage $S_i$ is a bargaining stage and by Lemma 1 the creditor receives the payoff $u_i = \alpha(V/N)\theta_i$, while the sovereign receives $v_i = (1 - \alpha)V$. By Lemma 2 all $i$ creditors randomize according to cdf $F_i = 1 - \exp\{-\lambda_i t\}$ where, after substitution, the hazard rate is $\lambda_i = r\theta_i/(i - 1)(1 - \theta_i)$, the expected duration of the stage is $E[t_i - t_{i-1}] = \left[(i\lambda_i)^{-1} = (i - 1)(1 - \theta_i)/ir\theta_i\right]$, and the expected discount factor is $\beta_i = i\theta_i[(i - 1) + \theta_i]^{-1}$. Thus $U_i = u_i$ and $V_i = \beta_iv_i = \delta_iv_i$.

Finally, suppose $i - 1 \geq N - M$, and assume that the above formulae hold for $i - 1$. We show that they hold for $i$. In this case, the settlement stage $S_i$ yields the creditor the payoff $u_i = \delta_{i-1}\alpha(V/N)\theta_i$, while the sovereign receives $v_i = \delta_{i-1}(1 - \alpha)V$. By Lemma 2 all $i$ creditors randomize according to cdf $F_i = 1 - \exp\{-\lambda_i t\}$ where after substitution the hazard rate is $\lambda_i = r\beta_{i-1}\theta_i/(i - 1)(\theta_{i-1} - \beta_{i-1}\theta_i)$, the expected duration of the stage is

$$E[t_i - t_{i-1}] = \frac{1}{i\lambda_i} = \frac{i - 1}{ri} \frac{\beta_{i-1}\theta_{i-1} - (1 - \alpha)\theta_i}{(1 - \alpha)\theta_i},$$

and the expected discount factor is $\beta_i = i\beta_{i-1}\theta_i[(i - 1)\theta_{i-1} + \beta_{i-1}\theta_i]^{-1}$. Using the expression for $\beta_{i-1}$ above we find

$$\beta_i = i\theta_i \left[ N - M + \sum_{k=N-(M-1)}^{i} \theta_k \right]^{-1}. $$

Substituting $\beta_{i-1}$ into the expression for expected delay we find

$$E[t_i - t_{i-1}] = \frac{1}{ri(1 - \alpha)\theta_i} \left[ N - M + \sum_{k=N-(M-1)}^{i-1} \theta_k - (i - 1)(1 - \alpha)\theta_i \right] $$

\[\Box\]

References


