Appendix to 'How Important is Human Capital? A Quantitative Theory Assessment of World Income Inequality'

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January 2010

1 Definition of a Steady-state Competitive Equilibrium

For the purposes of defining an equilibrium in a compact way, we suppress the individual state into a vector $x \equiv (q, h_p, z, \theta)$ and redefine the value functions and decision rules to be functions of the individual state x. Let the state space be given by $X \subset [0, \infty) \times [0, \infty) \times Z \times \Theta$, and denote by $\Omega(X)$ the Borel σ-algebra on X. Let $\Psi(X)$ be a probability measure of individuals with state $x \in X$.

DEFINITION. Given a productivity in the manufacturing sector A_M, share of manufactured goods in consumption γ, and income tax rate τ, a steady-state competitive equilibrium is

$$\{g_c(x), g_a(x), g_s(x), g_e(x), V(x), \Psi(X), w, r, P_S, P_c, p, K_M, K_S, H_M, H_S\}$$

such that

1. Given prices and income tax rate, $\{g_c(x), g_a(x), g_s(x), g_e(x), V(x)\}$ solve the dynamic programming problems of the households.
2. Prices are competitive:

$$\begin{aligned} r &= A_M \alpha \left(k\right)^{\alpha-1} - \delta \\ w &= A_M (1-\alpha) \left(k\right)^{\alpha} \\ P_S &= A_M^{1-\epsilon} \\ P_c &= \frac{(P_S)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}. \end{aligned}$$

3. Markets clear:
   (a) Capital: $\int_X g_a(x) d\Psi = K_M + K_S = K$.
   (b) Labor: $\int_X \left[(1-g_s(x))\psi_1 A_h z \left(g_s(x)^\eta g_e(x)^{1-\eta}\right)^\xi + \psi_2 h_p + \psi_3 h_p\right] d\Psi = H_M + H_S + \bar{l} \int_X g_s(x) d\Psi = H$.
   (c) Manufactured goods: $\int_X \gamma P_c g_c(x) d\Psi + K = A_M K_M^\alpha H_M^{1-\alpha} + (1-\delta)K$.

(d) Services: $\int_X \left[(1-\gamma)\frac{P_c}{P_S} g_c(x) + g_e(x)\right] d\Psi = A_S K_S^{\alpha} H_S^{1-\alpha}$.

4. Distribution of agents is invariant and consistent with household behavior:

$$\Psi(X_0) = \int_{X_0} \left\{ \int_X \Gamma(x, x') d\Psi \right\} dx',$$

for all $X_0 \in \Omega$, where $\Gamma(x, x')$ is the probability that an agent with current state x transits to state x' in the following period, defined as:

$$\Gamma(x, x') \equiv \sum_{\mu'} \Xi(z, \theta, z', \theta', \mu') I \left\{ x : q' = (1-\tau)\left[w\psi_3 h_p + r g_a(x)\right] + g_a(x), h'_p = \mu' A_h z \left(g_s(x)^{\eta} g_e(x)^{1-\eta}\right)^{\xi} \right\}$$

where $I\{.\}$ is an indicator function.

5. Public education budget balances: $\tau\,(wH + rK) = p \int_X g_s(x) d\Psi$.

2 Computational Method

The model is solved on a grid for (q, h_p, z, θ) with linear interpolation of the household decision rules on the (q, h) dimension.

2.1 Algorithm to Compute a Stationary Equilibrium

1. Guess the capital-labor ratio $k = \frac{K_M}{H_M}$ and public education subsidy p.

2. Obtain prices:

$$\begin{aligned} r &= A_M \alpha\,(k)^{\alpha-1} - \delta \\ w &= A_M (1-\alpha)\,(k)^{\alpha} \end{aligned}$$

3. Compute household decision rules (see below).

4. Starting from an arbitrary initial distribution of agents (population size 20000) across states, simulate dynasties for 10000 generations by using household decision rules.

5. Aggregate assets, human capital (labor), and public education subsidy across all households at period 10001.

6. Use market clearing conditions and government budget constraint to update the guesses for k and p and check for their convergence. Return to step 2 until convergence.

2.2 Algorithm to Solve Household Problems

1. For each grid point $x \equiv (q, h_p, z, \theta)$, guess household decision rules $g_c, g_a, g_s, g_e(x)$.
2. Using household decision rules, compute derivatives of the value function with respect to q and h_p for each x.
3. For each x, solve a system of equations composed of the household first-order conditions (FOCs) using Newton method. Use current decision rules as initial guesses. Linearly interpolate the derivatives of the value function between the grid points (q, h_p). Potential corners for assets $a' = 0$ and schooling time $s = 1$ are handled as follows:
   (a) start by solving constrained case $a' = 0$. If the FOC for savings holds, the optimal decision rule is found. Otherwise, go to the next step.
   (b) Solve the unconstrained case. If the optimal $s \leq 1$, the optimal decision rule is found. Otherwise, go to the next step.
   (c) Solve constrained case $s = 1$. If the optimal $a' \geq 0$, the optimal decision rule is found. Otherwise, go to the next step.
   (d) Solve constrained case $a' = 0$ and $s = 1$. Verify the FOCs for s and a' hold.
4. Go to step 2 till convergence of the household decision rules.